Question 1 (12 Marks) Begin a SEPARATE sheet of paper

Marks

(a) Evaluate $\dfrac{e+1}{\pi}$, correct to three decimal places. **2**

(b) Find θ, to the nearest degree, if $\sin\theta = \dfrac{4\sin 57^0}{6.7}$ **2**

(c) What is the centre and radius of a circle with equation $(x+2)^2+(y-3)^2 = 2{\cdot}25$. **2**

(d) The mean of 3, 5, 7, x is 6·75. What is the value of x? **1**

(e) If $x = 2{\cdot}35$, evaluate the expression $|-3-4x|$ **1**

(f) Factorise $3x^2 - 5x - 2$. **1**

(g) Express 2·32 radians as an angle in degrees, correct to the nearest minute. **1**

(h) Write down the domain and range for $y=\sqrt{x^2-9}$. **2**

Question 2 (12 Marks) Begin a SEPARATE sheet of paper

Marks

(a) Differentiate the following:

(i) $e^{0 \cdot 5}$. **1**

(ii) $\log_e (x^2 - 3x)$. **1**

(iii) $\dfrac{2x-1}{\cos x}$ **1**

(b) Integrate: $\displaystyle\int \sin\frac{x}{2}\,dx$ **2**

(c) Evaluate: $\displaystyle\int_0^1 \frac{5}{6}e^{3x}\,dx$ (Leave your answer in exact form). **2**

(d) Given $\log_m p = 1 \cdot 75$ and $\log_m q = 2 \cdot 25$.

Find the following:

(i) $\log_m pq$ **1**

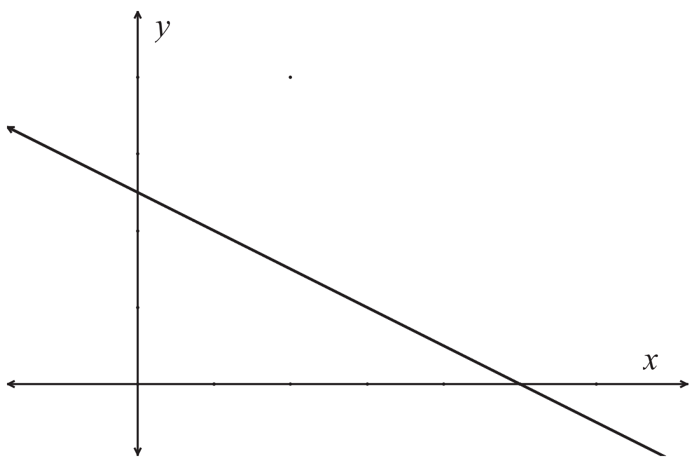
(ii) $\log_m \dfrac{q}{p}$ **1**

(iii) $\sqrt[5]{pq^2}$ in terms of m. **3**

Question 3 (12 Marks) Begin a SEPARATE sheet of paper

Marks

The number plane shows the line n with equation $x + 2y = 5$.
The point P is $(2, 4)$ and O is the origin.



(a) Find the coordinates of the midpoint M, of the interval OP. **1**

(b) Show that the point M lies on the line n. **2**

(c) Find the gradient of the line OP. **1**

(d) Show that the line n is the perpendicular bisector of the interval OP. **3**

(e) Line n meets the x-axis at Q. Find the co-ordinates of Q. **1**

(f) A line is drawn through O parallel to PQ and it meets line n in R. Find the co-ordinates of R. **2**

(g) What sort of quadrilateral is $PQOR$? Give reasons for your answer. **2**

Question 4 **(12 Marks) Begin a SEPARATE sheet of paper**

Marks

(a) Let m and n be positive whole numbers where $m > n$.

(i) Show that a triangle with sides $m^2 + n^2$, $m^2 - n^2$, $2mn$ obeys Pythagoras' Theorem. **2**

(ii) Which Pythagorean Triad is generated when $m = 3$ and $n = 2$? **1**

(b) Consider the function $f(x) = x - 2\log_e x$, for $x > 0$.

(i) Find the first **and** second derivative of $f(x)$. **2**

(ii) Find the co-ordinates of the turning point(s) and determine their nature. **2**

(iii) Show that there are no points of inflexion. **2**

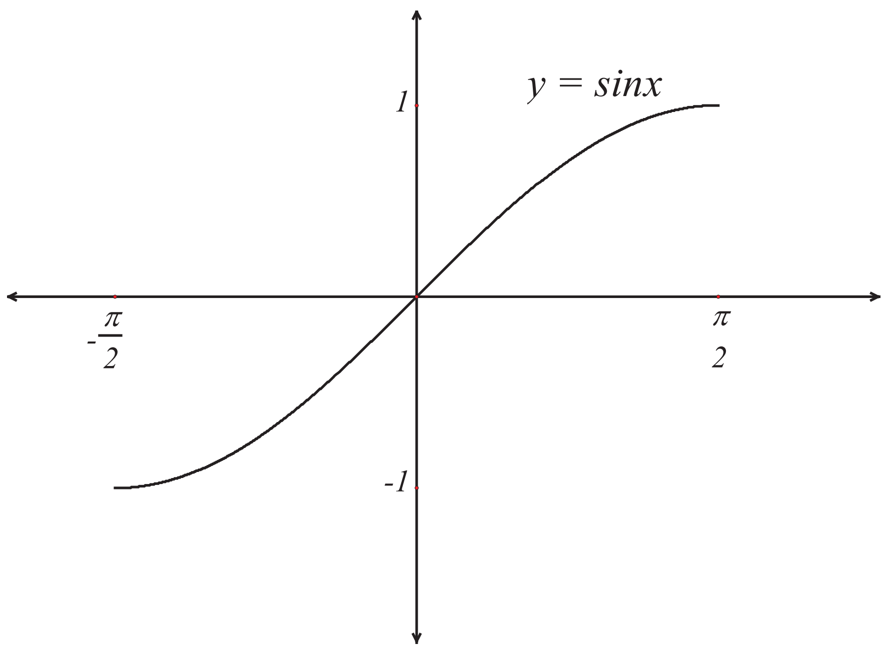
(iv) What is the maximum value of $x - 2\log_e x$ in the domain $1 \le x \le 5$? **1**

(v) Sketch the curve $y = x - 2\log_e x$, for $0 < x \le 5$. **2**

Question 5 (12 Marks) Begin a SEPARATE sheet of paper

Marks

(a)



The diagram above shows the graph of $y = \sin x$ for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$.

(i) Copy the diagram onto your exam paper. **2**

On the same set of axes, graph $y = \cos 2x$ for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$.

(ii) Show that the two graphs intersect where $x = \frac{\pi}{6}$. **1**

(iii) Calculate the exact area enclosed between the two curves. **3**

(b) Show that $\dfrac{\sec\theta - \sec\theta\cos^4\theta}{1+\cos^2\theta} = \sin\theta\tan\theta$. **3**

(c) (i) Find the value(s) of k for which $x^2 + (2 - k)x + 2{\cdot}25 = 0$ has equal roots . **2**

(ii) Find the value(s) of k for which $y = kx + 1$ is a tangent to $y = x^2 + 2x + 3{\cdot}25$. **1**

<u>Question 6</u> (12 Marks) Begin a SEPARATE sheet of paper

Marks

(a) Boat A is 15km from point P on a bearing of 055^0T.
Boat B is 25km from point P on a bearing of 135^0T.

(i) Draw a diagram showing the information above and find the angle APB. **2**

(ii) Calculate, to one decimal place, the distance that the two boats are apart. **2**

(b) Evaluate $\sum_{r=3}^{7} 2^r - 3r$. **2**

(c) Water enters an empty container where the rate of filling is $R = 6\left(e^{t} + e^{-t}\right)$ where $R = \dfrac{dV}{dt}$ in litres per minute. The volume (V) of water in the vessel is in litres and time (t) is in minutes.

(i) What is the initial rate of filling? **1**

(ii) Find V in terms of t. **2**

(iii) Show that, when the container holds 5 litres, then $6e^{2t} - 5e^{t} - 6 = 0$. **1**

(iv) Hence, find to the nearest second, the time taken for the volume to reach 5 litres. **2**

Question 7 (12 Marks) Begin a SEPARATE sheet of paper

Marks

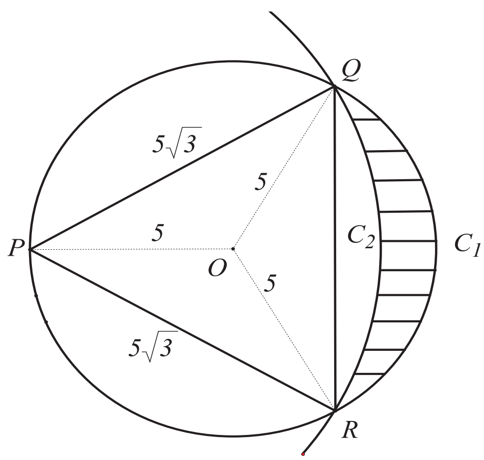
(a) Find y if $\dfrac{dy}{dx} = \dfrac{3\sec^2 x}{\tan x}$ and $y = 4$ when $x = \dfrac{\pi}{4}$. **3**

(b) *Twinkle Finance* offers its investors the opportunity to have interest credited to their investment "as often as you wish". Naturally, many investors decide for the "EVERY MINUTE" plan in which *Twinkle* offers 12% pa, compounded each minute.

(i) Tanya invests \$1000 for a year with *Twinkle* on the "EVERY MINUTE" plan. Theoretically, *Twinkle's* computers multiply Tanya's balance by approximately $1{\cdot}000\,000\,228$ every minute. Show why this is so. **2**

(ii) How much, to the nearest dollar, is Tanya's investment worth after 1 year? **1**

(c)



In the diagram above, an equilateral triangle PQR is inscribed in a circle C_1, with centre O, and radius 5 units. $\angle POQ = \angle QOR = \angle POR$.

An arc of another circle C_2, with centre P, and radius $5\sqrt{3}$ units, intersects C_1 at the points Q and R, as shown in the diagram.

(i) Show $\angle QPO = \dfrac{\pi}{6}$, giving reasons. **1**

(ii) Find the area of the sector QPR, of the circle C_2. **1**

(iii) Find the area of the sector QOR, of the circle C_1. **1**

(iv) Hence, or otherwise, show that the area of the shaded region is $25\left(\dfrac{\sqrt{3}}{2} - \dfrac{\pi}{6}\right)$ units^2. **3**

Question 8 (12 Marks) Begin a SEPARATE sheet of paper

Marks

(a) Let A be the point $(-2, 0)$ and B be the point $(6, 0)$.
At $P\ (x, y)$, PA meets PB at right angles.

(i) Show that the gradient of PA is $\dfrac{y}{x+2}$. **1**

(ii) Find the equation of the locus of P **and** give a geometric description of the locus. **3**

(b) The velocity of an object is given by the equation $v = 6t - 8 - t^2$, where time (t) is in seconds and velocity (v) is in metres/second. Initially, the object is 5 metres to the right of the origin.

(i) Find an equation for the displacement of the object. **2**

(ii) Find when the object is momentarily at rest. **1**

(iii) Find the distance travelled by the object in the first four seconds. **2**

(c) Two dice are biased so that, the probability of throwing a six face up is $\dfrac{3}{8}$ and the probability of throwing any other number is $\dfrac{1}{8}$.

Find the probability of:

(i) Rolling a double six face up. **1**

(ii) Rolling the dice so that neither is a six face up. **1**

(iii) Only one six appearing face up when the two dice are rolled. **1**

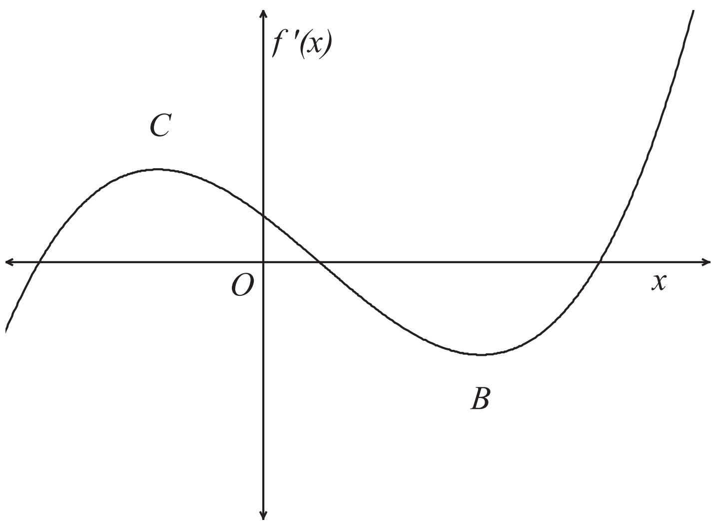
Question 9 (12 Marks) Begin a SEPARATE sheet of paper

Marks

(a) (i) Write $x^2 - 6x + 8 = 2y$ in the form $(x-h)^2 = 4a(y-k)$. **2**

(ii) Hence, state the co-ordinates of focus for the parabola. **1**

(b)



The graph of $y = f'(x)$ is shown above. The zeros of $f'(x)$ are $x = -2$, $0{\cdot}5$, and 3.
Point C has x co-ordinate of $-0{\cdot}9$ and point B has x co-ordinate of $1{\cdot}9$.

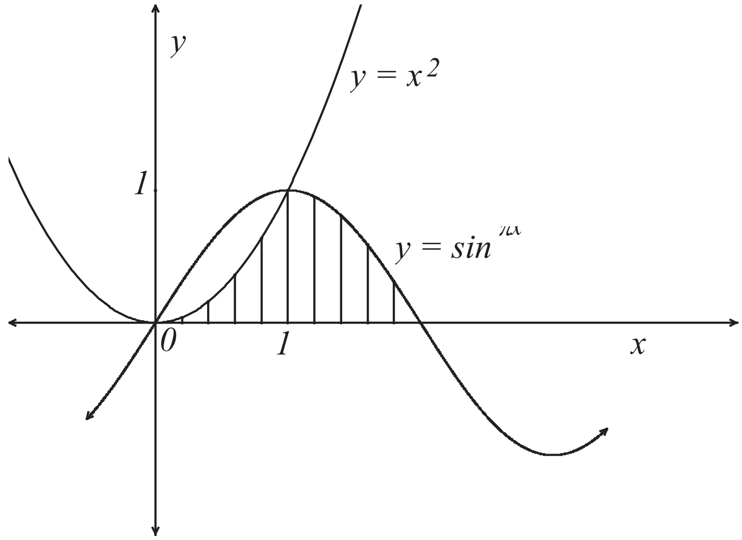
(i) For what range of values of x is $f(x)$ increasing? **1**

(ii) Point C is the local maximum of $f'(x)$. **2**
What type of point occurs on $y = f(x)$ when $x = -0{\cdot}9$. Justify your answer.

(iii) For what range of values of x is $f(x)$ concave down? **1**

Question 9(c) continued on next page

9(c)



The graphs of $y = \sin\frac{\pi}{2}x$ and $y = x^2$ are shown above.

The region bounded by these curves and the x-axis is shaded.

(i) Show that the two curves meet where $x = 1$. **1**

1

(ii) Write the definite integral(s) which will give the volume of the solid of revolution when this region is rotated about the x-axis. **DO NOT EVALUATE THESE INTEGRAL S**

3

(iii) Use Simpson's Rule with five function values to approximate this volume, leaving your answer in terms of π.

<u>Question 10</u> (12 Marks) Begin a SEPARATE sheet of paper

Marks

(a) The percentage of Carbon14 in an organism falls exponentially after the death of the organism. After 1845 years 80% of the original concentration of Carbon14 remains.
Using the equation $C = C_o\, e^{-kt}$ to represent the exponential fall of Carbon14:

(i) Find the exact value of k. **2**

(ii) An artefact containing this organism has 65% of the original concentration of Carbon14. How long has this organism been dead? Give answer to the nearest year. **2**

(iii) A sea sponge containing this organism has been dead for 12 000 years. What percentage (to 1 decimal place) of the original Carbon14 concentration does it have? **2**

(b) Two sailors are paid to bring a motor launch back to Sydney from Gilligan's Island, a total distance of 1 200 km. They are each paid \$25 per hour for the time spent at sea.

The launch uses fuel at a rate of $20 + \frac{v^2}{10}$ litres per hour where the speed of the launch is v km per hour. Diesel fuel costs \$1·25 per litre.

(i) Show that, to bring the launch back from Gilligan's Island, the total cost (\$$C$) to the owners is **3**

$$C = \frac{90\,000}{v} + 150v.$$

(ii) Find the speed that minimises the cost and determine this cost, giving your answer to the nearest dollar. **3**

END OF EXAMINATION

SOLUTIONS TO 2UNIT TRIAL MATHEMATICS

Question 1 (12 marks)

a) $\frac{e+1}{\pi} = 1.183565\ldots$ — 1

$= 1.184 \quad (3\,dp)$ — $\boxed{2}$ — 1 — −1 if not to 3 dp

b) $\sin\theta = 0.500698846$

$\theta = 30^\circ 3'$

$\theta = 30^\circ$ (nearest degree) — $\boxed{2}$ — 2 — −1 if not to nearest degree.

c) Centre $(-2, 3)$ — 1

radius $= \sqrt{2.25} = 1.5$ — $\boxed{2}$ — 1

d) $\frac{3+5+7+x}{4} = 6.75$

$15 + x = 27$

$x = 12$ — $\boxed{1}$ — 1

e) $|-3 - 4 \times 2.35| = |-12.4| = 12.4$ — $\boxed{1}$ — 1

f) $3x^2 - 5x - 2 = (3x+1)(x-2)$ — $\boxed{1}$ — 1

g) $2.32 \text{ rads} = 2.32 \times \frac{180^\circ}{\pi} = 132^\circ 56'$ — $\boxed{1}$ — 1

h) $x^2 - 9 \geq 0$

D: $x \leq -3$ or $x \geq 3$ — 1

R: $y \geq 0$ — $\boxed{2}$ — 1

Question 2 (12 marks)

a) (i) 0 — $\boxed{1}$ — 1

(ii) $\frac{2x-3}{x^2-3x}$ — $\boxed{1}$ — 1

(iii) $\frac{2\cos x + (2x-1)\sin x}{\cos^2 x}$ — $\boxed{1}$ — 1

b) $-2\cos\frac{x}{2} + c$ — $\boxed{2}$ — 1 for −2; 1 for $\cos\frac{x}{2}$; $-\frac{1}{2}$ for no c

c) $\int_0^1 \frac{5}{6} e^{3x}\,dx = \frac{5}{6}\left[\frac{1}{3}e^{3x}\right]_0^1$ — 1

$= \frac{5}{18}(e^3 - 1)$ — $\boxed{2}$ — 1

d) (i) $\log pq = \log p + \log q = 1.75 + 2.25 = 4$ — $\boxed{1}$ — 1

(ii) $\log\frac{q}{p} = \log q - \log p = 2.25 - 1.75 = 0.5$ — $\boxed{1}$ — 1

(iii) Let $\sqrt[5]{pq^2} = y \implies \log_m \sqrt[5]{pq^2} = \log_m y$

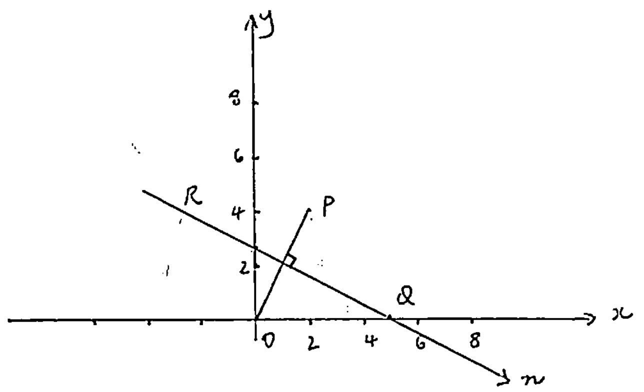
$\frac{1}{5}(\log p + 2\log q) =$ — 1

$\frac{1}{5}(1.75 + 2 \times 2.25) =$

$1.25 = \log_m y$ — 1

$\therefore m^{1.25} = y$

Question 3 (12 marks)



a) M is $(1, 2)$ — $\boxed{1}$ — 1

b) Sub. $M(1,2)$ into $x + 2y = 5$:

LHS $= 1 + 2 \times 2 = 5$

RHS $= 5$

$\therefore$ LHS = RHS

$\therefore M(1,2)$ lies on $x + 2y = 5$ — $\boxed{2}$ — 2 — −1 if not properly set out.

c) $m_{OP} = \frac{4}{2} = 2$ — $\boxed{1}$ — 1

d) $m_{\text{line } n} = -\frac{2\frac{1}{2}}{5} = -\frac{1}{2}$ — 1

$m_{OP} \times m_{\text{line } n} = 2 \times -\frac{1}{2} = -1$ — 1

$\therefore OP \perp$ line n

Also $(1,2)$ is the midpoint of OP $\therefore$ bisector — $\boxed{3}$ — 1

$\therefore$ line n is the perpendicular bisector of OP.

e) $Q(5, 0)$ — $\boxed{1}$ — 1

f) $m_{PQ} = -\frac{4}{3}$

$m_{OR} = m_{PQ} = -\frac{4}{3} \quad (PQ \parallel OR)$ — 1

By congruent triangles, and using diagram above,

R is $(-3, 4)$ — $\boxed{2}$ — 1

g) $OQ \parallel PR$ (gradients 0)

$PQ \parallel OR$ (from (f))

Diagonals meet at 90°

$\therefore$ PQOR is a rhombus (2 pairs of opposite sides parallel & diagonals meet at 90°) — $\boxed{2}$ — 2 — NB: Other reasons are also possible

Question 4 (12 marks)

a) (i) Required to prove $(m^2-n^2)^2+(2mn)^2=(m^2+n^2)^2$

$\underline{LHS}$: $(m^2-n^2)^2+(2mn)^2$

$= m^4-2m^2n^2+n^4+4m^2n^2$ — 1

$= m^4+2m^2n^2+n^4$ — 1

$= (m^2+n^2)^2$

$= RHS.$

$\therefore LHS = RHS$ $\boxed{2}$

(ii) If $m=3, n=2$: triad is 13, 5, 12 $\boxed{1}$ — 1

b) (i) $f(x)=x-2\log_e x$, $x>0$

$f'(x)=1-\frac{2}{x}$ or $1-2x^{-1}$ — 1

$f''(x)=2x^{-2}$ or $\frac{2}{x^2}$ $\boxed{2}$ — 1

(ii) For t.p. $(f'(x)=0)$: $1-\frac{2}{x}=0$

$x-2=0$

$x=2$

$\therefore$ Possible t.p. is $(2,\ \overset{\doteqdot 0.6}{2-2\ln 2})$ — 1

To determine its nature use either 1st or 2nd derivative

$\underline{\text{Using 1st derivative}}$:

x	1	2	3
$f'(x)$	-1	0	$\frac{1}{3}$
	\	—	/

$\therefore$ local minimum at $x=2$ — 1

$\underline{\text{Using 2nd derivative}}$: at $x=2$

$f''(2)=2\times 2^{-2}>0$

$\therefore$ concave up

$\therefore$ local minimum at $x=2$ $\boxed{2}$ — (1)

(iii) For possible points of inflexion, $f''(x)=0$

$\therefore \frac{2}{x^2}=0$ — 1

but $\frac{2}{x^2}\neq 0$

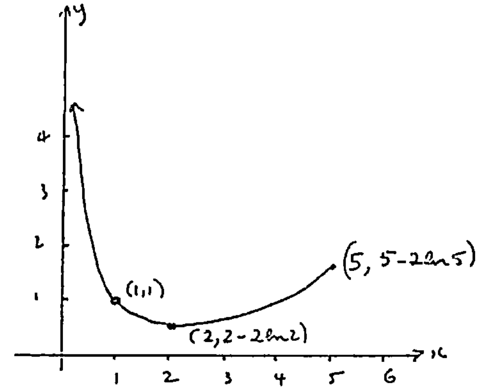
$\therefore$ There are no points of inflexion. $\boxed{1}$

(iv) at $x=1$, $f(1)=1-2\ln 1=1$ — ½

at $x=5$, $f(5)=5-2\ln 5\doteqdot 1.78$ — ½

$\therefore$ Maximum value is $\underline{5-2\ln 5}$ $\boxed{2}$ — 1

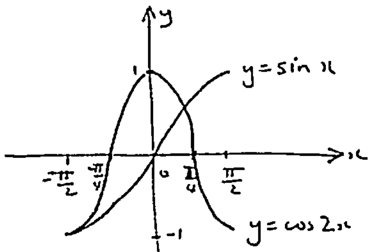
(v) $y=x-2\ln x$, $0<x\le 5$



$\boxed{2}$ — ½ shape; ½ for minimum; ½ for (1,1) & (5, 5−2ln5); ½ for scale

Question 5 (12 marks)

a) (i)



$\boxed{2}$ — 1 for shape; 1 for scale

(ii) at $x=\frac{\pi}{6}$, $\sin\frac{\pi}{6}=\frac{1}{2}$ — ½

$\cos 2\times\frac{\pi}{6}=\cos\frac{\pi}{3}=\frac{1}{2}$ $\boxed{1}$ — ½

$\therefore \sin x=\cos 2x$ at $x=\frac{\pi}{6}$.

(iii) $A=\int_{-\frac{\pi}{2}}^{\frac{\pi}{6}}(\cos 2x-\sin x)\,dx$ — 1

$=\left[\frac{1}{2}\sin 2x+\cos x\right]_{-\pi/2}^{\pi/6}$ — 1

$=\left(\frac{1}{2}\sin\frac{\pi}{3}+\cos\frac{\pi}{6}\right)-\left(\frac{1}{2}\sin(-\pi)+\cos\left(-\frac{\pi}{2}\right)\right)$

$=\frac{\sqrt{3}}{4}+\frac{\sqrt{3}}{2}=\frac{3\sqrt{3}}{4}$ units^2 $\boxed{3}$ — 1

Question 5 continued.

b) $LHS = \dfrac{\sec\theta - \sec\theta\cos^4\theta}{1+\cos^2\theta}$

$= \dfrac{\sec\theta(1-\cos^4\theta)}{1+\cos^2\theta}$

$= \dfrac{\sec\theta\,\cancel{(1+\cos^2\theta)}(1-\cos^2\theta)}{\cancel{1+\cos^2\theta}}$ 1

$= \dfrac{1}{\cos\theta}\cdot\sin^2\theta$ 1

$= \dfrac{\sin\theta}{\cos\theta}\cdot\sin\theta$ 1

$= \tan\theta\cdot\sin\theta$

$= RHS$ $\boxed{3}$

c)(i) $\Delta = (2-k)^2 - 4.1.(2\cdot 25)$

$= (2-k)^2 - 9$

For equal roots, $\Delta = 0$

ie $(2-k)^2 - 9 = 0$ 1

$(2-k)^2 = 9$

$2-k = \pm 3$

$\underline{k = 5 \text{ or } -1}$ $\boxed{2}$ 1

(ii) $y = kx+1$

$y = x^2+2x+3.25$

Solve: $x^2+2x+3\cdot 25 = kx+1$

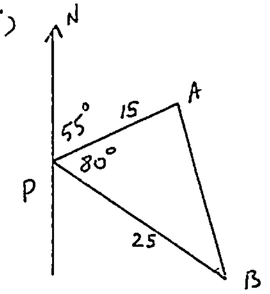
$x^2+(2-k)x+2\cdot 25 = 0$ 1

For line to be a tangent, $\Delta = 0$, then from (i)

$k = 5$ or -1. $\boxed{1}$

Question 6 (12 marks)

a) (i)



$\boxed{2}$ 1 for diagram; 1 for $\angle APB = 80°$

(ii) $AB^2 = 15^2+25^2 - 2.15.25.\cos 80°$ 1

$= 719.76$

$\underline{AB = 26.8 \text{ km}}$ $\boxed{2}$ 1 $-\frac{1}{2}$ if not to 1dp.

b) $\displaystyle\sum_{r=3}^{7} 2^r - 3r = 2^3 - 3\times 3 + 2^4 - 3\times 4 + 2^5 - 3\times 5 + 2^6 - 3\times 6 + 2^7 - 3\times 7$ 1

$= (2^3+2^4+2^5+2^6+2^7) - 3(3+4+5+6+7)$

$= 248 - 75 = 173$ $\boxed{2}$ 1

NB: It is not intended that rules for sum of AP & GP's be used here.

c) (i) When $t=0$, $R = 6(e^0+e^0) = 12$ L/min $\boxed{1}$ 1 $-\frac{1}{2}$ if no units are provided.

(ii) $V = \int 6(e^t+e^{-t})\,dt$

$V = 6(e^t - e^{-t}) + c$ 1

When $t=0$, $V=0 \Longrightarrow 0 = 6(e^0-e^0)+c$

$\underline{c = 0}$ 1

$\underline{V = 6(e^t - e^{-t})}$ $\boxed{2}$

(iii) $5 = 6\left(e^t - \frac{1}{e^t}\right)$

$5e^t = 6e^{2t} - 6$ 1

$\therefore\ \underline{6e^{2t} - 5e^t - 6 = 0}$ $\boxed{1}$

(iv) $(2e^t-3)(3e^t+2) = 0$

$e^t = 3/2$ or $e^t = -\frac{2}{3}$ 1

but $e^t \neq -\frac{2}{3}$ $\therefore$ no sol^n.

$\therefore\ t = \ln 3/2 \doteqdot \underline{24 \text{ seconds}}$ $\boxed{2}$ 1 $-1/2$ if not to nearest second.

Question 7 (12 marks)

a) $\frac{dy}{dx} = \frac{3\sec^2 x}{\tan x}$

$y = 3\ln(\tan x) + c$ 1

When $x = \frac{\pi}{4}$, $y = 4 \implies 4 = 3\ln(\tan\frac{\pi}{4}) + c$

$\therefore c = 4$ 1

$\therefore \underline{y = 3\ln(\tan x) + 4}$ [2]

b) (i) 1 year $= 365 \times 24 \times 60$ minutes

$= 525600$ minutes 1

$\therefore$ Incremental rate of interest $= \frac{12}{525600}$ % per minute

$= \frac{0.12}{525600}$ per minute

$\doteqdot 0.00002283$ per min. ½

Since interest is added onto the amount invested then interest is multiplied by approximately 1.00002283 every minute [2] ½

(ii) $1000\,(1.00002283)^{525600}$

$= \underline{\$1127}$ (nearest dollar) [1] 1

c) (i) $\angle QOP = \frac{2\pi}{3}$ (angle sum about a point is 360°) 1

$\angle QPO = \frac{2\pi - \frac{2\pi}{3}}{2} = \frac{\pi}{6}$ (angle sum of triangle is 360°; angles opposite equal sides are equal) [2] 1

(ii) $A_{\text{Sector }QPR} = \frac{1}{2} \times 5\sqrt{3} \times 5\sqrt{3} \times \frac{\pi}{3} = \frac{25\pi}{2}$ unit² [1] 1

(iii) $A_{\text{Sector }QOR} = \frac{1}{2} \times 5 \times 5 \times \frac{2\pi}{3} = \frac{25\pi}{3}$ units² [1] 1

(iv) There are several ways to do this which may involve areas of segments/triangles/sectors.

PTO.

Question 7 (continued)

(iv) Method 1

$A_{\text{Segment }QC_1R} = \frac{1}{2} \times (5\sqrt{3})^2\left(\frac{\pi}{3} - \sin\frac{\pi}{3}\right)$

$= 25 \times \frac{3}{2}\left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right)$

$= 25\left(\frac{\pi}{2} - \frac{3\sqrt{3}}{4}\right)$ 1

$A_{\text{Segment }QC_2R} = \frac{1}{2} \times 5^2\left(\frac{2\pi}{3} - \sin\frac{2\pi}{3}\right)$

$= 25 \times \frac{1}{2}\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$

$= 25\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)$ 1

$A_{\text{Shaded}} = 25\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right) - 25\left(\frac{\pi}{2} - \frac{3\sqrt{3}}{4}\right)$ 1

$= \underline{25\left(\frac{\sqrt{3}}{2} - \frac{\pi}{6}\right) \text{ units}^2}$ [3]

Alt. Method 2

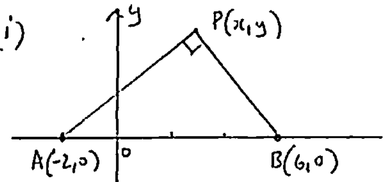
Shaded area $= (iii) - (ii) + 2\,\Delta's$

$= \frac{25\pi}{3} - \frac{25\pi}{2} + 2 \times \frac{25\sqrt{3}}{4}$

$= 25\left(\frac{\sqrt{3}}{2} - \frac{\pi}{6}\right)$ ([3])

Question 8 (12 marks)

a) (i)



$m_{PA} = \frac{y-0}{x-(-2)} = \frac{y}{x+2}$ 1

[1]

(ii) $m_{PA} \times m_{PB} = -1$

$\frac{y}{x+2} \times \frac{y}{x-6} = -1$ 1

$y^2 = -(x+2)(x-6)$

$= -x^2 + 4x + 12$

$x^2 - 4x + 4 + y^2 = 12 + 4$

$(x-2)^2 + y^2 = 16$ 1

$\therefore$ Locus is a circle, centre $(2,0)$, radius 4 units. [2] 1

Question 8 (continued)

b) $V = 6t - 8 - t^2 = \frac{dx}{dt}$

(i) $x = 3t^2 - 8t - \frac{t^3}{3} + c$ 1

When $t=0$, $x = +5 \implies \underline{5 = c}$ 1

$\therefore x = 3t^2 - 8t - \frac{t^3}{3} + 5$ [2]

(ii) at rest, $V=0 \implies 0 = 6t - 8 - t^2$

$0 = (t-4)(t-2)$

$\therefore$ at rest when $\underline{t = 2 \text{ and } t = 4}$ [1] 1

(iii) $x(0) = 5$

$x(2) = -1\frac{2}{3}$ $\quad \therefore$ In first 2 secs it travels $5 + 1\frac{2}{3} = 6\frac{2}{3}$ m ½

$x(4) = x = -\frac{1}{3}$ $\quad \therefore$ from $t=2$ to $t=4$ it travels $1\frac{1}{3}$ units. ½

$\therefore$ distance travelled $= 5 + 1\frac{2}{3} + 1\frac{1}{3} = \underline{8 \text{ units}}$ [2] 1

c) (i) $P(6,6) = \frac{3}{8} \times \frac{3}{8} = \frac{9}{64}$ [1] 1

(ii) $P(\tilde{6},\tilde{6}) = \frac{5}{8} \times \frac{5}{8} = \frac{25}{64}$ [1] 1

(iii) $P(6,\tilde{6}) + P(\tilde{6},6) = \frac{3}{8} \times \frac{5}{8} \times 2 = \frac{30}{64} = \frac{15}{32}$ [1] 1

Question 9 (12 marks)

a) (i) $x^2 - 6x + 8 = 2y$

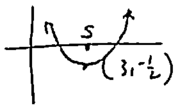
$x^2 - 6x + 9 - 1 = 2y$

$(x-3)^2 = 2y + 1$ 1

$= 2(y + \frac{1}{2})$

$(x-3)^2 = 4(\frac{1}{2})(y - (-\frac{1}{2}))$ [2] 1

(ii) Vertex $= (3, -\frac{1}{2})$ $\quad a = \frac{1}{2}$



$\underline{\text{Focus} = (3, 0)}$ [1] 1

Question 9 (continued)

b) (i) For increasing function, $f'(x) > 0$

ie $\{x : -2 < x < \frac{1}{2}\}$, $\{x : x > 3\}$ [1] 1 $\quad$ $-\frac{1}{2}$ if either is missing

(ii) $f'(-0.9)$ is a maximum, so $f''(-0.9) = 0$

$\therefore$ C is a possible point of inflexion. 1

for $x < -0.9$, $f'(x) > 0$

for $x > -0.9$, $f'(x) > 0$

$\therefore$ In the neighbourhood of -0.9, gradients > 0. 1

$\therefore$ $\underline{\text{C is a point of inflexion.}}$ [2]

(iii) For concave down, $f'(x)$ is decreasing.

$\therefore \{x : -0.9 < x < 1.9\}$ [1] 1

c) (i) at $x = 1$, $\sin\frac{\pi}{2}\times 1 = \sin\frac{\pi}{2} = 1$ 1

$1^2 = 1$

$\therefore \sin\frac{\pi}{2}x = x^2$ at $x = 1$ [1]

(ii) $V = \pi\int_0^1 (x^2)^2\,dx - \pi\int_1^2 (\sin\frac{\pi}{2}x)^2\,dx$

OR

$V = \pi\int_0^1 x^4\,dx - \pi\int_1^2 \sin^2\frac{\pi}{2}x\,dx$ [1] 1

(iii)

x	Weight	$f(x)^2$
0	1	0
$\frac{1}{2}$	4	$1/16$
1	2	1
$1\frac{1}{2}$	4	$\sin^2\frac{3\pi}{4} = \frac{1}{2}$
2	1	$\sin^2\pi = 0$
	12	

$V = \pi \times \frac{2-0}{12}\left[0 + 0 + 4\left(\frac{1}{16} + \frac{1}{2}\right) + 2(1)\right]$ 2

$= \frac{\pi}{6}\left[\frac{17}{4}\right]$

$= \frac{17\pi}{24}$ u^3 1 [3]

Question 10 (12 Marks)

a) (i) When $t=0$, $A=A_0$

When $t=1845$, $A=0.8(A_0)$,

$\therefore\ 0.8A_0 = A_0 e^{-1845k}$ 1

$0.8 = e^{-1845k}$

$\ln 0.8 = -1845k$

$k = -\dfrac{\ln 0.8}{1845}$ or $\dfrac{\ln 1.25}{1845}$ $\boxed{2}$ 1

(ii) $A = 0.65A_0$ when $t=T$

$\therefore\ 0.65\cancel{A_0} = \cancel{A_0}\, e^{+\frac{\ln 0.8}{1845}T}$ 1

$\ln 0.65 = \dfrac{\ln 0.8}{1845}T$

$T = \dfrac{1845 \times \ln 0.65}{\ln 0.8}$

$T = 3562$ years $\boxed{2}$ 1

(iii) When $t = 12000$:

$A = A_0\, e^{+12000 \times \frac{\ln 0.8}{1845}}$ 1

$A \doteqdot 0.234\, A_0$

$\therefore A \doteqdot 23.4\%$ of original amount is left $\boxed{2}$ 1

b) $S = \dfrac{D}{T}$

(i) $T = \dfrac{D}{S} = \dfrac{1200}{v}$ hours 1

Cost of wages $= 25 \times 2 \times \dfrac{1200}{v} = \dfrac{60000}{v}$ 1

Cost of fuel $= \left(20 + \dfrac{v^2}{10}\right) \times \dfrac{1200}{v} \times 1.25$ 1

$= \left(20 + \dfrac{v^2}{10}\right) \times \dfrac{1500}{v}$

$= \dfrac{30000}{v} + 150v$

Total cost, $C = \dfrac{60\,000}{v} + \dfrac{30\,000}{v} + 150v$

$C = \dfrac{90\,000}{v} + 150v$ $\boxed{3}$

Question 10 (continued)

(ii) $C = \dfrac{90\,000}{v} + 150v$

$\dfrac{dC}{dv} = 150 - 90\,000v^{-2}$

For min. cost, $\left(\dfrac{dC}{dv} = 0\right)$:

$150 - \dfrac{90\,000}{v^2} = 0$

$150v^2 - 90\,000 = 0$

$v^2 - 600 = 0$

$v = \sqrt{600}$ $(v > 0)$ 1

$\therefore\ v = 10\sqrt{6}$

Check $C(10\sqrt{6})$ is a minimum:

$\dfrac{d^2C}{dv^2} = 180000\,v^{-3}$

at $v = 10\sqrt{6}$, $\dfrac{d^2C}{dv^2} = \dfrac{180\,000}{(10\sqrt{6})^3} > 0$

$\Longrightarrow$ concave up

$\therefore$ local min at $v = 10\sqrt{6}$. 1

(May use 1st derivative to test nature of stationary point.)

$\therefore$ Minimum cost $= \dfrac{90000}{\sqrt{600}} + 150 \times \sqrt{600}$

$= \$7348$ $\boxed{3}$ 1

END OF SOLUTIONS