

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION 2011

MATHEMATICS

*Time Allowed – 3 Hours
(Plus 5 minutes Reading Time)*

- *All* questions may be attempted
- *All* questions are of equal value
- Department of Education approved calculators and templates are permitted
- In every question, show all necessary working
- Marks may not be awarded for careless or badly arranged work
- No grid paper is to be used unless provided with the examination paper

The answers to all questions are to be returned in separate *stapled* bundles clearly labeled Question 1, Question 2, etc. Each question must show your Candidate Number.

Question 1 - Start a new sheet of paper**Marks**

- a) Solve $x^2 - 3x - 18 = 0$ 1
- b) Convert $\frac{7\pi}{9}$ radians to degrees 1
- c) Write down a primitive of $\sec^2 3x$ 1
- d) Solve $|2x - 1| = 7$ 2
- e) Write $\frac{2}{\sqrt{17} - 3}$ with a rational denominator 1
- f) Given $f(x) = \begin{cases} \frac{1}{3^x} & x < 3 \\ (x - 3)^2 + 1 & x \geq 3 \end{cases}$,
evaluate $2f(4) - f(-1)$ 2
- g) Find the non-zero values of x for which the geometric series
 $2 - 6x + 18x^2 - \dots$ has a limiting sum 2
- h) i) Write down the equation of the locus of a point P that is 3 units
from the point $A(2, -4)$ 1
- ii) How many times does this locus cut the x axis? 1

Question 2 - Start a new sheet of paper**Marks**

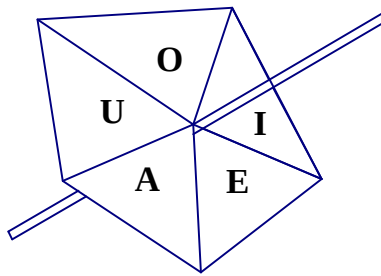
- a) Differentiate the following with respect to x :
- i) $\ln \sqrt{x^2 + 3}$ 2
- ii) $(2x^2 + 7)^3$ 1
- iii) $x^2 \cos x$ 2
- b) The equation of a parabola is given by $8y = x^2 + 4x + 12$
- i) Find the coordinates of the vertex 2
- ii) Write down the focal length of the parabola 1
- iii) Sketch the parabola, clearly showing the focus and directrix 2
- c) Solve the equation $\sec^2 x = 4$ for $0 \leq x \leq 2\pi$ 2

Question 3 - Start a new sheet of paper**Marks**

a) Given that $\log_k 5 = 1.838$ and $\log_k 2 = 0.792$, find the values of

- i) $\log_k 50$ ii) $\log_k (0.4)$ iii) $\log_{10} k$ **4**

b) An unbiased spinner, as shown, is equally likely to stop at any one of the letters **A, E, I, O** or **U**



- i) If the spinner is spun twice, find the probability that it stops on the same letter both times. **1**
- ii) How many times must the spinner be spun for it to be 99% certain that the spinner will have stopped on **E** at least once? **2**
- c) i) Sketch the graph of $y = 3\sin 2x$ for $0 \leq x \leq 2\pi$ (Your diagram should take up at least one quarter of a page.) **2**
- ii) Draw the line $y = \frac{x}{2}$ on the same diagram, noting particularly the point on the line where $x = 2\pi$ **1**
- iii) Hence determine the **total** number of solutions there are to the equation
- $$6\sin 2x - x = 0 \quad \text{for all } x \quad \mathbf{2}$$

(**NB** You are **not** required to solve the equation but you should include a short explanation of your answer)

Question 4 - Start a new sheet of paper**Marks**

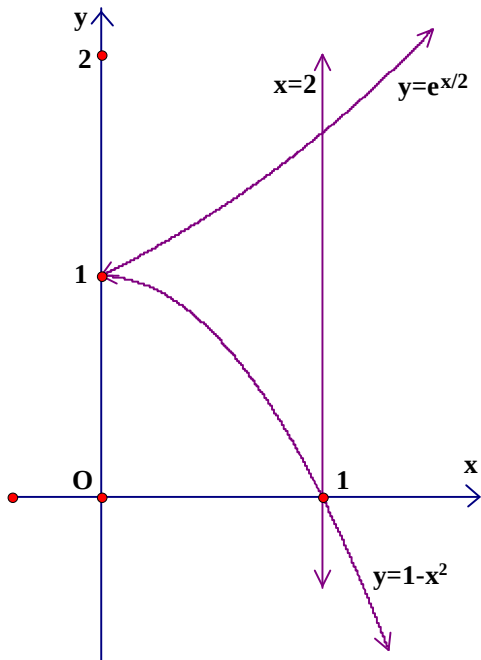
- a) The gradient of a curve is given by $\frac{dy}{dx} = 3x^2 - 4$
- i) Find $\frac{d^2y}{dx^2}$ 1
- ii) Find those values of x for which the curve is both increasing and concave downwards. Show your reasoning. 2
- iii) If the curve passes through the point $(1, -2)$, find the equation of the curve. 2
- b) The coordinates of A , B and C are $(4, -6)$, $(-18, 0)$ and $(0, 6)$ respectively. D is the midpoint of AB .
- i) Draw these points on a clear coordinate diagram and show that D has coordinates $(-7, -3)$. 1
- ii) Show that the equation of the line AC is $3x + y - 6 = 0$ 2
- iii) Show that the line BC is perpendicular to the line AC 1
- iv) Therefore AB is the diameter of a circle which passes through A , B and C . Find the equation of this circle. 2
- v) Find the circumference of the circle, accurate to 1 decimal place. 1

Question 5 - Start a new sheet of paper**Marks**

- a) A ship sails from O on a bearing of 130°T to the point P . P lies on a bearing of 030°T from M where M is 120 kms due south of O .
- i) Show this information on a clear diagram and confirm that $\angle OPM = 100^\circ$ 1
- ii) How far, to the nearest kilometre, is P from M ? 2
- iii) A lighthouse is located at L which is 40 kms due south of O . What was the closest that the ship came to L as it made its way from O to P . 2
- b) A substance decomposes such that the mass M (kgs) remaining at any time t (hrs) is given by $M = M_0 e^{-kt}$. If $\frac{3}{4}$ of the substance has decomposed in 4 hours, find:
- i) the exact value of k 2
- ii) the value of M_0 , correct to 2 decimal places, given that 4kg of the substance remains after the first 90 minutes. 2
- c) Simplify $\frac{1 + \cot \theta}{\operatorname{cosec} \theta} - \frac{\sec \theta}{\tan \theta + \cot \theta}$ 3

Question 6 - Start a new sheet of paper**Marks**

- a) i) Find $\int x \sin(x^2) dx$ 1
- ii) Evaluate $\int_1^{16} \sqrt{x} dx$ 2
- b) The quadratic equation $5x^2 - (21 + a)x + a = 0$ has two roots, with one of the roots being the reciprocal of the other. Find the value of a and hence find the two roots. 2
- c) A polygon has 60 sides. The lengths of the sides, starting with the smallest, form an arithmetic sequence. The perimeter of the polygon is 1770 cms and the length of the longest side is three times the length of the shortest side. Find the lengths of the two shortest sides. 4
- d)



The diagram shows the region enclosed between the two curves $y = e^{x/2}$ and $y = 1 - x^2$ and the line $x = 1$. Find the area of this enclosed region.

3**Question 7 - Start a new sheet of paper****Marks**

- a) $PQRS$ is a rhombus, angle P being acute. X is the point on PS such that QX is perpendicular to PS . QX intersects the diagonal PR at Y .
- i) Show this information on a clear diagram and explain why $\angle SRP = \angle QRP$ 1
- ii) Prove that triangle SYR is congruent to triangle QYR . 3
- iii) Show that $\angle RQY$ is 90° and hence find the size of $\angle YSR$ 2

Question 7 is continued on the next page.....

Question 7 (continued)**Marks**

- b) The velocity v (kms/min) of a train travelling from Epping to Eastwood, is given by $v = t^2(3-t)/3$, where t is the time in minutes since leaving Epping.
- i) If the first stop is at Eastwood, how long does the journey take? **1**
- ii) Find an expression for the distance (x km) travelled from Epping after time t (where $0 \leq t \leq 3$) **2**
- iii) Hence find the distance from Epping to Eastwood. **1**
- iv) Where and when, between the two stations, was the train travelling the fastest? **2**

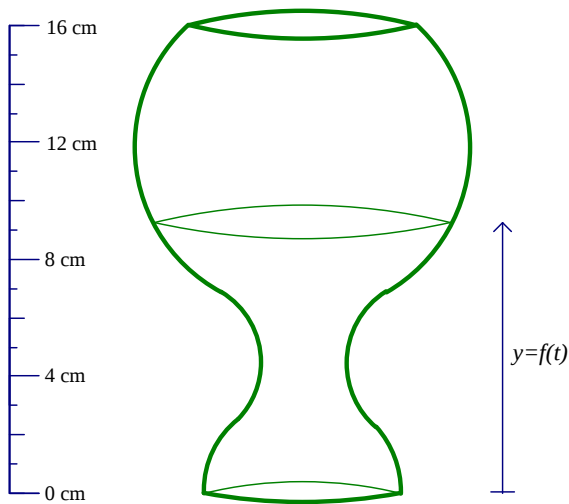
Question 8 - Start a new sheet of paper**Marks**

- a) Stella sets up a prize fund with a single investment of \$1000 to provide her school with an annual prize of \$72. The fund accrues interest at a rate of 6% per annum, compounded annually. The first prize is awarded one year after the investment is set up.
- i) Calculate the balance in the fund at the beginning of the second year, after the first prize has been awarded. **1**
- ii) Let $\$B_n$ be the balance in the fund at the end of n years (after the n th prize has been awarded and while funds are still available).
Show that $B_n = 1200 - 200 \times (1.06)^n$ **2**
- iii) At the end of the tenth year (after that prize has been awarded), it is decided that the prize will henceforward be increased to \$90. For how many more years can the fund be used to award the full prize? **3**
- b) i) Sketch the graph of $y = e^x$ for $0 \leq x \leq 2$ **1**
- ii) The portion of the curve $y = e^x$ between $y = 3$ and $y = 5$ is rotated about the y -axis to give a solid. Show that the volume of the solid formed is
given by $\pi \int_3^5 (\ln y)^2 dy$ cubic units. **2**
- iii) Use Simpson's Rule and 5 function values to find an approximation to this volume (answer to 1 decimal place). **3**

Question 9 - Start a new sheet of paper

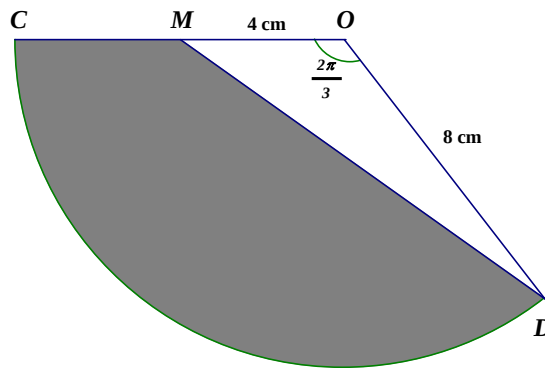
Marks

- a) The diagram shows a 16 cm high wine glass that is being filled with water at a constant rate (by volume). Let $y = f(t)$ be the depth of the liquid in the glass as a function of time.



- Write down the approximate depth y_1 at which $\frac{dy}{dt}$ is a minimum. 1
- Write down the approximate depth y_2 at which $\frac{dy}{dt}$ is a maximum. 1
- If the glass takes 8 seconds to fill, graph $y = f(t)$ and identify any points on your graph where the concavity changes. 2

- b) In the diagram, CD is an arc of a circle with radius 8 cm and centre O . M is the midpoint of OC and angle COD is $\frac{2\pi}{3}$ radians.



Find the perimeter of the shaded area CDM in exact form. 3

- Find the exact value of k (where $k > 2$) such that $\int_2^k \frac{2t}{3t^2 - 1} dt = \frac{1}{3} \log_e 13$ 3
- If a , b and c are consecutive positive terms of a geometric sequence, show that $\log a$, $\log b$ and $\log c$ are consecutive terms in an arithmetic sequence. 2

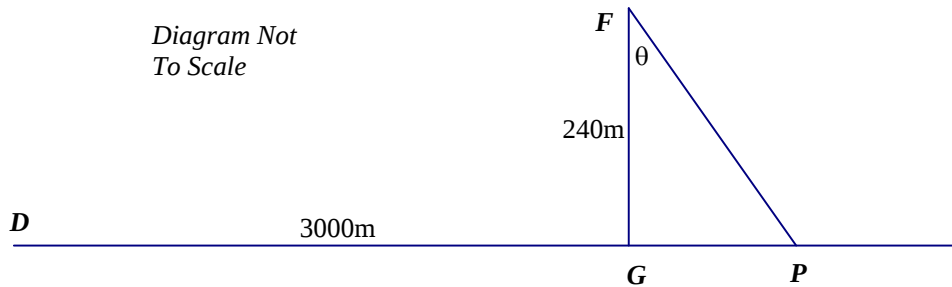
Question 10 - Start a new sheet of paper

Marks

a) State the Domain and Range of the function $y = 2\sqrt{25 - x^2}$ 2

b) Evaluate $\sum_{r=1}^{63} \frac{1}{\sqrt{r+1} + \sqrt{r}}$ 2

c)



The diagram shows a farmhouse F that is located 240m from a straight section of road, at the end of which is the bus depot D . The front gate G of the farmhouse is 3000m from the bus depot. The school bus leaves the depot at 8am and travels along the road at 15 ms^{-1} . Peter lives in the farmhouse and can run across the open paddock at a speed of 4 ms^{-1} . The bus will stop for Peter anywhere on the road but will not wait.

Assume that Peter catches the bus at the point P where $\angle GFP = \theta$.

- i) Show that the time, in seconds, taken for the bus to go from D to P is given by $200 + 16 \tan \theta$ 2
- ii) Find an expression, in terms of θ , for the time taken by Peter to run from F to P . 1
- iii) If Peter leaves home T seconds after 8am and he and the bus arrive at P at the same time, show that $T = 200 + 16 \tan \theta - 60 \sec \theta$ 1
- iv) What is the latest time, to the nearest second, that Peter can leave home and still catch the bus? 4

***** THIS IS THE END OF THE EXAM *****

2011 TRIAL HSC 2\MATHEMATICS: Question...1....

Suggested Solutions	Marks	Marker's Comments
1(a) $x^2 - 3x - 18 \geq 0$ $(x-6)(x+3) \geq 0$ $\therefore x = -3 \text{ or } 6$	1	$\frac{1}{2}$ mark for each value
(b) $\frac{7\pi}{9} = \frac{7\pi}{9} \times \frac{180}{\pi}$ $= 140^\circ$	1	
(c) $\int \sec^2 3x dx = \frac{1}{3} \tan 3x + C$	1	+C not penalized as a primitive of $\sec^2 3x$ is $\frac{1}{3} \tan 3x$
(d) $ 2x-1 = 7$ $2x-1 = 7$ or $-(2x-1) = 7$ $2x = 8$ or $-2x+1 = 7$ $x = 4$ or $x = -3$	2	1 mark for each answer
(e) $\frac{2}{\sqrt{11}-3} \times \frac{\sqrt{11}+3}{\sqrt{11}+3} = \frac{2(\sqrt{11}+3)}{8}$ $= \frac{\sqrt{11}+3}{4}$	1	no marks deducted for not simplifying
(f) $f(4) = (4-3)^2 + 1 = 2$ $f(-1) = \frac{1}{3+1} = \frac{1}{4}$ $2f(4) - f(-1) = 4 - \frac{1}{4} = \frac{15}{4}$	1 1	
(g) $r = \frac{T_2}{T_1} = -3x$ For limiting sum $ r < 1$ $\therefore 3x < 1$ $-\frac{1}{3} < x < \frac{1}{3}$ $x \neq 0$	1 1	$x \neq 0$ $\frac{1}{2}$ mark
(h) (i) $(x-2)^2 + (y+4)^2 = 9$ (ii) 0 times (the centre is 4 units below the axis and the radius is only 3)	1 1	

Suggested Solutions

Marks

Marker's Comments

(a) (i) $y = \ln \sqrt{x^2 + 3}$

$\therefore y = \frac{1}{2} \ln(x^2 + 3)$

$\frac{dy}{dx} = \frac{1}{2} \cdot \frac{2x}{x^2 + 3}$
 $= \frac{x}{x^2 + 3}$

(ii) $y = (2x^2 + 7)^3$

$\frac{dy}{dx} = 3(2x^2 + 7)^2 \cdot 4x$
 $= 12x(2x^2 + 7)^2$

(iii) $y = x^2 \cos x$

$\frac{dy}{dx} = 2x \cos x - x^2 \sin x$ (Product Rule) $\therefore$ for each part
 $= x(2 \cos x - x \sin x)$

(b) (i) $8y = x^2 + 4x + 12$

$8y = x^2 + 4x + 4 + 8$

$8y = (x + 2)^2 + 8$

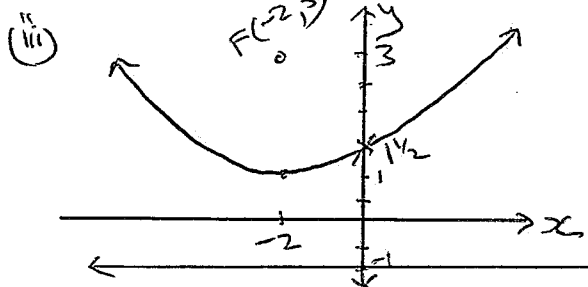
$8y - 8 = (x + 2)^2$

$8(y - 1) = (x + 2)^2$

$\therefore$ vertex is $(-2, 1)$

(ii) $4a = 8$

$a = 2 \therefore$ focal length is 2



(c) $\sec^2 x = 4$

$\cos^2 x = \frac{1}{4}$

$\cos x = \pm \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

$\frac{1}{2}$

1

$\frac{1}{2}$

$\frac{1}{2}$

$\frac{1}{2}$

$\frac{1}{2}$

$\frac{1}{2}$

1

1

2

2.

If they just wrote down

$(-2, 1)$ they got the marks

$\left\{ \begin{array}{l} \frac{1}{2} \text{ for } y\text{-intercept} \\ \frac{1}{2} \text{ for focus} \\ \frac{1}{2} \text{ for directrix} \\ \frac{1}{2} \text{ for shape} \end{array} \right.$

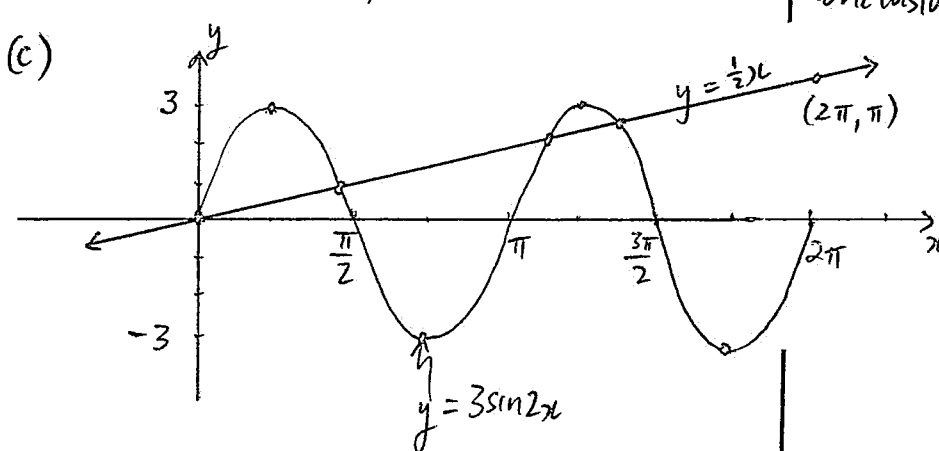
* If they only had 2 answers — 1 mark
 * If they had $\cos x = \frac{1}{2}$

and then 2 answers $\Rightarrow$ 1mk

QUESTION 3

EXPECTED ANSWERS	MARKS	COMMENTS
a) (i) $\log_R 50$ $= \log 5^2 \times 2$ $= 2 \log 5 + \log 2$ $= 2 \times 1.838 + 0.792$ $= \underline{4.468} \rightarrow$	$\frac{1}{2} \dots$ sum $\frac{1}{2}$	Fairly well done
(ii) $\log_R (0.4)$ $= \log_R \frac{2}{5}$ $= \log 2 - \log 5$ $= 0.792 - 1.838$ $= \underline{-1.046} \rightarrow$	$\frac{1}{2} \dots$ difference $\frac{1}{2}$	
(iii) $\log_{10} R$ $= \frac{1}{\log_R 10}$ $= \frac{1}{\log_R 5 + \log_R 2}$ $= \frac{1}{1.838 + 0.792}$ $= \frac{1}{2.63}$ $= \frac{100}{263}$ $= \underline{0.380 (3dp)} \rightarrow$	$\frac{1}{2} \dots$ quotient $\frac{1}{2} \dots$ sum $\frac{1}{2}$ $\left\{ \frac{1}{2} \right.$	• Generally not well done • Many provided lengthy solutions • Some "creative" students found R numerically and then resubstituted!! $(R=2.4)$ • max $1\frac{1}{2}$ if answer was left as $\frac{1}{2.63}$

QUESTION 3

EXPECTED ANSWERS	MARKS	COMMENTS
(b) (i) $P(\text{same letter, both times})$ $= \left(\frac{1}{5}\right)^2 \times 5$ $= \frac{1}{5}$ $\rightarrow$ (ii) $P(E) = \frac{1}{5} \Rightarrow P(\bar{E}) = \frac{4}{5}$ Let n = number of spins Then $\left(\frac{4}{5}\right)^n < 0.01$ $n \log(4/5) < \log \frac{1}{100}$ $n > \log \frac{1}{100} / \log(4/5)$ $n > 20.6$ (1dp) $\therefore$ Number of spins is 21	1 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ for correct conclusion	• No marks if $\frac{1}{25}$ only. • max $\frac{1}{2}$ if $n=2.75$ • generally poorly done
(c)  (iii) There are 4 solutions for $x \geq 0$ as can be seen on the graph and also due to $\pi > 3$, there would be no points of intersection beyond $x > 2\pi$. For $x < 0$ and $x > -2\pi$, there would be a further 3 PoI's. Hence total number of solutions is <u>7</u>	$\frac{1}{2}$ $\frac{1}{2}$ 1	(i) $\frac{1}{2}$ amplitude $\frac{1}{2}$ period $\frac{1}{2}$ scale $\frac{1}{2}$ shape (ii) $\frac{1}{2}$ for $(2\pi, \pi)$ $\frac{1}{2}$ for labelled line through the origin (iii) $(\frac{1}{2})$ for recognising that $6 \sin 2x - x = 0$ transform to $3 \sin 2x = \frac{1}{2}x$ $(\frac{1}{2})$ for 4 solutions; $x \geq 0$ (i) for the 3 solutions; $x < 0$ and total of 7

MATHEMATICS: Question 4

Suggested Solutions

Marks

Marker's Comments

a) i) $\frac{d^2y}{dx^2} = 6x$

1

Correctly differentiates

ii) For curve to be increasing, $y' > 0$.

i.e. $3x^2 - 4 > 0$

$x^2 > \frac{4}{3}$

$\therefore x > \frac{2}{\sqrt{3}}$ or $x < -\frac{2}{\sqrt{3}}$

$\frac{1}{2}$

Solves inequality for y'

For curve to be concave down, $y'' < 0$

i.e. $6x < 0$

$x < 0$

$\frac{1}{2}$

Solves inequality for y''

$\therefore x < -\frac{2}{\sqrt{3}}$ only.

1

Takes intersection

iii) $y = \int (3x^2 - 4) dx$

$y = x^3 - 4x + c$

1

Indefinite integral

Since curve passes through $(1, -2)$,

$-2 = 1 - 4 + c$

$c = 1$

1

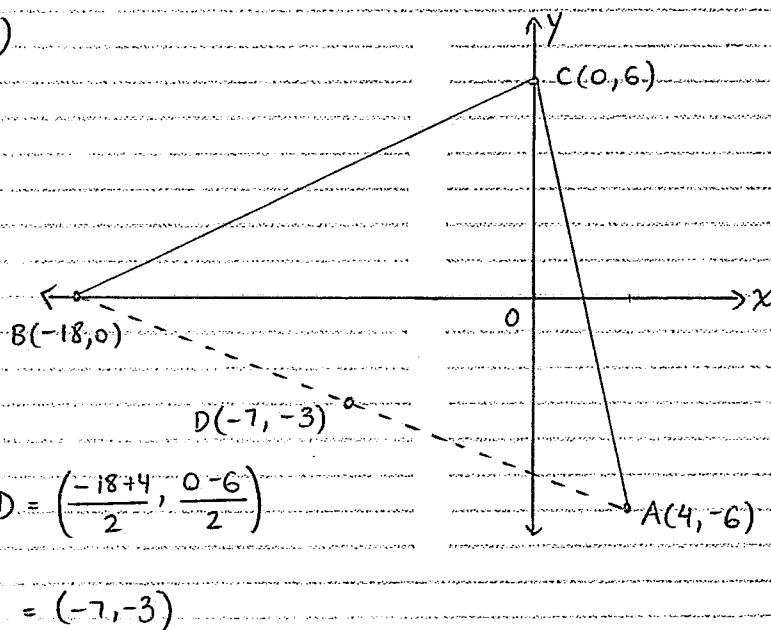
Correctly evaluates constant

$\therefore y = x^3 - 4x + 1$

b) i)

$\frac{1}{2}$

Accurate diagram



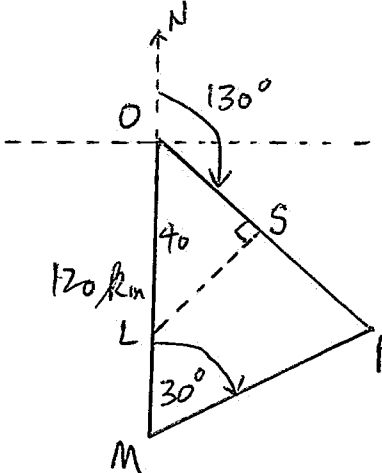
$\frac{1}{2}$

Calculate midpoint of AB

MATHEMATICS: Question 4

Suggested Solutions	Marks	Marker's Comments
b) Continued. ii) $m_{AC} = \frac{6 - (-6)}{0 - 4}$ $= -3$ $\therefore$ Equation of AC is $y - 6 = -3(x - 0)$ $y = -3x + 6$ $3x + y - 6 = 0$ iii) $m_{AC} = -3$ (shown above) $m_{BC} = \frac{6 - 0}{0 - (-18)}$ $= \frac{1}{3}$ $\therefore m_{AC} \times m_{BC} = -3 \times \frac{1}{3}$ $= -1$ $\therefore AC \perp BC$ iv) If AB is diameter, D(-7, -3) is centre. Radius AD $= \sqrt{(-7 - 4)^2 + (-3 + 6)^2}$ $= \sqrt{121 + 9}$ $= \sqrt{130}$ units $\therefore$ Circle equation: $(x + 7)^2 + (y + 3)^2 = 130$ v) Circumference $= 2\pi r$ $= 2\pi \times \sqrt{130}$ $= 71.639334 \dots$ ≈ 71.6 units (1dp)	2 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2}$ $\frac{1}{2}$	Correct use of point-gradient or two-point form Calculate both gradients Relate gradients Centre co-ordinates Radius Equation Exact value Correctly approximates

Q 5

Expected Answers	MARKS	COMMENTS
a) (i)  $\angle POM = 50^\circ \dots$ (adjacent supplement on a straight line add up to 180°) $\therefore \angle OPM = 180^\circ - (50 + 30)^\circ \dots$ (angles of $\triangle POM$ add to 180°) $= 100^\circ$ (ii) Using sine rule on $\triangle POM$: $\frac{PM}{\sin \angle POM} = \frac{OM}{\sin \angle OPM}$ $\Rightarrow PM = \frac{120 \times \sin 50^\circ}{\sin 100^\circ}$ $= 93.344 \dots \text{ km}$ $= \underline{93 \text{ km (nearest km)}}$ $\therefore P$ is 93 km away from M (iii) Shortest distance is $LS (\perp OP)$ $\therefore$ In $\triangle LOS$: $\sin 50^\circ = \frac{LS}{40}$ $\therefore LS = 40 \sin 50^\circ$ $= 30.641 \dots$ $\therefore$ closest the ship is from L is $\underline{31 \text{ km (nearest km)}}$	$\frac{1}{2}$ for correct diagram $\frac{1}{2}$ for confirmation $\frac{1}{2}$ for sine rule $\frac{1}{2}$ for correct substitution $\frac{1}{2}$ for nearest km $\frac{1}{2}$ for final conclusion 1 mark 1 mark	Many students just assumed $\angle POM = 50^\circ$ many students omitted the calculator display Some students omitted final conclusion, to answer Question Some made $SL \perp OM$

EXPECTED ANSWERS	MARKS	COMMENTS
b) (i) $M = M_0 e^{-kt}$ = amount remaining When $t = 4$, $M = \frac{1}{4} M_0$ $\therefore \frac{1}{4} = e^{-4k}$ $4k = \ln 4$ $k = \frac{1}{2} \ln 2 \rightarrow$ (ii) When $t = 90 \text{ minutes} = 1\frac{1}{2} \text{ hrs}$, then $M = 4 \text{ kg}$ $\therefore 4 = M_0 e^{-\frac{3}{2}k}$ $\therefore M_0 = 4 e^{\frac{3}{2}k}$ $= 4 e^{\frac{3}{2} \times \frac{1}{2} \ln 2}$ $= 4 e^{\frac{3}{4} \ln 2}$ $= 6.727... \text{ kg}$ $= 6.73 \text{ kg (2dp)} \rightarrow$ $\therefore M_0 \text{ is } 6.73 \text{ kg}$	1 for recognising $M = \frac{1}{4} M_0$ 1 for final step 1 for setting up correct equation $\frac{1}{2}$ for display $\frac{1}{2}$ for ans to 2dp	• A large number of students used $M = \frac{3}{4} M_0$, maximum then was 1 mark. In this case the exact value for k was $\frac{1}{4} \ln(4/3)$ • Many students are still not showing how the initial equations are obtained • Again, the calculator display not written down • no marks docked if concluding statement omitted

QUESTION 5

EXPECTED ANSWERS	MARKS	COMMENTS
(c) $\frac{1 + \cot \theta}{\operatorname{cosec} \theta} - \frac{\sec \theta}{\tan \theta + \cot \theta}$ $= \frac{1 + \frac{\cos \theta}{\sin \theta}}{\frac{1}{\sin \theta}} - \frac{\frac{1}{\cos \theta}}{\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}}$ $= \sin \theta \left(1 + \frac{\cos \theta}{\sin \theta} \right) - \frac{1}{\cos \theta \left(\frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta} \right)}$ $= \sin \theta + \cos \theta - \frac{1}{\cos \theta \left(\frac{1}{\cos \theta \sin \theta} \right)}$ $= \sin \theta + \cos \theta - 1/\sin \theta$ $= \sin \theta + \cos \theta - \sin \theta$ $= \cos \theta \rightarrow$	$\frac{1}{2}$ for expressing everything in terms of $\sin \theta$ and/or $\cos \theta$ $\frac{1}{2}$ for simplifying 2nd term $\frac{1}{2}$ for $\sin^2 \theta + \cos^2 \theta = 1$ $1\frac{1}{2}$ for general simplification	Students are still skipping many parts of their working $\div \frac{1}{2}$ if $\sin^2 \theta + \cos^2 \theta = 1$ was not stated or expressed accordingly in the expression

2011 TRIAL HSC 2U MATHEMATICS: Question 6

Suggested Solutions	Marks	Marker's Comments
Q06 (i) $\int x \sin(x^2) dx = -\frac{1}{2}(\cos(x^2)) + k$	1	$\cos(x^2) + k$ without the minus loses $\frac{1}{2}$ mark
(ii) $\int_1^{16} x^{\frac{1}{2}} dx = \left[\frac{2}{3} x^{\frac{3}{2}} \right]_1^{16}$ $= \frac{2}{3}(64-1)$ $= 42$	1 1	
(b) $5x^2 - (21+a)x + a = 0$ Let the roots be a and $\frac{1}{a}$ Product of roots $= a \times \frac{1}{a}$ $= 1$ $\therefore \frac{c}{a} = 1$ where $c=a$ $a=5$ $\therefore \frac{a}{5} = 1$ <u>$a = 5$</u> $5x^2 - 26x + 5 = 0$ $(5x-1)(x-5) = 0$ Roots are 5 and $\frac{1}{5}$	1 1	1 for each root
(c) let a be the first term and d the common difference $S_{60} = \frac{60}{2}[2a + 59d] = 1770$ $6a + 177d = 177 \dots\dots(1)$ Also $a + 59d = 3a$ $2a = 59d$ $\therefore 6a = 177d \dots\dots(2)$ Sub (2) in (1) $177d + 177d = 177 \therefore d = \frac{1}{2}$ $\therefore$ lengths of shortest sides are $14\frac{3}{4}, 15\frac{1}{4}$ cms	$\frac{1}{2}$ $\frac{1}{2}$ 1 2	1 mark for each length
(d) Area $= \int_0^1 e^{-x^2} - (1-x^2) dx$ $= \left[2e^{-\frac{x^2}{2}} - x + \frac{x^3}{3} \right]_0^1$ $= 2e^{-\frac{1}{2}} - 1 + \frac{1}{3} - 2$ $\therefore$ Area $= (2\sqrt{e} - \frac{8}{3}) \text{ units}^2$ $= 0.6307758 \dots$	1 1 1	

1/2

MATHEMATICS: Question 7...

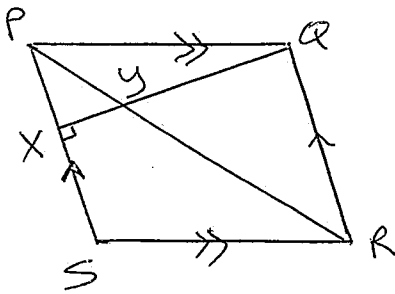
2011

Suggested Solutions

Marks

Marker's Comments

(a)



(i) $\angle SRP = \angle QRP$ (diagonals of a rhombus bisect the angles at the vertex).

(ii) In $\triangle YSR, YQR$

$\angle SRP = \angle QRP$ (proven above)

$QR = SR$ (sides of a rhombus are all equal).

YR is common

$\triangle YSR \equiv \triangle YQR$ (SAS)

(iii) $QR \parallel PS$ (opposite sides of a rhombus are parallel).

$\angle RQY = \angle QXP$ (alternate angles are equal, $QR \parallel PS$).

but $\angle QXP = 90^\circ$

$\therefore \angle RQY = 90^\circ$

$\therefore \angle YSR = \angle RQY$ (corresponding angles in congruent triangles are equal).

some students didn't use a ruler!!

1/2

1/2

1/2

1/2

1/2

1/2

1

1/2

1/2

1/2

1/2

Suggested Solutions

Marks

Marker's Comments

(b) (i) stops when $v=0$

$$\frac{t^2(t-3)}{3} = 0$$

$\therefore$ stops when $t=0$ or $t=3$

$\therefore$ Journey takes 3 minutes

(ii) $\frac{dx}{dt} = \frac{t^2(3-t)}{3} = \frac{3t^2}{3} - \frac{t^3}{3} = t^2 - \frac{t^3}{3}$

$$\therefore x = \frac{1}{3}t^3 - \frac{t^4}{12} + C$$

When $t=0$, $x=0 \therefore C=0$

$$\therefore x = \frac{1}{3}t^3 - \frac{1}{12}t^4$$

(iii) when $t=3$, $x = \frac{1}{3} \times 27 - \frac{1}{12} \times 3^4$
 $= 2\frac{1}{4}$

$\therefore$ It is $2\frac{1}{4}$ km from Epping to Eastwood $\frac{1}{2}$

(iv) $v = t^2 - \frac{t^3}{3}$

$$\frac{dv}{dt} = 2t - t^2$$

$$\frac{dv}{dt} = 0 \text{ at } t=0 \text{ or } t=2$$

$$\frac{d^2v}{dt^2} = 2 - 2t$$

when $t=2$, $\frac{d^2v}{dt^2} = 2 - 4 = -2 < 0$
 $\therefore$ concave down
 $\therefore$ relative maximum

when $t=2$, $x = \frac{1}{3}(2)^3 - \frac{1}{12}(2^4)$
 $= \frac{4}{3}$

$\therefore$ maximum speed at $x = \frac{4}{3}$ km
 after 2 minutes.

① right or wrong only

lost one mark if they forgot the "+ C".

$\frac{1}{2}$

$\frac{1}{2}$

$\frac{1}{2}$

If they didn't discuss why it was the fastest or test the nature at $t=2$, they lost $\frac{1}{2}$ a mark!!

MATHEMATICS: Question 8

Suggested Solutions

Marks

Marker's Comments

$$\begin{aligned} \text{a) i) } B_1 &= 1000 \times 1.06 - 72 \\ &= 988 \\ \therefore \text{Balance is } \$988. \end{aligned}$$

$\frac{1}{2}$

Interprets question

$\frac{1}{2}$

Evaluates

$$\begin{aligned} \text{ii) } B_2 &= B_1 \times 1.06 - 72 \\ &= 1000(1.06)^2 - 72(1.06) - 72 \\ &= 1000(1.06)^2 - 72(1.06 + 1) \end{aligned}$$

$$B_3 = 1000(1.06)^3 - 72(1.06^2 + 1.06 + 1)$$

$$\therefore B_n = 1000(1.06)^n - 72(1.06^{n-1} + \dots + 1.06 + 1)$$

1

Establish pattern

$$B_n = 1000(1.06)^n - 72 \left[\frac{1.06^n - 1}{1.06 - 1} \right] \text{ --- EQUATION ONE}$$

$\frac{1}{2}$

Identify GP

$$= 1000(1.06)^n - 1200(1.06^n - 1)$$

$$= 1000(1.06)^n - 1200(1.06)^n + 1200$$

$\frac{1}{2}$

Simplify

$$B_n = 1200 - 200(1.06)^n \text{ as required.}$$

$$\begin{aligned} \text{iii) } B_{10} &= 1200 - 200(1.06)^{10} \\ &= 843.83046 \dots \end{aligned}$$

$\frac{1}{2}$

Evaluating B_{10}

Let C_n be the balance at the end of n years after the prize increases.

$$\therefore C_n = 841.83(1.06)^n - 90 \left[\frac{1.06^n - 1}{1.06 - 1} \right] \text{ from EQUATION ONE}$$

$$= 841.83(1.06)^n - 1500(1.06^n - 1)$$

$$C_n = 1500 - 658.17(1.06)^n$$

$\frac{1}{2}$

Equation for new balance

The fund will run out when $C_n = 0$,

$$\text{i.e. } 1500 - 658.17(1.06)^n = 0$$

$$(1.06)^n = \frac{1500}{658.17}$$

$$n = \log_{1.06} \left[\frac{1500}{658.17} \right]$$

$$= \frac{\ln 1500 - \ln 658.17}{\ln 1.06}$$

$$n = 14.14 \dots$$

1

Make n the subject

$\therefore$ Full prize can be given for a further 14 years.

1

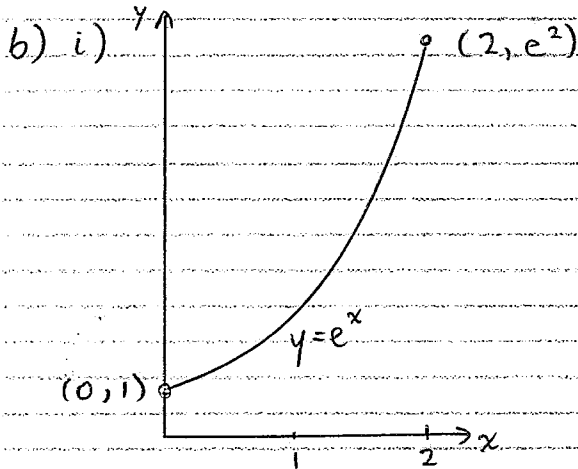
Final answer

MATHEMATICS: Question 8

Suggested Solutions

Marks

Marker's Comments



$\frac{1}{2}$

Accurate shape

$\frac{1}{2}$

Correct domain

ii) $y = e^x$

$$x = \ln y$$

$$x^2 = (\ln y)^2$$

$$V = \pi \int_a^b x^2 dy \quad \text{where } a=3, b=5$$

$$\therefore V = \pi \int_3^5 (\ln y)^2 dy$$

1

Make x the subject

1

Form integral

iii)

y	3	3.5	4	4.5	5
x	1.099	1.253	1.386	1.504	1.609
x^2	1.207	1.569	1.922	2.262	2.590

1

Function values

$$V = \pi \int_3^5 (\ln y)^2 dy$$

$$\approx \pi \times \frac{h}{3} (f_0 + 4f_1 + 2f_2 + 4f_3 + f_4)$$

$$\approx \frac{\pi}{6} [1.207 + 4(1.569) + 2(1.922) + 4(2.262) + 2.590]$$

$$\approx 12.02576 \dots$$

$$\approx 12.0 \text{ units}^3 \text{ (1dp)}$$

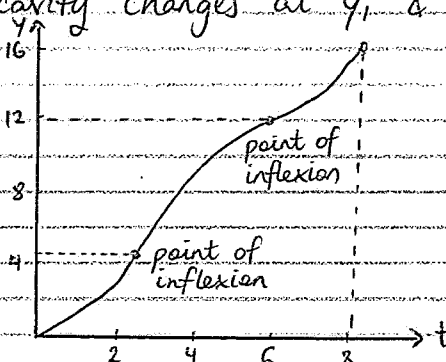
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Correct use of Simpson's formula

1

Approximated answer

MATHEMATICS: Question 9

Suggested Solutions	Marks	Marker's Comments
a) i) $\frac{dy}{dt}$ is minimum when glass is widest, i.e. $y_1 = 12$ ii) $\frac{dy}{dt}$ is maximum when glass is thinnest, i.e. $y_2 = 4\frac{1}{2}$ iii) Concavity changes at y_1 & y_2 	1 1 1 $\frac{1}{2}$ $\frac{1}{2}$	Accepted $\pm\frac{1}{2}$ Accepted $\pm\frac{1}{2}$ Points of inflexion Endpoint (8, 16) Shape (i.e. correct concavity)
b) As M bisects CO, $CM = 4\text{cm}$. Arc CD = $r\theta$ $= 8 \times \frac{2\pi}{3}$ $= \frac{16\pi}{3}$ or $5\frac{1}{3}\pi$ cm. In $\triangle DOM$, $MD^2 = 4^2 + 8^2 - 2(4 \times 8) \cos(\frac{2\pi}{3})$ $= 16 + 64 - (-32)$ $MD^2 = 112$ $MD = \sqrt{112}$ (as $MD > 0$) $= 4\sqrt{7}$ cm $\therefore$ Perimeter is $[4 + 4\sqrt{7} + \frac{16\pi}{3}]$ cm.	$\frac{1}{2}$ 1 1 $\frac{1}{2}$	Radius Arc length Cosine rule Simplify MD

MATHEMATICS: Question 9

Suggested Solutions	Marks	Marker's Comments
c) $\int_2^k \frac{2t}{3t^2-1} dt = \frac{1}{3} \int_2^k \frac{6t}{3t^2-1} dt$ $= \frac{1}{3} [\ln(3t^2-1)]_2^k$ $\frac{1}{3} \ln 13 = \frac{1}{3} [\ln(3k^2-1) - \ln(12-1)]$ $\ln 13 = \ln \left[\frac{3k^2-1}{11} \right]$ $\frac{3k^2-1}{11} = 13$ $3k^2-1 = 143$ $3k^2 = 144$ $k^2 = 48$ $k = \pm 4\sqrt{3}$ but $k > 2$ $\therefore k = 4\sqrt{3}$ only.	1 1 $\frac{1}{2}$ $\frac{1}{2}$	Integrate Eliminate log terms Simplify Consider restriction
d) Since a, b, c is a GP, $\frac{c}{b} = \frac{b}{a}$ $\therefore \log\left(\frac{c}{b}\right) = \log\left(\frac{b}{a}\right)$ $\log c - \log b = \log b - \log a$ $\therefore \log a, \log b, \log c$ is an AP. <u>ALTERNATIVE METHOD</u> Let r be the common ratio of the GP. $b = ar$ $\log b = \log(ar)$ $= \log a + \log r$ $c = ar^2$ $\log c = \log(ar^2)$ $= \log a + \log(r^2)$ $= \log a + 2\log r$ $\therefore \log a, \log b, \log c$ is an AP where the common difference is $\log r$.	1 1 1	Take logs of both sides Manipulate with log laws

2011 TRIAL HSC MATHEMATICS: Question...10

Suggested Solutions	Marks	Marker's Comments
10 (a) $25 - x^2 \geq 10$ $x^2 \leq 25$ $\therefore$ Domain: $-5 \leq x \leq 5$ Min value 0 at ends of range $\therefore$ Range: $0 \leq y \leq 10$	1 1	$\frac{1}{2}$ mark for each correct limit of domain or range
(b) $\sum_{r=1}^{63} \frac{1}{\sqrt{r+1} + \sqrt{r}} = \sum_{r=1}^{63} \frac{1}{\sqrt{r+1} + \sqrt{r}} \times \frac{\sqrt{r+1} - \sqrt{r}}{\sqrt{r+1} - \sqrt{r}}$ $= \sum_{r=1}^{63} \sqrt{r+1} - \sqrt{r}$ $= (\sqrt{1+1} - \sqrt{1}) + (\sqrt{2+1} - \sqrt{2}) + \dots + (\sqrt{63+1} - \sqrt{63})$ $= -\sqrt{1} + \sqrt{63+1}$ $= -1 + 8$ $= 7$	$\frac{1}{2}$ 1 $\frac{1}{2}$	
(c) (i) GP = $240 \tan \theta$ $\therefore$ DP = $3000 + 240 \tan \theta$ $\therefore$ Time = $\frac{DP}{\text{speed}} = \frac{3000 + 240 \tan \theta}{15}$ $= 200 + 16 \tan \theta$	$\frac{1}{2}$ $\frac{1}{2}$ 1	
(ii) FP = $240 \sec \theta$ $\therefore$ Time = $\frac{240 \sec \theta}{4}$ $= 60 \sec \theta$	1	$\frac{60}{\cos \theta}$, $\sqrt{3600(1 + \tan^2 \theta)}$ also received full marks
(iii) Time taken for Peter to reach P = $T + 60 \sec \theta$ If Peter and Bus arrive at P at the same time $T + 60 \sec \theta = 200 + 16 \tan \theta$ $\therefore T = 200 + 16 \tan \theta - 60 \sec \theta$	1	
(iv) $T = 200 + 16 \tan \theta - 60 \sec \theta$ $\frac{dT}{d\theta} = 16 \sec^2 \theta - 60 \sec \theta \tan \theta$ $\frac{dT}{d\theta} = 4 \sec \theta (4 \sec \theta - 15 \tan \theta)$ $\frac{dT}{d\theta} = 0$ for stationary pts As $4 \sec \theta \neq 0$	1	

$$(iv) \text{ cont. } \therefore 4 \sec \theta - 15 \tan \theta = 0$$

$$4 \sec \theta = 15 \tan \theta$$

$$\frac{4}{\cos \theta} = 15 \frac{\sin \theta}{\cos \theta}$$

$$\therefore \sin \theta = \frac{4}{15}$$

Test nature

θ	0.26	$\sin^{-1}(\frac{4}{15})$	0.27
$\frac{dT}{d\theta}$	0.6159	0	-0.0042
slope	/	-	/

There is a change in gradient from positive to negative hence a local max occurs at $\theta = \sin^{-1}(\frac{4}{15})$

As there is only one turning pt for $0.26 \leq \theta \leq 0.27$ then the local max. is also the absolute max.

$$\therefore \text{Maximum Time} = 200 + 16 \tan(\sin^{-1}(\frac{4}{15})) - 60 \sec(\sin^{-1}(\frac{4}{15}))$$

$$= 142.17 \text{ (2dp)}$$

$$\approx 142 \text{ seconds (nearest second)}$$

 $\frac{1}{2}$

or 2 minutes 22 seconds

OR Test nature

$$\frac{d^2T}{d\theta^2} = 4 \sec \theta (4 \tan \theta \sec \theta - 15 \sec^3 \theta) + (4 \sec \theta - 15 \tan \theta) \times 4 \tan \theta \sec \theta$$

$$= 16 \tan \theta \sec^3 \theta - 60 \sec^3 \theta + 16 \tan^2 \theta \sec^3 \theta$$

$$- 60 \tan^2 \theta \sec \theta$$

$$= 32 \tan^2 \theta \sec^3 \theta - 60 \sec^3 \theta - 60 \tan^2 \theta \sec \theta$$

$$= \frac{1}{\cos^3 \theta} [32 \sin^2 \theta - 60 - 60 \sin^2 \theta] \approx -62.254$$

$$\text{Now as } 60 > 32 \sin^2 \theta \quad \frac{d^2T}{d\theta^2} < 0$$

Hence the curve is concave down and a max exists when $\sin \theta = \frac{4}{15}$

$\therefore$ latest time that Peter can leave home and still catch the bus is

$$\underline{8:02:22 \text{ am}}$$

 $\frac{1}{2}$