
Student number

2025 **TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION**

Mathematics Advanced

General instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A laminated reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Write your Student Number at the top of this page

Total marks: 100

Section I – 10 marks

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks

- Attempt Questions 11–25 in Booklet A and 26 – 33 in Booklet B
- Allow about 2 hours and 45 minutes for this section

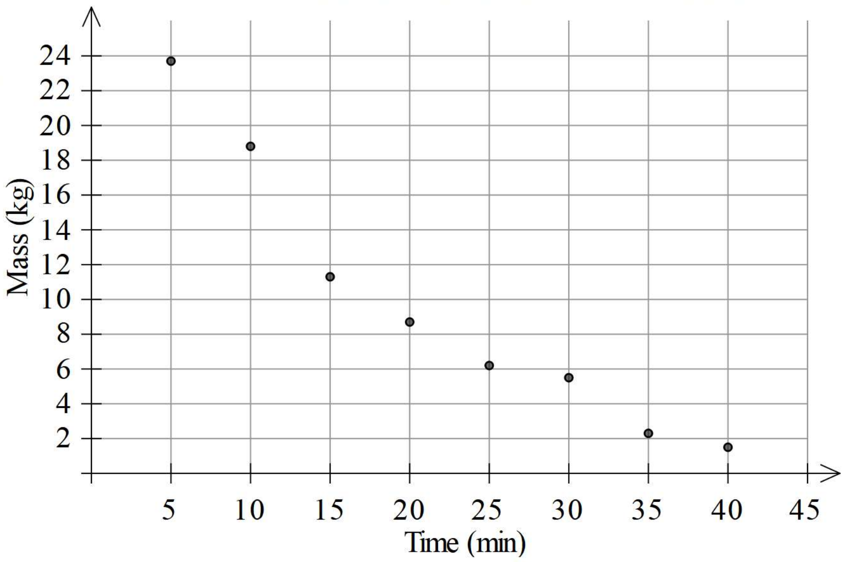
Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

---

1 A block of ice was taken out of a freezer and left to thaw. The results are shown below.



Which Pearson's correlation coefficient best describes the observations?

A. $r = 0.95$

B. $r = 0.5$

C. $r = -0.5$

D. $r = -0.95$

2 What is $\int (4x - 1)^5 dx$?

A. $\frac{1}{6}(4x - 1)^6 + C$

B. $\frac{1}{24}(4x - 1)^6 + C$

C. $24(4x - 1)^6 + C$

D. $6(4x - 1)^6 + C$

3 The probability that Young's soccer team wins this weekend is $\frac{4}{9}$. The probability that Young's rugby league team wins this weekend is $\frac{3}{4}$.

What is the probability that neither team wins this weekend?

A. $\frac{5}{36}$

B. $\frac{2}{3}$

C. $\frac{5}{12}$

D. $\frac{1}{9}$

4 A and B are two chance events such that $P(A) = 0.45$, $P(B) = 0.3$ and $P(A|B) = 0.6$.

What is the value of $P(B|A)$?

A. 0.4

B. 0.5

C. 0.8

D. 0.9

5 Given that $f(x) = \begin{cases} 5 - (x-3)^2, & x \leq 2 \\ mx + 3, & x > 2 \end{cases}$

What is the value of m for which $f(x)$ is continuous at $x = 2$?

A. $\frac{7}{2}$

B. $\frac{3}{2}$

C. $\frac{1}{2}$

D. 2

6 Let $h(x) = \frac{f(x)}{g(x)}$ where $f(2) = 2$, $f'(2) = 1$, $g(2) = 5$, $g'(2) = 4$.

What is the gradient of the tangent to the graph of $y = h(x)$ at $x = 2$?

A. $\frac{3}{25}$

B. 13

C. $-\frac{3}{25}$

D. $\frac{13}{25}$

7 Let $f(x) = \sqrt{2 + x}$ and $g(x) = 2x + 3$. What is the domain of the composite function $f(g(x))$?

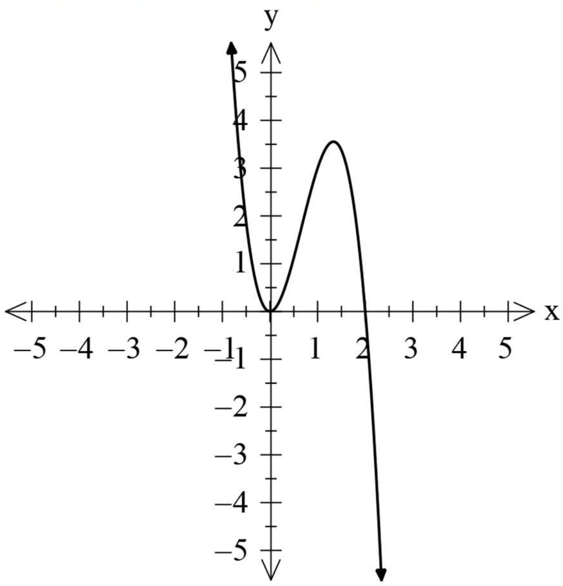
A. $x \in [-2.5, 2.5]$

B. $x \in [-2.5, \infty)$

C. $x \in \mathbb{R}$

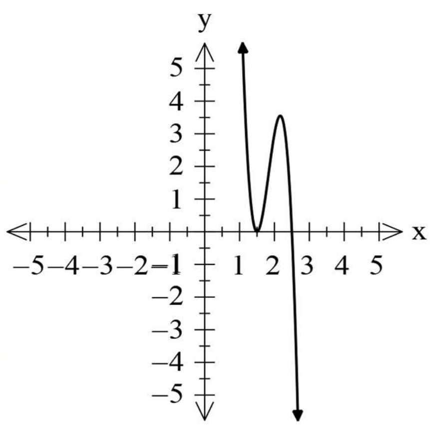
D. $x \in (-\infty, 2.5]$

8 The diagram shows the graph of $y = f(x)$.

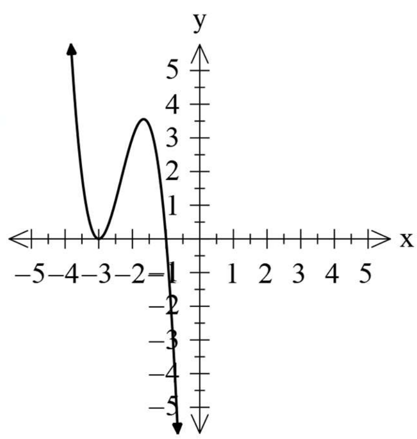


Which of the following best represents the graph of $y = f(2x + 3)$?

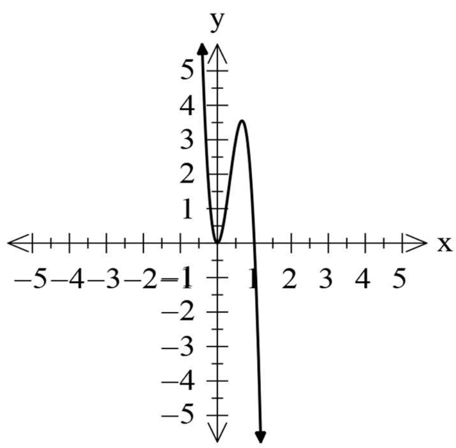
A.



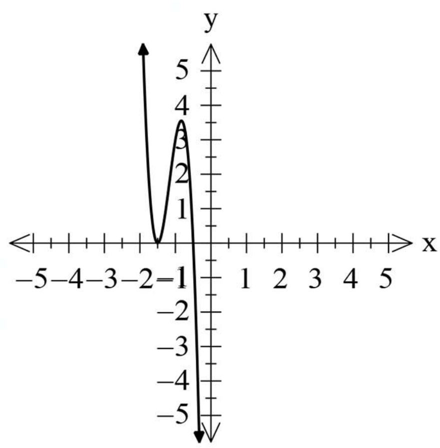
B.



C.



D.



9 Which expression is equal to $\int 6^{3x-2}\,dx$?

A. $\dfrac{6^{3x-2}}{3\ln(6)} + C$

B. $\dfrac{6^{3x-2}}{\ln(6)} + C$

C. $3\ln(6) \times 6^{3x-2} + C$

D. $\ln(6) \times 6^{3x-2} + C$

10 The z scores on a test are normally distributed such that $P(-N < z < N) = p$ for some constants $N > 0$ and $0 < p < 1$. What is the value of $P(z < N)$?

A. $\dfrac{1-p}{2}$

B. $1 - \dfrac{p}{2}$

C. $\dfrac{1+p}{2}$

D. $\dfrac{1+2p}{3}$

Student number

Mathematics Advanced
Section II Answer booklet A

50 marks
Attempt Questions 11–25
Allow about 1 hour and 30 minutes for this section

Instructions

- Write your Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Question 11 (2 marks)

Find the vertex of the parabola $y = x^2 - 6x + 10$. **2**

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

Question 12 (2 marks)

Solve the equation $\cos x = \dfrac{\sqrt{3}}{2}$ for $0 \leq x \leq 2\pi$. **2**

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

Question 13 (3 marks)

Solve the equation $\log_2(1 - x) + \log_2(3 - x) = 3$. **3**

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

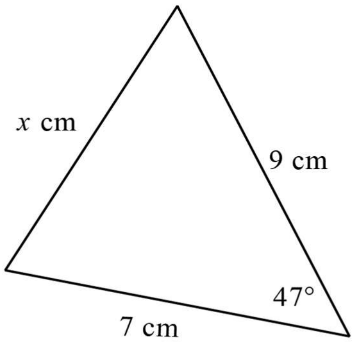
……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

Question 14 (2 marks)

Consider the diagram below.



Use the cosine rule to find the value of x, correct to two significant figures. **2**

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

Question 15 (2 marks)

Differentiate $x^2(3x+5)^3$ with respect to x. **2**

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

Question 16 (3 marks)

Find in simplest general form the equation of the tangent to the curve $y = e^x + \log_e(2x - 1)$ at the point where $x = 1$. **3**

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

Question 17 (2 marks)

A discrete random variable X with mean 2 has the following probability distribution where a and b are constants.

x	0	1	2	3
$p(x)$	a	0.1	b	0.3

Find the values of a and b. **2**

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

Question 18 (2 marks)

A probability density function is given by $f(x) = \begin{cases} \dfrac{3x^2}{117} & 2 \le x \le k \\ 0 & \text{for all other } x \end{cases}$.

Find the value of k. **2**

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

Question 19 (4 marks)

The number of years of teaching experience of the 4800 secondary teachers teaching in Australia are normally distributed with mean 20 years and standard deviation 6 years.

a) Use the Empirical Rules to determine how many more teachers with at most 14 years of experience there are than teachers with at least 32 years of experience. **2**

....

....

....

....

....

....

....

....

b) Teachers with more than 14 years of teaching experience are known as highly accomplished teachers. Use the Empirical rule to find, to the nearest whole number, the percentage of highly accomplished teachers who have more than 26 years teaching experience. **2**

....

....

....

....

....

....

....

Question 20 (6 marks)

A bag contains 11 blue lollies and 9 green lollies.

David takes out a lolly from the bag without looking and eats it. He then takes out another lolly without looking and eats it.

a) What is the probability of David choosing a blue lolly in his first selection? **1**

....

b) By drawing a tree diagram, or otherwise, find the probability that David eats two lollies of different colours. **2**

....

....

....

....

....

....

c) David opens a different bag of blue and green lollies. He notices that it contains 10 green lollies but does not count the blue lollies. He then eats two lollies without looking at their colour. If the probability that David eats two green lollies is $\frac{3}{8}$, how many blue lollies were in the bag? **3**

....

....

....

....

....

....

....

....

....

Question 21 (8 marks)

Consider the curve given by $y = x^3 - 3x^2$ for $-1 \leq x \leq 3.5$.

a) Find any stationary points and determine their nature. **3**

....

....

....

....

....

....

....

....

....

....

....

....

....

....

b) Find any points of inflection. **2**

....

....

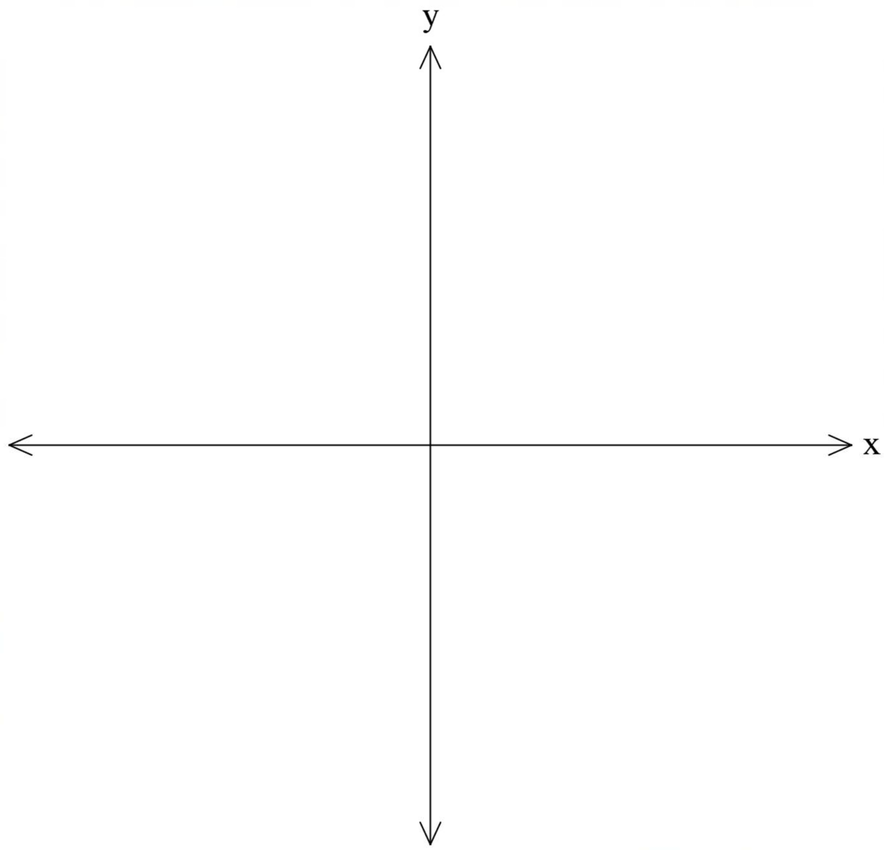
....

....

....

....

c) Sketch the curve over its given domain, clearly labelling any stationary points, points of inflection, intercepts and end points. **2**



d) Hence state the global maximum for $y = x^3 - 3x^2$ in the domain $-1 \leq x \leq 3.5$. **1**

Question 22 (3 marks)

Given that $y = \log_e(3^x - 2x)$

a) Complete the table below by finding the missing value of y correct to three decimal places. **1**

x	1	1.5	2	2.5	3
y	0	0.787	1.609		3.045

....

....

....

b) Using the table above and the trapezoidal rule, find an approximation for the value of $\int_1^3 \log_e(3^x - 2x)\, dx$ correct to three decimal places. **2**

....

....

....

....

....

....

....

....

....

Question 23 (4 marks)

Let $(\tan\theta - 1)\left(\sqrt{3}\sin\theta - \cos\theta\right)\left(\sqrt{3}\sin\theta + \cos\theta\right) = 0$

Find all solutions for $(\tan\theta - 1)\left(\sqrt{3}\sin\theta - \cos\theta\right)\left(\sqrt{3}\sin\theta + \cos\theta\right) = 0$, where $0 \leq \theta \leq \pi$. **4**

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

Question 24 (4 marks)

The table below shows the recorded height and the right foot length for 9 people (*A* to *I*). Only the height data is available for person *J*.

	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*
Height (cm)	142	164	183	172	154	158	149	181	175	177
Foot length (cm)	20	26	28	26	22	23	24	29	27	

a) Calculate Pearson's correlation coefficient between height and right foot length for *A* to *I* correct to three decimal places. Describe the nature of the correlation. **2**

....

....

....

....

b) Find the equation of the least-squares regression line of height on right foot length for *A* to *I*. Estimate the right foot length of person *J* to the nearest centimetre. **2**

....

....

....

....

....

....

Question 25 (3 marks)

Find the values of any constant A such that $\int_0^3 (x^2 + A)\, dx = 2A^2$. **3**

Student number

Mathematics Advanced
Section II Answer booklet B

40 marks
Attempt Questions 26–33
Allow about 1 hour and 15 minutes for this section

Instructions

- Write your Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Question 26 (6 marks)

A particle is moving in a straight line. At time t seconds it has displacement x metres from a fixed point O on the line and velocity $v(t) = 3\sqrt{t + 1} - 9$ metres per second. The particle starts at O.

a) Show that $x(t) = 2(t + 1)\sqrt{t + 1} - 9t - 2$. **2**

....

....

....

....

....

....

....

....

....

b) Find when and where the particle is at rest in relation to the fixed point O. **2**

....

....

....

....

....

c) Find the total distance travelled by the particle in the first 24 seconds. **2**

....

....

....

....

Question 27 (5 marks)

a) Prove that $(1 + \tan x)^2 = 2\tan x + \sec^2 x$. **2**

....

....

....

....

....

....

....

....

b) Hence find the area bounded by $y = (1 + \tan x)^2$ and the x-axis between $-\frac{\pi}{6} \leq x \leq \frac{\pi}{6}$ with a rational denominator. **3**

....

....

....

....

....

....

....

....

....

....

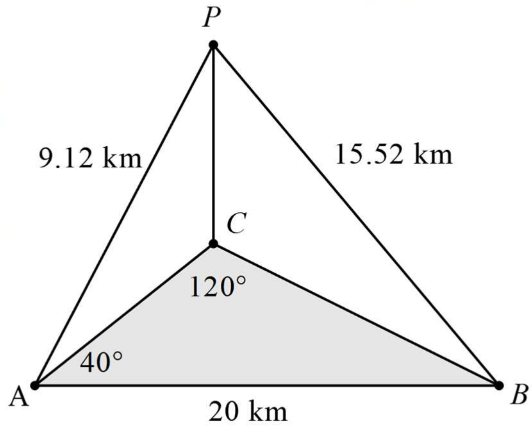
....

....

....

Question 28 (6 marks)

Consider the diagram below where A, B and C are three points on level ground and P is a point vertically above C.



a) Find the length of AC correct to the nearest 0.1 km. **2**

....

....

....

....

....

....

....

b) Hence find the angle of elevation of P from A correct to the nearest minute. **2**

....

....

....

....

....

....

c) Find the angle between PA and PB correct to the nearest minute. **2**

....

....

....

....

....

Question 29 (4 marks)

a) Show that $\frac{d}{dx}(\sin^3 x) = 3\cos x - 3\cos^3 x$. **2**

....

....

....

....

....

b) Hence, or otherwise, find the exact value of $\int_0^{\frac{\pi}{6}} \cos^3 x \, dx$. **2**

....

....

....

....

....

....

....

....

....

Question 30 (4 marks)

The weight, in grams, of beans in a tin is normally distributed with mean μ and standard deviation 7.8. It is known that 10.03% of tins contains less than 200 g of beans.

Probability table for calculating the standard normal distribution. The values represent the area to the left of (or less than) the z-score.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
–1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
–1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
–1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
–1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
–1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
–1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
–0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
–0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
–0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
–0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
–0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952

a) Use the probability table above to find the mean weight, μ, of beans in a tin. Answer correct to two decimal places. **2**

b) The machine settings are adjusted so that the weight, in grams, of beans in a tin is normally distributed with a mean of 205 and standard deviation σ. Given that 97.5% of tins contain between 200 g and 210 g of beans, draw a normal distribution graph to illustrate this information. Use the probability table on page 27 to find the value of the standard deviation, σ, that can be achieved with the new setting. Answer correct to two decimal places. **2**

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

Question 31 (2 marks)

Solve the equation $x^{\ln(2)} = 8$. **2**

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

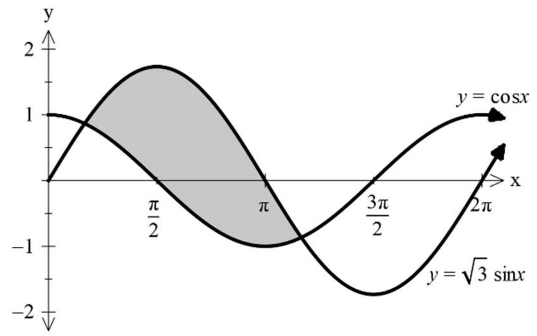
……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

Question 32 (6 marks)

The diagram shows the graphs of $y = \sqrt{3}\,\sin x$ and $y = \cos x$ for $0 \leq x \leq 2\pi$.



a) Show that the x coordinates, α and β, of the points of intersection of the two curves are the solutions of the equation $\tan x = \dfrac{1}{\sqrt{3}}$ for $0 \leq x \leq 2\pi$. **2**

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

b) Hence show that the region enclosed by the two curves has an area of 4 square units. **4**

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

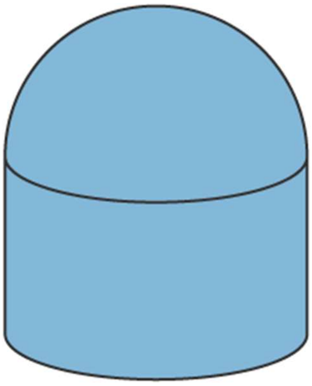
……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

Question 33 (7 marks)



A metal container of volume $0.66\pi\ \text{m}^3$ is to be made in the shape of a hollow cylinder of radius r metres and height h metres, closed at the bottom and capped at the top with a hemispherical shell of the same radius. The area of the metal required is $A\ \text{m}^2$.

a) Given that a sphere with radius r has volume $\dfrac{4}{3}\pi r^3$ and surface area $4\pi r^2$ and a cylinder of radius r and height h has volume $\pi r^2 h$ and surface area $2\pi r^2 + 2\pi rh$, show that $A = \dfrac{33\pi}{25r} + \dfrac{5\pi r^2}{3}$. **3**

....

....

....

....

....

....

....

....

....

....

....

....

....

....

b) Hence find the dimensions of such a container that uses the least amount of metal. **4**

End of paper

Student number

2025 **TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION**

Mathematics Advanced

SOLUTIONS & MARKING GUIDELINES

Section I

Multiple choice Answer Key

Question	Answer
1	D
2	B
3	A
4	A
5	C
6	C
7	B
8	D
9	A
10	C

Question 1

Strong, negative linear correlation

$-1 < r < -0.7$

$\Rightarrow D$

Question 2

$$\int (4x-1)^5 dx = \frac{(4x-1)^6}{6 \times 4} + C$$

$$= \frac{1}{24}(4x-1)^6 + C$$

$$\Rightarrow B$$

Question 3

Let S=soccer team wins and let R = *Rugby* team wins

$$P\left(\overline{S}\,\overline{R}\right) = \frac{5}{9} \times \frac{1}{4}$$

$$= \frac{5}{36}$$

$$\Rightarrow A$$

Question 4

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

$$= \frac{P(A|B) \times P(B)}{P(A)}$$

$$= \frac{0.6 \times 0.3}{0.45}$$

$$= 0.4$$

$\Rightarrow A$

Question 5

Let $x = 2$

$$5 - (2 - 3)^2 = 2m + 3$$

$$2m + 3 = 4$$

$$2m = 1$$

$$m = \frac{1}{2}$$

$\Rightarrow C$

Question 6

$$h(2) = \frac{f(2)}{g(2)}$$

$$h'(2) = \frac{g(2) \times f'(2) - g'(2) \times f(2)}{[g(2)]^2}$$

$$h'(2) = \frac{5 \times 1 - 4 \times 2}{[5]^2}$$

$$\therefore \; h'(2) = -\frac{3}{25}$$

$\Rightarrow C$

Question 7

$$f(g(x)) = \sqrt{2 + 2x + 3}$$

$$= \sqrt{2x + 5}$$

$$2x + 5 \geq 0$$

So $x \geq -2.5$

Domain: $x \in [-2.5, \infty)$

$\Rightarrow B$

Question 8

$$f(2x+3)=f\left(2\left(x+\frac{3}{2}\right)\right)$$

Horizontal dilation by scale factor $\frac{1}{2}$ and horizontal translation to the left by $\frac{3}{2}$ units.

First – for $f(2x)$, x intercepts are $x=0$ and $x=1$.

Second – $f(2x+3)$ moves the x intercepts to $x=-\frac{3}{2}$ and $x=-\frac{1}{2}$

$\Rightarrow D$

Question 9

$$\int 6^{3x-2}dx=\frac{1}{3}\times\frac{1}{\ln(6)}\times 6^{3x-2}+C$$

$$=\frac{6^{3x-2}}{3\ln(6)}+C$$

$\Rightarrow A$

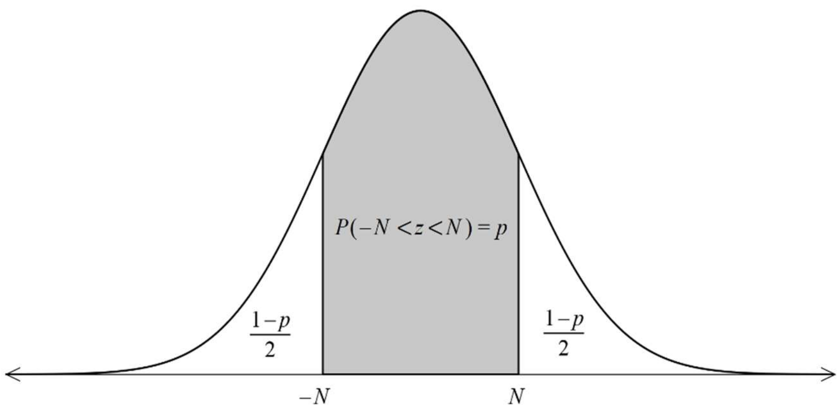
Question 10

$$P(z<N)=P(z\le -N)+P(-N<z<N)$$

$$=\frac{1-p}{2}+p$$

$$=\frac{1+p}{2}$$

$\Rightarrow C$



Section II

Question 11 (2 marks)

Find the vertex of the parabola $y = x^2 - 6x + 10$.

Sample answer:

Axis of symmetry $\left(x = -\dfrac{b}{2a}\right)$

$x = 3$

y-coordinate:

$y = (3)^2 - 6(3) + 10$

$y = 1$

$\therefore$ Vertex is $(3,1)$

Criteria	Marks
Provides correct solution	**2**
Finds the axis of symmetry or equivalent merit.	**1**

Question 12 (2 marks)

Solve the equation $\cos x = \dfrac{\sqrt{3}}{2}$ for $0 \leq x \leq 2\pi$.

Sample answer:

$\cos x = \dfrac{\sqrt{3}}{2}$

Since $\cos x > 0$, solutions are in Quadrants 1 and 4.

$x = \dfrac{\pi}{6}$ and $2\pi - \dfrac{\pi}{6}$

$\therefore \quad x = \dfrac{\pi}{6}$ and $\dfrac{11\pi}{6}$

Criteria	Marks
Provides two correct solutions	**2**
Provides one correct solution, or equivalent merit	**1**

Question 13 (3 marks)

Solve the equation $\log_2(1-x)+\log_2(3-x)=3$.

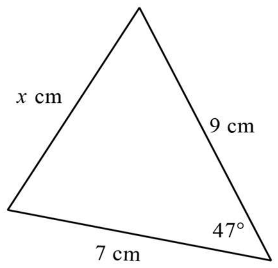
Sample answer:

$$\log_2(1-x)+\log_2(3-x)=3 \quad \text{and} \quad x<1$$
$$\log_2[(1-x)(3-x)]=3$$
$$(1-x)(3-x)=2^3$$
$$x^2-4x+3=8$$
$$x^2-4x-5=0$$
$$(x-5)(x+1)=0$$
$$\therefore \quad x=-1 \quad (\text{since } x<1)$$

Criteria	Marks
Provides correct solution, or equivalent merit.	**3**
Forms the correct quadratic equation, or equivalent merit.	**2**
Simplifies the equation using the addition/multiplication log law.	**1**

Question 14 (2 marks)

Consider the diagram below.



Use the cosine rule to find the value of x, correct to two significant figures.

Sample answer:

$$x^2=7^2+9^2-2(7)(9)\cos(47°)$$
$$x=\sqrt{7^2+9^2-2(7)(9)\cos(47°)}$$
$$x=6.638388858...$$
$$x\approx 6.6 \text{ cm } (2 \text{ sig. figs.})$$

Criteria	Marks
Provides correct solution, with correct rounding.	**2**
Writes the cosine rule with correct substitution.	**1**

Question 15 (2 marks)

Differentiate $x^2(3x+5)^3$ with respect to x.

Sample answer:

$$\frac{d}{dx}\left[x^2(3x+5)^3\right] = 2x(3x+5)^3 + x^2\left(3(3x+5)^2 \times 3\right)$$

$$= 2x(3x+5)^3 + 9x^2(3x+5)^2 \qquad \text{(Accept this answer)}$$

$$= x(3x+5)^2(9x+2(3x+5))$$

$$= x(3x+5)^2(15x+10)$$

$$= 5x(3x+5)^2(3x+2)$$

Criteria	Marks
Provides correct solution, accept unfactorised solution.	**2**
Attempts to use the product rule, or equivalent merit	**1**

Question 16 (3 marks)

Find in simplest general form the equation of the tangent to the curve $y = e^x + \log_e(2x-1)$ at the point where $x = 1$.

Sample answer:

$$y = e^x + \log_e(2x-1)$$

$$y' = e^x + \frac{2}{2x-1}$$

Gradient of the tangent when $x = 1$

$$m = y'(1)$$

$$m = e + 2$$

y-coordinate:

$$y(1) = e + \log_e(1)$$

$$y(1) = e$$

Equation of tangent:

$$y - e = (e+2)(x-1)$$

$$y - e = (e+2)x - e - 2$$

$$\therefore (e+2)x - y - 2 = 0$$

Criteria	Marks
Provides correct solution in general form	**3**
Finds the gradient of the tangent and the y-coordinate or equivalent merit	**2**
Correctly differentiates $y = e^x + \log_e(2x-1)$	**1**

Question 17 (2 marks)

A discrete random variable X with mean 2 has the following probability distribution where a and b are constants.

x	0	1	2	3
$p(x)$	a	0.1	b	0.3

Find the values of a and b.

Sample answer:

Sum of probabilities:

$$a + 0.1 + b + 0.3 = 1$$

$$a + b = 0.6 \qquad (1)$$

Expected Value:

$$0 \times a + 1 \times 0.1 + 2 \times b + 3 \times 0.3 = 2$$

$$2b + 1 = 2$$

$$2b = 1$$

$$b = 0.5$$

Substitute $b = 0.5$ into (1)

$$a + 0.5 = 0.6$$

$$\therefore \qquad a = 0.1$$

Criteria	Marks
Provides correct solution for a and b	**2**
Finds b using the expected value.	**1**

Question 18 (2 marks)

A probability density function is given by $f(x) = \begin{cases} \dfrac{3x^2}{117} & 2 \leq x \leq k \\ 0 & \text{for all other } x \end{cases}$.

Find the value of k.

Sample answer:

$$\int_2^k \frac{3x^2}{117}\,dx = 1$$

$$\left[\frac{x^3}{117}\right]_2^k = 1$$

$$\frac{k^3 - 8}{117} = 1$$

$$k^3 - 8 = 117$$

$$k^3 = 125$$

$$\therefore \quad k = 5$$

Criteria	Marks
Provides correct solution	**2**
Recognises that a PDF has an area under the curve of 1 square unit	**1**

Question 19 (4 marks)

The number of years of teaching experience of the 4800 secondary teachers teaching in Australia are normally distributed with mean 20 years and standard deviation 6 years.

a) Use the Empirical Rules to determine how many more teachers with at most 14 years of experience there are than teachers with at least 32 years of experience.

Sample answer:

14 years and 32 years correspond to z-scores of -1 and 2 respectively. Using the Empirical Rules,

$P(z \leq -1) = 16\%$ and $P(z \geq 2) = 2.5\%$

Teachers with at most 14 years experience = $0.16 \times 4\,800$
$= 768$

Teachers with at least 32 years experience = $0.025 \times 4\,800$
$= 120$

Hence there are 648 more teachers with at most 14 years experience than there are teacher with at least 32 years experience.

Criteria	Marks
Provides correct solution	**2**
Finds the probabilities for $P(z \leq -1)$ and $P(z \geq 2)$	**1**

b) Teachers with more than 14 years of teaching experience are known as highly accomplished teachers. Use the Empirical rule to find, to the nearest whole number, the percentage of highly accomplished teachers who have more than 26 years teaching experience.

Sample answer:

26 years experience corresponds to a z-score of 1.

$$\frac{P(z > 1)}{P(z > -1)} = \frac{16}{84}$$
$$\approx 19\%$$

Note that this can also be done using conditional probability

Criteria	Marks
Provides correct solution	**2**
Finds $P(z > 1)$ or $P(z > -1)$	**1**

Question 20 (6 marks)

A bag contains 11 blue lollies and 9 green lollies.

David takes out a lolly from the bag without looking and eats it. He then takes out another lolly without looking and eats it.

a) What is the probability of David choosing a blue lolly in his first selection?

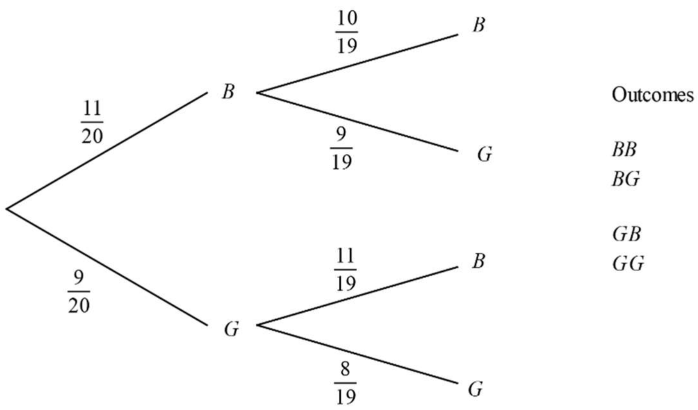
Sample answer:

$P(Blue) = \dfrac{11}{20}$

Criteria	Marks
Correct solution	**1**

b) By drawing a tree diagram, or otherwise, find the probability that David eats two lollies of different colours.

Sample answer:



$\text{P(different } colours) = P(BG) + P(GB)$

$\text{P(different } colours) = \left(\dfrac{11}{20} \times \dfrac{9}{19}\right) + \left(\dfrac{9}{20} \times \dfrac{11}{19}\right)$

$\text{P(different } colours) = \dfrac{99}{190}$

Criteria	Marks
Provides correct solution with or without the tree diagram	**2**
Significant progress towards solution with or without the tree diagram	**1**

c) David opens a different bag of blue and green lollies. He notices that it contains 10 green lollies but does not count the blue lollies. He then eats two lollies without looking at their colour. If the probability that David eats two green lollies is $\frac{3}{8}$, how many blue lollies were in the bag?

Sample answer:

Let x be the number of blue lollies in the bag

Told that $P(GG) = \frac{3}{8}$ and that there are originally 10 Green lollies.

To get $P(GG)$, we start with $\frac{10}{x+10}$ for the first selection.

The second selection will be $\frac{9}{x+9}$

$$\frac{10}{x+10} \times \frac{9}{x+9} = \frac{3}{8}$$

$$\frac{90}{(x+10)(x+9)} = \frac{3}{8}$$

$$720 = 3(x+10)(x+9)$$

$$(x+10)(x+9) = 240$$

$$x^2 + 19x + 90 = 240$$

$$x^2 + 19x - 150 = 0$$

$$(x+25)(x-6) = 0$$

$$\therefore \quad x = 6 \qquad (x > 0)$$

$\therefore$ there are 6 blue lollies in the bag.

Criteria	Marks
Provides correct solution	**3**
Forms a quadratic equation or makes significant progress or equivalent merit	**2**
Finds an equation for P(GG) using blue and green lollies	**1**

Question 21 (8 marks)

Consider the curve given by $y = x^3 - 3x^2$ for $-1 \leq x \leq 3.5$.

a) Find any stationary points and determine their nature.

Sample answer:

$y' = 3x^2 - 6x$

$y'' = 6x - 6$

For stationary points, $y' = 0$.

$3x^2 - 6x = 0$

$3x(x - 2) = 0$

$x = 0$ and $x = 2$

For the nature of the stationary points, consider the sign of y''.

When $x = \dot{0}$ $y'' = -6$ and $y = 0$

$\Rightarrow$ Maximum turning point at $(0,0)$

When $x = \dot{2}$ $y'' = 6$ and $y = -4$

$\Rightarrow$ Minimum turning point at $(2,-4)$

maximum turning point at $x = \frac{1}{3}$

Criteria	Marks
Provides correct solution	**3**
Provides correct x coordinates AND correct expression for y'', or equivalent merit	**2**
Provides correct x coordinates OR correct expression for y'', or equivalent merit	**1**

b) Find any points of inflection.

Sample answer:

Points of inflection occur when $y'' = 0$ **AND** concavity changes.

$y'' = 0$

$6x - 6 = 0$

$6x = 6$

$x = 1$

There is a possible point of inflection where $x = 1$

Test concavity:

x	0.5	1	1.5
y''	-3	0	3

Since concavity changes either side of $x = 1$, there is a point of inflection here.

y-coordinate of point of inflection:

$y(1) = 1^3 - 3(1)^2$

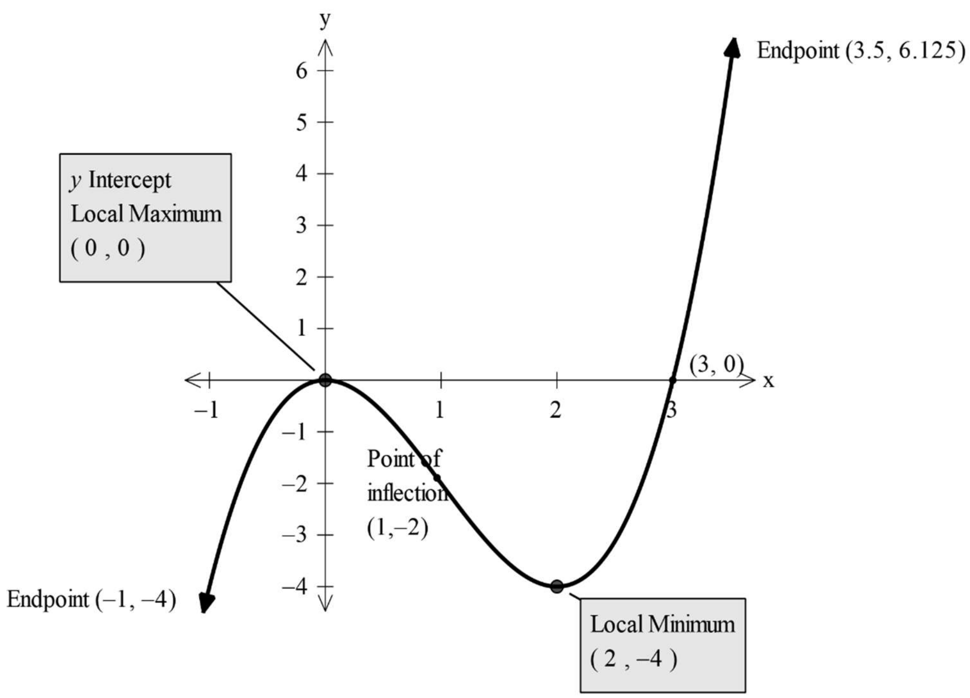
$y(1) = -2$

$\therefore$ Point of inflection at $(1,-2)$

Criteria	Marks
Provides correct solution with appropriate tests of the conditions for a point of inflection	**2**
Solves $y'' = 0$ but does not check concavity.	**1**

c) Sketch the curve over its given domain, clearly labelling any stationary points, points of inflection, intercepts and end points.

Sample answer:



Criteria	Marks
Provides correct graph with points indicated as required	**2**
Provides a graph with some correct features, or equivalent merit	**1**

d) Hence state the global maximum for $y = x^3 - 3x^2$ in the domain $-1 \leq x \leq 3.5$.

Sample answer:

Global maximum is $y = 6.125$ and occurs when $x = 3.5$

Criteria	Marks
Provides correct solution	**1**

Question 22 (3 marks)

Given that $y = \log_e(3^x - 2x)$

a) Complete the table below by finding the missing value of y correct to three decimal places.

Sample answer:

x	1	1.5	2	2.5	3
y	0	0.787	1.609	**2.360**	3.045

Criteria	Marks
Provides correct solution	**1**

b) Using the table above and the trapezoidal rule, find an approximation for the value of $\int_1^3 \log_e(3^x - 2x)\,dx$ correct to three decimal places.

Sample answer:

$$\int_1^3 \log_e(3^x - 2x)\,dx \approx \frac{3-1}{2(4)}[0 + 3.045 + 2(0.787 + 1.609 + 2.360)]$$

$$= 3.13925$$

$$\approx 3.139$$

Criteria	Marks
Provides correct solution with correct rounding	**2**
Makes a clear attempt at using the trapezoidal rule which is mostly correct or equivalent merit	**1**

Question 23 (4 marks)

Let $(\tan\theta - 1)\left(\sqrt{3}\sin\theta - \cos\theta\right)\left(\sqrt{3}\sin\theta + \cos\theta\right) = 0$

Find all solutions for $(\tan\theta - 1)\left(\sqrt{3}\sin\theta - \cos\theta\right)\left(\sqrt{3}\sin\theta + \cos\theta\right) = 0$, where $0 \leq \theta \leq \pi$.

Sample answer:

$$(\tan\theta - 1)\left(\sqrt{3}\ \sin\theta - \cos\theta\right)\left(\sqrt{3}\ \sin\theta + \cos\theta\right) = 0$$

$$\tan\theta - 1 = 0, \quad \sqrt{3}\ \sin\theta - \cos\theta = 0, \quad \sqrt{3}\ \sin\theta + \cos\theta = 0$$

$$\tan\theta = 1$$
$$\theta = \frac{\pi}{4}$$

$$\sqrt{3}\ \sin\theta - \cos\theta = 0$$
$$\sqrt{3}\ \sin\theta = \cos\theta$$
$$\tan\theta = \frac{1}{\sqrt{3}}$$
$$\theta = \frac{\pi}{6}$$

$$\sqrt{3}\ \sin\theta + \cos\theta = 0$$
$$\sqrt{3}\ \sin\theta = -\cos\theta$$
$$\tan\theta = -\frac{1}{\sqrt{3}}$$
$$\theta = \pi - \frac{\pi}{6}$$
$$\theta = \frac{5\pi}{6}$$

Criteria	Marks
Provides all correct solutions with appropriate working	**4**
Provides significant progress towards solution or does not heed the given domain.	**3**
Provides one correct solution, or equivalent merit	**2**
Finds an equation to solve in terms of tan.	**1**

Question 24 (4 marks)

The table below shows the recorded height and the right foot length for 9 people (*A* to *I*). Only the height data is available for person *J*.

	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*
Height (cm)	142	164	183	172	154	158	149	181	175	177
Foot length (cm)	20	26	28	26	22	23	24	29	27	

a) Calculate Pearson's correlation coefficient between height and right foot length for *A* to *I* correct to three decimal places. Describe the nature of the correlation.

Sample answer:

$r \approx 0.939$

This shows a strong, positive correlation between height and right foot length

Criteria	Marks
Provides correct solution with correct interpretation	**2**
Provides correct value with no interpretation provided	**1**

b) Find the equation of the least-squares regression line of height on right foot length for *A* to *I*. Estimate the right foot length of person *J* to the nearest centimetre

Sample answer:

Least-squares regression line:

$f = 0.1917173856h - 6.484255094$

For Person J

$f = 0.1917173856(177) - 6.484255094$

$f = 27.44972216$

$f \approx 27$ cm

Criteria	Marks
Provides correct equation and required foot length	**2**
Provides the correct equation, accept if values are rounded to 3 d.p. for the equation.	**1**

Question 25 (3 marks)

Find the values of any constant A such that $\int_0^3 (x^2 + A)\, dx = 2A^2$.

Sample answer:

$$\int_0^3 (x^2 + A)\, dx = 2A^2$$

$$\left[\frac{x^3}{3} + Ax\right]_0^3 = 2A^2$$

$$\frac{3^3}{3} + 3A = 2A^2$$

$$2A^2 - 3A - 9 = 0$$

$$2A^2 - 6A + 3A - 9 = 0$$

$$2A(A - 3) + 3(A - 3) = 0$$

$$(A - 3)(2A + 3) = 0$$

$$\therefore \quad A = -\frac{3}{2} \quad \text{or} \quad A = 3$$

Criteria	Marks
Provides correct solutions	**3**
Provides significant progress towards solutions	**2**
Provides correct anti-derivative	**1**

Question 26 (6 marks)

A particle is moving in a straight line. At time t seconds it has displacement x metres from a fixed point O on the line and velocity $v(t) = 3\sqrt{t+1} - 9$ metres per second. The particle starts at O.

a) Show that $x(t) = 2(t+1)\sqrt{t+1} - 9t - 2$.

Sample answer:

$$v(t) = 3\sqrt{t+1} - 9$$

$$v(t) = 3(t+1)^{\frac{1}{2}} - 9$$

$$x(t) = \int 3(t+1)^{\frac{1}{2}} - 9 \text{ dt}$$

$$x(t) = \frac{3(t+1)^{\frac{3}{2}}}{\frac{3}{2}} - 9t + C$$

$$x(t) = 2\sqrt{(t+1)^3} - 9t + C$$

$$x(t) = 2(t+1)\sqrt{t+1} - 9t + C$$

When $t = 0, x = 0$

$$0 = 2(1)\sqrt{1} + C$$

$\therefore \quad C = -2$

$\therefore x(t) = 2(t+1)\sqrt{t+1} - 9t - 2$ as required

Criteria	Marks
Provides correct solution with appropriate and logical steps shown	**2**
Correctly integrates v(t) but does not evaluate the constant of integration or makes an error	**1**

b) Find when and where the particle is at rest in relation to the fixed point O.

Sample answer:

Particle at rest when $v = 0$

$$3\sqrt{t+1} - 9 = 0$$

$$\sqrt{t+1} = 3$$

$$t + 1 = 9$$

$\therefore \quad t = 8$

$$x(8) = 2(8+1)\sqrt{8+1} - 9(8) - 2$$

$$= -20$$

$\therefore$ The particle is at rest after 8 seconds, 20 m away from O in the initial direction of the motion.

Criteria	Marks
Provides correct solution, accept 20 m away from the origin	**2**
Finds when the particle is at rest	**1**

c) Find the total distance travelled by the particle in the first 24 seconds.

Sample answer:

When $t = 24, x = 32$

Particle travels from $x = 0$ to $x = -20$ then to $x = 32$ in the first 24 seconds.

Hence distance travelled is $20 + 20 + 32 = 72\ m$

Criteria	Marks
Provides correct solution	**2**
Finds x when $t = 24$	**1**

Question 27 (5 marks)

a) Prove that $(1 + \tan x)^2 = 2\tan x + \sec^2 x$.

Sample answer:

$$\begin{aligned} LHS &= (1 + \tan x)^2 \\ &= 1 + 2\tan x + \tan^2 x \\ &= 2\tan x + 1 + \tan^2 x \\ &= 2\tan x + \sec^2 x \\ &= RHS \end{aligned}$$

Criteria	Marks
Correct solution with all steps shown	**2**
Expands correctly	**1**

b) Hence find the area bounded by $y = (1 + \tan x)^2$ and the x-axis between

$-\frac{\pi}{6} \leq x \leq \frac{\pi}{6}$ with a rational denominator.

Sample answer:

$y = (1 + \tan x)^2$ is positive for all x values

$y = 2\tan x + \sec^2 x$

$$A = \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} (2\tan x + \sec^2 x)\ dx$$

$$A = \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \left(\frac{2\sin x}{\cos x} + \sec^2 x\right) dx$$

$$A = [-2\ln|\cos x| + \tan x]^{\frac{\pi}{6}}_{-\frac{\pi}{6}}$$

$$A = \left(-2\ln\left|\cos\frac{\pi}{6}\right| + \tan\frac{\pi}{6}\right) - \left(-2\ln\left|\cos\left(-\frac{\pi}{6}\right)\right| + \tan\left(-\frac{\pi}{6}\right)\right)$$

$$A = -2\ln\left(\frac{\sqrt{3}}{2}\right) + \frac{1}{\sqrt{3}} - \left(-2\ln\left(\frac{\sqrt{3}}{2}\right) - \frac{1}{\sqrt{3}}\right)$$

$$A = -2\ln\left(\frac{\sqrt{3}}{2}\right) + \frac{1}{\sqrt{3}} + 2\ln\left(\frac{\sqrt{3}}{2}\right) + \frac{1}{\sqrt{3}}$$

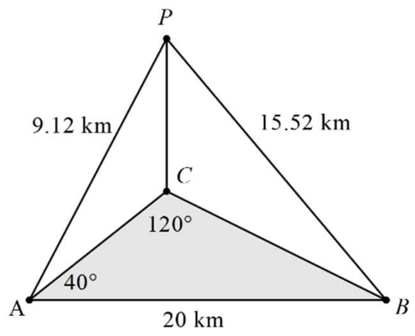
$$A = \frac{2}{\sqrt{3}}$$

$$A = \frac{2\sqrt{3}}{3}\ units^2$$

Criteria	Marks
Provides correct solution	**3**
Correctly integrates but makes an error in evaluating	**2**
Changes tanx into sinx/cosx before integrating	**1**

Question 28 (6 marks)

Consider the diagram below where A, B and C are three points on level ground and P is a point vertically above C.



a) Find the length of AC correct to the nearest 0.1 km.

Sample answer:

$$\angle ABC = 180° - 120° - 40°$$
$$= 20°$$

Using the Sine Rule:

$$\frac{AC}{\sin 20°} = \frac{20}{\sin 120°}$$

$$AC = \frac{20\sin 20°}{\sin 120°}$$

$$AC = 7.898616873 \ \ldots$$

$$AC \approx 7.9 \text{ km}$$

Criteria	Marks
Provides correct solution	**2**
Attempts to use the sine rule	**1**

b) Hence find the angle of elevation of P from A correct to the nearest minute.

Sample answer:

$$\cos\theta = \frac{7.9}{9.12}$$

$$\theta = \cos^{-1}\left(\frac{7.9}{9.12}\right)$$

$$\therefore \quad \theta \approx 29°59'$$

Criteria	Marks
Provides correct solution – accept if the exact value of AC was used instead	**2**
Finds correct expression to find the angle	**1**

c) Find the angle between *PA* and *PB* correct to the nearest minute.

Sample answer:

$$\cos\alpha = \frac{(9.12)^2 + (15.52)^2 - 20^2}{2 \times 9.12 \times 15.52}$$

$$\alpha = \cos^{-1}\left(\frac{(9.12)^2 + (15.52)^2 - 20^2}{2 \times 9.12 \times 15.52}\right)$$

$$\alpha \approx 105°34'$$

Criteria	Marks
Provides correct solution	**2**
Attempts to use the cosine rule	**1**

Question 29 (4 marks)

c) Show that $\frac{d}{dx}(\sin^3 x) = 3\cos x - 3\cos^3 x$.

Sample answer:

$$\begin{aligned} LHS &= \frac{d}{dx}(\sin^3 x) \\ &= 3\sin^2 x \cos x \\ &= 3\cos x(1 - \cos^2 x) \\ &= 3\cos x - 3\cos^3 x \\ &= RHS \text{ as required} \end{aligned}$$

Criteria	Marks
Provides correct solution with all appropriate steps shown	**2**
Differentiates correctly but does not make the required identity	**1**

d) Hence, or otherwise, find the exact value of $\int_0^{\frac{\pi}{6}} \cos^3 x \, dx$.

Sample answer:

$$\frac{d}{dx}(\sin^3 x) = 3\cos x - 3\cos^3 x$$

$$\therefore \quad \cos^3 x = \cos x - \frac{1}{3}\left[\frac{d}{dx}(\sin^3 x)\right]$$

$$\int_0^{\frac{\pi}{6}} \cos^3 x \, dx = \int_0^{\frac{\pi}{6}} \left(\cos x - \frac{1}{3}\left[\frac{d}{dx}(\sin^3 x)\right]\right) dx$$

$$= \int_0^{\frac{\pi}{6}} \cos x \, dx - \frac{1}{3}\int_0^{\frac{\pi}{6}} \left[\frac{d}{dx}(\sin^3 x)\right] dx$$

$$= [\sin x]_0^{\frac{\pi}{6}} - \frac{1}{3}[\sin^3 x]_0^{\frac{\pi}{6}}$$

$$= \sin\left(\frac{\pi}{6}\right) - \sin(0) - \frac{1}{3}\left(\sin^3\left(\frac{\pi}{6}\right) - \sin^3(0)\right)$$

$$= \frac{1}{2} - 0 - \frac{1}{3}\left(\frac{1}{8} - 0\right)$$

$$= \frac{11}{24}$$

Criteria	Marks
Provides correct solution	**2**
Correctly rearranges the answer to part a to integrate	**1**

Question 30 (4 marks)

The weight, in grams, of beans in a tin is normally distributed with mean μ and standard deviation 7.8. It is known that 10.03% of tins contains less than 200 g of beans.

Probability table for calculating the standard normal distribution. The values represent the area to the left of (or less than) the z-score.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
–1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
–1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
–1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
–1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
–1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
–1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
–0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
–0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
–0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
–0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
–0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952

a) Use the probability table above to find the mean weight, μ, of beans in a tin. Answer correct to two decimal places. **2**

Sample answer:

$$P(X < 200) = 10.03\,\%$$
$$= 0.1003$$

From the table, this corresponds to a z-score of $z = -1.28$

Using z-score formula:

$$-1.28 = \frac{200 - \mu}{7.8}$$

$$\mu = 200 + 1.28 \times 7.8$$

$$\therefore \quad \mu \approx 209.98\ g$$

Criteria	Marks
Provides correct solution	**2**
Identifies the correct z-score for 200 g. Uses an incorrect z-score to find a μ	**1**

b) The machine settings are adjusted so that the weight, in grams, of beans in a tin is normally distributed with a mean of 205 and standard deviation σ. Given that 97.5% of tins contain between 200 g and 210 g of beans, draw a normal distribution graph to illustrate this information. Use the probability table on page 27 to find the value of the standard deviation, σ, that can be achieved with the new setting. Answer correct to two decimal places.

Sample answer:

$P(200 \leq X \leq 210) = 0.975$

$P(X \leq z_{210}) = 0.975 + 0.0125$

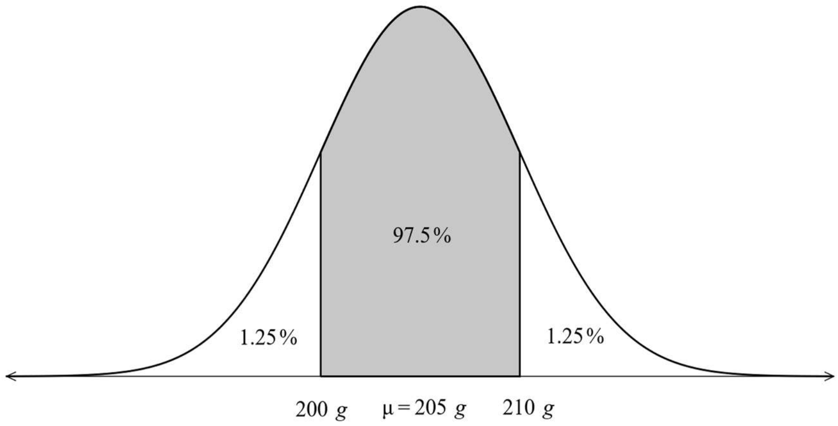
$= 0.9875$

$z_{210} = 2.24$ (from table)

$2.24 = \dfrac{210 - 205}{\sigma}$

$\sigma = \dfrac{5}{2.24}$

$\sigma \approx 2.23$



Criteria	Marks
Provides correct solution with diagram	**2**
Draws a relevant diagram or finds the z-score for 210 g	**1**

Question 31 (2 marks)

Solve the equation $x^{\ln(2)} = 8$.

Sample answer:

$x^{\ln(2)} = 8$

$x^{\ln(2)} = 2^3$

$\ln(x^{\ln(2)}) = \ln(2^3)$

$\ln(2)\ln x = 3\ln(2)$

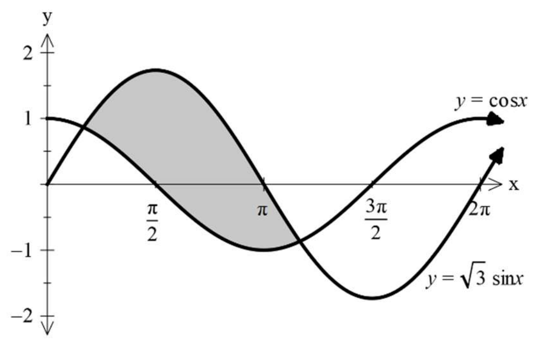
$\ln x = 3$

$x = e^3$

Criteria	Marks
Provides correct solution (exact or rounded form)	**2**
Applies one log law	**1**

Question 32 (6 marks)

The diagram shows the graphs of $y = \sqrt{3}\sin x$ and $y = \cos x$ for $0 \leq x \leq 2\pi$.



a) Show that the x coordinates, α and β, of the points of intersection of the two curves are the solutions of the equation $\tan x = \frac{1}{\sqrt{3}}$ for $0 \leq x \leq 2\pi$.

Sample answer:

$$\sqrt{3}\sin x = \cos x$$
$$\sqrt{3}\tan x = 1$$
$$\tan x = \frac{1}{\sqrt{3}}$$

Criteria	Marks
Provides correct solution	**2**
Recognises point of intersection requires that the equations must equal each other	**1**

b) Hence show that the region enclosed by the two curves has an area of 4 square units.

Sample answer:

$$\tan x = \frac{1}{\sqrt{3}}$$

$$x = \frac{\pi}{6}, \frac{7\pi}{6}$$

$$Area = \int_{\frac{\pi}{6}}^{\frac{7\pi}{6}} \left(\sqrt{3}\sin x - \cos x\right) dx$$

$$= \left[-\sqrt{3}\cos x - \sin x\right]_{\frac{\pi}{6}}^{\frac{7\pi}{6}}$$

$$= \left(-\sqrt{3}\cos\left(\frac{7\pi}{6}\right) - \sin\left(\frac{7\pi}{6}\right)\right) - \left(-\sqrt{3}\cos\left(\frac{\pi}{6}\right) - \sin\left(\frac{\pi}{6}\right)\right)$$

$$= \left(\sqrt{3}\cos\left(\frac{\pi}{6}\right) + \sin\left(\frac{\pi}{6}\right)\right) - \left(-\sqrt{3}\cos\left(\frac{\pi}{6}\right) - \sin\left(\frac{\pi}{6}\right)\right)$$

$$= \left(\sqrt{3} \times \frac{\sqrt{3}}{2} + \frac{1}{2}\right) - \left(-\sqrt{3} \times \frac{\sqrt{3}}{2} - \frac{1}{2}\right)$$

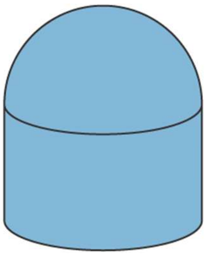
$$= \frac{3}{2} + \frac{1}{2} + \frac{3}{2} + \frac{1}{2}$$

$$= \frac{8}{2}$$

$$= 4\ units^2$$

Criteria	Marks
All prior steps completed, shows correct substitution leading to the correct solution.	**4**
Both prior steps completed, finds the correct primitive function	**3**
Forms an appropriate integral for finding the area	**Allow 1**
Solves the equation $\tan x = \frac{1}{\sqrt{3}}$ to find intersection points	**Allow 1**

Question 33 (7 marks)



A metal container of volume $0.66\pi\ \text{m}^3$ is to be made in the shape of a hollow cylinder of radius r metres and height h metres, closed at the bottom and capped at the top with a hemispherical shell of the same radius. The area of the metal required is $A\ \text{m}^2$.

a) Given that a sphere with radius r has volume $\frac{4}{3}\pi r^3$ and surface area $4\pi r^2$ and a cylinder of radius r and height h has volume $\pi r^2 h$ and surface area $2\pi r^2 + 2\pi rh$, show that $A = \frac{33\pi}{25r} + \frac{5\pi r^2}{3}$.

Sample answer:

Volume

$$\frac{1}{2} \times \frac{4}{3}\pi r^3 + \pi r^2 h = 0.66\pi$$

$$\frac{2}{3}r^3 + r^2 h = 0.66$$

$$2r^3 + 3r^2 h = \frac{99}{50}$$

$$3r^2 h = \frac{99}{50} - 2r^3$$

$$h = \frac{33}{50r^2} - \frac{2r}{3}$$

Area

$$A = \frac{1}{2} \times 4\pi r^2 + \pi r^2 + 2\pi rh$$

$$= 2\pi r^2 + \pi r^2 + 2\pi rh$$

$$= 3\pi r^2 + 2\pi r\left(\frac{33}{50r^2} - \frac{2r}{3}\right)$$

$$= 3\pi r^2 + \frac{33\pi}{25r} - \frac{4\pi r^2}{3}$$

$$= \frac{33\pi}{25r} + \frac{5\pi r^2}{3} \text{ as required}$$

Criteria	Marks
Provides correct solution	**3**
Finds an equation for the surface area in terms of r (eliminating h)	**2**
Writes equation for volume incorporating the given volume.	**1**

b) Hence find the dimensions of such a container that uses the least amount of metal.

Sample answer:

$$A = \frac{33\pi}{25r} + \frac{5\pi r^2}{3}$$

$$= \frac{33\pi}{25} r^{-1} + \frac{5\pi}{3} r^2$$

$$A' = -\frac{33\pi}{25} r^{-2} + \frac{10\pi}{3} r$$

$$A'' = \frac{66\pi}{25} r^{-3} + \frac{10\pi}{3}$$

Min/Max when $A' = 0$

$$\frac{10\pi r}{3} - \frac{33\pi}{25r^2} = 0$$

$$\frac{10\pi r^3}{3} - \frac{33\pi}{25} = 0$$

$$\frac{10\pi r^3}{3} = \frac{33\pi}{25}$$

$$r^3 = \frac{99}{250}$$

$$\therefore \qquad r = \sqrt[3]{\frac{99}{250}}$$

Test whether this will give a minimum or maximum value:

$$A'' = \frac{66\pi}{25}\left(\sqrt[3]{\frac{99}{250}}\right)^{-3} + \frac{10\pi}{3}$$

$$= 10\pi$$

Since $A'' > 0 \Rightarrow Min$ at $r = \sqrt[3]{\frac{99}{250}}$

Answer continued over page.

$$h = \frac{33}{50r^2} - \frac{2r}{3}$$

$$h = \frac{99 - 100r^3}{150r^2}$$

$$h = \frac{99 - 100\left(\sqrt[3]{\frac{99}{250}}\right)^3}{150\left(\sqrt[3]{\frac{99}{250}}\right)^2}$$

$$h = \frac{99 - 100\left(\frac{99}{250}\right)}{150\left(\frac{99}{250}\right)^{\frac{2}{3}}}$$

$$h = \frac{99 - \frac{990}{25}}{150} \times \left(\frac{99}{250}\right)^{-\frac{2}{3}}$$

$$h = \frac{297}{750} \times \left(\frac{99}{250}\right)^{-\frac{2}{3}}$$

$$h = \left(\frac{99}{250}\right)^1 \times \left(\frac{99}{250}\right)^{-\frac{2}{3}}$$

$$h = \left(\frac{99}{250}\right)^{\frac{1}{3}}$$

$$\therefore h = \sqrt[3]{\frac{99}{250}}$$

$\therefore$ Dimensions are $r = h = \sqrt[3]{\frac{99}{250}}\ m$

Alternate end of solution:

$$2r^3 + 3r^2h = 0.66$$

$$h = \frac{1.98 - 2r^3}{3r^2}$$

$$h = \frac{1.98 - 2\left(\sqrt[3]{\frac{99}{250}}\right)^3}{3\left(\sqrt[3]{\frac{99}{250}}\right)^2}$$

$$h = \frac{1.98 - \frac{99}{125}}{3} \times \left(\frac{99}{250}\right)^{-\frac{2}{3}}$$

$$h = \frac{99}{250} \times \left(\frac{99}{250}\right)^{-\frac{2}{3}}$$

$$h = \left(\frac{99}{250}\right)^{\frac{1}{3}}$$

$$\therefore \quad h = \sqrt[3]{\frac{99}{250}}$$

Criteria	Marks
Provides correct dimensions with relevant working including justification that the given r value gives a minimum SA	**4**
Provides correct dimensions or justifies that the determined r value gives a minimum.	**3**
Finds the value of r that makes A'=0	**2**
Differentiates A with respect to r	**1**