HORNSBY GIRLS HIGH SCHOOL

Mathematics Advanced

Year 12 Higher School Certificate Trial Examination
Term 3 2025

STUDENT NUMBER: ______________________________

General Instructions:	• Reading time – 10 minutes • Working time – 3 hours • Write using black pen • Calculators approved by NESA may be used • For questions in Section II, show relevant mathematical reasoning and/ or calculations • A NESA reference sheet is provided
Total Marks: 100	**Section I – 10 marks** (pages 3–6) • Attempt Questions 1– 10 • Allow about 15 minutes for this section **Section II – 90 marks** (pages 8 - 29) • Attempt Questions 11 - 33 • Allow about 2 hours and 45 minutes for this section

Q1 - 10	Q11	Q12	Q13	Q14	Q15	Q16	Q17	Q18	Q19	Q20	Q21	
/10	**/2**	**/3**	**/2**	**/4**	**/5**	**/4**	**/2**	**/6**	**/4**	**/3**	**/3**	
Q22	Q23	Q24	Q25	Q26	Q27	Q28	Q29	Q30	Q31	Q32	Q33	**Total**
/2	**/2**	**/2**	**/6**	**/6**	**/4**	**/4**	**/4**	**/5**	**/5**	**/5**	**/7**	**/100**

Outcomes assessed: MA 12 – 1, 3, 4, 5, 6, 7, 8, 10

This assessment task constitutes 30% of the Higher School Certificate Course School Assessment

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the Objective Response answer sheet for Questions 1 – 10

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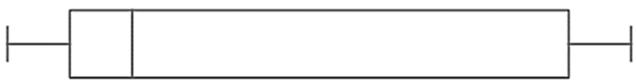
1 The stem-and-leaf plot below shows the distribution of the hourly wages ($) of a manufacturing company.

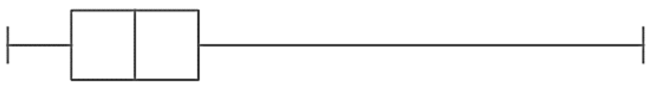
4	0 0 2 2 2 4 4 5 7
5	0 0 1 2 3 6 8 9 9
6	3 6 6 8
7	1
8	3 7
9	6

Which if the following box-and-whisker plots may represent the distribution of their hourly wages?

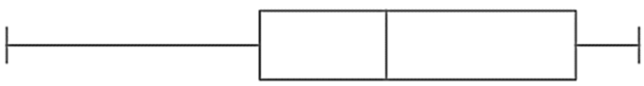
A.



B.



C.



D.

2 A and B are two independent events with $P(A \cup B) = \frac{5}{8}$ and $P(A) = \frac{1}{6}$.
What is the value of $P(B)$?

A. $\frac{5}{48}$

B. $\frac{11}{24}$

C. $\frac{11}{20}$

D. $\frac{15}{4}$

3 Consider the function $f(x) = \ln(x-2) + \sqrt{3-x}$. What is the domain of the function?

A. $(-\infty, \infty)$

B. $(0, \infty)$

C. $(3, 2]$

D. $(2, 3]$

4 What is the solution of the equation $x = \log_2 2\sqrt{2} + \log_2 \sqrt{2}$?

A. $x = 2$

B. $x = 3$

C. $x = 4$

D. $x = 5$

5 Given that $\int_2^8 f(x)\,dx = C$ and $\int_6^8 f(x)\,dx = D$. What is the value of $\int_2^6 f(x)\,dx$?

A. $C + D$

B. $C + D - 2$

C. $C - D$

D. $D - C$

6 For what values of x is the curve $f(x) = 2x^3 + x^2$ concave down?

A. $x < \frac{-1}{6}$

B. $x > \frac{-1}{6}$

C. $x < 6$

D. $x > 6$

7 If $f(x) = \tan\left(g\left(x^2\right)\right)$, where $g(x)$ is a differentiable function, what is $f'(x)$?

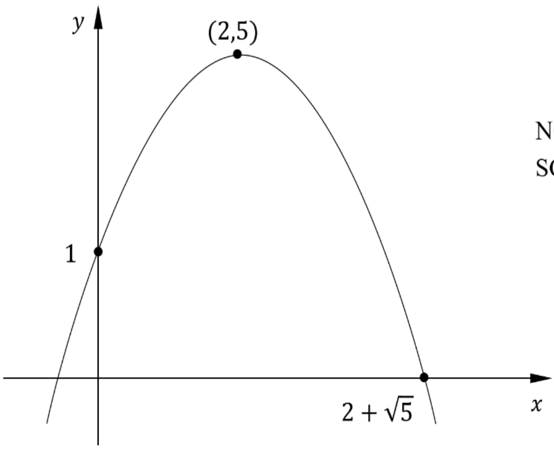
A. $2x\sec^2\left(g\left(x^2\right)\right)$

B. $2xg'\left(x^2\right)\sec^2\left(g(2x)\right)$

C. $2xg\left(x^2\right)\sec^2\left(g\left(x^2\right)\right)$

D. $2xg'\left(x^2\right)\sec^2\left(g\left(x^2\right)\right)$

8 Consider the diagram of the parabola $y = -x^2 + bx + c$.



NOT TO SCALE

What are the values of b and c?

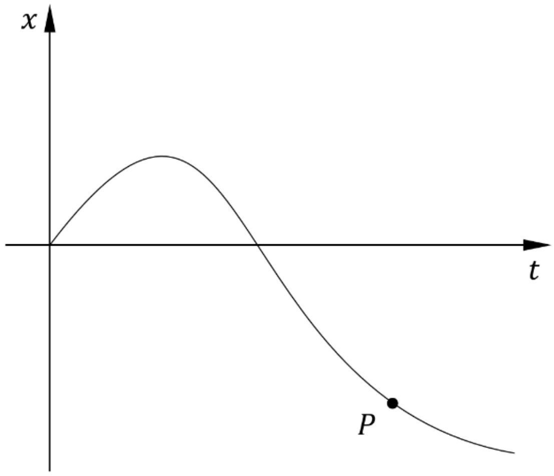
A. $b = 4,\ c = 1$

B. $b = 2,\ c = 2 + \sqrt{5}$

C. $b = -2,\ c = 2 + \sqrt{5}$

D. $b = -4,\ c = 1$

9 The displacement of a car to the east of its starting point along a straight road that runs east-west is x kilometres. At time t hours the car's velocity is $\frac{dx}{dt}$ and its acceleration is $\frac{d^2x}{dt^2}$.
The graph of $x = f(t)$ is shown.



When the car is at point P, which statement reflects its motion?

A. The car is west of its starting point, it is moving east, and its speed is decreasing.

B. The car is west of its starting point, it is moving east, and its speed is increasing.

C. The car is west of its starting point, it is moving west, and its speed is decreasing.

D. The car is west of its starting point, it is moving west, and its speed is increasing.

10 The function $y = 2\sin x + 1$ is translated up m units and then vertically dilated by a factor of n from the x-axis, where m and n are positive numbers.
What is the range of the new curve?

A. $[mn - 2n,\ mn + n]$

B. $[mn - n,\ mn + 3n]$

C. $[mn + n,\ mn + 3n]$

D. $[mn + 2n,\ mn + n]$

End of Section

Section II

90 marks
Attempt questions 11 to 34
Allow about 2 hours and 45 minutes for this section

Answer each question in the spaces provided.
Your responses should include relevant mathematical reasoning and/or calculations.
Extra writing space is provided at the back of the examination paper.

Question 11 (2 marks)

The first three terms of an arithmetic sequence are $x + 6$, $2x - 5$ and $4x - 11$. **2**
Find the value of x.

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Question 12 (3 marks)

The table shows the probability distribution of a discrete random variable.

x	1	2	3	4	5
$P(X = x)$	0	0.2	0.5	0.2	0.1

(a) Show that the expected value $E(X) = 3.2$. **1**

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(b) Calculate the standard deviation, correct to 2 decimal places. **2**

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Question 13 (2 marks)

Let $P(t)$ be a function such that $\dfrac{dP}{dt} = 900e^{3t}$. **2**

When $t = 0,\ P = 2000$.

Find an expression for $P(t)$

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Question 14 (4 marks)

Differentiate the following with respect to *x*.

(a) $y = (3x+1)^4$ **2**

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(b) $y = \ln(\sin x)$ **2**

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Question 15 (5 marks)

A ball is dropped from a height of 2 metres onto a hard floor and bounces. After each bounce, the maximum height reached by the ball is 75% of the previous maximum height. Thus, after it first hits the floor, it reaches a height of 1.5 metres before falling again, and after the second bounce, it reaches a height of 1.125 metres before falling again. Correct answers to 4 significant figures.

(a) Find the maximum height reached after the fifth bounce. **2**

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(b) Find the total distance travelled by the ball from the time after reaching the maximum height with the 5th bounce. **3**

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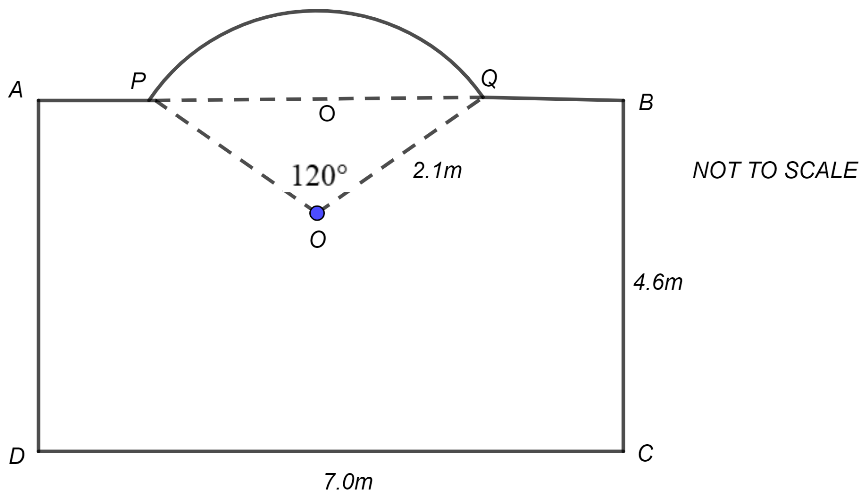
Question 16 (4 marks)

The diagram shows a shape $APQBCD$. The shape consists of a rectangle $ABCD$ with an arc PQ on the side AB. $BC = 4.6$ m and $DC = 7.0$ m.
The arc PQ is an arc of a circle with centre O and radius 2.1 m and $\angle POQ = 120^0$.



What is the area of the shape $APQBCD$? Give your answer correct to one decimal place. **4**

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Question 17 (2 marks)

Find $\int \sin x \cos^3 x \, dx$. **2**

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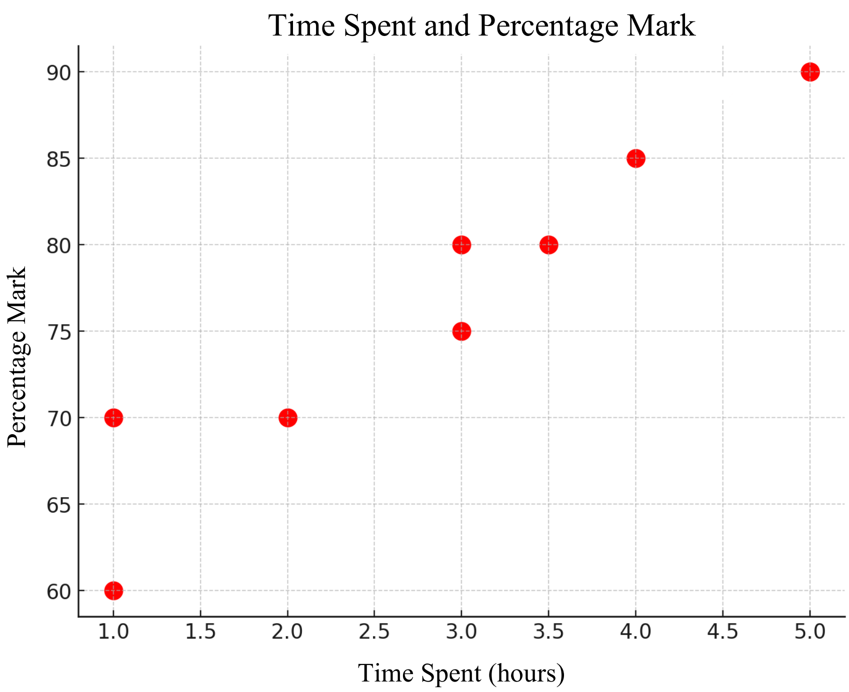
Question 18 (6 marks)

A set of bivariate data is collected by recording the time (x) that a student spent per week on a particular subject and the marks as a percentage (y) that they achieved, as shown on the scatter plot below.



(a) Calculate Pearson's correlation coefficient for the data, correct to two decimal places. **1**

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Question 18 continues on the next page

Question 18 continued

(b) Describe the linear association between time spent and mark. **2**

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(c) Find the equation of the regression line, expressing values correct to two decimal places. **2**

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(d) In the context of the dataset, identify ONE problem with using the regression line to predict the mark when the time spent is 10 hours. **1**

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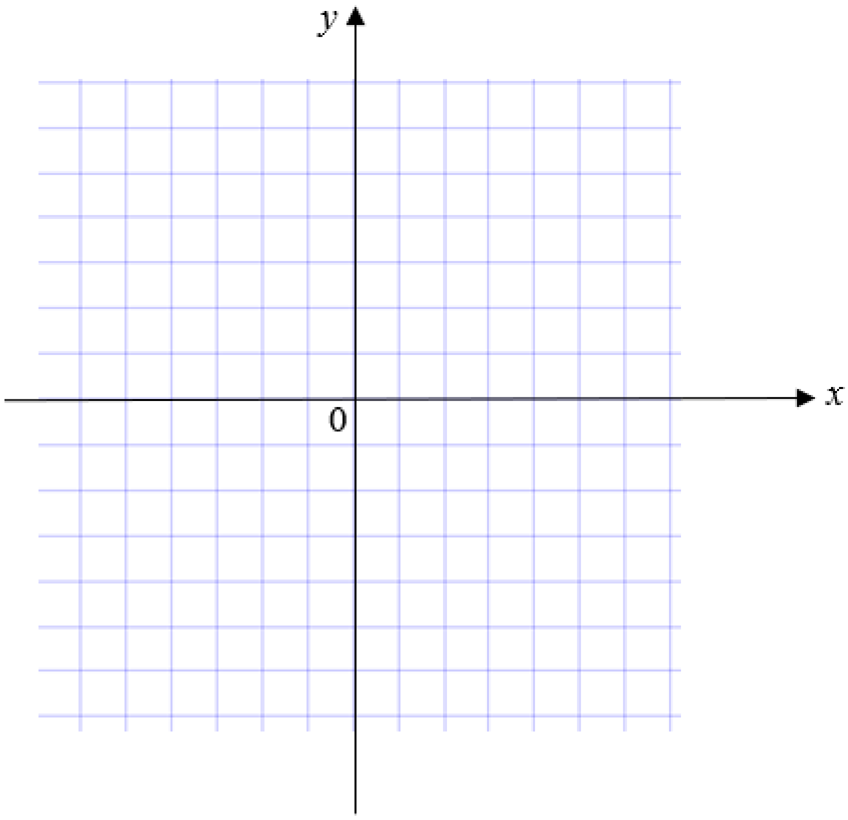
Question 19 (4 marks)

(a) Sketch the graphs of the functions $f(x) = x - 3$ and $g(x) = (3 - x)(x + 1)$ showing the x-intercepts. **2**



(b) Hence, or otherwise, solve the inequality $x - 3 > (3 - x)(x + 1)$. **2**

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Question 20 (3 marks)

Find all values of θ, where $0^{\circ} \leq \theta \leq 360^{\circ}$, such that $\cos(\theta - 30^{\circ}) = \frac{\sqrt{3}}{2}$. **3**

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Question 21 (3 marks)

The sum of the first *n* terms of an arithmetic sequence is $S_n = 3n^2 - 4n$, where $n > 1$. **3**
Find the *n*th term, T_n, **and** state the common difference.

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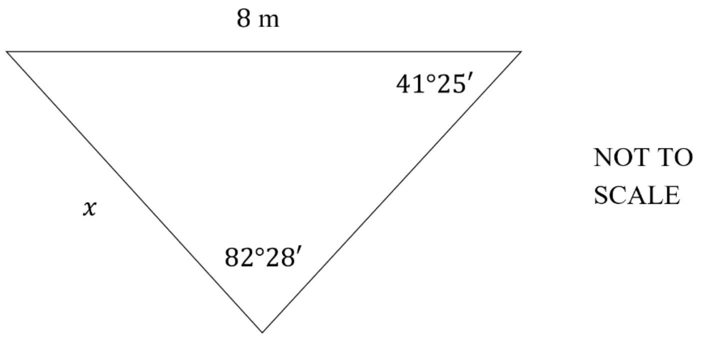
Question 22 (2 marks)

A triangle has one side that is 8 metres in length with an opposite angle of $82°28'$. **2**

A second angle is $41°25'$, with an opposite side of x metres.



Find the length of x, correct to the nearest centimetre.

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Question 23 (2 marks)

Sketch a graph of a continuous function with the following properties: **2**

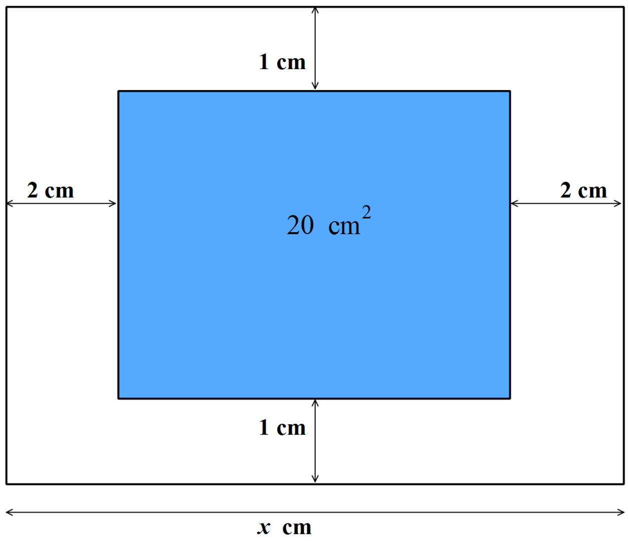
- $f(x)$ is odd
- $f(3)=0$ and $f'(1)=0$
- $f''(x)>0$ for $x>0$

Question 24 (2 marks)

Show that $\sec^2 A \equiv \dfrac{\operatorname{cosec} A}{\operatorname{cosec} A - \sin A}$. **2**

Question 25 (6 marks)

A photograph with an area of 20 cm^2 is to be framed. The frame has a width of 1 cm at the top and bottom and a width of 2 cm on each side, as shown below. Let the frame have length x cm.



NOT TO SCALE

(a) Show that the area, A, of the frame and photograph combined is $A = \dfrac{2x^2 + 12x}{x - 4}$. **2**

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Question 25 continues on the next page

Question 25 continued

(b) Find the exact value of x such that the area of the frame and photograph combined is a minimum. Show that your answer gives a minimum area. **4**

The assessment continues on the next page

Question 26 (6 marks)

(a) Show that $\frac{d}{dx}\left((2x+4)e^x\right)=(2x+6)e^x$. **1**

(b) Hence, or otherwise, evaluate $\int_0^1 2xe^x \, dx$. **2**

(c) Using the trapezoidal rule with 3 function values, find an approximation to the area between the curve $y=2xe^x$ and the x axis from $x=0$ to $x=1$. **2**

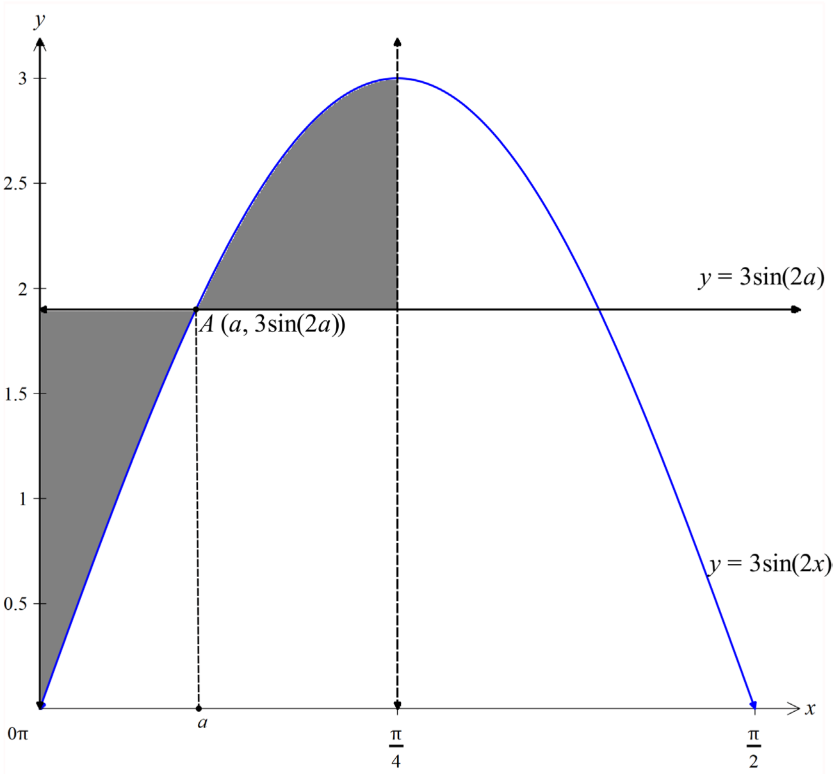
Express your answer correct to three significant figures.

(d) Comparing your answers for parts (b) and (c), comment on the concavity of $y=2xe^x$. **1**

Question 27 (4 marks)

A is a point on the curve $y = 3\sin(2x)$ with coordinates $\left(a, 3\sin(2a)\right)$. By considering the area enclosed between the curve $y = 3\sin(2x)$, the horizontal line through A with equation $y = 3\sin(2a)$ and the lines $x = 0$ and $x = \dfrac{\pi}{4}$, find the value of a, correct to 3 decimal places, such that **4**

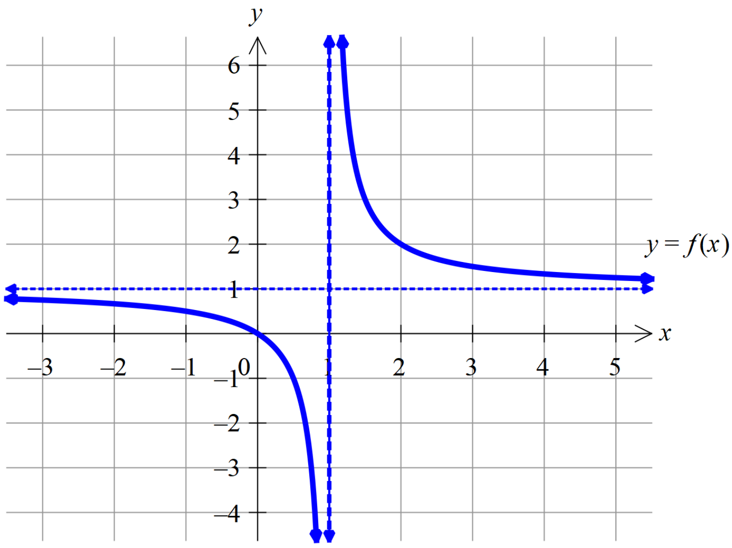
$\int_0^{\frac{\pi}{4}} \left(3\sin 2x - 3\sin 2a\right) dx = 0$.



NOT TO SCALE

Question 28 (4 marks)

Consider the graph of a hyperbola $y = f(x)$, as shown below.



(a) Justify that the equation of $y = f(x)$ is $f(x) = \dfrac{x}{x-1}$. Show full working. **2**

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(b) Find the exact value of m for which the equation $f(x) = mx$ is a tangent to $f(x) = \dfrac{x}{x-1}$. **2**

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Question 29 (4 marks)

The line $y = c - \frac{x}{2}$ is a normal to the curve $y = x^3 + ax^2 + bx - 4$ at the point (2, 7). **4**

What are the values of a, b and c?

Question 30 (5 marks)

Sketch the curve $y=\dfrac{x^3}{6}+\dfrac{x^2}{4}-3x+1$ by finding all stationary points and testing their nature. **5**

Also, find the points of inflection and the y intercept.

The assessment continues on the next page

Question 31 (5 marks)

A particle is moving along the x-axis such that at a time t seconds its displacement from the origin is x metres. The acceleration of the particle is given by $a = \frac{8}{e^{2t}} + \frac{3}{e^{t}}$.

Initially its velocity is -6 m/s and its displacement is 5 m.

(a) Find the equation for the displacement (x) of the particle in terms of t. **2**

(b) At what time and where does the particle come to rest? **3**

Question 32 (5 marks)

Four students want to organise a netball game. All four students have the same probability, $P(T)$, of being available next Tuesday. All four students have the same probability, $P(W)$, of being available next Wednesday. Josie is one of the four students.

It is known that $P(T)=\frac{2}{5}$, $P(W \mid T)=\frac{1}{4}$ and $P(T \mid W)=\frac{1}{3}$.

(a) Is Josie's availability next Tuesday independent from her availability next Wednesday? Justify your answer. **1**

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(b) Show that the probability that Josie is available next Wednesday is $\frac{3}{10}$. **2**

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(c) What is the probability that at least one of the four students is NOT available next Wednesday? **2**

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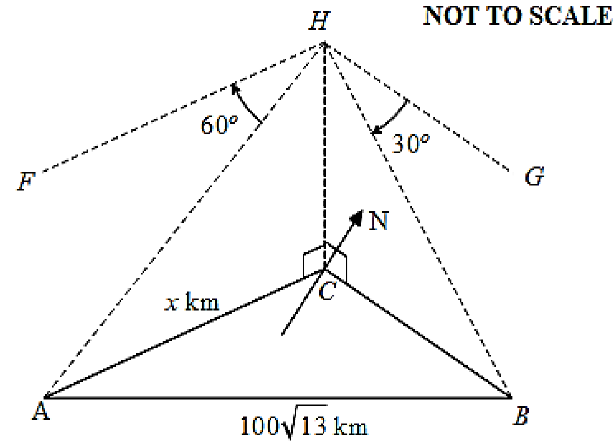
Question 33 (7 marks)

The figure below shows three buoys at sea at A, B and C. A helicopter flies along FH and then HG, where F, G and H are three points of the same height vertically above A, B and C, respectively. When the helicopter reaches H, the angles of depression of A and B from the helicopter are 60° and 30°, respectively. The true bearings of A and B from C are 250° and 130°, respectively. The distance between A and B is $100\sqrt{13}$ km.



(a) Show that the distance between A and C is 100 km. **3**

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Question 33 continues on the next page

At 9 a.m., two destroyers start travelling from A and B in the direction of C and continue past C in the same straight lines. The destroyer at A travels along AC with a uniform speed of $20\ \text{km/h}$, while the destroyer at B travels along BC with a uniform speed of $50\ \text{km/h}$.

(b) Show that the distance between the two destroyers after travelling for t hours is $10\sqrt{1300-450t+39t^2}$. **2**

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(c) The destroyers are known to be "on alert" when they are less than $40\ \text{km}$ apart. **2**

During what times will the two destroyers be "on alert"?

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End of paper

HORNSBY GIRLS HIGH SCHOOL

Mathematics Advanced

Year 12 Higher School Certificate Trial Examination
Term 3 2025

STUDENT NUMBER: solutions and markers feedback

General Instructions:
- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- For questions in Section II, show relevant mathematical reasoning and/ or calculations
- A NESA reference sheet is provided

Total Marks: 100

Section I – 10 marks (pages 3–6)
- Attempt Questions 1– 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 8 - 29)
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- Allow about 2 hours and 45 minutes for this section

Q1 - 10	Q11	Q12	Q13	Q14	Q15	Q16	Q17	Q18	Q19	Q20	Q21	
/10	/2	/3	/2	/4	/5	/4	/2	/6	/4	/3	/3	
Q22	Q23	Q24	Q25	Q26	Q27	Q28	Q29	Q30	Q31	Q32	Q33	**Total**
/2	/2	/2	/6	/6	/4	/4	/4	/5	/5	/5	/7	/100

Outcomes assessed: MA 12 – 1, 3, 4, 5, 6, 7, 8, 10

This assessment task constitutes 30% of the Higher School Certificate Course School Assessment

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the Objective Response answer sheet for Questions 1 – 10

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1 The stem-and-leaf plot below shows the distribution of the hourly wages ($) of a manufacturing company.

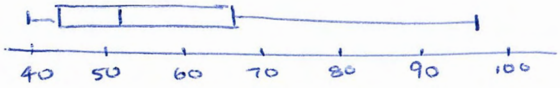
26 scores

median $\frac{13th + 14th}{2}$

Q_1 is 7th score

Q_3 is 20th score

4	0 0 2 2 2 4 4 5 7
5	0 0 1 2 3 6 8 9 9
6	3 6 6 8
7	1
8	3 7
9	6



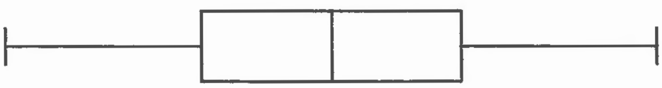
Distribution is negatively skewed, majority of scores are between 40 and 60, hence the box will be smaller on the LHS

Which if the following box-and-whisker plots may represent the distribution of their hourly wages?

A.


B.


C.


D.


2 A and B are two independent events with $P(A \cup B) = \frac{5}{8}$ and $P(A) = \frac{1}{6}$.

What is the value of $P(B)$?

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

but A and B are independent

$\therefore P(A \cap B) = P(A) \times P(B)$

$= P(A) + P(B) - P(A) \times P(B)$

$P(A \cup B) = P(A) + P(B)[1 - P(A)]$

$\frac{5}{8} = \frac{1}{6} + P(B)[1 - \frac{1}{6}]$

$\frac{5}{8} - \frac{1}{6} = P(B) \times \frac{5}{6}$

$\frac{15-4}{24} = P(B) \times \frac{5}{6}$

$P(B) = \frac{11}{24} \times \frac{6}{5} = \frac{11}{20}$ $\therefore$ C.

A. $\frac{5}{48}$

B. $\frac{11}{24}$

C. $\frac{11}{20}$

D. $\frac{15}{4}$

3 Consider the function $f(x) = \ln(x-2) + \sqrt{3-x}$. What is the domain of the function?

D: $x > 2$ D: $x \leq 3$

$2 < x \leq 3$

$(2, 3]$ $\therefore$ D

A. $(-\infty, \infty)$

B. $(0, \infty)$

C. $(3, 2]$

D. $(2,3]$

4 What is the solution of the equation $x = \log_2 2\sqrt{2} + \log_2 \sqrt{2}$?

$x = \log_2 2 + \log_2 \sqrt{2} + \log_2 \sqrt{2}$

$= 1 + \log_2 \sqrt{2} \times \sqrt{2}$

$= 1 + \log_2 2$

$= 1 + 1$

$x = 2$ $\therefore$ A

or

$x = \log_2 (2\sqrt{2} \times \sqrt{2})$

$= \log_2 2 \times 2$

$= \log_2 2^2$

$= 2 \times \log_2 2$

$= 2 \times 1$

$= 2$

A. $x = 2$

B. $x = 3$

C. $x = 4$

D. $x = 5$

5 Given that $\int_2^8 f(x)\,dx = C$ and $\int_6^8 f(x)\,dx = D$. What is the value of $\int_2^6 f(x)\,dx$?

$\int_2^6 f(x).dx = \int_2^8 f(x).dx - \int_6^8 f(x).dx$

$= C - D$

A. $C + D$

B. $C + D - 2$

C. $C - D$

D. $D - C$

6 For what values of x is the curve $f(x) = 2x^3 + x^2$ concave down?

A. $x < \frac{-1}{6}$

B. $x > \frac{-1}{6}$

C. $x < 6$

D. $x > 6$

$f'(x) = 6x^2 + 2x$

$f''(x) = 12x + 2 < 0$ for concave down

$12x < -2$

$x < -\frac{1}{6}$ $\therefore$ A

7 If $f(x) = \tan\left(g\left(x^2\right)\right)$, where $g(x)$ is a differentiable function, what is $f'(x)$?

A. $2x\sec^2\left(g\left(x^2\right)\right)$

B. $2xg'\left(x^2\right)\sec^2\left(g(2x)\right)$

C. $2xg\left(x^2\right)\sec^2\left(g\left(x^2\right)\right)$

D. $2xg'\left(x^2\right)\sec^2\left(g\left(x^2\right)\right)$

$f(x) = \tan\left(g(x^2)\right)$

$f'(x) = \sec^2\left(g(x^2)\right) \times \frac{d}{dx}\left(g(x^2)\right)$

$f'(x) = \sec^2\left(g(x^2)\right) \times g'(x^2) \times 2x$

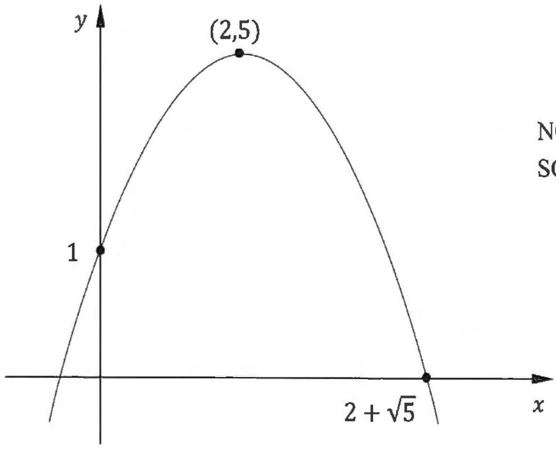
$= 2x \times g'(x^2) \times \sec^2\left(g(x^2)\right)$

8 Consider the diagram of the parabola $y = -x^2 + bx + c$.

The y-intercept is 1

$\therefore c = 1$

Eliminate B and C



NOT TO SCALE

What are the values of b and c?

A. $b = 4,\ c = 1$

B. $b = 2,\ c = 2 + \sqrt{5}$

C. $b = -2,\ c = 2 + \sqrt{5}$

D. $b = -4,\ c = 1$

Axis of symmetry, $x = \frac{-b}{2a}$

passes through vertex $\therefore x = 2$

$2 = \frac{-b}{2 \times -1}$

$2 = \frac{-b}{-2}$

$-4 = -b$

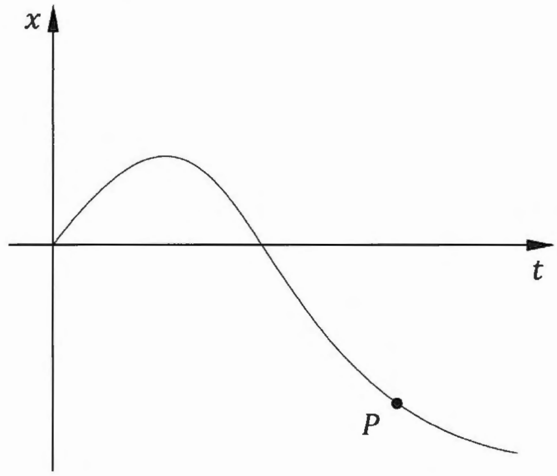
$b = 4$

$c = 1$

$\therefore$ A

9 The displacement of a car to the east of its starting point along a straight road that runs east-west is x kilometres. At time t hours the car's velocity is $\frac{dx}{dt}$ and its acceleration is $\frac{d^2x}{dt^2}$.
The graph of $x = f(t)$ is shown.

• The particle has a negative x-value, so it is west of its starting point.
• The gradient of the curve is negative, so the car is moving to the west
• The curve is concave up so acceleration is positive but velocity is negative $\therefore$ it is slowing down



$x < 0$
$\therefore$ P is west of O and moving west
curve is getting flatter (less steep)

When the car is at point P, which statement reflects its motion?

A. The car is west of its starting point, it is moving east, and its speed is decreasing.

B. The car is west of its starting point, it is moving east, and its speed is increasing.

C. The car is west of its starting point, it is moving west, and its speed is decreasing.

D. The car is west of its starting point, it is moving west, and its speed is increasing.

10 The function $y = 2\sin x + 1$ is translated up m units and then vertically dilated by a factor of n from the x-axis, where m and n are positive numbers.
What is the range of the new curve?

A. $[mn - 2n,\ mn + n]$

B. $[mn - n,\ mn + 3n]$

C. $[mn + n,\ mn + 3n]$

D. $[mn + 2n,\ mn + n]$

For $y = \sin x$ $R: -1 \le y \le 1$
$y = 2\sin x$ $R: -2 \le y \le 2$
$y = 2\sin x + 1$ $R: -1 \le y \le 3$
$y = 2\sin x + 1 + m$ $R: -1 + m \le y \le 3 + m$
$\frac{y}{n} = 2\sin x + 1 + m$ $R: n(-1+m) \le y \le n(3+m)$
$(mn - n) \le y \le (mn + 3n)$

End of Section

Section II

90 marks
Attempt questions 11 to 34
Allow about 2 hours and 45 minutes for this section

Answer each question in the spaces provided.
Your responses should include relevant mathematical reasoning and/or calculations.
Extra writing space is provided at the back of the examination paper.

Question 11 (2 marks)

The first three terms of an arithmetic sequence are $x + 6$, $2x - 5$ and $4x - 11$. **2**
Find the value of x.

* evidence of $T_3 - T_2 = T_2 - T_1$ was helpful
* fairly well answered

Arithmetic sequence $\therefore d = T_3 - T_2 = T_2 - T_1$

$4x - 11 - (2x-5) = 2x - 5 - (x+6)$ 1 or $T_3 - T_1 = 2T_2$

$4x - 11 - 2x + 5 = 2x - 5 - x - 6$

$2x - 6 = x - 11$

$x = -5$ 1

Question 12 (3 marks)

The table shows the probability distribution of a discrete random variable.

x	1	2	3	4	5	Sum
$p(x) = P(X = x)$	0	0.2	0.5	0.2	0.1	= 1
$x \cdot p(x)$	0	0.4	1.5	0.8	0.5	= 3.2
$x^2 \cdot p(x)$	0	0.8	4.5	3.2	2.5	= 11

(a) Show that the expected value $E(X) = 3.2$. **1**

$E(x) = 1\times0 + 2\times0.2 + 3\times0.5 + 4\times0.2 + 5\times0.1$ 1.

$\Sigma x.p(x) = 0 + 0.4 + 1.5 + 0.8 + 0.5 = 3.2 = \mu$

Fairly well answered however many showed no working here - understandable for 1 mark question but would have helped in part(b).

(b) Calculate the standard deviation, correct to 2 decimal places. **2**

Not as well answered and too many showed no working here. Incorrect bold answers unfortunately 0/2 marks.

$\Sigma x^2.p(x) = E(X^2) = 0 + 0.8 + 4.5$

$= 0 + 0.8 + 4.5 + 3.2 + 2.5$

$= 11$ 1.

Standard deviation $= \sqrt{Var(X)}$

$= \sqrt{E(X^2) - \mu^2}$

$= \sqrt{11 - 3.2^2}$

$= \sqrt{0.76}$ 1.

$= 0.8717797887$

$= 0.87$ to 2d.p.

Question 13 (2 marks)

Let $P(t)$ be a function such that $\frac{dP}{dt} = 900e^{3t}$. **2**

When $t = 0$, $P = 2000$.

Find an expression for $P(t)$

$P(t) = \int 900e^{3t} \, dt$

$P(t) = 900 \int e^{3t} \, dt$

$P(t) = 900 \times \frac{e^{3t}}{3} + c$ 1

looked for attempt at integrating then substitution to find C.

When $t=0$, $P=2000$

$2000 = 900 \times \frac{e^0}{3} + c$

$2000 = 300 + c$

$c = 1700$

* quite well answered, a few had trouble finding c → made it harder than necessary dividing rather than subtracting

$P(t) = 300e^{3t} + 1700$ 1 ← needed to show expression as asked.

Question 14 (4 marks)

Differentiate the following with respect to x.

(a) $y = (3x+1)^4$ **2**

$y' = 4(3x+1)^3 \times 3$ 1. 1.

$y' = 12(3x+1)^3$

* very well answered.

(b) $y = \ln(\sin x)$ **2**

$y' = \frac{\cos x}{\sin x}$ 1. 1.

$y' = \cot x$

* very well answered

→ some did not change to cot x but no penalty

Question 15 (5 marks)

A ball is dropped from a height of 2 metres onto a hard floor and bounces. After each bounce, the maximum height reached by the ball is 75% of the previous maximum height. Thus, after it first hits the floor, it reaches a height of 1.5 metres before falling again, and after the second bounce, it reaches a height of 1.125 metres before falling again. Correct answers to 4 significant figures.

(a) Find the maximum height reached after the fifth bounce. **2**

Let h_n = height after nth bounce

$h_1 = 2 \times 0.75 = 1.5$

$h_2 = 2 \times 0.75^2 = 1.125$

$h_3 = 2 \times 0.75^3$

$\vdots$

$h_5 = 2 \times 0.75^5 = 0.474609375$

$= 0.4746$ to 4 sig fig

Not well done – many students gave the height after the 4th bounce

(b) Find the total distance travelled by the ball from the time after reaching the maximum height with the 5th bounce. **3**

Downward motion $= 2 + 2 \times 0.75 + 2 \times 0.75^2 + \dots + \underbrace{2 \times 0.75^5 + 2 \times 0.75^6 + \dots}$

This is a geometric series with $a = 2 \times 0.75^5$, $r = 0.75$, $n \to \infty$

$|r| < 1 \therefore$ limiting sum exists

Upward motion $= 2 \times 0.75 + 2 \times 0.75^2 + \dots + 2 \times 0.75^5 + \underbrace{2 \times 0.75^6 + \dots}$

This is a geometric series with $a = 2 \times 0.75^6$, $r = 0.75$, $n \to \infty$

$|r| < 1$

$\therefore$ limiting sum exists

Distance after 5th bounce $= \dfrac{2 \times 0.75^5}{1 - 0.75} + \dfrac{2 \times 0.75^6}{1 - 0.75}$

$= \dfrac{2 \times 0.75^5 + 2 \times 0.75^6}{0.25}$

$= 3.3222\ldots$ 3.3222...

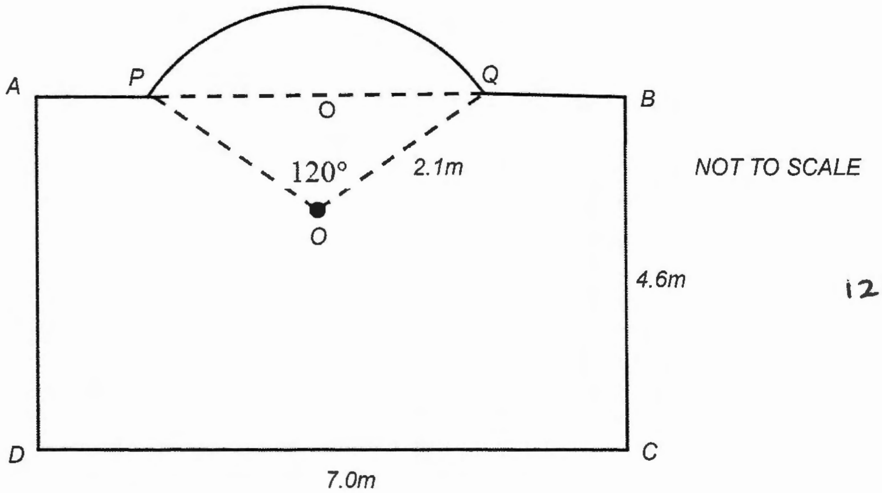
$\doteqdot 3.322$ to 4 sig fig

Not well done. Many students found sum of the first 5 bounces. Some attempted to find the total sum and subtract the first 5 bounces but this was poorly done.

Question 16 (4 marks)

The diagram shows a shape $APQBCD$. The shape consists of a rectangle $ABCD$ with an arc PQ on the side AB. $BC = 4.6\text{ m}$ and $DC = 7.0\text{ m}$.

The arc PQ is an arc of a circle with centre O and radius 2.1 m and $\angle POQ = 120^0$.



$120^\circ = \frac{2\pi}{3}$

What is the area of the shape $APQBCD$? Give your answer correct to one decimal place. **4**

A segment OPQ = A sector OPQ − A $\triangle$OPQ

$= \frac{1}{2} \times 2.1^2 \times \frac{2\pi}{3} - \frac{1}{2} \times 2.1^2 \times \sin \frac{2\pi}{3}$ in radians

or or

$\frac{1}{2} \times 2.1^2 \times \frac{120^\circ}{360^\circ} - \frac{1}{2} \times 2.1^2 \times \sin 120^\circ$ in degrees

$= 4.618... - 1.909586$

$= 2.708555185$

A APQBCD = A rectangle + A segment

$= 4.6 \times 7 + 2.708555185$

$= 32.2 + 2.7...$

$= 34.90855519$

$= 34.9\text{ m}^2$ to 1d.p.

Quite well done. Most students knew to find area of sector and subtract the triangle. Some poor calculations led to errors.

Question 17 (2 marks)

Find $\int \sin x \cos^3 x \, dx$. **2**

$f(x) = \cos x$

$f'(x) = -\sin x$

$\int f'(x) \times [f(x)]^n . dx = \frac{1}{n+1} [f(x)]^{n+1} + c$

$= -\int (-\sin x)(\cos x)^3 . dx$

$= \frac{-(\cos x)^4}{4} + c$

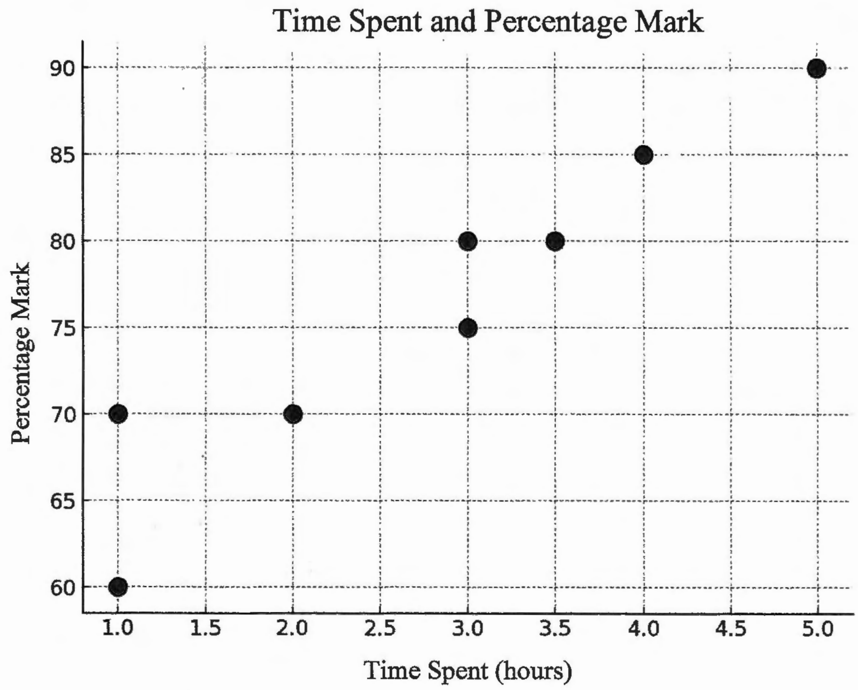
There were still many students who couldn't integrate this correctly using reverse chain rule. more practising is needed.

Some students gave $\frac{-(\sin x)^4}{4} + C$

Need avoid this mistake!

Question 18 (6 marks)

A set of bivariate data is collected by recording the time (x) that a student spent per week on a particular subject and the marks as a percentage (y) that they achieved, as shown on the scatter plot below.



(a) Calculate Pearson's correlation coefficient for the data, correct to two decimal places. **1**

$r = 0.947102946 8$

$r = 0.95$ to 2 d.p.

well done

Question 18 continues on the next page

Question 18 continued

(b) Describe the linear association between time spent and mark. **2**

Strong positive linear correlation

well done!

(c) Find the equation of the regression line, expressing values correct to two decimal places. **2**

$A = 58.25503356$ $B = 6.398210291$

$y = A + Bx$

$y = 58.26 + 6.40x$

well done!

or

$y = 6.40x + 58.26$

(d) In the context of the dataset, identify ONE problem with using the regression line to predict the mark when the time spent is 10 hours. **1**

Extrapolation, which is predicting outside the range of data, shoud be done with caution as the relationship between time and mark might not hold true for very low or very high study times.

In this case, if $x = 10$, $y = 58.26 + 6.40 \times 10$

$= 122.26$ which is beyond the maximum 100%.

well done!

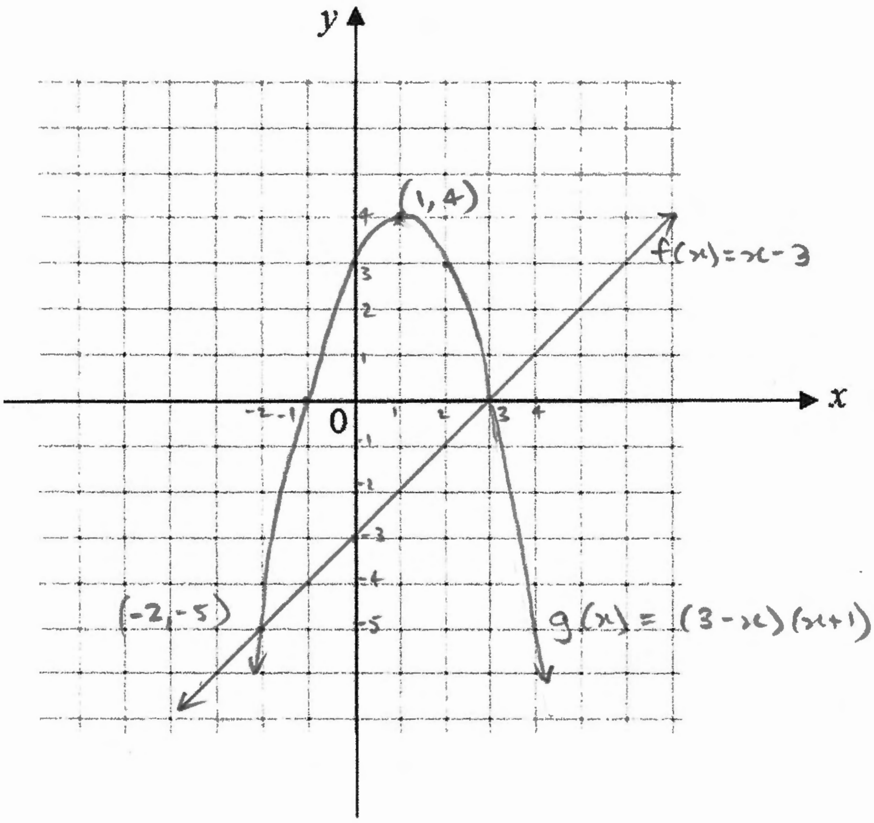
It would be better to show the calculation

Only saying inaccurate is not acceptable.

We are estimating/predicting using the regression line.

Question 19 (4 marks)

(a) Sketch the graphs of the functions $f(x) = x - 3$ and $g(x) = (3 - x)(x + 1)$ showing the x-intercepts. **2**



Very well done!

(b) Hence, or otherwise, solve the inequality $x - 3 > (3 - x)(x + 1)$. **2**

from graph
$x < -2,\ x > 3$

Points of intersection when

$x - 3 = (3 - x)(x + 1)$

$(x - 3) = -(x - 3)(x + 1)$

$(x - 3) + (x - 3)(x + 1) = 0$

$(x - 3)[1 + x + 1] = 0$

$(x - 3)(x + 2) = 0$

$x = 3, -2$

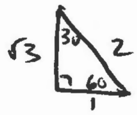
$x - 3 > (3 - x)(x + 1)$

when

$x < -2, x > 3$ from graph

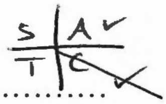
Very well done however some students didn't understand which region was needed.

Question 20 (3 marks)



Find all values of θ, where $0^\circ \le \theta \le 360^\circ$, such that $\cos(\theta - 30^\circ) = \frac{\sqrt{3}}{2}$. **3**

$-30^\circ \le \theta - 30^\circ \le 330^\circ$ related angle is 30°



$(\theta - 30^\circ) = 0^\circ - 30^\circ,\ 0 + 30^\circ,\ 360^\circ - 30^\circ$

$\theta = 0^\circ, 60^\circ, 360^\circ$

Generally well done however some students had one angle missing.

Question 21 (3 marks)

The sum of the first n terms of an arithmetic sequence is $S_n = 3n^2 - 4n$, where $n > 1$. **3**
Find the nth term, T_n, **and** state the common difference.

$T_n = S_n - S_{n-1}$
$= 3n^2 - 4n - [3(n-1)^2 - 4(n-1)]$
$= 3n^2 - 4n - 3(n^2 - 2n + 1) + 4(n-1)$
$= \cancel{3n^2} - \cancel{4n} - \cancel{3n^2} + 6n - 3 + \cancel{4n} - 4$
$T_n = 6n - 7$

Poorly done. Many students didn't use the formula $T_n = S_n - S_{n-1}$ which made their working out confusing.

$T_1 = 6 - 7 = -1$
$T_2 = 12 - 7 = 5$
$T_3 = 18 - 7 = 11$
$d = 11 - 5 = 5 - -1$
$d = 6$

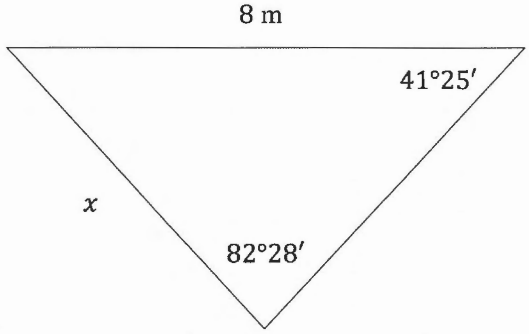
or

$d = T_n - T_{n-1}$
$= 6n - 7 - ((6n-1) - 7)$
$= \cancel{6n} - \cancel{7} - \cancel{6n} + 6 + \cancel{7}$
$d = 6$

Question 22 (2 marks)

A triangle has one side that is 8 metres in length with an opposite angle of $82°28'$. **2**

A second angle is $41°25'$, with an opposite side of x metres.



NOT TO SCALE

Find the length of x, correct to the nearest centimetre.

$$\frac{x}{\sin 41°25'} = \frac{8}{\sin 82°28'}$$

$$x = \frac{8 \times \sin 41°25'}{\sin 82°28'}$$

$x = 5.338316597$

$x = 5.34$ m to nearest cm.

or 534 cm NOT 5cm 1 mark out of 2 awarded

Sine Rule was done well. About 20% of students had errors with nearest cm.

Question 23 (2 marks)

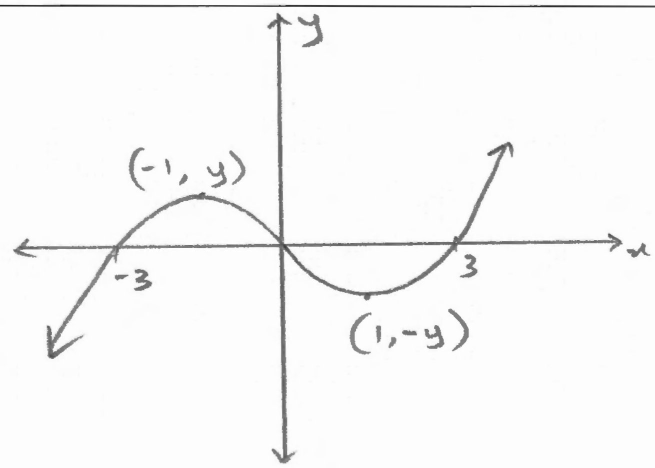
Sketch a graph of a continuous function with the following properties: **2**

- $f(x)$ is odd
- $f(3)=0$ and $f'(1)=0$
- $f''(x)>0$ for $x>0$

passes through $(3,0)$; but odd $\therefore$ also passes through $(-3,0)$

stationary point at $x=1$, but odd $\therefore$ stat pt at $x=-1$ also

concave up for $x>0$, but odd $\therefore$ concave down for $x<0$



Most students got 1 out of 2. Most had a curve cutting the x-axis at 3, a turning point at $x=1$ and concave up for $x>0$ but many did not have an odd function

Question 24 (2 marks)

Show that $\sec^2 A \equiv \dfrac{\operatorname{cosec} A}{\operatorname{cosec} A - \sin A}$. 2

Several students did not recognise the triple bar ($\equiv$) symbol for identically equal.
It is often used in the same way as a regular equals sign ($=$) in contexts where the two expressions are equivalent for <u>all</u> possible values of their variables, not just one (or more) value(s).

start here

$$RHS = \frac{\operatorname{cosec} A}{\operatorname{cosec} A - \sin A}$$

not here

$$= \operatorname{cosec} A \div (\operatorname{cosec} A - \sin A)$$

$$= \frac{1}{\sin A} \div \left(\frac{1}{\sin A} - \sin A\right)$$

$$= \frac{1}{\sin A} \div \frac{1 - \sin^2 A}{\sin A}$$

but $\cos^2 A + \sin^2 A = 1$

$\therefore \cos^2 A = 1 - \sin^2 A$

$$= \frac{1}{\cancel{\sin A}} \times \frac{\cancel{\sin A}}{\cos^2 A}$$

$$= \frac{1}{\cos^2 A}$$

Reasonably well done

$$= \sec^2 A$$

$RHS = LHS$

or $LHS = \sec^2 A$

$$= 1 + \tan^2 A$$

$$= 1 + \frac{\sin^2 A}{\cos^2 A}$$

$$= 1 + \frac{\sin^2 A}{1 - \sin^2 A}$$

$$= \frac{1 - \sin^2 A + \sin^2 A}{1 - \sin^2 A}$$

$$= \frac{1}{1 - \sin^2 A} \qquad \begin{matrix} \div \sin A \\ \div \sin A \end{matrix}$$

$$= \frac{\frac{1}{\sin A}}{\frac{1}{\sin A} - \frac{\sin^2 A}{\sin A}}$$

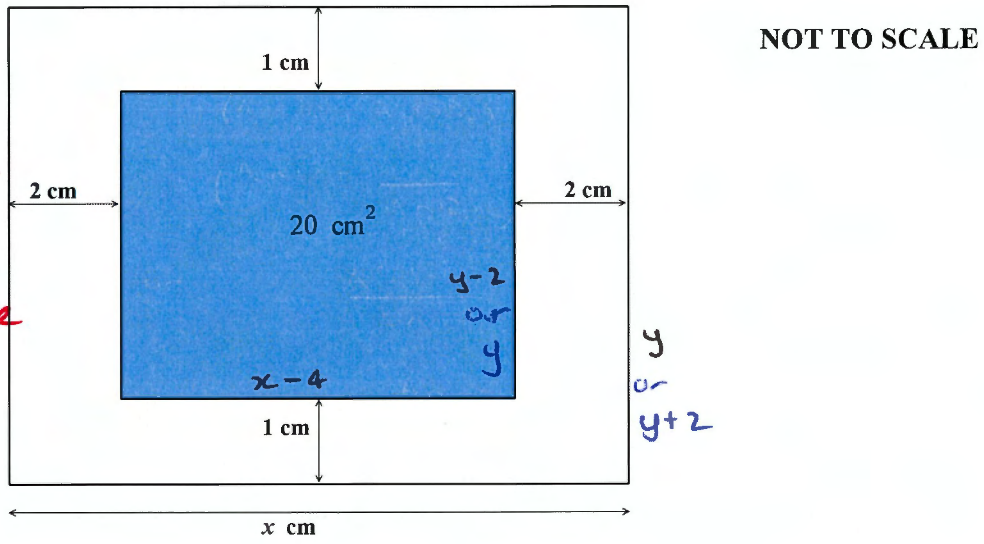
$$= \frac{\operatorname{cosec} A}{\operatorname{cosec} A - \sin A}$$

$= RHS$

Question 25 (6 marks)

A photograph with an area of 20 cm^2 is to be framed. The frame has a width of 1 cm at the top and bottom and a width of 2 cm on each side, as shown below. Let the frame have length x cm.

Quite well done. Students are advised to write a formula for the required area and then substitute an expression for y.



(a) Show that the area, A, of the frame and photograph combined is $A = \dfrac{2x^2+12x}{x-4}$. **2**

Let the width of the frame be y cm **OR** Let the width of the photo be y

$A_{photo} = (x-4)(y-2) = 20$

$y - 2 = \dfrac{20}{x-4}$

$y = \dfrac{20}{x-4} + 2$

$= \dfrac{20+2(x-4)}{x-4}$

$= \dfrac{20+2x-8}{x-4}$

$y = \dfrac{12+2x}{x-4}$ 1

$A = x \times y$

$= x \times \dfrac{12+2x}{x-4}$

$= \dfrac{12x+2x^2}{x-4}$ 1

$= \dfrac{2x^2+12x}{x-4}$

$A_{photo} = (x-4) \times y = 20$

$y = \dfrac{20}{x-4}$ 1

$A = x(y+2)$

$= x\left(\dfrac{20}{x-4}+2\right)$

$= x \times \dfrac{20+2(x-4)}{x-4}$

$= x \times \dfrac{20+2x-8}{x-4}$

$= \dfrac{x(12+2x)}{x-4}$ 1

$= \dfrac{12x+2x^2}{x-4}$

$= \dfrac{2x^2+12x}{x-4}$

Question 25 continues on the next page

Question 25 continued

(b) Find the exact value of x such that the area of the frame and photograph combined is a minimum. Show that your answer gives a minimum area. **4**

$$\frac{dA}{dx} = \frac{(x-4)(4x+12) - (2x^2+12x)\times 1}{(x-4)^2}$$

$$= \frac{4x^2 + \cancel{12x} - 16x - 48 - 2x^2 - \cancel{12x}}{(x-4)^2}$$

$$= \frac{2x^2 - 16x - 48}{(x-4)^2}$$

$$\frac{dA}{dx} = \frac{2(x^2 - 8x - 24)}{(x-4)^2}$$

Min/Max area occurs when $\frac{dA}{dx} = 0$

$$x^2 - 8x - 24 = 0$$

$$x = \frac{8 \pm \sqrt{64 - 4\times 1\times(-24)}}{2}$$

$$= \frac{8 \pm \sqrt{64+96}}{2}$$

$$= \frac{8 \pm \sqrt{160}}{2}$$

$$= \frac{8 \pm 4\sqrt{10}}{2}$$

$$= 4 \pm 2\sqrt{10} \quad \text{but } x > 0$$

$$x = 4 + 2\sqrt{10}$$

$$\doteqdot 4 + 6.32$$

$$x \doteqdot 10.32$$

x	10	10.32	11
$\frac{dA}{dx}$	$-\frac{8}{36}$	0	$\frac{18}{49}$

$\therefore$ Minimum area when $x = (4 + 2\sqrt{10})$ m

Reasonably well done.

Students are advised to:

1) write the derivatives they are using
2) show all substitutions into the Quotient rule (including 1)
3) Set $\frac{dA}{dx} = 0$
4) Hence solve for the exact values of x
5) Discard any solutions
6) Test by showing values in a first derivative table.
7) Give the required answer.

The assessment continues on the next page

Question 26 (6 marks)

(a) Show that $\frac{d}{dx}\left((2x+4)e^x\right) = (2x+6)e^x$. **1**

$\frac{d}{dx}\left((2x+4)e^x\right) = e^x \times 2 + (2x+4) \times e^x$

$= e^x\left[2 + 2x + 4\right]$ Done well

$= e^x(2x+6)$

$\therefore \int e^x(2x+6)\,dx = (2x+4)e^x + c$

(b) Hence, or otherwise, evaluate $\int_0^1 2xe^x\,dx$. **2**

$\int (2xe^x + 6e^x)\,dx = (2x+4)e^x + c$

$\int_0^1 2xe^x\,dx = \left[(2x+4)e^x\right]_0^1 - \int_0^1 6e^x\,dx$

Many students $= (2+4)e^1 - 4e^0 - 6\left[e^x\right]_0^1$

could improve their $= 6e - 4 - 6\left[e - 1\right]$

setting out $= \cancel{6e} - 4 - \cancel{6e} + 6$

$= 2$

(c) Using the trapezoidal rule with $\boxed{\text{3 function values,}}$ find an approximation **2**

means $n = 2$ not $n = 3$

to the area between the curve $y = 2xe^x$ and the x axis from $x = 0$ to $x = 1$.

Express your answer correct to three significant figures.

$a = 0$ $\quad y_1 = 2 \times 0 \times e^0 = 0$

$b = 1$ $\quad y_2 = 2 \times \frac{1}{2} \times e^{\frac{1}{2}} = e^{\frac{1}{2}}$

$\boxed{n = 2}$ $\quad y_3 = 2 \times 1 \times e^1 = 2e$

$h = \frac{b-a}{n} = \frac{1-0}{2} = \frac{1}{2}$

$A \doteqdot \frac{1-0}{2 \times 2}\left[0 + 2e + 2 \times e^{\frac{1}{2}}\right]$

$A \doteqdot \frac{1}{4}\left[2e + 2e^{\frac{1}{2}}\right]$

$A \doteqdot 2.1835$

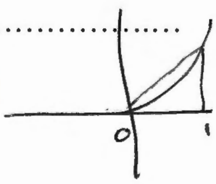
$A \doteqdot 2.18$ to 3 sig fig

(d) Comparing your answers for parts (b) and (c), comment on the concavity of $y = 2xe^x$. **1**

Since area using trapezoidal rule > area using integration

$y = 2xe^x$ must be concave up between $x = 0$ and $x = 1$

not positive concavity



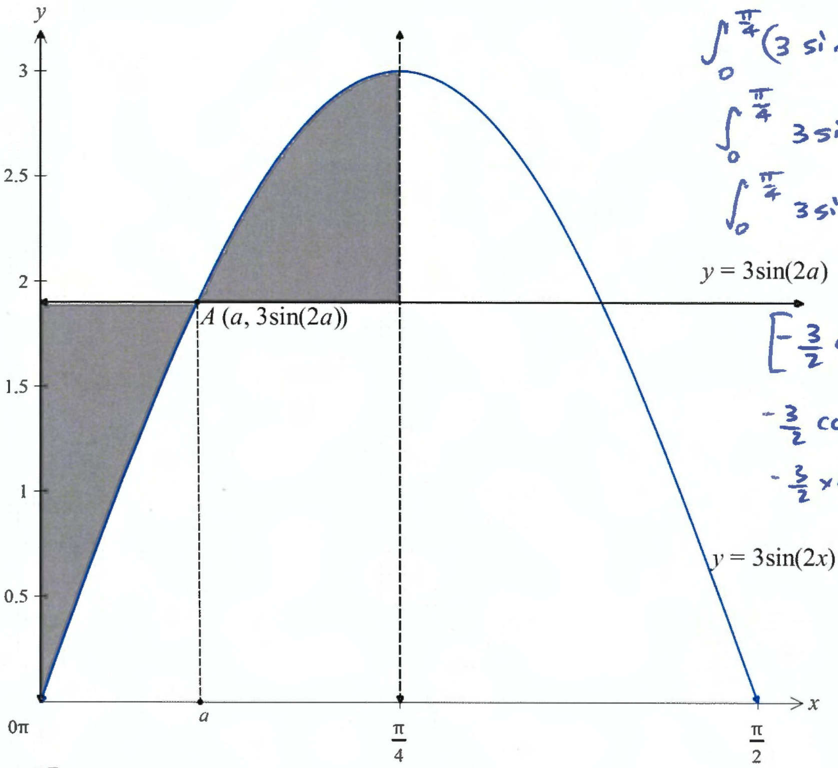
Reasonably well done

Question 27 (4 marks)

A is a point on the curve $y = 3\sin(2x)$ with coordinates $(a, 3\sin(2a))$. By considering the area enclosed between the curve $y = 3\sin(2x)$, the horizontal line through A with equation $y = 3\sin(2a)$ and the lines $x = 0$ and $x = \frac{\pi}{4}$, find the value of a, correct to 3 decimal places, such that **4**

$$\int_0^{\frac{\pi}{4}} (3\sin 2x - 3\sin 2a)\,dx = 0.$$

Many students got the correct answer without considering the area. They were awarded 3/4.



$\int_0^{\frac{\pi}{4}} (3\sin 2x - 3\sin 2a)\,dx = 0$

$\int_0^{\frac{\pi}{4}} 3\sin 2x\,.dx - \int_0^{\frac{\pi}{4}} 3\sin 2a\,dx$

$\int_0^{\frac{\pi}{4}} 3\sin 2x\,.dx = \int_0^{\frac{\pi}{4}} \underbrace{3\sin 2a}_{\text{this is a constant}}\,.dx$

$\left[-\frac{3}{2}\cos 2x\right]_0^{\frac{\pi}{4}} = \left[3x\sin 2a\right]_0^{\frac{\pi}{4}}$

$-\frac{3}{2}\cos\frac{\pi}{2} - -\frac{3}{2}\cos 0 = \frac{3\pi}{4}\sin 2a - 0$

$-\frac{3}{2}\times 0 + \frac{3}{2}\times 1 = \frac{3\pi}{4}\sin 2a$

$\frac{3}{2} = \frac{3\pi}{4}\sin 2a$

$\sin 2a = \frac{6}{3\pi}$

$2a = \sin^{-1}\left(\frac{2}{\pi}\right)$

$2a = 0.69\ldots$

$a = 0.345$

For $\int_0^{\frac{\pi}{4}} (3\sin 2x - 3\sin 2a)\,.dx = 0$ it is required that

$\int_0^{a} (3\sin 2a - 3\sin 2x)\,.dx = \int_a^{\frac{\pi}{4}} (3\sin 2x - 3\sin 2a)\,.dx$ 1 for correct area statement

$\left[3x\sin 2a - -\frac{3}{2}\cos 2x\right]_0^a = \left[-\frac{3}{2}\cos 2x - 3x\sin 2a\right]_a^{\pi/4}$

$3a\sin 2a + \frac{3}{2}\cos 2a - \left(0 + \frac{3}{2}\cos 0\right) = -\frac{3}{2}\cos\frac{\pi}{2} - \frac{3\pi}{4}\sin 2a - \left(-\frac{3}{2}\cos 2a - 3a\sin 2a\right)$ 1 for correct integration

$\cancel{3a\sin 2a} + \cancel{\frac{3}{2}\cos 2a} - \frac{3}{2} = -\frac{3}{2}\times 0 - \frac{3\pi}{4}\sin 2a + \cancel{\frac{3}{2}\cos 2a} + \cancel{3a\sin 2a}$

$-\frac{3}{2} = -\frac{3\pi}{4}\sin 2a$

Lots of problems with $-6 = -3\pi\sin 2a$

$\int 3\sin 2a\,.dx$ $\sin 2a = \frac{6}{3\pi} = \frac{2}{\pi}$

$3\sin 2a$ is a constant (no x) $2a = \sin^{-1}\left(\frac{2}{\pi}\right)$ 1.

so $\int 3\sin 2a\,.dx = 3x\sin 2a + c$ $2a = 0.6901070914$

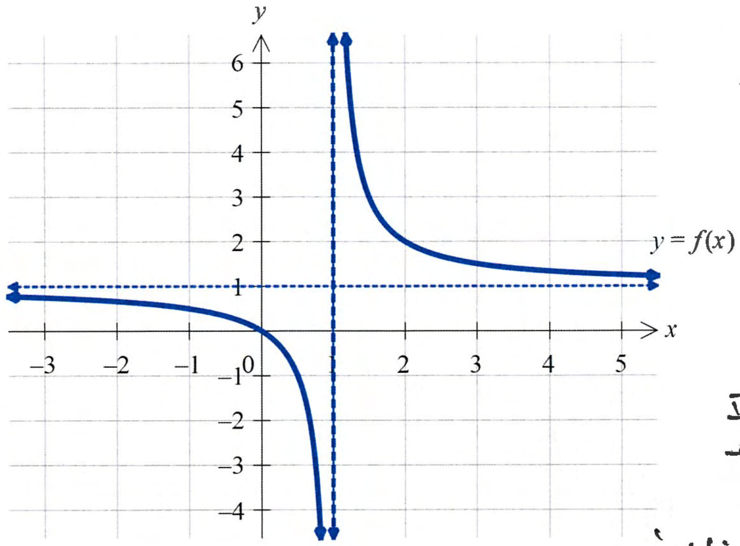
Some students wrote this as $a = 0.3450535457$

$3\sin 2a\,x$ meaning $3(\sin 2a)\times x$ $a = 0.345$ to 3 d.p. 1.

but they then worked with $3\sin(2ax)$ which is incorrect.

Question 28 (4 marks)

Consider the graph of a hyperbola $y = f(x)$, as shown below.



Many students could use transformation to find $y=1+\frac{1}{x-1}$ and organise the equation into $y=\frac{x}{x-1}$
Some could rearrange $y=\frac{x}{x-1}$ to $y=1+\frac{1}{x-1}$ by splitting the fraction.
Either way, they still need to justify the asymptotes.
It's not sufficient if they only stated $x \neq 1$, $y \neq 1$ without justification.

(a) Justify that the equation of $y = f(x)$ is $f(x) = \dfrac{x}{x-1}$. Show full working. **2**

$y - 1 = \frac{k}{x-1}$

$y = 1 + \frac{k}{x-1}$

when $x = 0$, $y = 0$

$0 = 1 + \frac{k}{-1}$

$\therefore k = 1$

$y = 1 + \frac{1}{x-1}$

$= \frac{x-1+1}{x-1}$

$y = \frac{x}{x-1}$

$\lim\limits_{x \to \infty} 1 + \frac{1}{x-1} = 1 + 0 = 1$

hence, horizontal asymptote is $y = 1$

Domain: $x \neq 1$, vertical asymptote is $x = 1$

(b) Find the exact value of m for which the equation $f(x) = mx$ is a tangent to $f(x) = \dfrac{x}{x-1}$. **2**

$\frac{x}{x-1} = mx$ needs to have one solution i.e. $\Delta = 0$

$x = mx(x-1)$

$x = mx^2 - mx$

$0 = mx^2 - mx - x$

$0 = mx^2 - (m+1)x$

$\Delta = b^2 - 4ac$

$= (-(m+1))^2 - 4 \times m \times 0$

$\Delta = (m+1)^2$

for $\Delta = 0$, $m = -1$

Most of the students use calculus to answer this question

$f'(x) = \frac{1(x-1) - x \cdot 1}{(x-1)^2}$

$= \frac{-1}{(x-1)^2}$

$m = f'(0) = \frac{-1}{(-1)^2} = -1$

$\therefore m = -1$

This approach is only correct if you know $f(x)$ passes $(0,0)$ which happens to be the case for this question.
The correct approach should be using Δ.
However, students using calculus to correctly answer this question get full marks.

Question 29 (4 marks)

The line $y = c - \frac{x}{2}$ is a normal to the curve $y = x^3 + ax^2 + bx - 4$ at the point (2, 7). **4**

What are the values of a, b and c?

$y = x^3 + ax^2 + bx - 4$

passes through $(2,7)$

$7 = 8 + 4a + 2b - 4$

$3 = 4a + 2b$ —— ①

Some students didn't find the gradient of the tangent correctly.

$y' = 3x^2 + 2ax + b$ since $y = -\frac{x}{2} + c$ is a normal

$m_N = -\frac{1}{2}$

$\therefore m_T = 2$ at $(2,7)$

$2 = 3 \times 2^2 + 2a \times 2 + b$

$2 = 12 + 4a + b$

$-10 = 4a + 2b$ —— ②

① − ② $13 = b$

$\boxed{b = 13}$

sub into ①

$3 = 4a + 2 \times 13$

$4a = 3 - 26$

$4a = -23$

$a = \frac{-23}{4}$

$\boxed{a = -5\frac{3}{4}}$

$(2,7)$ is on $y = -\frac{x}{2} + c$

$7 = \frac{-2}{2} + c$

$7 = -1 + c$

$\boxed{c = 8}$

Mostly well done.

$a = -5\frac{3}{4}$, $b = 13$, $c = 8$

Question 30 (5 marks)

Sketch the curve $y = \frac{x^3}{6} + \frac{x^2}{4} - 3x + 1$ by finding all stationary points and testing their nature. **5**

Also, find the points of inflection and the y intercept.

$y = \frac{x^3}{6} + \frac{x^2}{4} - 3x + 1$

mostly well done. Some students didn't test the nature or draw the conclusion for the nature of the points.

$y' = \frac{3x^2}{6} + \frac{2x}{4} - 3$

Stationary points occur when $y' = 0$

$0 = \frac{x^2}{2} + \frac{x}{2} - 3$

$0 = \frac{1}{2}(x^2 + x - 6)$

$0 = \frac{1}{2}(x+3)(x-2)$

$x = -3 \qquad x = 2$

$y = \frac{-27}{6} + \frac{9}{4} + 9 + 1 \qquad y = \frac{8}{6} + \frac{4}{4} - 6 + 1$

$= \frac{31}{4} \qquad = -\frac{8}{3}$

$= 7\frac{3}{4} \qquad = -2\frac{2}{3}$

$y'' = x + \frac{1}{2}$

When $x = -3$, $y'' = -3 + \frac{1}{2} = -2\frac{1}{2} < 0$

Maximum turning point at $(-3, 7\frac{3}{4})$

When $x = 2$, $y'' = 2 + \frac{1}{2} = 2\frac{1}{2} > 0$

Minimum turning point at $(2, -2\frac{2}{3})$

Possible points of inflexion when $y'' = 0$

$x + \frac{1}{2} = 0$

$x = -\frac{1}{2}$, $y = \frac{-1}{8 \times 6} + \frac{1}{4 \times 4} - 3(-\frac{1}{2}) + 1$

$= \frac{-1}{48} + \frac{1}{16} + \frac{3}{2} + 1$

$= \frac{61}{24}$

$= 2\frac{13}{24} \doteqdot 2.5416$

x	-1	$-\frac{1}{2}$	0
y''	$-\frac{1}{2}$	0	$\frac{1}{2}$

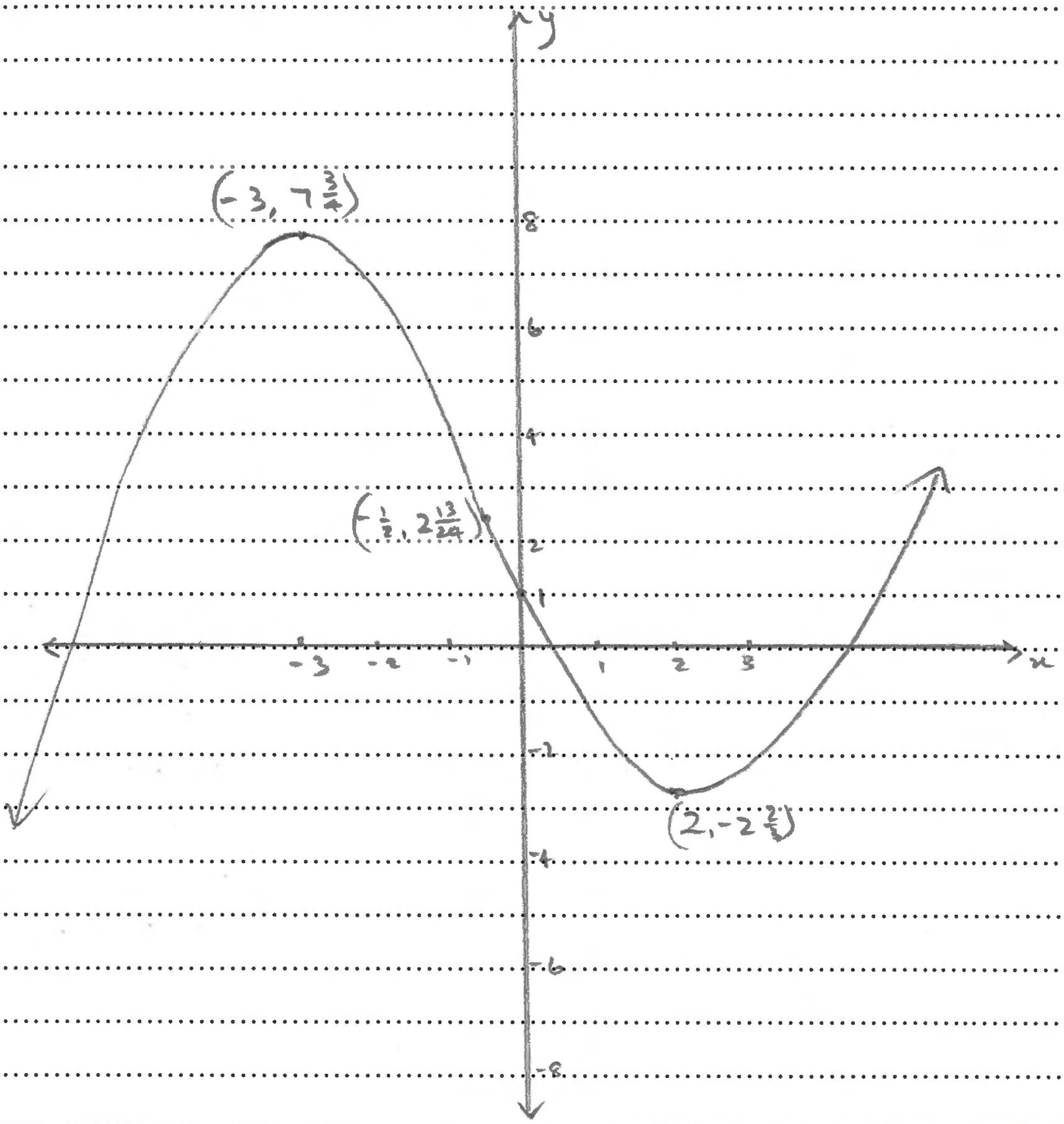
There is a change in concavity

$\therefore$ there is a point of inflexion at $(-\frac{1}{2}, 2\frac{13}{24})$

y-intercept occurs when $x = 0$

$y = 0 + 0 - 0 + 1$

$y = 1$



Some students did it draw the part of the curve near the point of inflection correctly

The assessment continues on the next page

Question 31 (5 marks)

A particle is moving along the x-axis such that at a time t seconds its displacement from the origin is x metres. The acceleration of the particle is given by $a = \frac{8}{e^{2t}} + \frac{3}{e^{t}}$.

Initially its velocity is -6 m/s and its displacement is 5 m.

(a) Find the equation for the displacement (x) of the particle in terms of t. **2**

$a = 8e^{-2t} + 3e^{-t}$

$v = \frac{8e^{-2t}}{-2} + \frac{3e^{-t}}{-1} + c$

When $t=0$, $v=-6$

$-6 = -4e^{0} - 3e^{0} + c$

$-6 = -7 + c$

$c = 1$

$v = -4e^{-2t} - 3e^{-t} + 1$

$x = \frac{-4e^{-2t}}{-2} - \frac{3e^{-t}}{-1} + t + k$

When $t=0$, $x=5$

$5 = 2e^{0} + 3e^{0} + 0 + k$

$5 = 5 + k$

$k = 0$

$x = 2e^{-2t} + 3e^{-t} + t = \frac{2}{e^{2t}} + \frac{3}{e^{t}} + t$

Generally well done however a number of students wrote '6' instead of '−6' for the velocity which created carry on errors.

(b) At what time and where does the particle come to rest? **3**

At rest when $v=0$

$0 = -4e^{-2t} - 3e^{-t} + 1$

$0 = 4e^{-2t} + 3e^{-t} - 1$

$0 = 4(e^{-t})^2 + 3e^{-t} - 1$

Let $m = e^{-t}$

$0 = 4m^2 + 3m - 1$

$0 = (4m-1)(m+1)$

$m = \frac{1}{4}$ or $m = -1$

$e^{-t} = \frac{1}{4}$ $\quad$ $e^{-t} = -1$

$\ln e^{-t} = \ln \frac{1}{4}$ $\quad$ but $e^{-t} > 0$

$-t = \ln 4^{-1}$ $\quad$ $\therefore$ no solution

$-t = -\ln 4$

$t = \ln 4$

$t = 1.386294361$

When $t = \ln 4$

$x = 2e^{-2\ln 4} + 3e^{-\ln 4} + \ln 4$

$= 2e^{\ln 4^{-2}} + 3e^{\ln 4^{-1}} + \ln 4$

$= 2 \times 4^{-2} + 3 \times 4^{-1} + \ln 4$

$= \frac{2}{16} + \frac{3}{4} + \ln 4$

$= \frac{7}{8} + \ln 4$

$= 2.261294361$

At rest when $t = \ln 4$

$\doteqdot 1.386$ seconds

at $x \doteqdot 2.26$ m

Generally well done! Most students realised that the velocity eq became a quadratic eqn.

Question 32 (5 marks)

Four students want to organise a netball game. All four students have the same probability, $P(T)$, of being available next Tuesday. All four students have the same probability, $P(W)$, of being available next Wednesday. Josie is one of the four students.

It is known that $P(T) = \frac{2}{5}$, $P(W \mid T) = \frac{1}{4}$ and $P(T \mid W) = \frac{1}{3}$.

(a) Is Josie's availability next Tuesday independent from her availability next Wednesday? Justify your answer. **1**

if T and W are independent then $P(T \mid W) = P(T)$
and $P(W \mid T) = P(W)$

but $P(T \mid W) = \frac{1}{3}$ and $P(T) = \frac{2}{5}$

$\therefore P(T \mid W) \neq P(T)$

$\therefore$ T and W are not independent

OK. Many students do not know the formula $P(T \mid W) = P(T)$ for independence.

(b) Show that the probability that Josie is available next Wednesday is $\frac{3}{10}$. **2**

$P(W \mid T) = \frac{P(W \cap T)}{P(T)}$

$\frac{1}{4} = \frac{P(W \cap T)}{\frac{2}{5}}$

$P(W \cap T) = \frac{1}{4} \times \frac{2}{5}$

$= \frac{2}{20}$

$= \frac{1}{10}$

$P(T \mid W) = \frac{P(T \cap W)}{P(W)} = \frac{P(W \cap T)}{P(W)}$

$\frac{1}{3} = \frac{\frac{1}{10}}{P(W)}$

$P(W) = \frac{1}{10} \div \frac{1}{3}$

$= \frac{1}{10} \times \frac{3}{1}$

$= \frac{3}{10}$

Well done!

(c) What is the probability that at least one of the four students is NOT available next Wednesday? **2**

$1 - P(\text{all available on W})$

$= 1 - \left(\frac{3}{10}\right)^4$

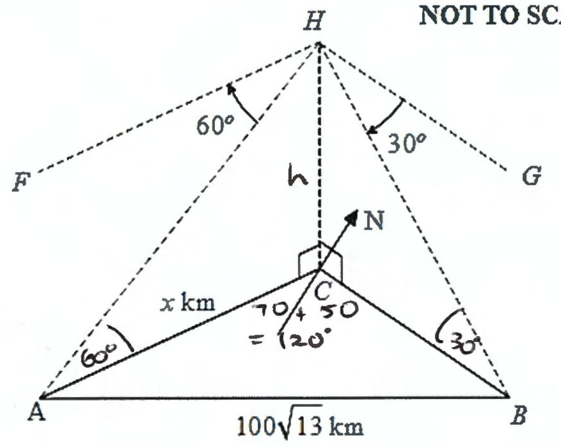
$= 1 - \frac{81}{10000}$

$= \frac{9919}{10000}$

Well done!

Question 33 (7 marks)

The figure below shows three buoys at sea at A, B and C. A helicopter flies along FH and then HG, where F, G and H are three points of the same height vertically above A, B and C, respectively. When the helicopter reaches H, the angles of depression of A and B from the helicopter are 60° and 30°, respectively. The true bearings of A and B from C are 250° and 130°, respectively. The distance between A and B is $100\sqrt{13}$ km.



NOT TO SCALE

$\angle ACB = 250 - 130$
$= 120^\circ$

(a) Show that the distance between A and C is 100 km. **3**

In $\triangle AHC$, $\tan 60^\circ = \frac{h}{x}$

$AC = x = \frac{h}{\tan 60^\circ}$

$AC = x = \frac{h}{\sqrt{3}}$ so $h = \sqrt{3}x$

In $\triangle BHC$, $\tan 30^\circ = \frac{h}{BC}$

$BC = \frac{h}{\tan 30^\circ} = \frac{h}{\frac{1}{\sqrt{3}}} = \sqrt{3}h$

but $h = \sqrt{3}x$

$\therefore BC = \sqrt{3} \times \sqrt{3}x$
$= 3x$

In $\triangle ABC$ $AB^2 = AC^2 + BC^2 - 2 \times AC \times BC \times \cos 120^\circ$

$(100\sqrt{13})^2 = \left(\frac{h}{\sqrt{3}}\right)^2 + (\sqrt{3}h)^2 - 2 \times \frac{h}{\sqrt{3}} \times \sqrt{3}h \times \left(-\frac{1}{2}\right)$

$10000 \times 13 = \frac{h^2}{3} + 3h^2 + h^2$

$130\,000 = \frac{h^2}{3} + 4h^2$

$130\,000 = \frac{13h^2}{3}$

$13h^2 = 3 \times 130\,000$

$h^2 = 3 \times 10\,000$

$h = \pm\sqrt{30\,000}$ but $h > 0$

$h = \sqrt{30\,000}$

$\therefore AC = x = \frac{h}{\sqrt{3}}$

$= \frac{\sqrt{30\,000}}{\sqrt{3}}$

$= \sqrt{10\,000}$

$= 100$ km

OR

$AB^2 = x^2 + (3x)^2 - 2 \times x \times 3x \times \cos 120$

$(100\sqrt{13})^2 = x^2 + 9x^2 - 6x^2 \times \left(-\frac{1}{2}\right)$

$10000 \times 13 = 10x^2 + 3x^2$

$130\,000 = 13x^2$

$x^2 = 10\,000$

$x = \pm 100$ but $x > 0$

$x = 100$

and $BC = 3x$
$= 300$ m

Question 33 continues on the next page

and $BC = \sqrt{3}h$
$= \sqrt{3} \times \sqrt{30\,000}$
$= \sqrt{90000} = 300$ m

Question 33 continued

At 9 a.m., two destroyers start travelling from A and B in the direction of C and continue past C in the same straight lines. The destroyer at A travels along AC with a uniform speed of 20 km/h, while the destroyer at B travels along BC with a uniform speed of 50 km/h.

(b) Show that the distance between the two destroyers after travelling for t hours is $10\sqrt{1300-450t+39t^2}$. **2**

$100+20t$, $300+50t$ accepted

After t hours the distances AC and BC will be $100-20t$ and $300-50t$ respectively

$AB^2 = (100-20t)^2 + (300-50t)^2 - 2(100-20t)(300-50t)\cos 120$

$= 10000 - 4000t + 400t^2 + 90000 - 30000t + 2500t^2$

$- 2(3000 - 5000t - 6000t + 1000t^2) \times (-\frac{1}{2})$

$= 130000 - 45000t + 3900t^2$

$AB^2 = 100(1300 - 450t + 39t^2)$ Not well done

$AB = \pm\sqrt{100(1300-450t+39t^2)}$ but $AB > 0$

$AB = 10\sqrt{1300 - 450t + 39t^2}$

(c) The destroyers are known to be "on alert" when they are less than 40 km apart. **2**

During what times will the two destroyers be "on alert"?

$\frac{450 \pm \sqrt{2196}}{78} = \frac{75 \pm \sqrt{61}}{13}$

$10\sqrt{1300 - 450t + 39t^2} < 40$

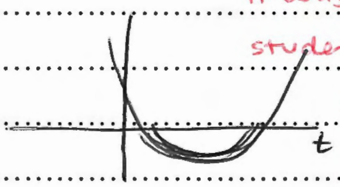
$\sqrt{1300 - 450t + 39t^2} < 4$

$1300 - 450t + 39t^2 < 16$

$39t^2 - 450t + 1284 < 0$

If $39t^2 - 450t + 1284 = 0$

$t = \frac{450 \pm \sqrt{450^2 - 4\times 39 \times 1284}}{2\times 39}$



It was good to see that students who could not do part (b) had a good attempt at part (c).

$\frac{450 - \sqrt{450^2 - 4\times39\times1284}}{78} < t < \frac{450 + \sqrt{450^2 - 4\times39\times1284}}{78}$ Quite well done to this stage.

$5.168442333 < t < 6.370019206$

5 h 10 min 6.39 s $< t <$ 6 h 22 min 12.07 s where t is time after 9am

$\therefore$ on alert between 2:10pm and 3:22pm

Poorly done in finding times.

End of paper