HORNSBY GIRLS HIGH SCHOOL

Mathematics Advanced

Year 12 Higher School Certificate
Trial Examination Term 3 2020

STUDENT NUMBER: ____________________

General Instructions:	• Reading time – 10 minutes • Working time – 3 hours • Write using black pen • Calculators approved by NESA may be used • For questions in Section II, show relevant mathematical reasoning and/ or calculations
Total Marks: 100	**Section I – 10 marks** (pages 2–5) • Attempt Questions 1–10 • Allow about 15 minutes for this section **Section II – 90 marks** (pages 6–21) • Attempt Questions 11–34 • Allow about 2 hour and 45 minutes for this section

Section I

10 marks
Attempt questions 1 - 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for questions 1-10

1. What is the value of $f'(x)$ if $f(x) = 3x^4(4-x)^3$?

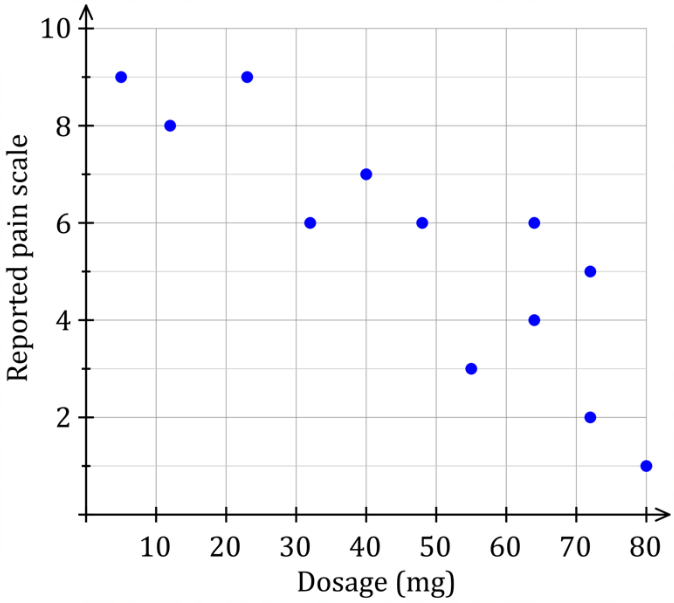
(A) $3x^3(4-x)^3(7x-16)$

(B) $3x^3(4-x)^3(16-7x)$

(C) $3x^3(4-x)^2(7x-16)$

(D) $3x^3(4-x)^2(16-7x)$

2. A scatterplot of pain (as reported by patients) compared to the dosage (in mg) of a drug is shown below.



How could you describe the correlation between the pain and the dosage?

(A) A moderate negative correlation

(B) A moderate positive correlation

(C) A weak positive correlation.

(D) No correlation.

3. What values of x is the curve $f(x) = 2x^3 + x^2$ concave down?

(A) $x < -\dfrac{1}{6}$

(B) $x > -\dfrac{1}{6}$

(C) $x < -6$

(D) $x > 6$

4. The probability density function for the continuous random variable X is:

$$f(x) = \begin{cases} x^3 - x + 4 & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

The value of $E(X)$ is:

(A) $\dfrac{17}{4}$

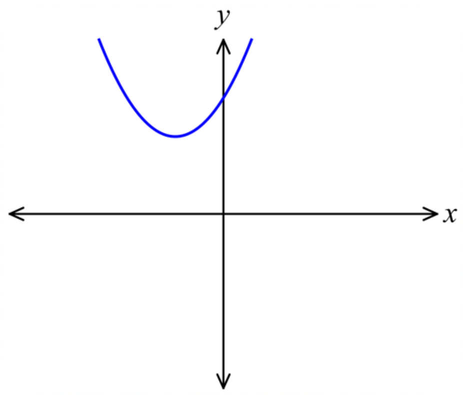
(B) $\dfrac{28}{15}$

(C) 0

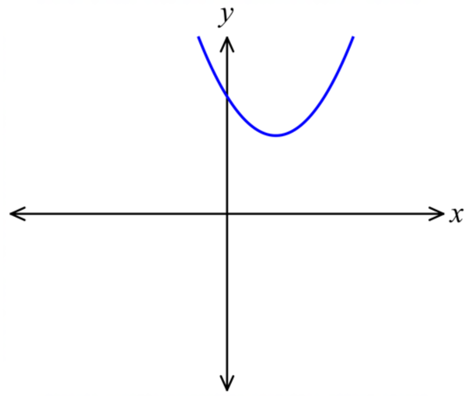
(D) 4

5. Which diagram best shows the graph of the parabola $y = 2 - (x+1)^2$

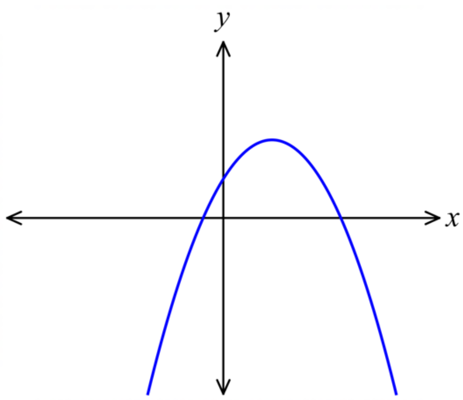
(A)



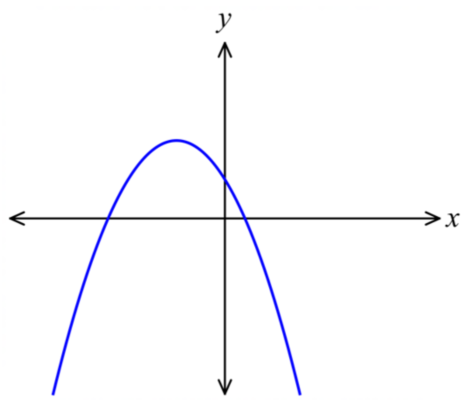
(B)



(C)



(D)



6. Which of the following is the set of all solutions to $2x^2 - 5x + 2 \geq 0$?

(A) $\left[\frac{1}{2}, 2\right]$

(B) $\left(\frac{1}{2}, 2\right)$

(C) $\left(-\infty, \frac{1}{2}\right) \cup (2, \infty)$

(D) $\left(-\infty, \frac{1}{2}\right] \cup [2, \infty)$

7. Which statement is true for an ungrouped data set with no outliers?

(A) The largest possible range is 2 times the interquartile range.

(B) The largest possible range is 3 times the interquartile range.

(C) The largest possible range is 4 times the interquartile range.

(D) The largest possible range is 5 times the interquartile range.

8. A and B are events of a sample space.

Given that $\mathrm{P}(A \mid B) = 4p$, $\mathrm{P}(B) = p^3$ and $\mathrm{P}(A) = 2p^2$, $\mathrm{P}(B \mid A)$ is equal to

(A) $2p^2$

(B) $2p^6$

(C) $\dfrac{p^2}{2}$

(D) $\dfrac{2}{p^2}$

9. The continuous random variable X has a normal distribution with a mean 11 and standard deviation 2. If the random variable Z has the standard normal distribution, then the probability that X is greater than 16 is equal to:

(A) $P(Z > 2)$

(B) $P(Z < -2.5)$

(C) $P(Z < 2.5)$

(D) $P(Z > 5)$

10. What are the solutions to the equation $2\sin x + \sqrt{3} = 0$, where $\{x: 0 \le x \le 2\pi\}$?

(A) $\dfrac{\pi}{3}, \dfrac{2\pi}{3}$

(B) $\dfrac{2\pi}{3}, \dfrac{5\pi}{3}$

(C) $\dfrac{4\pi}{3}, \dfrac{5\pi}{3}$

(D) $\dfrac{7\pi}{3}, \dfrac{11\pi}{3}$

Section II

90 marks
Attempt all questions
Allow about 2 hours and 45 minutes for this section

Answer each question in the spaces provided.
Your responses should include relevant mathematical reasoning and/or calculations.
Extra writing space is provided at the back of the examination paper.

Question 11 (2 marks) **Marks**

Find the primitive function of $\frac{1}{1-2x}$ with respect to x. **2**

Question 12 (3 marks)

Let $f(x) = \frac{(x+3)(2x+1)}{\sqrt{x}}, x > 0$

(a) By expressing $f(x)$ in the form **2**

$$Ax^{\frac{3}{2}} + Bx^{\frac{1}{2}} + Cx^{-\frac{1}{2}}$$

find the values of A, B and C.

(b) Find $f'(x)$ **1**

Question 13 (3 marks) **Marks**

The discrete random variable, X, has the following probability distribution.

X	0	1	2	3	4
$P(X = x)$	0.1	0.2	0.4	0.2	0.1

(a) Find $P(1 < X \leq 3)$ **1**

(b) Find the variance of X. **2**

Question 14 (2 marks)

Find $\int (6x^2 + 2 + x^{-\frac{1}{2}})\, dx,$ giving each term in its simplest form. **2**

Question 15 (2 marks)

In an arithmetic sequence, the fifth term is 9 and the twenty-first term is 425. **2**
Find the fiftieth term.

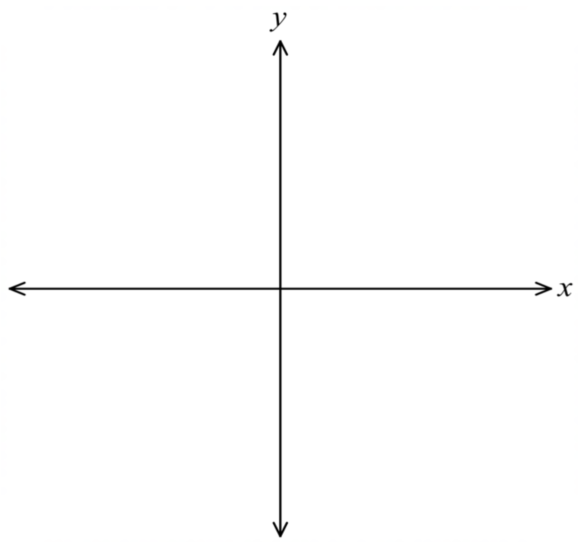
Question 16 (8 marks) **Marks**

Let $f(x) = (x-2)(x^2+1)$

(a) Find where the graph of $y = f(x)$ cuts the x-axis and y axis. **2**

(b) Find the coordinates of the stationary points on the curve with the equation $y = f(x)$ and determine their nature. **3**

(c) Sketch the graphs of $y = f(x)$ and $y = -f(x)$ on the same diagram. **3**



Question 17 (2 marks)

Find the exact value of $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos x \, dx$. **2**

Question 18 (5 marks) **Marks**

Given that $f(x) = (x^2 - 6x)(x - 3) + 2x$.

(a) Express $f(x)$ in the form $x(ax^2 + bx + c)$, where a, b and c are constants. **2**

(b) Hence factorise $f(x)$ completely. **1**

(c) Sketch the graph of $y = f(x)$, showing the coordinates of each point at which the graph meets the axes. **2**

Question 19 (3 marks)

Differentiate with respect to x

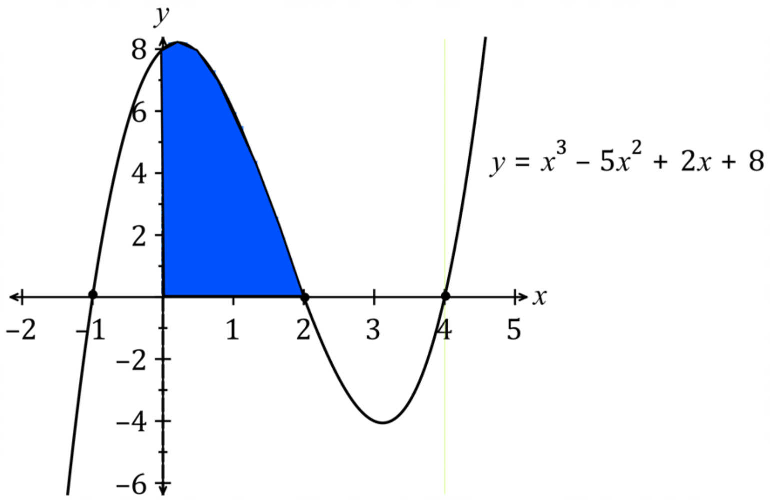
(a) $\ln(x^2 + 2)$ **1**

(b) $\dfrac{\sin x}{x^2}$ **2**

Question 20 (2 marks)

Marks

2



Find the shaded area enclosed by the curve $y = x^3 - 5x^2 + 2x + 8$ and the coordinate axes.

Question 21 (3 marks)

The probability density function for the continuous random variable X is given by:

$$f(x) = \begin{cases} |1 - x| & 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find $P(X \leq 1.5)$ **2**

(b) Find $P(1 \leq X \leq 2)$ **1**

Question 22 (5 marks) **Marks**

An arithmetic series is defined as 7+5+3+……………

(a) What is the common difference, d, of this series? **1**

(b) The sum of n terms of this series is -48. **2**
Form an equation in terms of n, and show that your equation may be written as $n^2 - 8n - 48 = 0$.

(c) Find the solution(s) to the equation in part (b). **2**

Question 23 (2 marks)

If $\tan\theta = \dfrac{2}{3}$, and θ is acute, find the exact value of $\sin\theta$? **2**

Question 24 (6 marks) **Marks**

A curve with the equation $y = f(x)$, has $\dfrac{dy}{dx} = x^3 + 2x - 7$.

(a) Find $\dfrac{d^2y}{dx^2}$ **1**

(b) Show that $\dfrac{d^2y}{dx^2} \geq 2$ for all values of *x*. **1**

(c) The point *P* (2, 4) lies on the curve. Find *y* in terms of *x*. **2**

(d) Find an equation for the normal to the curve at *P*, expressing your answer in the form $ax + by + c = 0$, where *a*, *b* and *c* are integers. **2**

Question 25 (4 marks) **Marks**

The students in a school were surveyed on the number of hours of sleep per week. The results were normally distributed. The survey indicated that 95% of students had between 42 and 54 hours of sleep per week.

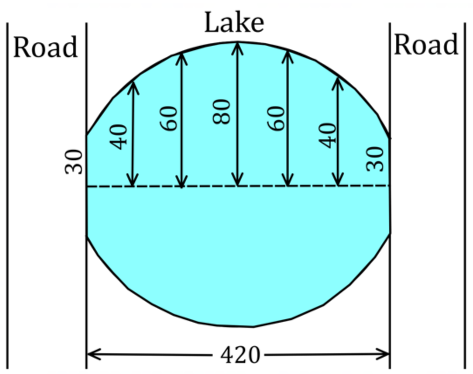
(a) Determine the mean number of hours of sleep per week. **1**

(b) What was the standard deviation? **1**

(c) What percentage of students would have indicated they had between 51 and 57 hours of sleep per week? **2**

Question 26 (3 marks)

A symmetrical lake has two roads, 420 metres apart, forming two of its sides.



Not to scale

Equally spaced measurements of the lake, in metres, are shown on the above diagram. Use the trapezoidal rule to estimate the area of the lake. **3**

Question 27 (7 marks) **Marks**

A particle moves in a straight line. At time t seconds, its distance x metres from a fixed point 0 on the line is given by $x = 1 - \cos 2t$.

(a) Sketch the graph of x as a function of t for $0 \leq t \leq \pi$. **2**

(b) Using your graph, or otherwise, to find the times when the particle is at rest and the position of the particle at these times. **2**

(c) Find the velocity of the particle when $t = \frac{\pi}{4}$. **1**

(d) When is the particle's velocity greater than 1 m/s? **2**

Question 28 (3 marks)

Show that the curve $y = 2x^2 - \ln\left(\frac{x}{2}\right) - 4$ has a minimum value at $\left(\frac{1}{2}, \ln 4 - 3\frac{1}{2}\right)$ **3**

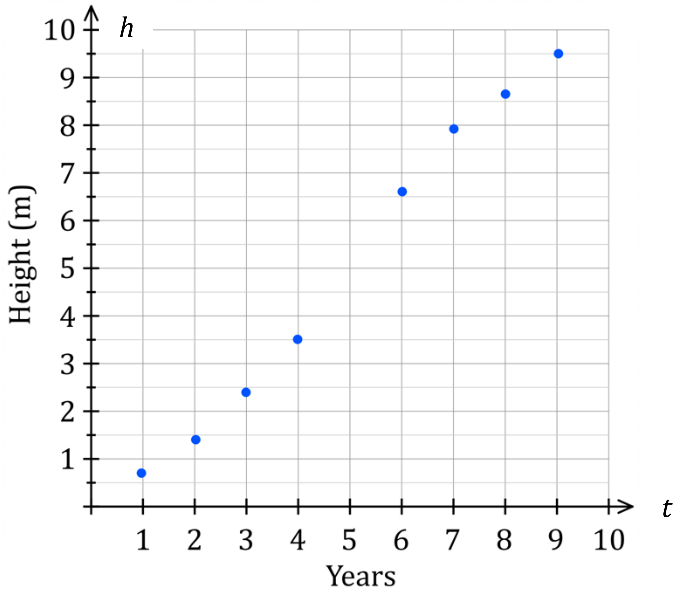
Question 29 (3 marks) **Marks**

Hayden is an agricultural scientist studying the growth of a particular tree over several years. The data he recorded is shown in the table below.

Years since planting, t	1	2	3	4	5	6	7	8	9
Height of tree, h metres	0.7	1.4	2.4	3.5	$\boldsymbol{H}$	6.6	7.9	8.7	9.5

$\boldsymbol{H} > 0$

A scatterplot of the data is shown below.



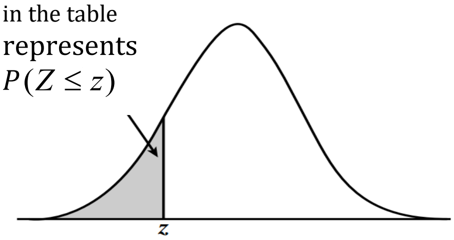
(a) What is Pearson's correlation coefficient? Answer correct to 4 decimal places. **1**

(b) Find the equation of the least-squares line of best fit in terms of years (t) and height (h). Answer correct to 2 decimal places. **1**

(c) Hayden did not record the tree's height after five years. Predict the height, $\boldsymbol{H}$, after five years, correct to one decimal place. **1**

Question 30 (2 marks) **Marks**

A section of the standard normal table is shown below:



Z	.00	.01	.02	.03	.04	.05
−2.4	.0082	.0080	.0078	.0075	.0073	.0071
−2.3	.0107	.0104	.0102	.0099	.0096	.0094
−2.2	.0139	.0136	.0132	.0129	.0125	.0122
−2.1	.0179	.0174	.0170	.0166	.0162	.0158
−2.0	.0228	.0222	.0217	.0212	.0207	.0202
−1.9	.0287	.0281	.0274	.0268	.0262	.0256
−1.8	.0359	.0351	.0344	.0336	.0329	.0322
−1.7	.0446	.0436	.0427	.0418	.0409	.0401
−1.6	.0548	.0537	.0526	.0516	.0505	.0495

Find $P(-2.3 \le z \le 1.72)$ **2**

Question 31 (6 marks) **Marks**

The height, $h(t)$, metres of the tide above the **mean sea level** on 1st April is given by the equation:

$$h(t) = 4\sin\left(\frac{\pi}{8}t\right)$$

where t is the number of hours after midnight.

(a) Draw a graph of $y = h(t)$ for $0 \le t \le 24$. **2**

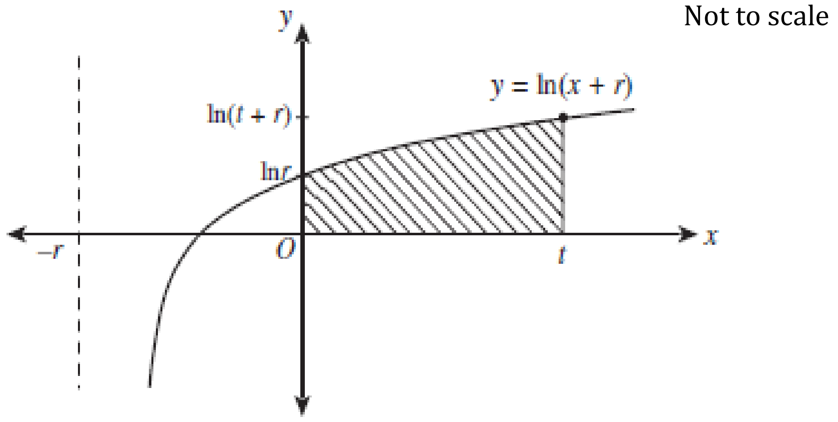
(b) When was high tide? **1**

(c) What was the height of the high tide? **1**

(d) What was the height of the tide at 10 am? Answer correct to one decimal place. **2**

Question 32 (3 marks) **Marks**

Consider the graph of $y = \ln(x+r)$ shown below. Let $r > 1$.



The region bound by the curve $y = \ln(x+r)$, the coordinate axes and the line $x = t$ is used as a model for a garden, where $t > 0$.

(a) Show that $x = e^y - r$ **1**

(b) Show that the area of this region, A, in terms of r and t is **2**

$$A = (t+r)\ln(t+r) - r\ln r - t$$

Question 33 (3 marks) **Marks**

The average life cycle of an insect is one month. A viable nest of this insect has between 100 000 to 500 000 insects. The population, *P*, of a nest of this insect grows exponentially so that: **3**

$$\frac{dP}{dt} = 1200e^{0\cdot 3t}$$

A nest of these insects had a population of 5 000 after one month.
Determine how long it will take the nest to reach the viable stage (i.e. when the population has reached 100 000). Answer correct to the nearest month.

Question 34 (8 marks) **Marks**

The acceleration on a particle, P_1, in m s^{-2} is $\frac{d^2x}{dt^2} = e^{-t} + e^{-2t}$, after t seconds. Initially, the particle is $\frac{3}{4}$ m to the right of the origin, travelling at velocity $\frac{dx}{dt} = -\frac{3}{2}$ m/s.

(a) Show that the displacement of the particle is given by **2**

$$\boldsymbol{x} = e^{-t} + \frac{1}{4}e^{-2t} - \frac{1}{2}.$$

(b) Find the limiting displacement of P_1 **and** hence state the limiting distance that P_1 travels. **2**

Question 34 (continued) **Marks**

(c) Another particle, P_2, moves simultaneously with the first particle, P_1. 4
The acceleration P_2 experiences is given by

$$\frac{d^2x}{dt^2} = -e^{-t} - e^{-2t}$$

This particle is also $\frac{3}{4}$ m to the right of the origin, travelling at velocity $\frac{dx}{dt} = -\frac{3}{2}$ m/s

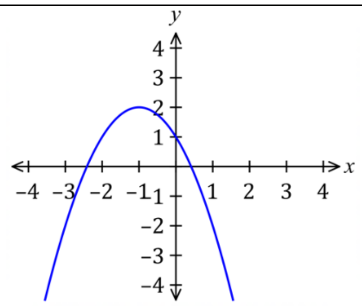
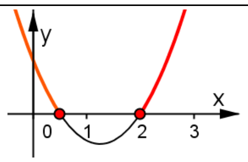
when $t = \ln 3$ seconds.

Determine the exact time when P_1 and P_2 are travelling at the same velocity. Do NOT simplify your answer.

End of paper

Year 12 Mathematics Advanced Trial Term 3 2020 Solutions

MULTIPLE CHOICE

Solution	Comment
1. $f(x)=3x^4(4-x)^3$ $f'(x)=12x^3(4-x)^3+\left[-3(4-x)^2\right](3x^4)$ $=12x^3(4-x)^3-9x^4(4-x)^2$ $=3x^3(4-x)^2\left[4(4-x)-3x\right]$ $=3x^3(4-x)^2(16-7x)$ **(D)**	Easier with product rule, otherwise you have to expand perfect cube first. $\frac{55}{77}$ obtained the correct answer
2. A moderate negative correlation. **(A)**	$\frac{72}{77}$ obtained the correct answer – negative gradient.
3. $f(x)=2x^3+x^2$ $f'(x)=6x^2+2x$ $f''(x)=12x+2<0$ $12x<-2$ $x<-\frac{1}{6}$ **(A)**	$\frac{66}{77}$ obtained the correct answer.
4. $E(x)=\int_0^1 x(x^3-x+4)\ dx$ $=\int_0^1 (x^4-x^2+4x)\ dx$ $=\left[\frac{x^5}{5}-\frac{x^3}{3}+\frac{4x^2}{2}\right]_0^1$ $=\left[\frac{x^5}{5}-\frac{x^3}{3}+2x^2\right]_0^1$ $=\frac{1}{5}-\frac{1}{3}+2-0$ $=\frac{28}{15}$ **(B)**	$\frac{56}{77}$ obtained the correct answer.
5.  **(D)**	$\frac{72}{77}$ obtained the correct answer.
6. $2x^2-5x+2\geq 0$ $(2x-1)(x-2)\geq 0$ $\therefore\left(-\infty,\ \frac{1}{2}\right]\cup[2,\ \infty)$  **(D)**	$\frac{59}{77}$ obtained the correct answer.

Solution	Comment
7. No outliers: $Q_1 - 1.5IQR < x < Q_3 + 1.5IQR$  **(C)**	** $\frac{28}{77}$ obtained the correct answer. Students did not realised that the IQR is one portion whilst the outliers are 1.5 portions away from the lower and upper quartiles which gives a total of 4 IQR for the range.
8. $P(A\|B)=\frac{P(A\cap B)}{P(B)}$ and $P(B\|A)=\frac{P(A\cap B)}{P(A)}$ $P(A\cap B)=P(A\|B).P(B)$ $\quad P(A\cap B)=P(B\|A).P(A)$ $P(A\|B).P(B)=P(B\|A).P(A)$ $4p\left(p^3\right)=P(B\|A)\left(2p^2\right)$ $\therefore P(B\|A)=\frac{4p^4}{2p^2}$ $\therefore P(B\|A)=2p^2$ **(A)**	$\frac{59}{77}$ obtained the correct answer.
9. $z=\frac{x-\mu}{\sigma}=\frac{16-11}{2}=2.5$ $P(X>16)=P(Z>2.5)$ $=P(Z<-2.5)$ **(B)**	$\frac{43}{77}$ obtained the correct answer.
10. $2\sin x+\sqrt{3}=0, \quad 0\le x\le 2\pi$ $\sin x=-\frac{\sqrt{3}}{2}$ **(QIII, IV)** $x=\pi+\frac{\pi}{3},\ 2\pi-\frac{\pi}{3}$ $\therefore x=\frac{4\pi}{3}, \frac{5\pi}{3}$ **(C)**	$\frac{69}{77}$ obtained the correct answer.

Section II Solutions	
11. $\int \frac{1}{1-2x}\,dx=-\frac{1}{2}\int\frac{-2}{1-2x}\,dx$ $=-\frac{1}{2}\ln\|1-2x\|+C$	Students forgot the absolute value, and + C.
12.(a) $f(x)=\frac{(x+3)(2x+1)}{\sqrt{x}}$ $=\frac{2x^2+7x+3}{\sqrt{x}}$ $=2x^{\frac{3}{2}}+7x^{\frac{1}{2}}+3x^{-\frac{1}{2}}$ $\therefore A=2, B=7$ and $C=3$ (b) $f(x)=2x^{\frac{3}{2}}+9x^{\frac{1}{2}}+3x^{-\frac{1}{2}}$ $f'(x)=3x^{\frac{1}{2}}+\frac{9}{2}x^{-\frac{1}{2}}-\frac{3}{2}x^{-\frac{3}{2}}$	Requested to: (1) Express in the form of $Ax^{\frac{3}{2}}+Bx^{\frac{1}{2}}+Cx^{-\frac{1}{2}}$ (2) Find the values of A, B and C Students did not answer as requested.

Section II Solutions

13.(a) $P(1 < X \leq 3) = 0.4 + 0.2$
$= 0.6$

Students included 0.2 when X = 1.

(b) $\mu = 0 \times 0.1 + 1 \times 0.2 + 2 \times 0.4 + 3 \times 0.2 + 4 \times 0.1$
$= 2$

$Var(X) = E(X^2) - \mu^2$
$= 0^2 \times 0.1 + 1^2 \times 0.2 + 2^2 \times 0.4 + 3^2 \times 0.2 + 4^2 \times 0.1 - 2^2$
$= 1.2$
$\therefore$ Variance is 1.2

Few students who got this wrong because they did not know the formula for Var(X), Or did not know how to get $E(X^2)$

14. $\int 6x^2 + 2 + x^{-\frac{1}{2}}\,dx = \frac{6x^3}{3} + 2x + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C$
$= 2x^3 + 2x + 2x^{\frac{1}{2}} + C$

15. $T_5 = a + 4d = 9$ — ①
$T_{21} = a + 20d = 425$ — ②
②–① $16d = 416$
$d = 26$ sub into ①
$\therefore a = -95$
$\therefore T_{50} = a + 49d$
$= -95 + 49(26)$
$\therefore T_{50} = 1179$

There is a mixed up with the word fiftieth (50^{th}) and fifteenth (15^{th}).

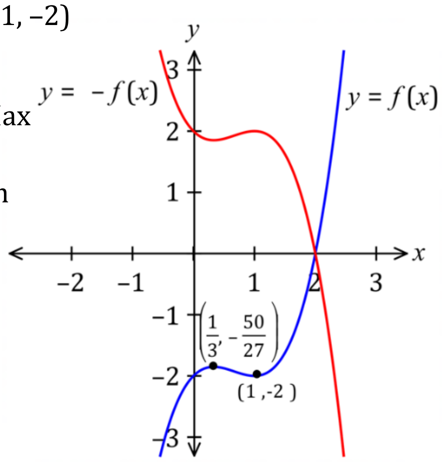
16.(a) x-intercept then $y = 0$
$0 = (x-2)(x^2+1)$
$x = 2$
y-intercept then $x = 0$
$y = (0-2)(0^2+1)$
$= -2$
$\therefore$ Intercepts are (2, 0) and (0, –2)

1 mark for each
Some students mistakenly included $x = \pm 1$

(b) $f(x) = (x-2)(x^2+1)$
$f'(x) = (x-2)2x + (x^2+1) \times 1$
$= 3x^2 - 4x + 1$
$f''(x) = 6x - 4$
Stationary points occur when first derivative is equal to zero.
$3x^2 - 4x + 1 = 0$
$(3x-1)(x-1) = 0$
$x = \frac{1}{3}$ or $x = 1$
When $x = \frac{1}{3}$ then $y = -\frac{50}{27}$ and
when $x = 1$ then $y = -2$
$\therefore$ Stationary points $(\frac{1}{3}, -\frac{50}{27})$ $(1, -2)$
At $(\frac{1}{3}, -\frac{50}{27})$
$f''(x) = 6 \times \frac{1}{3} - 4 = -2 < 0$ Max
At $(1, -2)$
$f''(x) = 6 \times 1 - 4 = 2 > 0$ Min

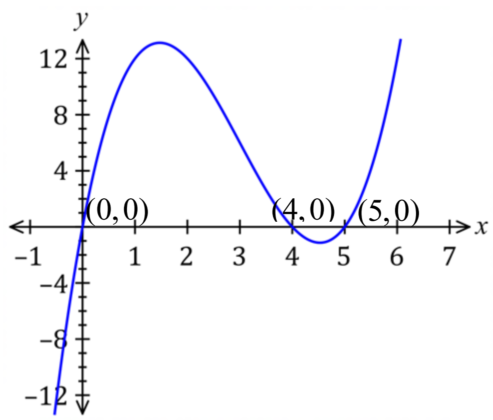
b) Done well
When determining nature, full working needs to be shown and the nature stated as maximum turning point or local maximum NOT concave down and similarly for minimum.

(c)



Most students found the maximum turning point $(\frac{1}{3}, -\frac{50}{27})$ but could not plot it well. $-\frac{50}{27} = -1\frac{23}{27} = -1\cdot\dot{8}5\dot{1} \cong -1\cdot 9$. On many graphs, it looked like $(\frac{1}{3}, -\frac{1}{3})$ or $(\frac{1}{3}, -1)$. Make sure it looks reasonable for your scale.

Many students misread $y = -f(x)$ as $y = f'(x)$ or simply did not know the relationship between $y = -f(x)$ and $y = -f(x)$

Section II Solutions	
17. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos x\, dx = [\sin x]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$ $= \sin\left(\frac{\pi}{2}\right) - \sin\left(\frac{\pi}{4}\right)$ $= 1 - \frac{1}{\sqrt{2}}$ $= \frac{\sqrt{2}-1}{\sqrt{2}}$ $= \frac{2-\sqrt{2}}{2}$	Done well 1 for correct integration 1 for correct substitution and evaluation
18.(a) $f(x) = (x^2 - 6x)(x-3) + 2x$ $= x[(x-6)(x-3)+2]$ $= x(x^2 - 9x + 20)$	Several students missed the $+2x$. Take more care
(b) $f(x) = x(x^2 - 9x + 20)$ $= x(x-4)(x-5)$	Done well
(c) 	The question clearly asks for "showing the coordinates of each point" and this was also an issue in the previous task. Students need to pay more attention to the scale of their graphs.
19.(a) $\frac{d}{dx}\ln\left(x^2+2\right) = \frac{2x}{x^2+2}$	1. Done well by most students
(b) $\frac{d}{dx}\left(\frac{\sin x}{x^2}\right) = \frac{u'v - v'u}{v^2}$ where $u = \sin x$ and $v = x^2$ $u' = \cos x \qquad v' = 2x$ $= \frac{\cos x\left(x^2\right) - (2x)\sin x}{\left(x^2\right)^2}$ $= \frac{x^2\cos x - 2x\sin x}{x^4}$ $= \frac{x\left(x\cos x - 2\sin x\right)}{x^4}$ $= \frac{x\cos x - 2\sin x}{x^3}$	1 for correct quotient rule 1 for correct differentiation of sinx and accuracy Students who used product rule with $\frac{d}{dx}x^{-2}\sin x$ had trouble expressing their answer as a single fraction

Section II Solutions

20. $A = \int_0^2 x^3 - 5x^2 + 2x + 8dx$

$$= \left[\frac{1}{4}x^4 - \frac{5}{3}x^3 + x^2 + 8x\right]_0^2$$

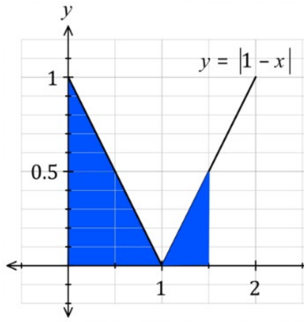
$$= \left(\frac{1}{4}(2)^4 - \frac{5}{3}(2)^3 + 2^2 + 8\times 2\right) - 0$$

$$= \frac{32}{3} = 10\frac{2}{3}\ sq.units$$

21.(a) $P(X \leq 1.5)$

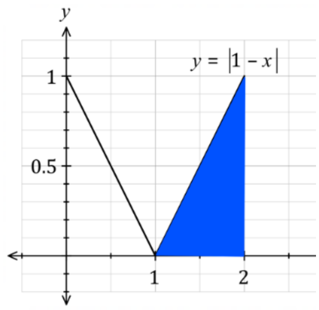
$$= \frac{1}{2}\times 1\times 1 + \frac{1}{2}\times 0.5\times 0.5$$

$$= 0.625$$



(b) $P(1 \leq X \leq 2) = \frac{1}{2}\times 1\times 1$

$$= 0.5$$



** Many students did not graph the function and integrated straight off to obtain the incorrect answer.
Students need to recognise the absolute value functions and signal them to draw the graphs. They can get the solution by applying area of triangle geometrically without integrating.

22.(a) AP: 7 + 5 + 3 + …

$d = T_2 - T_1$

$\therefore d = 7 - 5$

$= -2.$

$\therefore$ the common difference is -2.

(b) $S_n = -48$, $a = 7$ and $d = -2$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$-48 = \frac{n}{2}\times[2\times 7 + (n-1)\times(-2)]$$

$$-96 = n\times(14 - 2n + 2)$$

$$= 16n - 2n^2$$

$$2n^2 - 16n - 96 = 0$$

$$n^2 - 8n - 48 = 0$$

(c) $n^2 - 8n - 48 = 0$

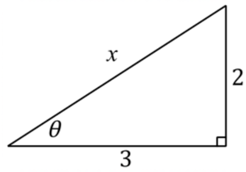
$(n-12)(n+4) = 0$

$n = 12$ or $n = -4$, but $n \in N$

$\therefore n = 12$

Generally well done.

Some students forget to reject $n = -4$.

Section II Solutions	
23.  $x^2 = 2^2 + 3^2$ $x = \sqrt{13}$ $\sin\theta = \dfrac{2}{\sqrt{13}}$ or $\dfrac{2\sqrt{13}}{13}$	Well done.
24.(a) $\dfrac{dy}{dx} = x^3 + 2x - 7$ $\dfrac{d^2y}{dx^2} = 3x^2 + 2$ (b) Since $x^2 \geq 0$ (for all values of x) then $3x^2 \geq 0$ then $3x^2 + 2 \geq 2$ for all x $\therefore \dfrac{d^2y}{dx^2} \geq 2$ (c) Finding the anti-derivative. $y = \dfrac{x^4}{4} + x^2 - 7x + C$ $P(2, 4)$ is on the curve and satisfies the equation. $4 = \dfrac{2^4}{4} + 2^2 - 14 + C$ $C = 10$ $\therefore y = \dfrac{x^4}{4} + x^2 - 7x + 10$ (d) Gradient of the tangent at $P(2, 4)$ $m = \dfrac{dy}{dx} = 2^3 + 2 \times 2 - 7 = 5$ Gradient of the normal at $P(2, 4)$ $m_1 m_2 = -1$ $m = -\dfrac{1}{5}$ Equation of the normal at $P(2, 4)$ $y - y_1 = m(x - x_1)$ $y - 4 = -\dfrac{1}{5}(x - 2)$ $\therefore x + 5y - 22 = 0$	Generally well done. (b) Some students tried to solve $3x^2 + 2 \geq 2$ which is the wrong approach. (c) Some students try to find the equation of the tangent. Read the question carefully.
25.(a) 95% of the data lie within two standard deviations of the mean. 42 hours is a z-score of –2 and 54 hours has a z-score of 2. The mean is midway between 42 and 54 hours. $\therefore$ Mean is 48 hours. (b) There are 4 standard deviations between 42 and 54 hours. Standard deviation $= \dfrac{54 - 42}{4}$ $= 3$ hours (c) To find the z-score of 51 and 57 $P(51 \leq X \leq 57) = P(1 \leq Z \leq 3) = \dfrac{x - \bar{x}}{s} = \dfrac{51 - 48}{3} = 1$ $z = \dfrac{x - \bar{x}}{s} = \dfrac{57 - 48}{3} = 3$ Percentage $= \dfrac{99.7\%}{2} - \dfrac{68\%}{2}$ $= 15.85\%$ $\therefore$ 15.85% have sleep between 51 and 57 hours of sleep per week	Generally well done. Some students forget to half (99.7%-68%).

Section II Solutions

26. Width of the strip

$$h = \frac{b-a}{n} = \frac{420-0}{6} = 70$$

Top half of the lake.

$$A \approx \frac{h}{2}[y_0 + y_6 + 2(y_1 + y_2 + y_3 + y_4 + y_5)$$

$$\approx \frac{70}{2}[30 + 30 + 2(40 + 60 + 80 + 60 + 40)]$$

$$\approx 21\,700$$

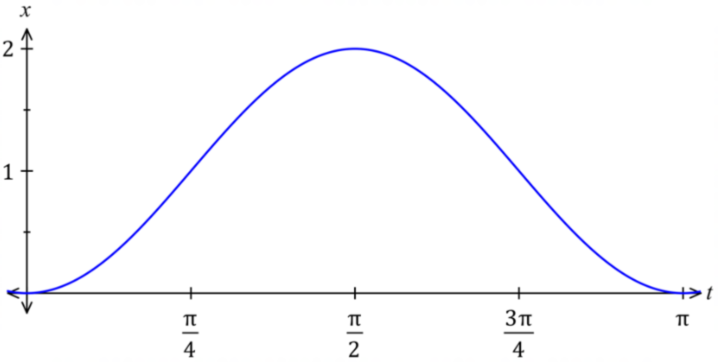
Area of the entire lake

$$A \approx 2 \times 21\,700 \approx 43\,400 \text{ m}$$

∴ Area of the lake is about 43 400 metres.

Generally well done.
Some students forget to double the estimated area.

27.(a) Amplitude = 1, Period $= \frac{2\pi}{1} = 2\pi$



27(a) Generally well done, however there are some students who didn't get the period correct and realise using their function work and graphing that it was a basic cosine curve turned upside down and raised 1 unit up. Some students need to work on the general shape and take more care as it did in some cases cost them in subsequent questions.

(b) The particle is a rest when $v = 0$ or $\frac{dx}{dt} = 0$

$$\therefore t = 0, \frac{\pi}{2}, \pi$$

∴ Position of the particle at these times: $x = 0, 2, 0$

27(b) Well done by students who read the whole question and drew their graph correctly above. Many did NOT write the position of the particle when it was stationary as the question asked.

(c)

$$x = 1 - \cos 2t$$

$$v = \frac{dx}{dt} = 2\sin 2t$$

At $t = \frac{\pi}{4}$. $\quad v = 2\sin\left(2 \times \frac{\pi}{4}\right)$

$$= 2 \text{ m/s}$$

∴ Velocity of the particle is 2 m/s.

27(c) Well done. Some students divided the derivative by 2 rather than multiplying by 2, or had a negative for the derivative of –cos which it should not have.

(d)

$$2\sin 2t = 1$$

$$\sin 2t = \frac{1}{2}$$

$$2t = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$t = \frac{\pi}{12}, \frac{5\pi}{12}$$

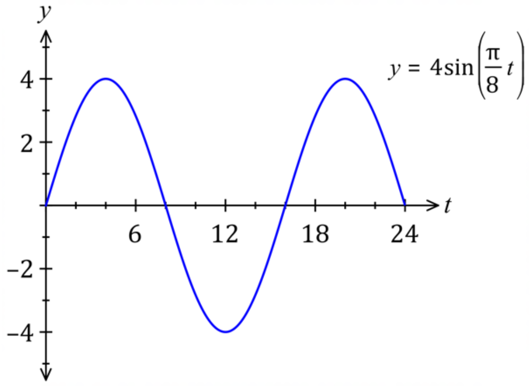
Using the graph

$$\therefore \frac{\pi}{12} < t < \frac{5\pi}{12}$$

27(d) NOT well done in general. A lot of students were only able to find a single solution mostly pi/12 and have everything greater than that (which also goes beyond the domain of pi).

Solving Trig inequalities needs to be done by finding the critical values of equality (Due to multiple quadrant answers ASTC) and **then** interpreting them to the inequality.

Section II Solutions	
28. Minimum value occurs when $\frac{dy}{dx} = 0$ $$\frac{dy}{dx} = 4x - \frac{0.5}{\frac{x}{2}}$$ $$0 = 4x - \frac{1}{x}$$ $$4x^2 = 1$$ $$x^2 = \frac{1}{4}$$ $$x = \pm\frac{1}{2}$$ Since x cannot take a negative value, $x = 0.5$. $$y = 2(0.5)^2 - \ln\left(\frac{0.5}{2}\right) - 4 = \ln 4 - 3\frac{1}{2}$$ $$\therefore \left(\frac{1}{2}, \ln 4 - 3\frac{1}{2}\right)$$ Check if a minima $$\frac{d^2y}{dx^2} = 4 + \frac{1}{x^2}$$ When $x = 0.5$ $$\frac{d^2y}{dx^2} = 4 + \frac{1}{0.5^2} = 8 > 0 \text{ Minima}$$	28. Mostly well done. Some students need to check their method of differentiating their log functions as this did cause some confusion. Another thing that was ignored in the marking, but may not be ignored in the HSC is the realising of the natural domain being x>0 only because of the log component in the function. (x=-1/2 is not possible) Again as in the last assessment some students did not do all 3 components to the question. Diff and set to zero, check nature and show the value of y.
29.(a) $$r = 0.995193611\ldots$$ $$\approx 0.9952$$ (b) $$y = mx + c$$ $$= BX + A$$ $$h = 1.19t - 0.85$$ (c) When $t = 5$ years $$H = 1.19t - 0.85$$ $$= 1.19 \times 5 - 0.85$$ $$\approx 5.1 \text{ m}$$ ∴Height of the tree after 5 years is 5.1 metres.	29. This was a very easy 3 marks for those students who knew how to find these results by using their calculator. If you don't know how to do this yet, make sure you do (see your teacher for help if you need to).
30. From the table: $P(Z < -2.3) = 0.0107$ $$P(Z < -1.72) = 0.0427$$ $$\therefore P(Z > 1.72) = 0.0427$$ $$\therefore P(Z < 1.72) = 1 - 0.0427$$ $$= 0.9573$$ $$\therefore P(-2.3 \le Z \le 1.72) = 0.9573 - 0.0107$$ $$= 0.9466$$	Generally done well. Some students didn't use the symmetry of the curve to calculate the area.

Section II Solutions	
31.(a)  (b) High tide occurs when $h(t) = 4$ Using the graph $\therefore t = 4$ or $t = 20$ (c) The high tide has height 4 metres above the mean height. (d) $h(t) = 4\sin\left(\frac{\pi}{8}t\right)$ $h(8) = 4\sin\left(\frac{\pi \times 10}{8}\right) = 4\sin\left(\frac{5\pi}{4}\right)$ $= -2.8284\ldots$ ≈ -2.8 $\therefore$ Water is about 2.8 metres below mean height.	The entire question was done well.
32.(a) $y = \log_e(x+r)$ $e^y = x + r$ $\therefore x = e^y - r$ (b) $A = \int_0^t \ln(x+r)\ dx$ $=$ Area of rectangle $- \int_{\ln r}^{\ln(t+r)} (e^y - r)\ dy$ $= t \times \ln(t+r) - \left[e^y - ry\right]_{\ln r}^{\ln(t+r)}$ $= t\ln(t+r) - \left[e^{\ln(t+r)} - r\ln(t+r)\right] + \left[e^{\ln r} - r\ln r\right]$ $= t\ln(t+r) - \left[t + r - r\ln(t+r)\right] + \left[r - r\ln r\right]$ $= t\ln(t+r) - t - r + r\ln(t+r) + r - r\ln r$ $= t\ln(t+r) - t + r\ln(t+r) - r\ln r$ $\therefore A = (t+r)\ln(t+r) - r\ln r - t$	32(a) Mostly well done 32(b) Mostly poorly done. Students must realise that we cannot integrate ln or log functions directly at the advanced level. This can only be done by looking at the areas with respect to (w.r.t) the y axis, rather than the x axis. This was the reason part (a) was given to you to prompt the change from y= to a x=
33. $\frac{dP}{dt} = 1200e^{0.3t}$ $\int dP = 1200\int e^{0.3t}\ dt$ $P = 1200\left[\frac{e^{0.3t}}{0.3}\right] + C$ $P = 4000\,e^{0.3t} + C$ When $t = 1$, $P = 5000$	33. Mostly well done! Some students did however multiply by 0.3 rather than divide by 0.3 in the integration. This led to a carry through error in some answers.

Section II Solutions	
33.cont. $5000 = 4000e^{0.3(1)} + C$ $C = 5000 - 4000e^{0.3}$ $\therefore C = -399.44$ $= -399$ (to the nearest whole number) $P = 4000e^{0.3t} - 399$ When $P = 100\,000$, $100\,000 = 4000e^{0.3t} - 399$ $100\,399 = 4000e^{0.3t}$ $e^{0.3t} = \frac{100\,399}{4000}$ $0.3t = \ln\left(\frac{100\,399}{4000}\right)$ $t = \frac{1}{0.3}\ln\left(\frac{100\,399}{4000}\right)$ $\therefore t = 10.74...$ months $\therefore t = 11$ months Hence it will take 11 months for the nest to reach the viable stage.	
34.(a) $\ddot{x} = e^{-t} + e^{-2t}$ $\dot{x} = \int e^{-t} + e^{-2t}\ dt$ $\dot{x} = -e^{-t} - \frac{e^{-2t}}{2} + C_1$ When $t = 0$, $\dot{x} = -\frac{3}{2}$, $-\frac{3}{2} = -e^{-(0)} - \frac{e^{-2(0)}}{2} + C_1$ $-\frac{3}{2} = -\frac{3}{2} + C_1$ $C_1 = 0$ $\therefore \dot{x} = -e^{-t} - \frac{e^{-2t}}{2}$ $x = \int\left(-e^{-t} - \frac{e^{-2t}}{2}\right) dt$ $x = e^{-t} + \frac{e^{-2t}}{4} + C_2$ When $t = 0$, $x = \frac{3}{4}$, $\frac{3}{4} = e^{-(0)} + \frac{e^{-2(0)}}{4} + C_2$ $\frac{3}{4} = 1 + \frac{1}{4} + C_2$ $C_2 = -\frac{1}{2}$ $\therefore x = e^{-t} + \frac{e^{-2t}}{4} - \frac{1}{2}$ (as required) (b) As $t \to \infty$, $e^{-t} \to 0$, $e^{-2t} \to 0$,	34.(a) Done well.

Section II Solutions	
34.(b)cont. $\therefore \lim_{t \to \infty} x = \lim_{t \to \infty}\left(e^{-t} + \frac{e^{-2t}}{4} - \frac{1}{2}\right)$ $= 0 + 0 - \frac{1}{2}$ $= -\frac{1}{2}$ Hence particle starts at $\frac{3}{4}$ m o right of origin and moves to the left to $-\frac{1}{2}$ of the origin. $\therefore$ Distance travelled $= \frac{3}{4} + \frac{1}{2}$ $= 1\frac{1}{4}$ m	34. (b) Poorly done, especially the second part. 34.(c) Poorly done. Few students realised that there was an equation reducible to a quadratic at the end.

34.

(c) For P_2, $\dfrac{d^2x}{dt^2} = e^{-t} + \dfrac{1}{2}e^{-2t} + c_3$.

When $t = \ln 3$, $\dfrac{dx}{dx} = -\dfrac{3}{2}$.

$$\therefore -\frac{3}{2} = e^{-\ln 3} + \frac{1}{2}e^{-2\ln 3} + c_3$$

$$= e^{\ln(3^{-1})} + \frac{1}{2}e^{\ln(3^{-2})} + c_3$$

$$= 3^{-1} + \frac{1}{2}(3^{-2}) + c_3$$

$$c_3 = -\frac{17}{9}$$

Hence we require:

$$-e^{-t} - \frac{1}{2}e^{-2t} = e^{-t} + \frac{1}{2}e^{-2t} - \frac{17}{9}$$

$$0 = e^{-2t} + 2e^{-t} - \frac{17}{9}$$

$$0 = 9(e^{-t})^2 + 18e^{-t} - 17$$

$$\therefore e^{-t} = \frac{-18 \pm \sqrt{18^2 - 4(9) - 17}}{2(9)}$$

$$e^{-t} = \frac{-18 \pm \sqrt{936}}{18}$$

As $e^{-t} > 0$ for all t, the only candidate solution is:

$$e^{-t} = \frac{-18 + \sqrt{936}}{18}$$

$$t = -\ln\left(\frac{-18 + \sqrt{936}}{18}\right)$$

Hence the particles move at the same velocity at

$-\ln\left(\dfrac{-18 + \sqrt{936}}{18}\right)$ seconds.