Student Number

Hunters Hill High School

2025 TRIAL EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Write your Student Number at the top of this page

Total marks: 100

Section I – 10 marks (pages 3–7)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 8–29)

- Attempt Questions 11–33
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1. Which of the following represents an inverse variation relationship?

(A) $y = \frac{4}{x}$

(B) $y = 4x$

(C) $y = 4^x$

(D) $y = -4x$

2. Liam's scores in various physical fitness tests are shown below. The results for each test were normally distributed among all participants.

Test	Liam's Score	Group Mean	Group Standard Deviation
Push-up (in 1 min)	48	35	5
Sit-up (in 1 min)	52	40	6
High Jump (cm)	62	50	8
Sprint Time (sec)	13	13	2

In which test did Liam perform best in comparison with the rest of the group?

(A) Push-up

(B) Sit-up

(C) High Jump

(D) Sprint Time

3. The parabola $y = (x-2)^2 + 3$ is reflected about the x-axis.
What is the equation of the resulting parabola?

(A) $y = (x+2)^2 - 3$

(B) $y = (x-2)^2 - 3$

(C) $y = -(x-2)^2 - 3$

(D) $y = -(x-2)^2 + 3$

4. Find the limiting sum of the series $2\pi, \pi, \frac{\pi}{2}, \ldots$

(A) $\frac{\pi}{2}$

(B) π

(C) 2π

(D) 4π

5. A function is given by $f(x) = \begin{cases} \frac{3x^2}{125}, & 0 \le x \le 5 \\ 0, & \text{elsewhere} \end{cases}$

If this function is a probability density function, what is the area under the curve?

(A) -1

(B) 0.51

(C) 1

(D) 2

6. What is the amplitude and period for the function $f(x) = 4\sin\left(\frac{x+\pi}{3}\right)$?

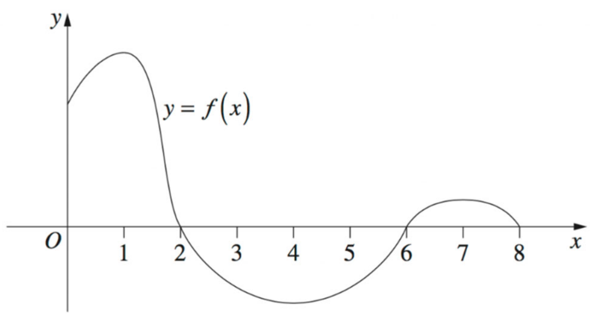
(A) Amplitude 3 and period $\frac{\pi}{2}$

(B) Amplitude 3 and period 6π

(C) Amplitude 4 and period $\frac{\pi}{2}$

(D) Amplitude 4 and period 6π

7. The graph $y = f(x)$ been drawn to scale for $0 \leq x \leq 8$



Which of the following integrals has the least value?

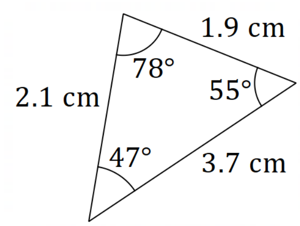
(A) $\int_0^1 f(x)\,dx$

(B) $\int_2^6 f(x)\,dx$

(C) $\int_6^8 f(x)\,dx$

(D) $\int_2^4 f(x)\,dx$

8. A triangle has side lengths 2.1 cm, 3.7 cm, and 1.9 cm and has angles opposite each of these sides of 55°, 78°, and 47° respectively.



NOT TO SCALE

Which of the following would give the area of this triangle?

(A) $\frac{1}{2} \times 2.1 \times 3.7 \times \sin 78°$

(B) $\frac{1}{2} \times 2.1 \times 1.9 \times \sin 78°$

(C) $\frac{1}{2} \times 1.9 \times 3.7$

(D) $\frac{1}{2} \times 2.1 \times 3.7$

9. Given $\log_a x = 2.139$ and $\log_a y = 8.74$, which of the following is closest to the value of $\log_a x^2y$?

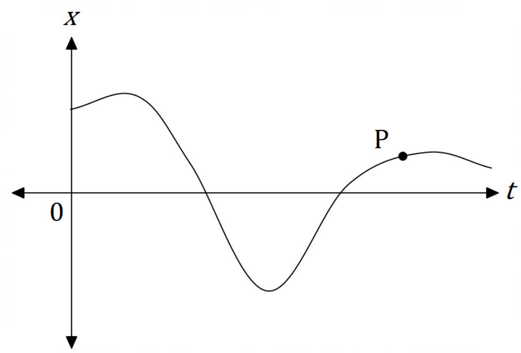
(A) 13.315

(B) 21.758

(C) 39.998

(D) 13.018

10. The graph below shows the displacement of a particle from the origin along the x-axis over time.



Which of the following best describes the motion of the particle at point P?

(A) Displacement is decreasing at an increasing rate

(B) Displacement is decreasing at a decreasing rate

(C) Displacement is increasing at an increasing rate

(D) Displacement is increasing at a decreasing rate

End of Section I

Section II

90 marks
Attempt Questions 11–33
Allow about 2 hours and 45 minutes for this section

Write each response in the spaces provided. Extra writing space is provided at the back of this paper.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Marks

Question 11 (2 marks)

For which values of k does the equation $x^2 + (k+2)x + 4 = 0$ have one solution? **2**

Question 12 (2 marks)

Bag A contains 2 white and 3 black balls. Bag B contains 3 white and 2 black balls.
If one ball is drawn from each bag, what is the probability that they are different colours? **2**

Marks

Question 13 (3 marks)

Find the equation of the normal to the curve $y = x^2 - 9x$ at the point (1, –8). 3

Question 14 (3 marks)

Consider the functions $f(x) = \dfrac{1}{x} + 3$ and $g(x) = x - 2$. 3

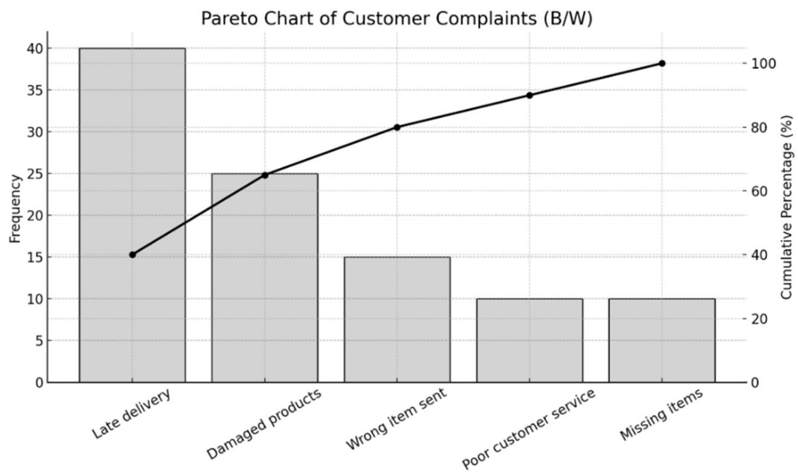
Let $h(x) = f\left(g(x)\right)$.

Find the domain and range of $h(x)$.

Marks

Question 15 (2 marks)

A company received customer complaints over the past month. The results are shown in the Pareto chart below.



a) Approximately what percentage of issues were due to late delivery or damaged products? 1

b) Approximately what percentage of complaints were due to poor customer service? 1

Marks

Question 16 (4 marks)

The tenth term of an arithmetic series is 29 and the fifteenth term is 44.

a) Find the value of the common difference and the value of the first term. 2

b) Find the sum of the first 75 terms. 2

Marks

Question 17 (2 marks)

Differentiate $f(x) = \dfrac{x^2 + 4x}{x + 2}$ 2

Question 18 (3 marks)

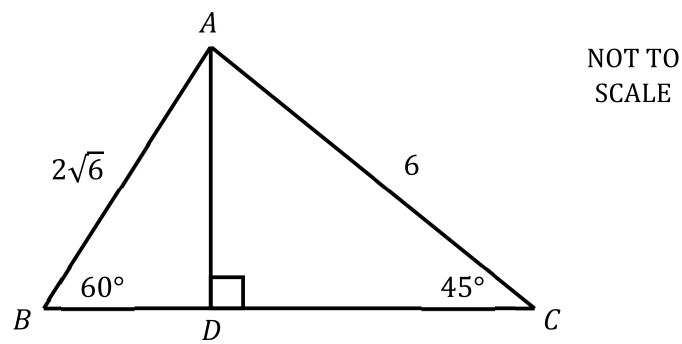
Differentiate $f(x) = e^{x \sin 3x}$ 3

Marks

Question 19 (4 marks)

In the diagram, ABC is a triangle in which $AB = 2\sqrt{6}$, $AC = 6$, $\angle ABC = 60^o$ and $\angle ACB = 45^o$.

AD is the perpendicular from A to BC.



a) Show that $BC = \sqrt{6} + 3\sqrt{2}$. **2**

b) Hence use the sine rule to show that $\sin 75° = \dfrac{\sqrt{6}+\sqrt{2}}{4}$. **2**

Marks

Question 20 (9 marks)

Data was gathered from eight students on the average number of hours, h, of sleep they get on school nights, and the number of times they got distracted in class, d.

The data is displayed in the table below.

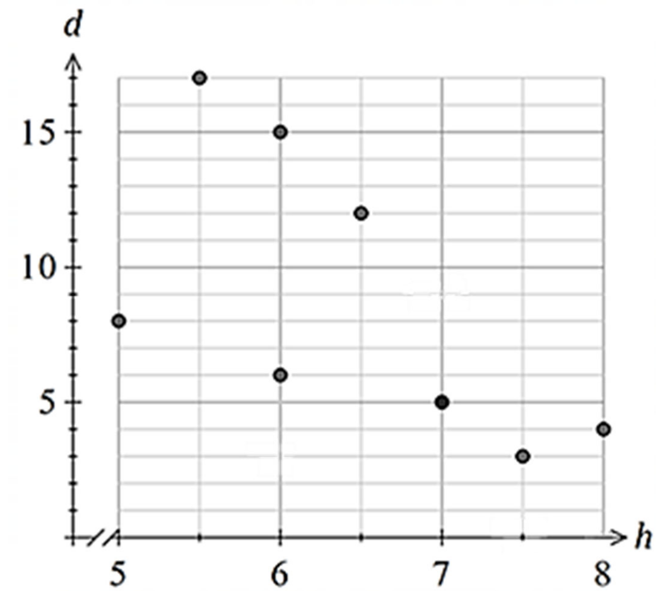
$\boldsymbol{h}$	7.5	8	7	6	5	6.5	6	5.5
$\boldsymbol{d}$	3	4	5	6	8	12	15	17

a) Use a calculator to find Pearson's correlation coefficient, r, to two decimal places and describe the strength and direction of the relationship. **3**

b) Use a calculator to find the equation of the least-squares line of best fit. **1**

Question 20 continued **Marks**

c) The data is plotted on the scatterplot below. 2



Sketch the graph of the least-squares regression line on the scatterplot.

d) Use the equation of the least-squares regression line to predict the average hours of sleep on school nights for a student who has 20 distractions in class. 2

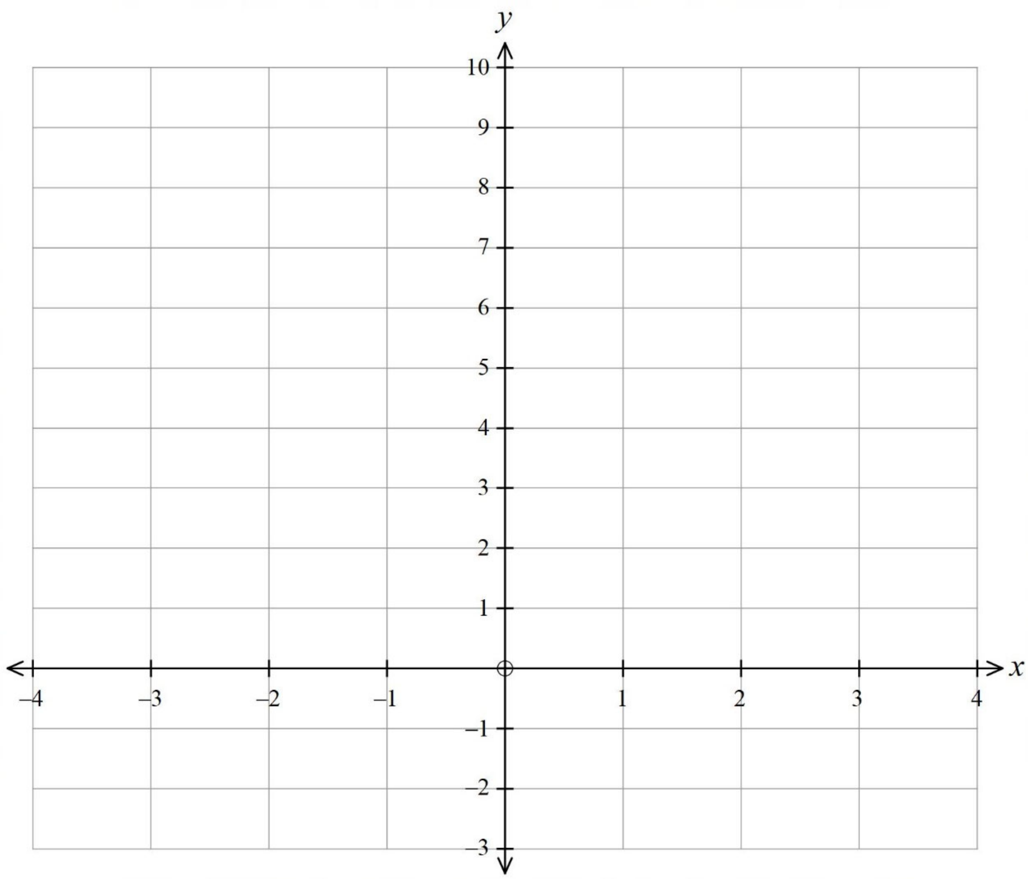
e) Classify the prediction in part (d) as interpolation or extrapolation. Give reasoning for your answer. 1

Marks

Question 21 (3 marks)

a) Show that $\dfrac{4x-3}{x} = -\dfrac{3}{x} + 4$ **1**

b) Hence or otherwise, sketch the graph of $y = \dfrac{4x-3}{x}$ showing any asymptotes and the x-intercept. **2**



Marks

Question 22 (4 marks)

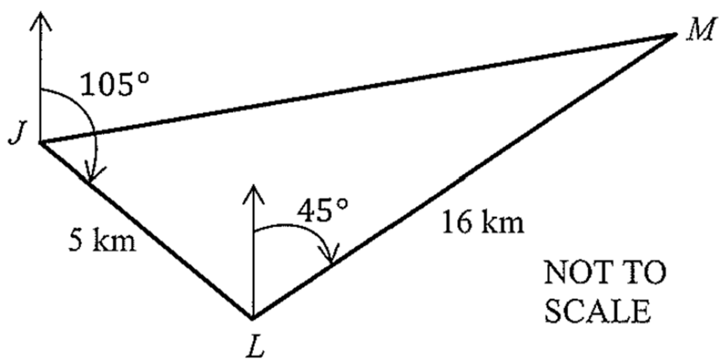
a) Show that the derivative of $\ln\left(\dfrac{3+x}{3-x}\right)$ is $\dfrac{6}{9-x^2}$. **2**

b) Hence, or otherwise, find $\displaystyle\int \frac{1}{9-x^2}\,dx$. **2**

Marks

Question 23 (4 marks)

Sally is training for the swim leg of a triathlon and decides to go for a training swim in the harbour. She starts at a jetty J and swims on a bearing of $105°$ for 5 km out to a buoy L. She then changes direction and swims on a bearing of $045°$ for a further 16 km to buoy M.



a) Find $\angle JLM$. 1

b) Find the distance JM that Sally will swim back to the jetty. 2

c) Find the bearing of jetty J from the buoy L. 1

Marks

Question 24 (9 marks)

Consider the curve $y = x^3 - 12x^2 + 36x$.

a) Find the x and y intercepts. 2

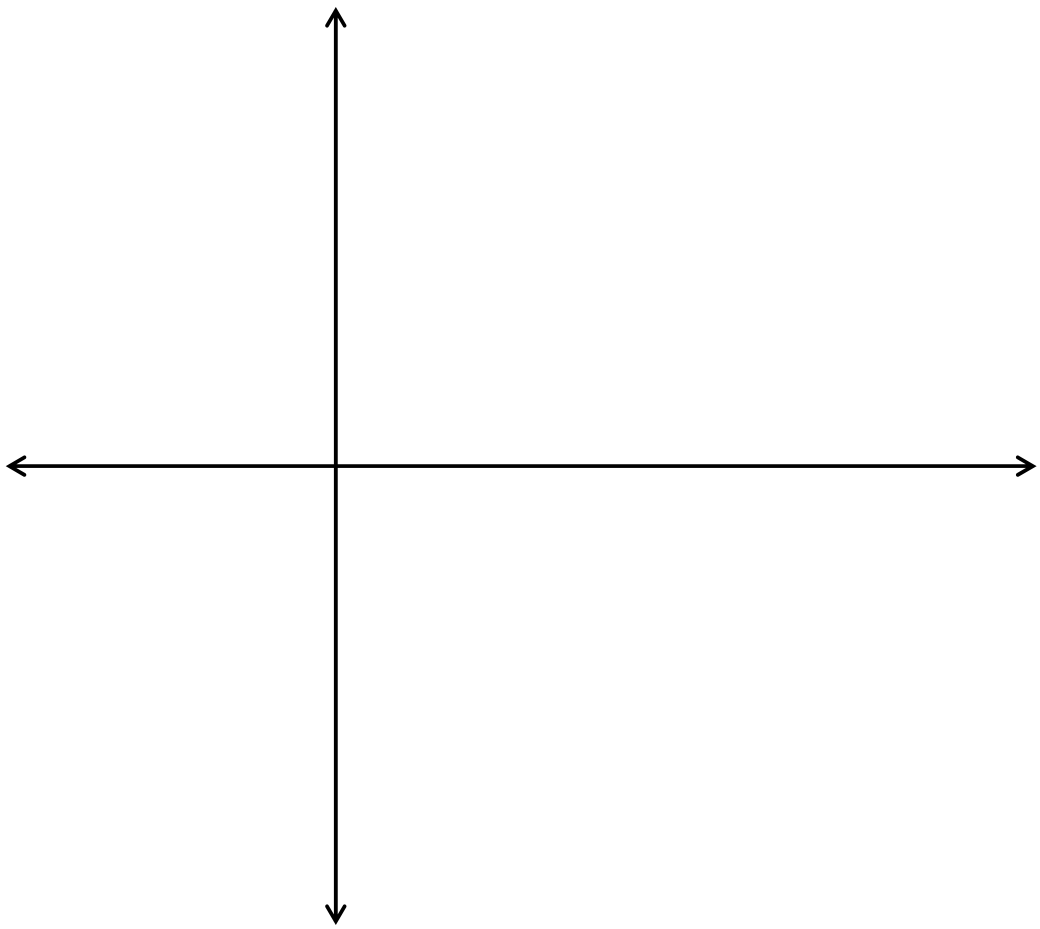
b) Find the location and nature of any stationary points. 3

c) Find the point of inflection. 2

Question 24 continued **Marks**

d) Sketch the curve $y = x^3 - 12x^2 + 36x$ for $-1 \leq x \leq 7$. **2**

Show all essential features.



Marks

Question 25 (4 marks)

A discrete random variable, X, has a probability distribution as follows. 4

x	0	0.5	1	1.5	2	2.5
$P(X = x)$	0.05	0.1	0.15	a	0.2	b

Given that $E(X) = 1.75$, find the values of a and b.

Marks

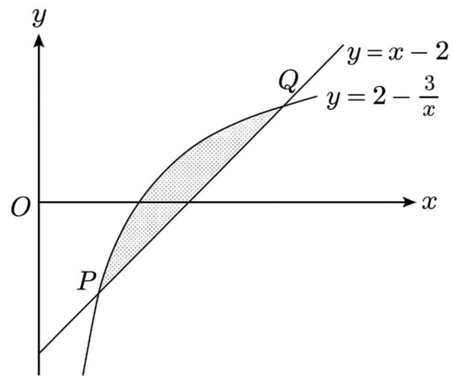
Question 26 (3 marks)

Evaluate $256 + 128 + 64 + \cdots + \frac{1}{8}$. 3

Marks

Question 27 (4 marks)

The diagram shows the graphs $y = 2 - \dfrac{3}{x}$ and $y = x - 2$ for $x \geq 0$.



a) Find the x-coordinates of the two points P and Q where the two graphs intersect. **2**

b) Hence, find in simplest exact form the area of the shaded region contained between. **2**

Marks

Question 28 (2 marks)

The completion time for the Oztown triathlon race was normally distributed with a mean time of 60 minutes and standard deviation of 5 minutes. 2

Find Dan's completion time if he finished ahead of 84% of competitors.

Question 29 (3 marks)

Prove that $\sin x + 1 + \cos x \cot x - \operatorname{cosec} x = 1$ 3

Marks

Question 30 (3 marks)

The function $f(x) = \ln(x+1)$ is continuous on the interval $[0, 4]$. **3**

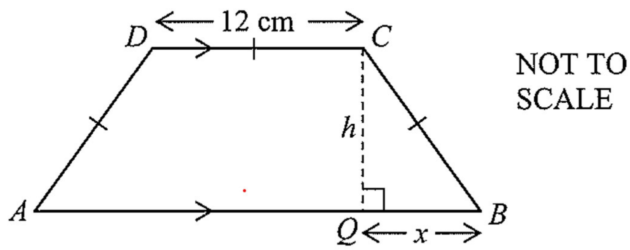
Use the Trapezoidal rule with 4 subintervals to estimate the value of:

$$\int_0^4 \ln(x+1)\ dx$$

Marks

Question 31 (5 marks)

$ABCD$ is an isosceles trapezium, with three equal sides, $AD = DC = CB = 12$ cm, as shown in the diagam below. The length of $BQ = x$.



a) Show that the area of $ABCD$ is given by $A = (12 + x)\sqrt{144 - x^2}$ cm^2. 2

b) Hence find the value of x when the area of the trapezium is a maximum. 3

Marks

Question 32 (5 marks)

The continuous random variable X has probability density function

$$f(x) = \frac{1}{2}\sin x \text{ for } 0 \le x \le \pi.$$

a) Find the cumulative distribution function. **3**

b) Find the first quartile of the distribution. **2**

Marks

Question 33 (7 marks)

The population of a certain bacteria culture grows exponentially. At 12:00 pm, the population is 1000 bacteria. After 3 hours, the population reaches 2500 bacteria.

a) The exponential function modelling the bacteria culture growth is of the form **2**

$$P(t) = P_0 a^t$$

where $P(t)$ is the population and t is the time in hours since 12:00 pm.

P_0 and a are constants.

Find the value of P_0 and a.

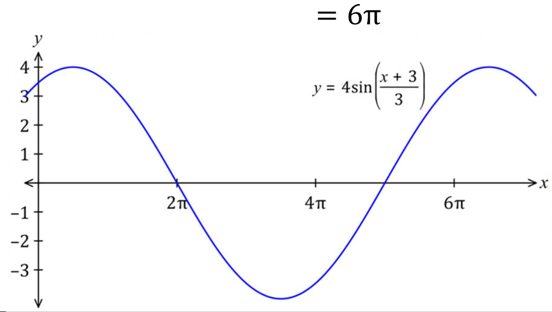
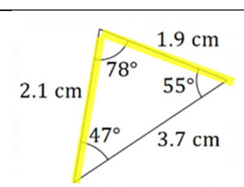
b) Find the average rate of change of the population between $t = 1$ hours and $t = 4$ hours. Include units in your answer. **1**

c) Find the instantaneous rate of change of the population at $t = 2$ hours. **2**

Question 33 continued **Marks**

d) Is the average rate of change constant over time in this scenario? Explain why or why not, based on the nature of exponential growth. **2**

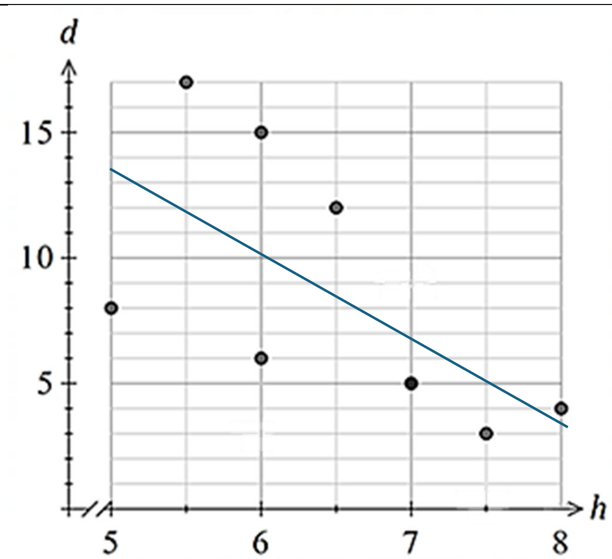
End of Examination

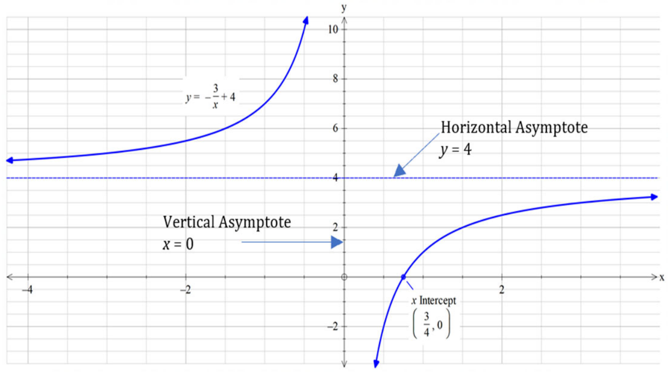
2025 Mathematics Advanced – Trial Solutions		
Section I		
1	Inverse variation is of the form $y = \frac{k}{x}$, where k is the constant of variation.	A
2	1. Push-ups $z = \frac{48-35}{5} = \frac{13}{5} = 2.6$ 2. Sit-ups $z = \frac{52-40}{6} = \frac{12}{6} = 2.0$ 3. Jump Height $z = \frac{62-50}{8} = \frac{12}{8} = 1.5$ 4. Sprint Time (lower is better) $z = \frac{13-13}{2} = \frac{0}{2} = 0$	A
3	$y = -(x-2)^2 - 3$	C
4	This is a geometric series with common ratio $r = \frac{1}{2}$ and first term $a = 2\pi$. $S = \frac{2\pi}{1 - \frac{1}{2}}$ $= 4\pi$	D
5	1	C
6	$\text{Amplitude} = 4$ $\text{Period} = \frac{2\pi}{\frac{1}{3}}$ $= 6\pi$ 	D
7	Since the integral measures the net area under a graph and above the x-axis (below the x-axis is a negative value.	B
8	we use the formula $A = \frac{1}{2}ab\sin C$. The angle C must be between the sides a and b. The expression in option B fits this configuration. 	B
9	$\log_a x^2 y = \log_a x + \log_a x + \log_a y$ $= 2(2.139) + 8.74$ $= 13.018$	D

10	At point P, the function is increasing (tangent is positive), so its displacement is increasing. At point P, the function is concave down, so its rate is decreasing.	D

Section II		
11	$\Delta = b^2 - 4ac$ $= (k+2)^2 - 4 \times 1 \times 4$ $= k^2 + 4k - 12$ Equal roots $\Delta = 0$ $k^2 + 4k - 12 = 0$ $(k+6)(k-2) = 0$ $\therefore k = -6$ or $k = 2$	2: Correct solution with working 1: uses the discriminant correctly
12	$P(E) = P(WB) + P(BW)$ $= \frac{2}{5} \times \frac{2}{5} + \frac{3}{5} \times \frac{3}{5}$ $= \frac{13}{25}$	2: correct solution with working 1: for one combination
13	$\frac{dy}{dx} = 2x - 9$ At $(1, -8)$ $\quad m_T = \frac{dy}{dx}$ $= 2 \times 1 - 9$ $= -7$ $m_N = \frac{1}{7}$ By point gradient formula $y - (-8) = \frac{1}{7}(x - 1)$ $y = \frac{x}{7} - 8\frac{1}{7}$	3: Correct equation with working 2: finding the gradient of the normal 1: derivative function
14	$h(x) = \frac{1}{x-2} + 3$ Graph of $y = h(x)$ is a hyperbola with vertical asymptote at $x = 2$ and horizontal asymptote at $y = 3$ $\therefore$ *Domain is* $(-\infty, 2) \cup (2, \infty)$*and* *range is* $(-\infty, 3) \cup (3, \infty)$	3: correct solution with working 2: Correct function for $h(x)$ and either correct domain or range 1: correct function for $h(x)$, 1: correct domain 1: range without function given.
15 a.	Approximately 65%	1: correct answer
b.	Approximately 10%	1: correct answer

16	a.	$T_{10}: \quad a + 9d = 29$ ① $T_{15}: a + 14d = 44$ ② ② − ① $5d = 15$ $d = 3$ Substitute ① $a + 27 = 29$ $a = 2$ $\therefore a = 2, d = 3$	2: Correct answer with working 1: Finds the two equations 1: Shows some understanding.
	b.	Given $n = 75, a = 2, d = 3$ $S_n = \frac{n}{2}[2a + (n-1)d]$ $S_{75} = \frac{75}{2}(2 \times 2 + (75-1) \times 3)$ $= 8475$ $\therefore$ Sum of the first 75 terms is 8475.	2: Correct answer with working. 1: Uses the formula for the sum of n terms with one correct value.
17		let $u = x^2 + 4x \qquad v = x + 2$ $u' = 2x + 4 \qquad v' = 1$ $f'(x) = \frac{(2x+4)(x+2) - (x^2+4x)}{(x+2)^2}$	2: correct solution with working 1: correctly differentiates u or v
18		$\frac{d}{dx}\left(e^{x \sin 3x}\right) = e^{x \sin 3x} \times \frac{d}{dx}(x \sin 3x)$ Finding $\frac{d}{dx}(x \sin 3x)$ let $u = x \qquad v = \sin 3x$ $u' = 1 \qquad v' = 3 \cos 3x$ so $\frac{d}{dx}(x \sin 3x) = \sin 3x\,(1) + x.3 \cos 3x$ hence $\frac{d}{dx}\left(e^{x \sin 3x}\right) = e^{x \sin 3x} \times (\sin 3x + 3x \cos 3x)$	3: correct solution with working 2: applies product rule 1: determines derivatives of v and u
19	a.	$BC = BD + DC$ $= 2\sqrt{6} \cos 60° + 6 \cos 45°$ $= 2\sqrt{6} \times \frac{1}{2} + 6 \times \frac{1}{\sqrt{2}}$ $= \sqrt{6} + 6 \times \frac{\sqrt{2}}{2}$ $= \sqrt{6} + 3\sqrt{2}$	2: correct solution with working 1: Finds exact value of BD or DC

	b.	$\text{In } \Delta ABC, \angle BAC = 180° - 60° - 45°$ $= 75°$ $\dfrac{\sin 75°}{\sqrt{6}+3\sqrt{2}} = \dfrac{\sin 60°}{6}$ (sine rule) $\sin 75° = \dfrac{\sqrt{3}}{12}(\sqrt{6}+3\sqrt{2})$ $= \dfrac{3\sqrt{2}+3\sqrt{6}}{12}$ $= \dfrac{3(\sqrt{2}+\sqrt{6})}{12}$ $= \dfrac{\sqrt{6}+\sqrt{2}}{4}$	2: correct solution with working 1: applies sine rule and substitutes values into formula
20	a.	$r = -0.629\ldots$ ≈ -0.63 Moderate, negative relationship	3: correct answers 2: correct correlation coefficient and either correct strength or direction 1: correct correlation coefficient
	b.	Equation: $d = -3.273h + 29.818$	1: correct equation
	c.		2: correct graph 1: one error in graph 1: line passes through one correct coordinate
	d.	When $d = 20$: $20 = -3.273h + 29.818$ $-9.818 = -3.273h$ $h = 3$	2: correct solution with working 1: correct substitution into the rule 1: correct answer without showing substitution
	e.	It is extrapolation because the data gathered only included distractions from 0 to 17 and 20 is outside this range.	1: correct answer with reason given
21	a.	$\dfrac{4x-3}{x} = \dfrac{4x}{x} - \dfrac{3}{x}$ $= 4 - \dfrac{3}{x}$ $= -\dfrac{3}{x} + 4$	1: correct solution

	b.		2: for diagram with correct shape, asymptotes and x-intercept 1: shape, asymptotes or x-intercept are incorrect
22	a.	$$\begin{aligned}\frac{d}{dx}\ln\left(\frac{3+x}{3-x}\right) &= \frac{d}{dx}(\ln(x+3) - \ln(3-x)) \\ &= \frac{1}{x+3} - \frac{-1}{3-x} \\ &= \frac{(3-x)-(-1)(x+3)}{(x+3)(3-x)} \\ &= \frac{3-x+x+3}{(3+x)(3-x)} \\ &= \frac{6}{9-x^2}\end{aligned}$$	2: correct solution 1: - correct use of log laws - correct derivative of a log - uses $\frac{d}{dx}\ln(f(x)) = \frac{f'(x)}{f(x)}$ - equivalent merit
	b.	$$\begin{aligned}\int \frac{1}{9-x^2}\,dx &= \frac{1}{6}\int \frac{6}{9-x^2}\,dx \\ &= \frac{1}{6}\ln\left(\frac{3+x}{3-x}\right) + c\end{aligned}$$	2: for correct solution 1: takes out the common factor to get part (a) answer and +C
23	a.	$$\begin{aligned}\angle JLM &= 75 + 45 \\ &= 120^o\end{aligned}$$	1: correct answer
	b.	$$JM^2 = 5^2 + 16^2 - 2 \times 5 \times 16 \times \cos 120^o$$ $JM = 19\ km$	2: correct solution with working 1: Applies the cos formula.
	c.	$$\begin{aligned}\text{Bearing} &= 360 - 75 \\ &= 285°\end{aligned}$$	1: correct answer
24	a.	y intercept ($x = 0$) is 0 x intercept ($y = 0$) $$\begin{aligned}y &= x^3 - 12x^2 + 36x \\ 0 &= x(x^2 - 12x + 36) \\ x(x-6)^2 &= 0\end{aligned}$$ $\therefore x = 0$ or $x = 6$ (x intercepts are 0 and 6)	2: Correct answer. 1: Finds the x or y intercepts
	b.	$$\begin{aligned}y &= x^3 - 12x^2 + 36x \\ y' &= 3x^2 - 24x + 36 \\ y'' &= 6x - 24\end{aligned}$$ Stationary points occur when $y' = 0$ $$\begin{aligned}3x^2 - 24x + 36 &= 0 \\ x^2 - 8x + 12 &= 0 \\ (x-2)(x-6) &= 0 \\ \therefore\ x = 2 &\text{ or } x = 6\end{aligned}$$	3: Correct answer. 2: Finds the location of the two stationary points 1: Finds $\frac{dy}{dx}$ and equates it to zero.

<table>
<tr>
<td></td>
<td>For $x = 2$
$$\begin{aligned} y &= 2^3 - 12(2)^2 + 36(2) \\ &= 32 \\ y'' &= 6(2) - 24 \\ &= -12 < 0 \rightarrow \text{conc down} \end{aligned}$$
For $x = 6$
$$\begin{aligned} y &= 6^3 - 12(6)^2 + 36(6) \\ &= 0 \\ y'' &= 6(6) - 24 \\ &= 12 > 0 \rightarrow \text{conc up} \end{aligned}$$
$\therefore$ Stationary points at $(2, 32)$ and $(6, 0)$ are a local maximum and a local minimum, respectively.</td>
<td></td>
</tr>
<tr>
<td>c.</td>
<td>Possible inflection when $y'' = 0$
$$\begin{aligned} 6x - 24 &= 0 \\ x &= 4 \end{aligned}$$
<table><tr><td>x</td><td>3</td><td>4</td><td>5</td></tr><tr><td>y''</td><td>-4
< 0</td><td>0</td><td>6
> 0</td></tr></table>
When $x = 4$, $y = 16$
As concavity changes across the point, (4, 16) is a point of inflection</td>
<td>2: proves point of inflection

1: correct point obtained</td>
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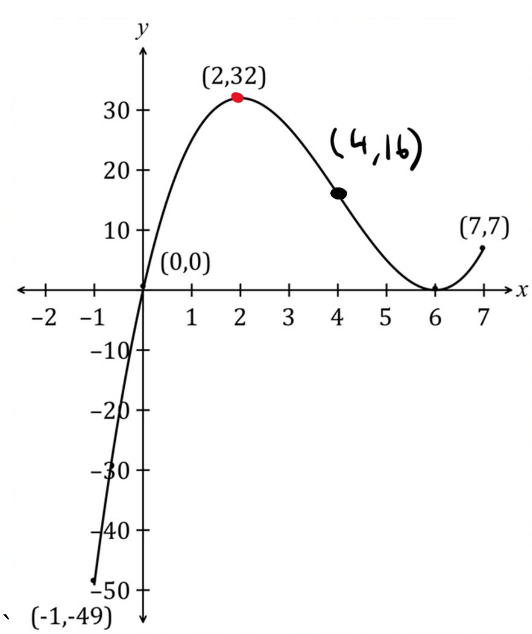
<tr>
<td>d.</td>
<td>
</td>
<td>2: Correct answer including end points.

1: Basic shape of the curve or the end values or shows some understanding.</td>
</tr>
</table>

25	The sum of probabilities is 1 $$0.05+0.1+0.15+a+0.2+b=1$$ $$0.5+a+b=1$$ $$a+b=0.5 \quad (1)$$ $E(X)=1.75$: $$0(0.05)+0.5(0.1)+1(0.15)+1.5a+2(0.2)+2.5b=1.75$$ $$0.6+1.5a+2.5b=1.75$$ $$1.5a+2.5b=1.15 \quad (2)$$ Solve (1) and (2) simultaneously: (1)$\times$ 2.5 $\rightarrow$ $2.5a+2.5b=1.25$ (3) (3)$-$(2): $$a=0.1$$ Substitute (1): $$0.1+b=0.5$$ $$b=0.4$$ $$\therefore\ a=0.1, b=0.4$$	4: Provides the correct solution set 3: Finds correct simultaneous equations and attempts to solve them 2: Finds correct simultaneous equations 1: Finds a correct equation based on the sum of the probabilities or the expected values
26	$$a=256, r=\frac{1}{2}, T_n=\frac{1}{8}$$ $$256\left(\frac{1}{2}\right)^{n-1}=\frac{1}{8}$$ $$\left(\frac{1}{2}\right)^{n-1}=\frac{1}{2048}$$ $$n=\frac{\ln\frac{1}{2048}}{\ln\frac{1}{2}}+1$$ $$=12$$ $$S_{12}=\frac{256\left(1-\left(\frac{1}{2}\right)^{12}\right)}{1-\frac{1}{2}}$$ $$=511.875$$	3: correct solution with working 2: finds the number of terms 1: finds first term and common ratio
27 a.	$$y=2-\frac{3}{x}$$ $$y=x-2$$ $$x-2=2-\frac{3}{x}$$ $$x^2-4x+3=0$$ $$(x-3)(x-1)=0$$ $$\therefore x=1 \text{ at } P$$ $$\text{and } x=3 \text{ at } Q$$	2: correct solution with working 1: correct procedure with one error.

<table>
<tr>
<td>b.</td>
<td>$$\int_1^3 \left(2-\frac{3}{x}\right)dx-\int_1^3 (x-2)dx$$
$$=\int_1^3 \left(\left(2-\frac{3}{x}\right)-(x-2)\right)dx$$
$$=\int_1^3 \left(4-\frac{3}{x}-x\right)dx$$
$$=\left[4x-3\ln x-\frac{x^2}{2}\right]_1^3$$
$$=4(3)-3\ln 3-\frac{3^2}{2}-\left(4(1)-3\ln 1-\frac{1^2}{2}\right)$$
$$=4-3\ln 3 \text{ units}^2$$</td>
<td>2: correct solution with working

1: finds correct integral</td>
</tr>
<tr>
<td>28</td>
<td>Using the empirical rule for a normal distribution
$$50\ \% + \frac{1}{2}\times 68\ \% = 84\%$$
of the times are greater than $\mu-\sigma$.
Dan's completion time is $60-5=55$ minutes.</td>
<td>2: correct answer by using the empirical rule

1: relates the 84% to an area under the curve.</td>
</tr>
<tr>
<td>29</td>
<td>$$\begin{aligned}\text{LHS} &= \sin x+1+\cos x\cot x-\operatorname{cosec} x\\ &= \sin x+1+\cos x\times\frac{\cos x}{\sin x}-\frac{1}{\sin x}\\ &= \frac{\sin^2 x+\sin x+\cos^2 x-1}{\sin x}\\ &= \frac{\sin^2 x+\cos^2 x+\sin x-1}{\sin x}\\ &= \frac{1+\sin x-1}{\sin x}\\ &= \frac{\sin x}{\sin x}\\ &= 1\\ &= \text{RHS}\end{aligned}$$</td>
<td>3: Correct answer.

2: Makes significant progress towards the solution.

1: Correctly uses one trig identity.</td>
</tr>
<tr>
<td>30</td>
<td><table>
<tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>$f(x)$</td><td>$\ln 1$
$=0$</td><td>$\ln 2$
≈ 0.6931</td><td>$\ln 3$
≈ 1.0986</td><td>$\ln 4$
≈ 1.3863</td><td>$\ln 5$
≈ 1.6094</td></tr>
</table>
$$\int_0^4 \ln(x+1)\,dx \approx \frac{1}{2}[\ln 1+\ln 5+2(\ln 2+\ln 3+\ln 4)]$$
$$\approx 3.9827$$</td>
<td>3: correct solution with working

2: uses correct formula

1: completes the table of $f(x)$ values correctly
- substituting with one error</td>
</tr>
<tr>
<td>31 a.</td>
<td>$$h=\sqrt{144-x^2}$$
$$AB=12+2x$$
$$A=\frac{1}{2}\times\sqrt{144-x^2}\times(12+12+2x)$$
$$=\frac{\sqrt{144-x^2}}{2}(24+2x)$$
$$=\sqrt{144-x^2}\times(12+x)$$
as required</td>
<td>2: correct working

1: shows h or AB Correctly</td>
</tr>
</table>

<table>
<tr><td></td><td>b.</td><td>$$\frac{dA}{dx} = \sqrt{144 - x^2} - \frac{x(12+x)}{\sqrt{144 - x^2}}$$
Stationary at $\frac{dA}{dx} = 0$
$$\sqrt{144 - x^2} = \frac{x(12+x)}{\sqrt{144 - x^2}}$$
$$144 - x^2 = x(12+x)$$
$$144 - x^2 = 12x + x^2$$
$$2x^2 + 12x - 144 = 0$$
$$x^2 + 6x - 72 = 0$$
$$(x+12)(x-6) = 0$$
$$\therefore x = -12, x = 6$$
Check concavity
<table><tr><td>x</td><td>5</td><td>6</td><td>7</td></tr><tr><td>$\frac{dA}{dx}$</td><td>3.2</td><td>0</td><td>-4.0</td></tr><tr><td>slope</td><td>> 0</td><td></td><td>< 0</td></tr></table>
So, the maximum turning point at $x = 6$
$\therefore$ trapezium will have a maximum area when $x = 6$cm</td><td>3: Correct solution with working

2: finds correct stationary points from correct derivative

1: progress to correct derivative function.</td></tr>
<tr><td>32</td><td>a.</td><td>$$\int_0^x \frac{1}{2}\sin t \; dt = -\frac{1}{2}\left[\cos t\right]_0^x$$
$$= -\frac{1}{2}(\cos x - 1)$$
The cumulative distribution function is
$F(x) = \frac{1}{2}(1 - \cos x), \qquad 0 \le x \le \pi$</td><td>3: correct solution with working

2: find integral
$$= -\frac{1}{2}(\cos x - 1)$$
1: integrates correctly</td></tr>
<tr><td></td><td>b.</td><td>$$F(x) = 0.25$$
$$\frac{1}{2}(1 - \cos x) = 0.25$$
$$(1 - \cos x) = 0.5$$
$$\cos x = \frac{1}{2}$$
First quartile is $\frac{\pi}{3}$</td><td>2: Correct solution with working

1: makes $F(x) = 0.25$</td></tr>
<tr><td>33</td><td>a.</td><td>$P(t) = 1000(a)^t$
Given $P(3) = 2500$
$$2500 = 1000(a)^3$$
$$a^3 = 2.5$$
$$a \approx 1.3572$$
So, the function is $P(t) = 1000(1.3572)^t$</td><td>2: Correct function with working

1: Correct substitution into the model</td></tr>
</table>

b.	$P(1) = 1000(1.3572)^1$ $P(4) = 1000(1.3572)^4$ $\text{Average rate } = \dfrac{3386 - 1357.2}{4 - 1}$ $\approx \dfrac{2028.8}{3}$ $\approx$ 676.27 bacteria/hour	1: Correct solution with working
c.	$P(t) = 1000(1.3572)^t$ $P'(t) = 1000(1.3572)^t \ln 13572$ $P'(2) = 1000(1.3572)^2 \ln 13572$ ≈ 560.1 Interpretation: At 2 hours, the population is increasing at approximately 560.1 bacteria/hour	2: Correct solution with working 1: Correct differentiation of function
d.	No, the average rate of change is not constant. In exponential growth, the rate increases over time because the population grows by a constant percentage, not a constant amount.	2: States the rate is not constant and the reason relates to exponential function 1: States the rate is not constant