Hunters Hill High School

Student Number

2024 TRIAL EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total Marks: 100

Section I – 10 marks (pages 3 – 6)

- Attempt Questions 1–10 on the multiple-choice answer sheet
- Allow about 15 minutes for this section

Section II – 90 marks (pages 7 – 27)

- Attempt Questions 11 – 33 in this booklet
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1 The function $f(x) = \sqrt{x}$ is transformed to $g(x) = \sqrt{x+1} - 3$ by a horizontal translation of 1 unit followed by a vertical translation of 3 units.

Which row of the table shows the directions of the translations?

	Horizontal translation of 1 unit	*Vertical translation of 3 units*
A.	Left	Up
B.	Right	Up
C.	Left	Down
D.	Right	Down

2 Which of the following statements is true of the function $f(x) = -x(x+2)(x-2)$

A. $f(x)$ is a parabola that has been reflected and translated

B. $f(x)$ is a one-to-one function

C. $f(x)$ is an even function

D. $f(x)$ is an odd function

3 What is the derivative of $\tan(\ln x)$, where $x > 0$?

A. $\sec^2\left(\ln\frac{1}{2}\right)$

B. $\dfrac{1}{\sec^2(\ln x)}$

C. $\dfrac{\sec^2(\ln x)}{x}$

D. $\dfrac{\sec^2\frac{1}{x}}{x}$

4 Find the gradient of the tangent to the graph of $y = x^3 - 2x^2 - 5$ at $x = 3$.

A. 6

B. 15

C. 21

D. 10

5 What is the solution of the equation $\log_a x^4 - \log_a x^2 = b$, where a and b are positive?

A. $x = b^{\frac{a}{2}}$

B. $x = a^{\frac{b}{2}}$

C. $x = \dfrac{b^a}{2}$

D. $x = \dfrac{a^b}{2}$

6 What is $\int \frac{1}{\sqrt{3x-1}}\,dx$?

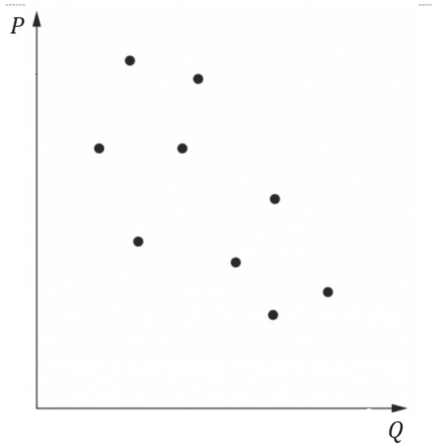
A. $\frac{2\sqrt{3x-1}}{3}+C$

B. $\frac{-2\sqrt{3x-1}}{3}+C$

C. $\frac{\sqrt{3x-1}}{6}+C$

D. $\frac{-\sqrt{3x-1}}{6}+C$

7 A scatterplot is shown.



Which of the following best describes the correlation between P and Q?

A. Positively skewed

B. Negatively skewed

C. Positive

D. Negative

8 The function with equation $f(x) = 2\sin\left(\frac{1}{2}x + 2\pi\right)$ has period.

A. 4π

B. 2π

C. π

D. 2

9 There are 2000 participants sitting an IQ test of intelligence. If the results form a normal distribution, how many participants would be expected to score a result between 1 and 3 standard deviations below the mean?

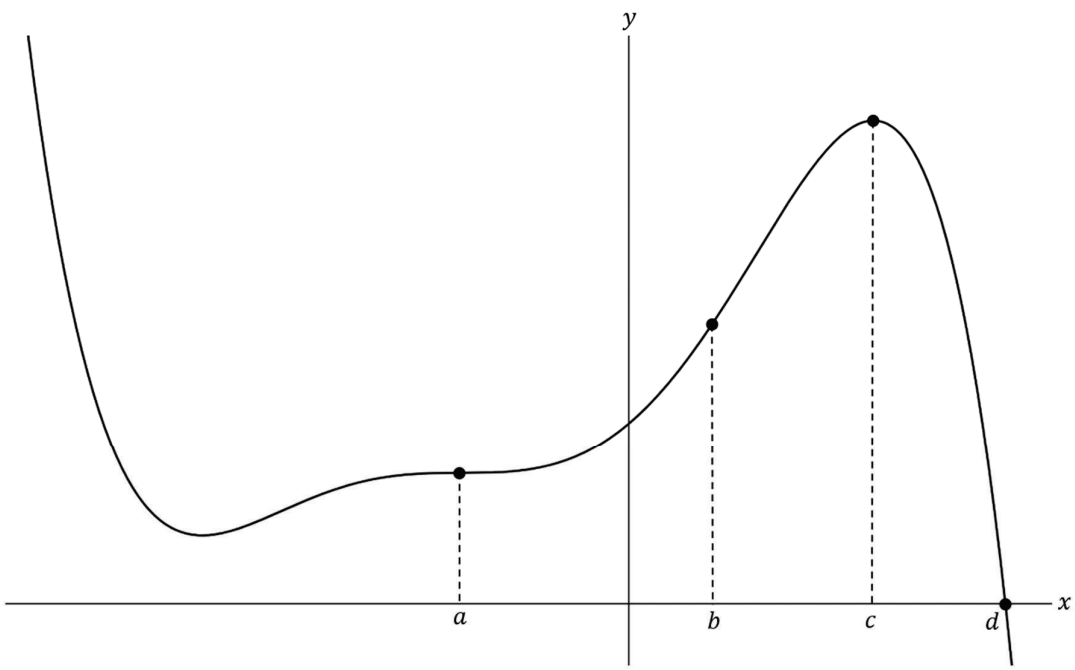
A. 314

B. 317

C. 350

D. 634

10 The diagram shows the graph of $f'(x)$, the derivative of a function.



For what value of x does the graph of the function $f(x)$ have a point of inflection?

A. $x = a$

B. $x = b$

C. $x = c$

D. $x = d$

End of Section I

Section II

90 marks
Attempt Questions 11 – 33
Allow about 2 hours and 45 minutes for this section

Write each response in the spaces provided. Extra writing space is provided at the back of this paper.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (3 marks)

Calculate the sum of the infinite geometric series $81 - 27 + 9 - 3 + \cdots$. **3**

Question 12 (3 marks)

Evaluate $\int_{-1}^{1} e^{3x}\,dx$ in exact form. **3**

Question 13 (6 marks)

Jeremy has three \$5 notes, two \$10 notes, four \$20 notes and one \$50 note. He decides to give one at random to his friend as a birthday present.

(a) Complete the following probability distribution table showing the amount given. **2**

x	5	10	20	50	Sum
$P(X = x)$					
$xp(x)$					
$x^2p(x)$					

(b) Show that the expected value $E(X) = 16.5$. **1**

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(c) Hence or otherwise, calculate the standard deviation, correct to one decimal place. **3**

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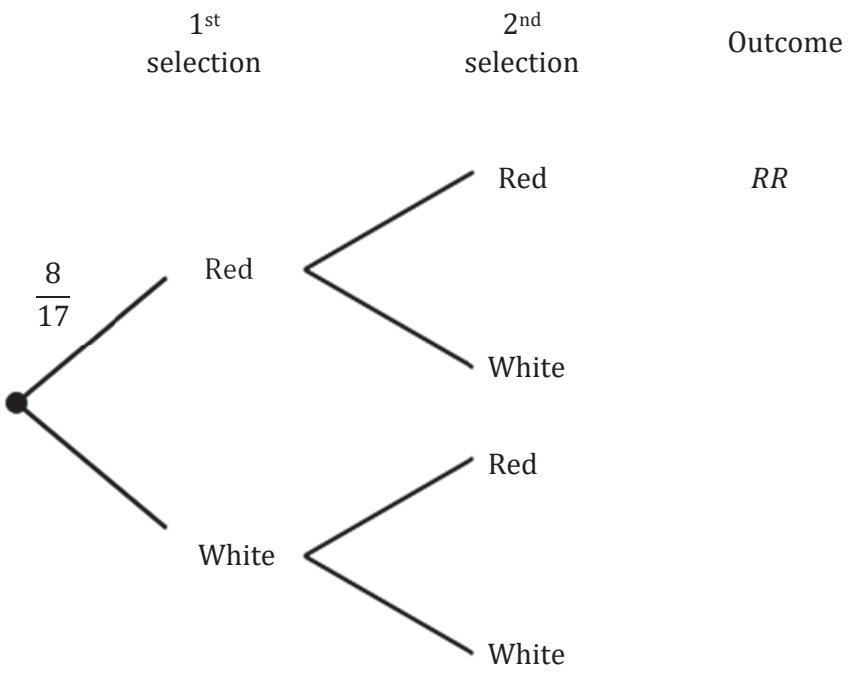
Question 14 (4 marks)

A bag contains 17 marbles. Eight of them are red, and the others are white. Sarah randomly takes out one marble from the bag, and without replacing it, selects a second marble.

(a) A partially completed tree diagram is shown. **2**

Complete the probability tree diagram, by including all outcomes and probabilities on the branches.



(b) Find the probability that both marbles are red. **1**

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(c) Find the probability that both marbles selected are different colours. **1**

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Question 15 (2 marks)

Differentiate $\dfrac{x^2 - x}{x + 2}$. **2**

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Question 16 (5 marks)

(a) Show that the derivative of $f(x) = (x - 1)\sin 2x$ is **2**

$$f'(x) = \sin 2x + 2(x - 1)\cos 2x.$$

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(b) Hence, find the exact value of $\displaystyle\int_0^{\frac{\pi}{4}} \left(\frac{1}{2}\sin 2x + (x - 1)\cos 2x\right) dx$. **3**

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Question 17 (2 marks)

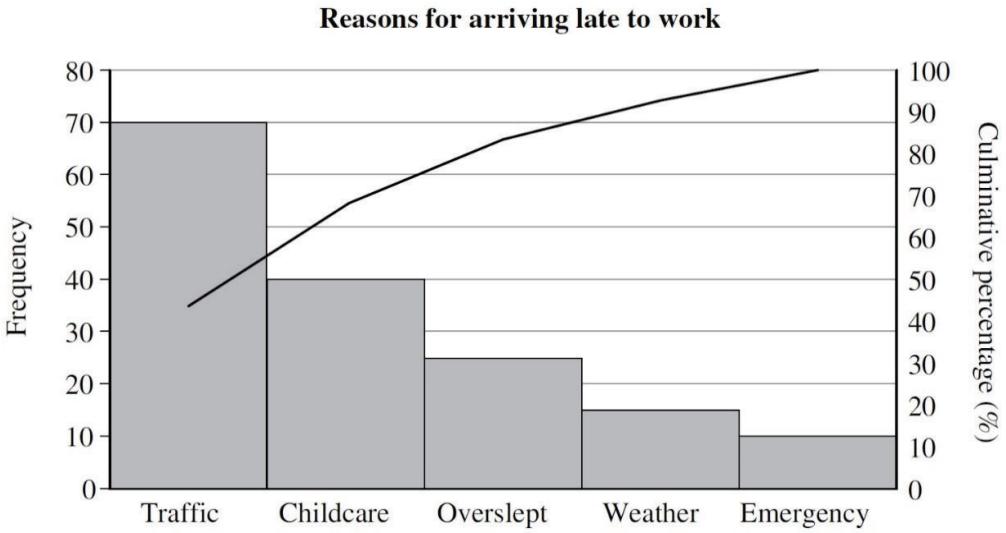
Prove $\sin x \tan x + \cos x = \sec x$. **2**

Question 18 (3 marks)

Find the equation of the normal to the curve $y = (2x - 1)^3$ at the point where $x = 1$. **3**

Question 19 (2 marks)

A company records the reasons for its employees arriving late to work. **2**
The results are shown in the Pareto chart below.



Approximately what percentage of issues were due to “Childcare” or “Overslept”?

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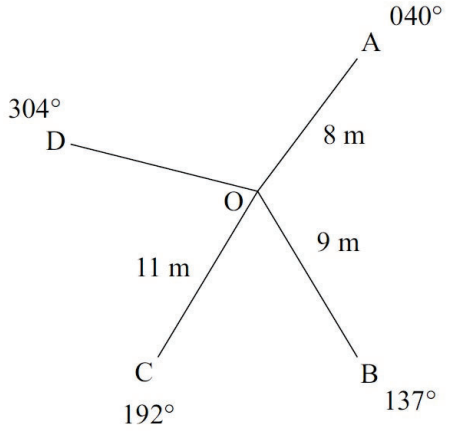
Question 20 (3 marks)

The diagram below shows a compass survey of the field $ABCD$. **3**



A surveyor was unable to measure the distance OD, but was able to locate the area of the park sections.

If the area of ΔDOA is 24 m^2, what is the length OD? Give your answer correct to the nearest metre.

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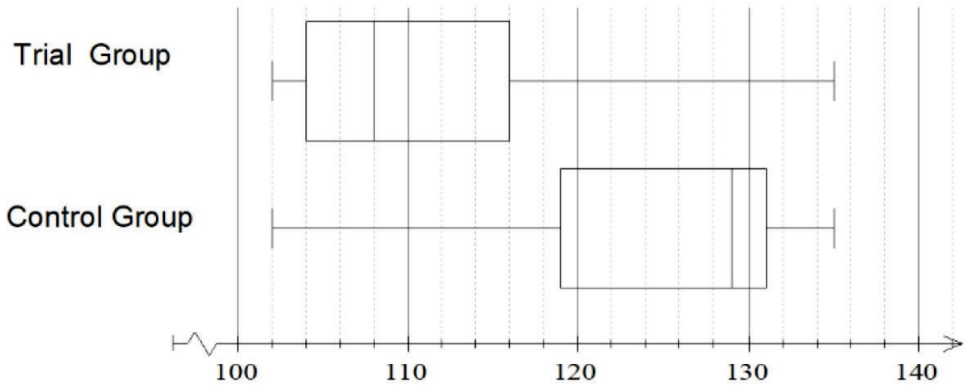
Question 21 (4 marks)

The graphs below compare the blood pressure of two groups of volunteers in a medical study. The trial group undertook a structured program of exercise and diet, while the control group maintained their previous habits. **4**



Compare and describe the data provided in both box plots, with reference to the context given.

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Question 22 (8 marks)

(a) Find the vertical and horizontal asymptotes of $y = \dfrac{3x-3}{(x+3)(x-2)}$. **2**

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(b) Hence sketch the graph of $y = \dfrac{3x-3}{(x+3)(x-2)}$, showing all intercepts and asymptotes. **3**

c. Hence or otherwise, find all values x for which **3**

$$\frac{3x-3}{(x+3)(x-2)} \geq 1.$$

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Question 23 (4 marks)

A probability density function is defined as:

$$f(x) = \begin{cases} hx(5-x), & 0 \le x \le 5 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Show that $h = \frac{6}{125}$. **2**

(b) Find the mode of $f(x)$. **2**

Question 24 (2 marks)

Find the sum of the first 15 terms of the arithmetic series $124 + 117.5 + 111 + 104.5 + \cdots$ **2**

Question 25 (2 marks)

Given that $a = \log_2 7$ and $b = \log_7 16$, show that $ab = 4$. **2**

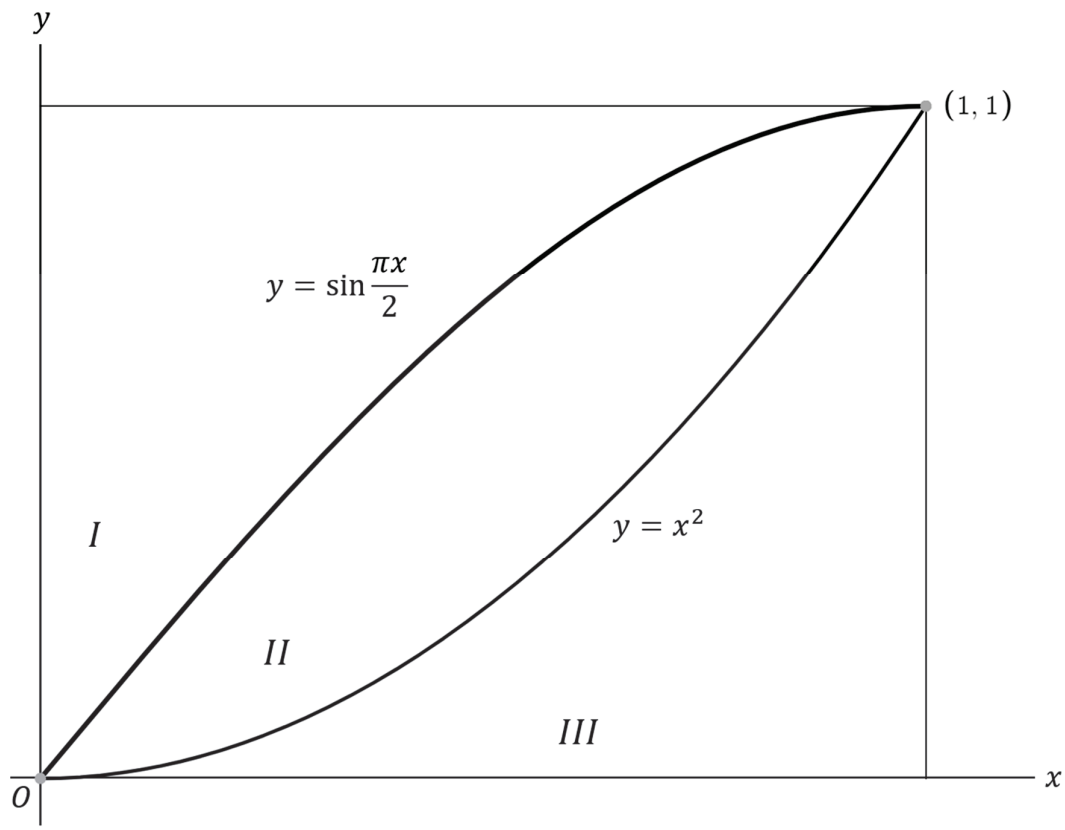
Question 26 (4 marks)

Find all solutions of $2\sin^2 x + \sin x - 1 = 0$, where $0 \leq x \leq 2\pi$. **4**

Question 27 (3 marks)

The diagram below is divided into three regions, I, II and III by the curves

$$y = \sin\frac{\pi x}{2} \text{ and } y = x^2.$$



Find the areas of each of the three regions correct to two decimal places. **3**

Question 28 (5 marks)

A company wanted to investigate the strength of the relationship between the amount of money spent on advertising each week and the number of visitors the company's website receives each week. Data was collected over a 15-week period.

(a) Pearson's correlation coefficient for the data set is $r = 0.8895$. Describe the strength and direction of this linear correlation. **1**

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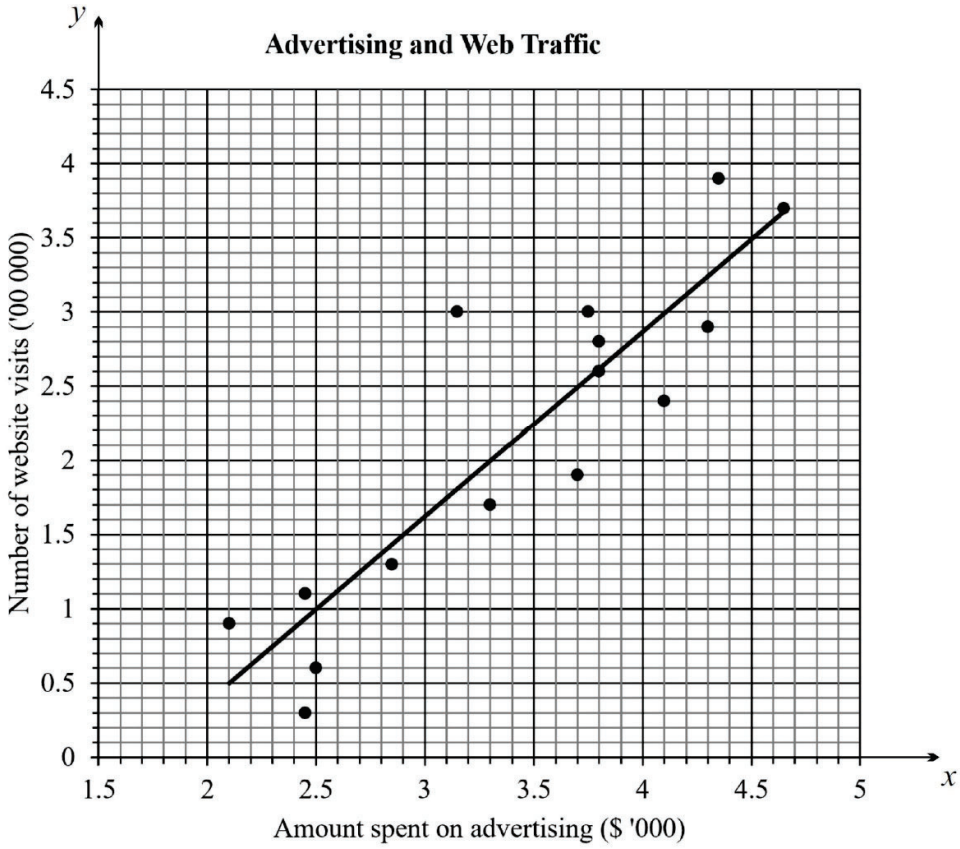
(b) The graph below shows a scatterplot and the regression line for the data. **2**



Use the graph to calculate the equation of the regression line.

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(c) Predict the number of visitors the website will receive for a week in which \$3000 was spent on advertising. **1**

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(d) Explain why the model is not useful for predicting the number of visitors for weeks in which \$1500 was spent on advertising. **1**

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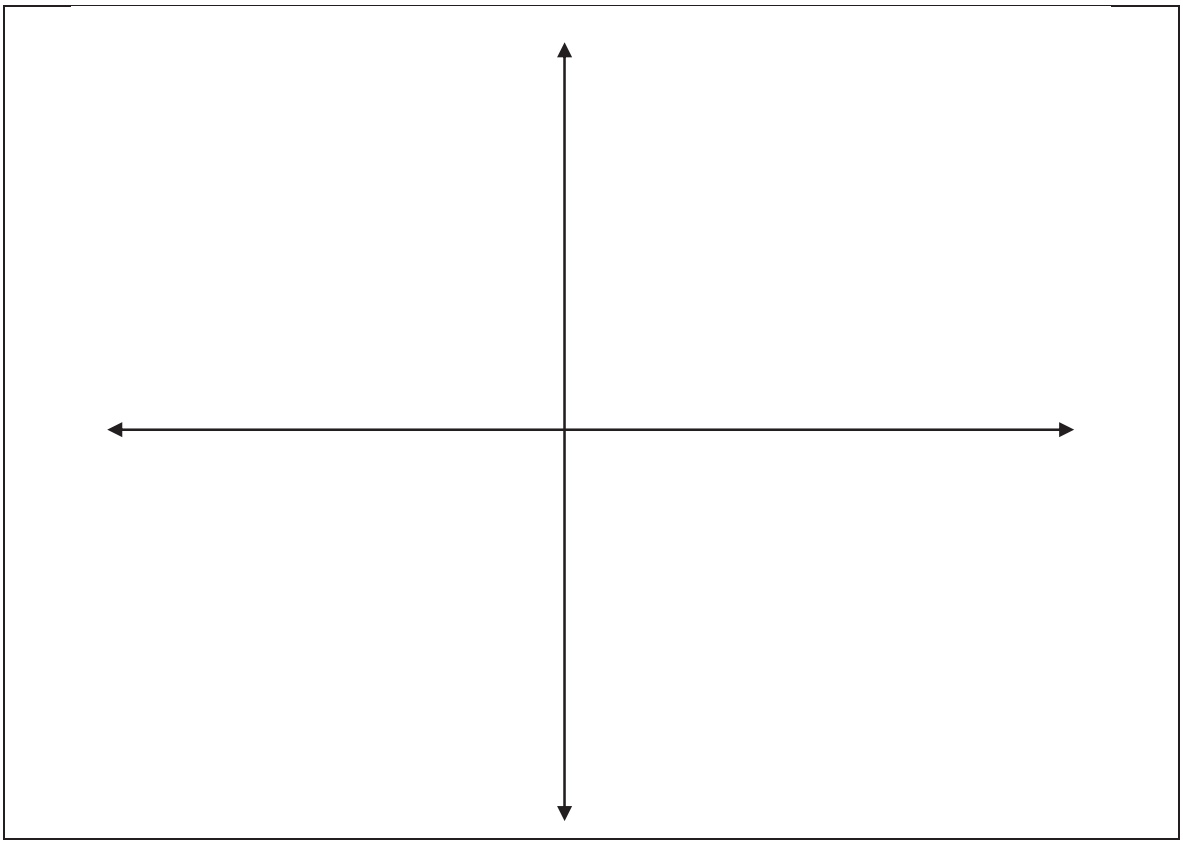
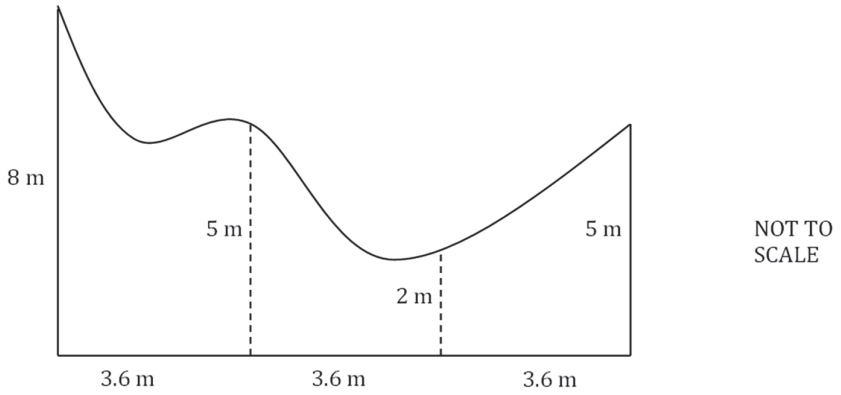
Question 29 (2 marks)

Sketch the curve $y = 3e^{-x} + 1$. **2**



Question 30 (4 marks)

In a garden, a pond is to be installed with the dimensions shown in the diagram.



(a) Using three applications of the trapezoidal rule, calculate the area of the pond. **2**

(b) The pond is initially empty. During a storm, 20 mm of rain falls into the pond. **2**

Calculate the amount of water in the pond immediately after the storm. Give your answer in cubic metres.

Question 31 (5 marks)

A game involves flipping a coin, then rolling two six-sided dice. If the coin lands heads, the sum of the dice is recorded. If the coin lands tails, the difference between the dice is recorded instead.

To win the game, the number recorded must be less than 4.

(a) What is the probability of winning given the coin landed tails? **1**

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(b) What is the probability of winning the game? **2**

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(c) Given the recorded number is 4, what is the probability that the coin landed heads? **2**

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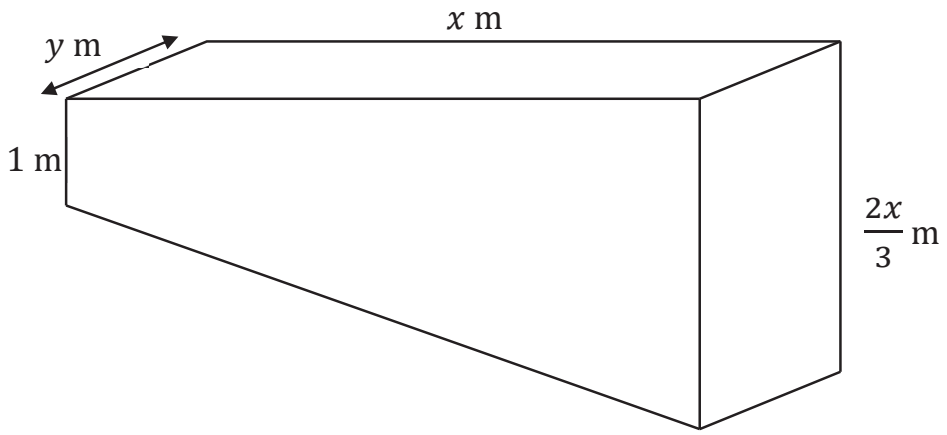
Question 32 (7 marks)

A homeowner intends to construct a rectangular pool in their backyard with a length of x m and a width of y m.

The depth will steadily increase from 1 m to $\frac{2x}{3}$ m, as seen in the diagram below.

The homeowner wishes to put tiles around the edge of the surface of the pool, and has enough to tile a perimeter of 27 m.



(a) Show that the volume of the pool is $V(x) = -\frac{x^3}{3} + 4x^2 + \frac{27}{4}x$. **2**

(b) Hence, find the maximum volume of the pool. **3**

(c) Confirm that the volume in part (b) is a maximum. **2**
Include reasoning and calculations in your response.

Question 33 (7 marks)

The number of daylight hours on the tth day of the year in Sydney can be modelled by the following equation.

$$L(t) = 12 + k\cos\left[\frac{2\pi}{365}(t + 10)\right]$$

(a) Given that there were 14 hours of daylight 10 days before the start of the year, show that $k = 2$. **1**

(b) How many days into the year will the number of daylight hours be increasing the fastest? (Answer to the nearest day) **3**

(c) Hence, find the number of daylight hours from that day until the end of the year. **3**

End of Examination

2024 12MAA Task 4 - Trial Solutions

Thursday, 25 July 2024 1:09 PM

1.) $g(x) = f(x+1) - 3$

$\therefore$ It is translated left 1 unit
down 3 units

Answer: C

2.) $f(-x) = -(-x)(-x+2)(-x-2)$
$= x\,(\cancel{-1})(x-2)(\cancel{-1})(x+2)$
$= x\,(x+2)(x-2)$
$\therefore f(-x) = -f(x)$
$\therefore$ odd function

Answer: D

3.) $\frac{d}{dx} \ln x = \frac{1}{x}$

$\frac{d}{dx} \tan x = \sec^2 x$

$\therefore \frac{d}{dx} \tan(\ln x) = \frac{1}{x} \sec^2(\ln x)$

$= \frac{\sec^2(\ln x)}{x}$

Answer: C

4.) $y' = 3x^2 - 4x$

let $x = 3$
$y' = 3(3)^2 - 4(3)$
$= 27 - 12$
$= 15$

$= 27 - 12$

$= 15$

Answer: B

5.) $\log_a x^4 - \log_a x^2 = b$

$4\log_a x - 2\log_a x = b$

$2\log_a x = b$

$\log_a x = \frac{b}{2}$

$\therefore x = a^{\frac{b}{2}}$

Answer: B

6.) $\int (3x-1)^{-\frac{1}{2}}\,dx$

$= \frac{1}{3}\int 3(3x-1)^{-\frac{1}{2}}\,dx$

$= \frac{1}{3}\left(\frac{1}{\left(\frac{1}{2}\right)}(3x-1)^{\frac{1}{2}}\right) + C$

$= \frac{2\sqrt{3x-1}}{3} + C$

Answer: A

7.) Negative

Answer: D

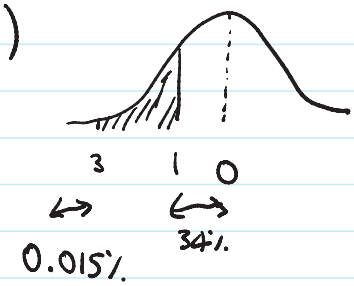
8.) $f(x) = 2\sin\left(\frac{1}{2}x + 2\pi\right)$

$b = \frac{1}{2}$, period $= \frac{2\pi}{b}$

$= 4\pi$

Answer: A

9.)



$50\% - 34\% - 0.15\% = 15.85\%$

$2000 \times 15.85\% = 317$

Answer: B

10.) Point of inflection occurs when concavity changes, which is a turning point for $f'(x)$

Answer: C

11.) $S = \dfrac{a}{1-r}$

$a = 81,\ r = -\frac{1}{3}$

$S = \dfrac{81}{1-(-\frac{1}{3})}$

$= \dfrac{243}{4}$ OR $60\frac{3}{4}$ OR 60.75

① Selecting the correct formula

① Identifying that $r = -\frac{1}{3}$

① Correct answer

12.) $\int_{-1}^{1} e^{3x}\,dx$

$= \left[\frac{1}{3}e^{3x}\right]_{-1}^{1}$

$= \frac{1}{3}e^{3} - \frac{1}{3}e^{-3}$

$= \dfrac{e^3}{3} - \dfrac{1}{3e^3}$

① Correct integration or correct subst. into incorrect integral

① Correct substitution into integral, inc. difference

① Correct answer in exact form

13.) a.)

x	5	10	20	50	Sum

② Completed

13.) a.)

x	5	10	20	50	Sum
$P(X=x)$	0.3	0.2	0.4	0.1	1
$xp(x)$	1.5	2	8	5	16.5
$x^2p(x)$	7.5	20	160	250	437.5

(2) Completed with no errors

(1) 2 lines correct

(0) no lines correct

b.) $E(X) = \sum xp(x)$
$= 1.5 + 2 + 8 + 5$
$= 16.5$

(1) Working sufficiently shows how 16.5 was calculated or $\sum xp(x)$

c.) $Var(X) = E(X^2) - E(X)^2$
$= 437.5 - 16.5^2$
$= 165.25$

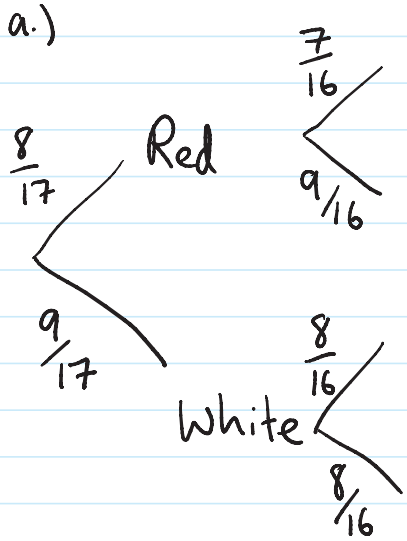
(1) Correctly finds variance

$\sigma = \sqrt{165.25}$
$= 12.85496\ldots$
$= 12.9$ (1d.p.)

(1) Square root taken

(1) Correctly rounded to 1 d.p.

14.) a.)



RR $\frac{56}{272} = \frac{7}{34}$

RW $\frac{72}{272} = \frac{9}{34}$

WR $\frac{72}{272} = \frac{9}{34}$

WW $\frac{72}{272} = \frac{9}{34}$

(2) Correct diagram with labels

(1) Correct outcomes OR probabilities

b.) $P(RR) = \frac{7}{34}$

(1) Correct answer

c.) $P(RWUWR) = \frac{9}{34} + \frac{9}{34}$
$= \frac{18}{34}$
$= \frac{9}{17}$

(1) Correct answer

15.) $u = x^2 - x$ $\quad v = x + 2$
$u' = 2x - 1$ $\quad v' = 1$

$\frac{u'v - uv'}{v^2} = \frac{(2x-1)(x+2) - (x^2 - x)}{(x+2)^2}$

(1) Correctly finds u' and v'

(1) Correctly substitutes

$$\frac{u'v - uv'}{v^2} = \frac{(2x-1)(x+2) - (x-x^2)}{(x+2)^2}$$

$$= \frac{2x^2+4x-x-2-x^2+x}{(x+2)^2}$$

$$= \frac{x^2+4x-2}{(x+2)^2}$$

① Correctly substitutes into the formula

16.)

a.) $u = x-1 \qquad v = \sin 2x$

$u' = 1 \qquad v' = 2\cos 2x$

$$f'(x) = u'v + uv'$$
$$= \sin 2x + (x-1)(2\cos 2x)$$
$$= \sin 2x + 2(x-1)\cos 2x$$

① Correctly finds u' and v' and substitutes into formula

① Sufficiently shows the required result

b.)

$$= \frac{1}{2}\int_0^{\frac{\pi}{4}} \left(\sin 2x + 2(x-1)\cos 2x\right) dx$$

$$= \frac{1}{2}\left[(x-1)\sin 2x\right]_0^{\frac{\pi}{4}}$$

$$= \frac{1}{2}\left(\left(\frac{\pi}{4}-1\right)\sin\frac{\pi}{2} - (0-1)\sin 0\right)$$

$$= \frac{1}{2}\left(\frac{\pi}{4}-1\right)$$

$$= \frac{\pi}{8} - \frac{1}{2}$$

① Correct integration

① Correct substitution
(Could be on calculator, not shown)

① Correct final answer

17.) $LHS = \sin x\left(\frac{\sin x}{\cos x}\right) + \cos x\left(\frac{\cos x}{\cos x}\right)$

$$= \frac{\sin^2 x + \cos^2 x}{\cos x}$$

$$= \frac{1}{\cos x}$$

$$= \sec x$$

$$= RHS$$

$\therefore$ LHS = RHS as required

① Correctly uses either $\sin^2 x + \cos^2 x = 1$, $\tan x = \frac{\sin x}{\cos x}$ or $\sec x = \frac{1}{\cos x}$

① Satisfactory proof

18.) $y = (2x-1)^3$

$y' = 2 \cdot 3(2x-1)^2$

at $x=1$: $y = (2(1)-1)^3$

$= 1$

$y' = 6(2(1)-1)^3$

$= 6$

$m_2 = -\frac{1}{m_1}$ | $y - 1 = -\frac{1}{6}(x-1)$

① Correctly finds the derivative

① Correct method to find gradient of normal
OR
method to find equation

$= 6$

$m_2 = -\frac{1}{m_1}$
$= -\frac{1}{6}$

$y - 1 = -\frac{1}{6}(x-1)$
$\therefore y = -\frac{1}{6}x + 1\frac{1}{6}$
or $x + 6y - 7 = 0$

OR
method to find equation

① Correct answer

19.) Total $\doteqdot 70 + 40 + 25 + 15 + 10$
$= 160$

Childcare is 40, overslept is 25

$\frac{40+25}{160} = 0.40625$
$= 40.625\%$

$\therefore$ Approximately 40% of issues were due to childcare or overslept.

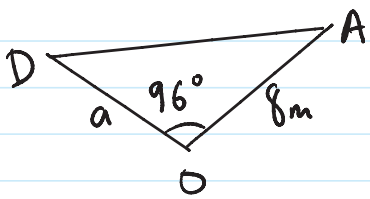
OR Traffic is at 45%
Overslept is at 88%
$\Big\}$ $88\% - 45\% = 43\%$

$\therefore$ Approximately 43% ...

② Correct answer with working

① For either
- correctly finding % of one reason
- seperate answers for both reasons
- total frequency of both reasons

20.) $\angle DOA = (360° - 304°) + 40°$
$= 96°$



$A = 24\ m^2$

$A = \frac{1}{2}ab\sin C$
$24 = \frac{1}{2}a \cdot 8 \sin 96°$
$6 = a \sin 96°$
$a = \frac{6}{\sin 96°}$
$= 6.033049677$

$\therefore OD = 6$ m (nearest metre)

③ Correct answer with required working

② Progress made towards solution

① Only states the angle and/or formula to use

21.) Trial group median: 108
Control group median: 129

④ Comprehensive comparisons made including values

Control group median: 129

The trial group is positively skewed, while the control group is negatively skewed.

They both have the same interquartile range of 12, but the control group has a higher centre.

made including values

③ Sound comparisons including at least one value

② Multiple reasonable comparisons made

① Relevant accurate information provided

22.) a.) Vertical at $(x+3)(x-2)=0$

$\therefore x=-3, 2$

$$y = \frac{(3x-3)\div x^2}{(x^2+x-6)\div x^2}$$

$$= \frac{\frac{3}{x}-\frac{3}{x^2}}{1+\frac{1}{x}-\frac{6}{x^2}}$$

$\therefore y=0$ as $x \to \pm\infty$

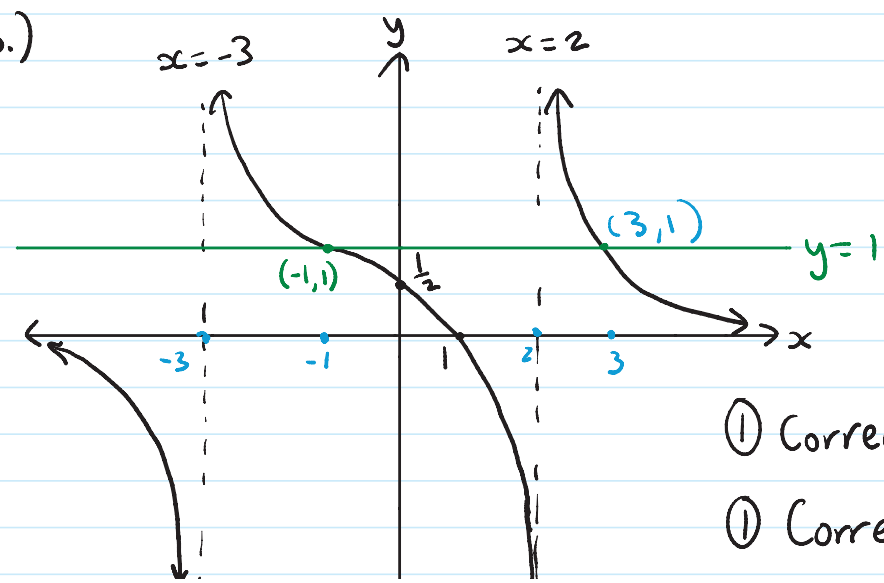
Asymptotes at $x=-3, 2$ and $y=0$

② All 3 asymptotes correctly found

① Two correct asymptotes OR correct method attempted for all 3

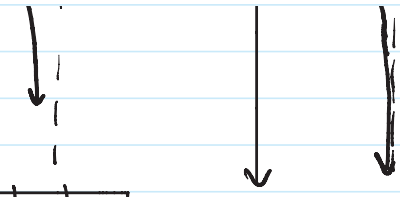
⓪ Less than 2 correct without correct method attempted for all 3

b.)



① Correct shape

① Correct intercepts



x	-4	-1	0	1	3
y	$-\frac{5}{2}$	1	$\frac{1}{2}$	0	1

① Correct intercepts

① Correct asymptotes

c.) let $\dfrac{3x-3}{(x+3)(x-2)} = 1$

$$3x-3 = (x+3)(x-2)$$
$$= x^2+x-6$$
$$0 = x^2-2x-3$$
$$= (x-3)(x+1)$$

$\therefore x = 3, -1$ intersect (see green line)

$-3 < x \leq -1$ and $2 < x \leq 3$

OR $(-3,-1] \cup (2,3]$

① Range from -3 to -1 given

① Range from 2 to 3 given

① Correct inclusive/exclusive symbols used

23.)

a.) $\displaystyle\int_0^5 (5hx - hx^2)\,dx = 1$

$$\left[\frac{5h}{2}x^2 - \frac{h}{3}x^3\right]_0^5 = 1$$

$$\frac{125}{6}h - 0 = 1$$

$$\therefore h = \frac{6}{125}$$

① Sets the integral from 0 to 5 to equal 1

① Sufficiently shows that $h = \frac{6}{125}$

b.) $f(x) = \dfrac{6}{125}\cdot 5x - \dfrac{6}{125}\cdot x^2$

$$f'(x) = \frac{6}{25} - \frac{12}{125}x$$

let $f'(x) = 0$

$$\frac{12}{125}x = \frac{6}{25}$$

$$x = 2.5$$

$\therefore$ the mode is at $x = 2.5$

① Correctly finds the derivative

① Lets the derivative equal zero and shows correct working to find x from there

24.) $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$n = 15, \quad a = 124, \quad d = -6.5$

$S_{15} = \frac{15}{2}\left[2(124) + (15-1)(-6.5)\right]$

$= 1177.5$

① Identifies that the common difference is -6.5 (could have been on calculator, not shown)

① Correct answer

25.) $ab = \log_2 7 \times \log_7 16$

$= \log_2 7 \; \frac{\log_2 16}{\log_2 7}$

$= \log_2 16$

$\therefore ab = 4$ (as req.)

① Makes a correct change of base (or other log law that makes progress)

① Sufficiently shows result

26.) $2\sin^2 x + \sin x - 1 = 0$

let $u = \sin x$

$2u^2 + u - 1 = 0$

$2u^2 + 2u - u - 1 = 0$

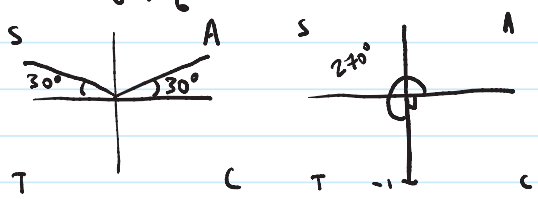
$2u(u+1) - (u+1) = 0$

$(2u-1)(u+1) = 0$

$u = \frac{1}{2}$ $\qquad$ $u = -1$

$\sin x = \frac{1}{2}$ $\qquad$ $\sin x = -1$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$ $\qquad$ $x = \frac{3\pi}{2}$



$\therefore x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

① Find both solutions to $\sin x$

① Find correct quadrants for solutions of x ($\pi \le x \le 2\pi$ and/or $0 \le x \le \pi$ for $\sin x = -\frac{1}{2}$ for $\sin x = 1$)

① Uses correct angles from exact value triangles

① Correct final answer in radians

27.) $A_{III} = \int_0^1 x^2 \, dx$

$= \left[\frac{1}{3}x^3\right]_0^1$

$= \frac{1}{3} - 0$

$= 0.33$ (2 d.p.)

$A_{II} = \int_0^1 \sin\frac{\pi x}{2} \, dx - \frac{1}{3}$

$= \left[-\frac{2}{\pi}\cos\frac{\pi x}{2}\right]^1 - \frac{1}{3}$

③ All areas calculated correctly

② Two correct areas

① One correct area

$A_{II} = \int_0^1 \sin\frac{\pi x}{2}\,dx - \frac{1}{3}$

$= \left[-\frac{2}{\pi}\cos\frac{\pi x}{2}\right]_0^1 - \frac{1}{3}$

$= 0 - \left(-\frac{2}{\pi}\right) - \frac{1}{3}$

$= 0.303\ldots$

$= 0.30$ (2 d.p.)

$A_I = 1 - \frac{2}{\pi}$

$= 0.363\ldots$

$= 0.36$ (2 d.p.)

① One correct area
OR
Correct integration of $\sin\frac{\pi x}{2} \rightarrow -\frac{2}{\pi}\cos\frac{\pi x}{2}$

⓪ Insufficient progress

28.) a.) It is a strong, positive correlation.

① Correct answer

b.) Using points $(2.5, 1)$ and $(4.5, 3.5)$
$(2500, 100\,000)$ and $(4500, 350\,000)$

$m = \frac{350000 - 100000}{4500 - 2500}$
$= 125$

$y - 100\,000 = 125(x - 2500)$
$= 125x - 312\,500$
$\therefore y = 125x - 212\,500$

$(2.5, 1)$ and $(4.5, 3.5)$
$m = \frac{3.5 - 1}{4.5 - 2.5}$
$= 1.25$

$y - 1 = 1.25(x - 2.5)$
$= 1.25x - 3.125$
$\therefore y = 1.25x - 2.125$

[ALSO ACCEPTED]

② Correct equation provided with working shown

① Progress made

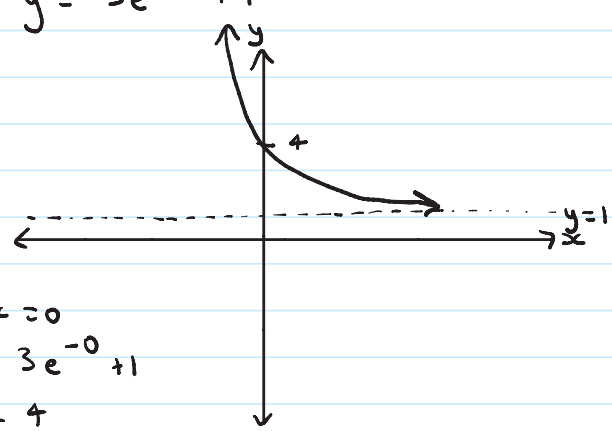
c.) $y = 125(3000) - 212\,500$
$= 162\,500$ visitors
OR
From graph: 160 000 to 165 000

① Between 160 000 and 165 000

d.) $y = 125(1500) - 212\,500$
$= -25\,000$ visitors
There cannot be a negative number of visitors though.

① States that the number of visitors would be negative

29.) $y = 3e^{-x} + 1$



① Correct shape, including asymptote

① Correct y-intercept

let $x = 0$
$y = 3e^{-0} + 1$
$= 4$

30.) a.) $\frac{b-a}{2n}\left(f(a)+f(b)+2\left[f(x_1)+f(x_2)\right]\right)$

$= \frac{10.8-0}{2(3)}\left(8+5+2\left[5+2\right]\right)$

$= 48.6\ m^2$

② Correct method and answer

① Progress made with correct method

b.) $20mm = 0.02$

$V = 48.6m^2 \times 0.02$

$= 0.972\ m^3$

① Correct change of units

① Correct answer

31.)

a.)

	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

① Correct answer

$P(win|T) = \frac{30}{36}$

$= \frac{5}{6}$

b.)

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

② Correct answer

① Progress beyond part (a) such as finding $P(win|H) = \frac{3}{36}$

$P(win) = \frac{1}{2}\left(\frac{30}{36}\right) + \frac{1}{2}\left(\frac{3}{36}\right)$

$= \frac{11}{24}$

⓪ Insufficient progress

c.) $P(H|4) = \frac{P(H \cap 4)}{P(4)}$

$= \frac{\frac{1}{2}\left(\frac{3}{36}\right)}{\frac{1}{2}\left(\frac{4}{36}\right)+\frac{1}{2}\left(\frac{3}{36}\right)}$

$= \frac{3}{7}$

② Correct answer

① Finds total number of ways to get 4

OR

Finds how many ways to get 4 after heads

[illegible] how many [illegible]
to get 4 after heads

(0) Insufficient progress

32.) a.) $V = \frac{x}{2}\left(1+\frac{2x}{3}\right)y$ (eq. 1)

$2x + 2y = 27$

$y = \frac{27}{2} - x$ (eq. 2)

sub. eq.2 into eq. 1:

$V = \frac{x}{2}\left(1+\frac{2x}{3}\right)\left(\frac{27}{2} - x\right)$

$= \frac{x}{2}\left(\frac{27}{2} - x + 9x - \frac{2x^2}{3}\right)$

$= \frac{x}{2}\left(\frac{27}{2} + 8x - \frac{2x^2}{3}\right)$

$\therefore V(x) = -\frac{x^3}{3} + 4x^2 + \frac{27}{4}x$

(1) Correctly substitutes eq.2 into eq. 1 or equivalent

(1) Sufficiently shows result

b.) $V'(x) = -x^2 + 8x + \frac{27}{4}$

$0 = x^2 - 8x - \frac{27}{4}$

$x = \frac{8 \pm \sqrt{64 - 4(1)(-\frac{27}{4})}}{2}$

$= \frac{8 \pm \sqrt{91}}{2}$

$\doteqdot 8.7697, -0.77$ (ignore negative length)

$V(8.7697) = \frac{(8.\cdots)^3}{3} + 4(8.\cdots)^2 + \frac{27}{4}(8.\cdots)$

$= 142\ m^3$ (3 s.f.)

(1) Correctly differentiates $V(x)$

(1) Correctly solves the quadratic

(1) Correctly uses original formula to find volume

c.) $V''(x) = -2x + 8$

$V''(8.7697) \doteqdot -9.5$

Since the first derivative is zero at that point its gradient is zero. Since the second derivative is negative, it is concave down. Therefore it must be a local maximum.

The domain is $0 < x < 13.5$ for the pool due to the perimeter of 27.

(1) Correctly finds the sign of the second derivative at the point

(1) Sufficient reasoning given to at least show it's a local maximum

The domain is $0 < x < 13.5$ for the pool due to the perimeter of 27. Both $V(0)$ and $V(13.5)$ equal zero.

show it's a local maximum

33.)

a.) $L(t) = 14, \quad t = -10$

$$14 = 12 + k\cos\left[\frac{2\pi}{365}(-10+10)\right]$$
$$= 12 + k$$
$$\therefore k = 2 \quad \text{(as req.)}$$

① Substitutes in both values, then shows that $k=2$

b.) $L'(t) = -\frac{4\pi}{365}\sin\left[\frac{2\pi}{365}(t+10)\right]$

$$L''(t) = -\frac{8\pi^2}{365}\cos\left[\frac{2\pi}{365}(t+10)\right]$$

let $L''(t) = 0$

$$0 = \cos\left[\frac{2\pi}{365}(t+10)\right]$$
$$\frac{2\pi}{365}(t+10) = \frac{\pi}{2}, \frac{3\pi}{2}$$
$$t+10 = 91.25, 273.75$$
$$t = 81.25, 263.75$$
$$L'(81.25) \doteqdot -0.03, \quad L'(263.75) \doteqdot 0.03$$

$\therefore$ Daylight hours are increasing the fastest 264 days into the year.

① Correct method used to find steepest gradient

① Correctly finds first OR second derivative

① Correct answer, clearly stating 264, not 81

c.) $$\int_{264}^{365}\left(12 + 2\cos\left[\frac{2\pi}{365}(t+10)\right]\right)dt$$
$$= \left[12t + \frac{365}{\pi}\sin\left[\frac{2\pi}{365}(t+10)\right]\right]_{264}^{365}$$
$$= 4399.90137 - 3051.817967$$
$$= 1348 \text{ hours (nearest hour)}$$

① Correct bounds of integral chosen

① Correct integration

① Correct answer