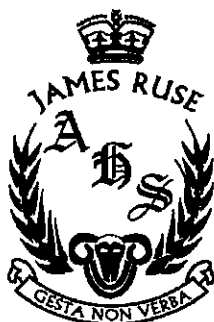


Name:	
Class:	



TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION 2019 MATHEMATICS

General Instructions:

- Reading Time: 5 minutes.
- Working Time: 3 hours.
- Write in black pen.
- Board approved calculators & templates may be used
- A Reference Sheet is provided.
- In Question 11 - 16, show all relevant mathematical reasoning and/or calculations.
- Marks may not be awarded for careless or badly arranged working.

Total Marks 100

Section I: 10 marks

- Attempt Question 1 – 10.
- Answer on the Multiple Choice answer sheet provided.
- Allow about 15 minutes for this section.

Section II: 90 Marks

- Attempt Question 11 - 16
- Answer on lined paper provided. Start a new sheet for each new question.
- Allow about 2 hours & 45 minutes for this section.

The answers to all questions are to be returned in separate *stapled* bundles clearly labelled Question 11, Question 12, etc. Each question must show your Candidate Number.

Section I

Multiple Choice (10 marks)

Attempt Question 1 – 10 (1 mark each)

Allow approximately 15 minutes for this section.

1. The graph of $y = e^{-x}$ is:

- A. Monotonically increasing
- B. Monotonically decreasing
- C. Neither monotonically increasing nor monotonically decreasing
- D. Cannot be determined

2. What is the 6th term of the series $10 - 15 + 22.5 - 33.75 \dots$

- A. -75.9375 B. 75.9375 C. -113.90625 D. 113.90625

3. Evaluate $\int_{-2}^2 4x^3 - 4x \, dx$:

- A. 0 B. 8 C. 16 D. 24

4. $\frac{3+\sqrt{2}}{3-\sqrt{2}} - 5$ is equivalent to:

- A. $\frac{-21(2+\sqrt{2})}{7}$ B. $\frac{2(23+3\sqrt{2})}{7}$ C. $\frac{6(5\sqrt{2}-8)}{7}$ D. $\frac{6(\sqrt{2}-4)}{7}$

5. The axis of symmetry for the parabola $y = -x^2 + 8x + 3$ is:

- A. y -axis B. $x = -4$ C. $x = 2$ D. $x = 4$

6. John has 10 marbles in a bag, 3 of which are green and the rest are yellow. He randomly draws 1 out of the bag, notes its colour and then puts it back before drawing another out of the bag. What is the probability that both marbles drawn are the same colour?

- A. 0.42 B. 0.58 C. 0.7 D. 3

7. The parabola with vertex $(-\frac{3}{2}, 0)$, focal length $\frac{1}{2}$, and exists only on the negative side of the y -axis is given by:

- A. $y^2 = 2(x + \frac{3}{2})$ B. $y^2 = -2(x + \frac{3}{2})$
C. $x^2 = 2(y + \frac{3}{2})$ D. $x^2 = -2(y + \frac{3}{2})$

8. The quadrilateral formed by $(1, 2), (3, 3), (1, 7), (-1, 3)$ is: (Give the most specific answer)

- A. Parallelogram B. Rhombus
C. Square D. Kite

9. For what values of m will the geometric series $1 + 2m^2 + 4m^4 + \dots$ have a limiting sum?

- A. $m < \frac{1}{2}$ B. $-\frac{1}{2} < m < \frac{1}{2}$ C. $-\frac{1}{4} < m < \frac{1}{4}$ D. $-\frac{1}{\sqrt{2}} < m < \frac{1}{\sqrt{2}}$

10. Suppose that the point $P(a, f(a))$ lies on the curve $y = f(x)$. If $f'(a) = 0$ and $f''(a) > 0$, which of the following statements describes the point P on the graph of $y = f(x)$?

- A. P is a maximum turning point B. P is a horizontal point of inflexion
C. P is a minimum turning point D. P is a point of inflexion

End of Section I

Section II**Total Marks is 90****Attempt Questions 11 – 16.****Allow approximately 2 hours & 45 minutes for this section.**

Answer all questions, starting each new question on a new sheet of paper with your **student ID number** in the top right hand corner and the question number on the left hand side of your paper.

All necessary working must be shown in each and every question.

Question 11 (15 marks)

a) Find the derivative of:

i. $(x^2 + 9x - 3)^3$ 1

ii. $\frac{x}{x^2+1}$ (Simplify your answer) 1

iii. $x^2\sqrt{x+1}$ (Simplify your answer fully over one denominator) 2

b) If $f(x) = \sqrt{x}$, show that $f'(x) = \frac{1}{2\sqrt{x}}$ using first principles. 2

c) A remote-controlled toy car travels in a straight line from an origin according to the equation $x = (t - 7)(t - 2)$ where x is in metres and t is in seconds.

i. Find the displacement of the car after 5 seconds. 1

ii. Find the velocity of the car in terms of t . 1

iii. According to the velocity expression, when will the car first change direction? 1

iv. The car unfortunately will malfunction if it reaches a speed of exactly 5 ms^{-1} , when will this happen first? 2

d) Let α and β be the roots of the quadratic equation $2x^2 + 4x - 9 = 0$. Find value of:

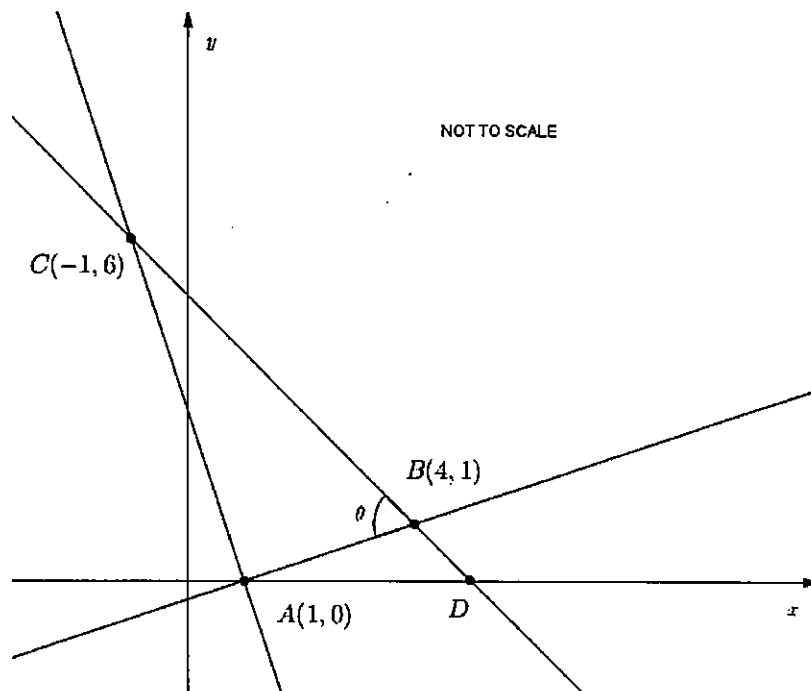
i. $\alpha + \beta$ 1

ii. $\alpha\beta$ 1

iii. $\alpha^3 + \beta^3$ 2

Question 12 (15 marks) Start a new sheet of paper

a)



The diagram shows points $A(1, 0)$ and $B(4, 1)$ and $C(-1, 6)$ in the Cartesian plane. $\angle ABC = \theta$ and D is where CB cuts the x -axis.

Copy or trace this diagram into your answer sheet.

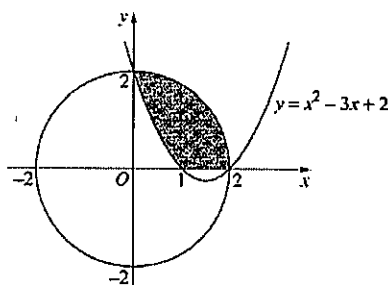
- i. Show that the line AC is given by $3x + y - 3 = 0$. 2
- ii. Hence, find the angle AC makes with the positive x -axis to the nearest degree. 1
- iii. Show that $AB \perp AC$. 2
- iv. Find the length of AB and AC . 2
- v. Hence, by considering the right angle $\triangle ABC$, find the value of θ to the nearest degree. 1
- vi. Find the coordinates of E , the midpoint of AC . 1
- vii. Find the area of $\triangle CEB$. 2
- viii. Suppose F lies on BC such that $EF \parallel AB$. Prove that $\triangle CEF \sim \triangle CAB$. 2
- ix. Hence, find the length of EF . Give reasons. 2

Question 13 (15 marks) Start a new sheet of paper

- a) Find $\frac{d}{dx}(\sin^2 x)$ 1
- b) Solve $\sqrt{3} \sin^2 \theta = \sin \theta \cos \theta$ for $0 \leq \theta \leq 2\pi$ (Give your answer in radians) 3
- c) Solve $\log_2 x + \log_2 (x - 3) = 2$ 3
- d) In a group of 10 birds, 5 are red and 5 are green. If three birds are selected at random, Find the probability that:
- i. They are all red 1
 - ii. At least one is green 1
- e) A coin is tossed continually, until for the first time successive tosses give the same result. Find the probability that the experiment finishes before the 4^{th} throw. 2
- f) The sum to n^{th} term of a series is given by $S_n = 3n^2 - 2n + 1$.
- i. Use the formula $T_n = S_n - S_{n-1}$ to show that the n^{th} term of the series is given by $T_n = 6n - 5$ 2
 - ii. It can be shown that the sum of this series is actually given by $S_n = C + T_1 + T_2 + \dots + T_n$ where C is a constant (DO NOT PROVE THIS FORMULA). Find the value of C . 2

Question 14 (15 marks) Start a new sheet of paper

a)



The shaded region in the diagram is bounded by the circle of radius 2 centred at the origin, the parabola $y = x^2 - 3x + 2$, and the x -axis.

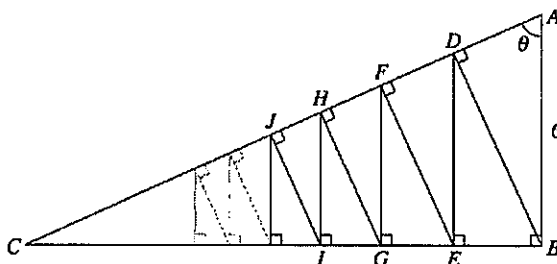
Find the area of the shaded region.

3

- b) Use Simpsons Rule with 5 function values to approximate $\int_0^{\pi} x \sin x \, dx$ correct to 2 decimal places.

3

c)

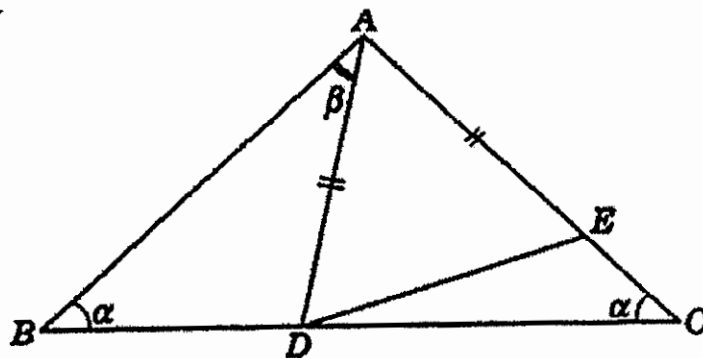


The triangle ABC has a right angle at B , $\angle BAC = \theta$ and $AB = 6$. The line BD is drawn perpendicular to AC . The line DE is then drawn perpendicular to BC . This process continues indefinitely as shown in the diagram. You may assume other angles are also θ without proof.

- i. Find the length of the interval BD , and hence show that the length of the interval EF is $6 \sin^3 \theta$. 2
 - ii. Show that the limiting sum $BD + EF + GH + \dots$ is given by $6 \sec \theta \tan \theta$ 3
- d) A restaurant offers choices of 3 vegetarian options, 3 meat options and 2 carb options. I wish to select **2 different** dishes.
- i. Draw a grid diagram (or dot diagram) to illustrate the sample space 2
 - ii. Hence or otherwise, find the probability that both my dishes belong to the same category. 1
 - iii. Amy wishes to also order 2 different dishes at random. Find the probability that we will have ordered exactly the same dishes. 1

Question 15 (15 marks) Start a new sheet of paper

a)



In the isosceles triangle ABC , $\angle ABC = \angle ACB = \alpha$. The points D and E lie on BC and AC respectively, so that $AD = AE$, as shown in the diagram. Let $\angle BAD = \beta$.

Copy the diagram onto your answer sheet.

- i. Find $\angle DAC$ in terms of α and β . Give reasons. 1
- ii. Hence or otherwise, find $\angle EDC$ in terms of β . Give reasons. 3

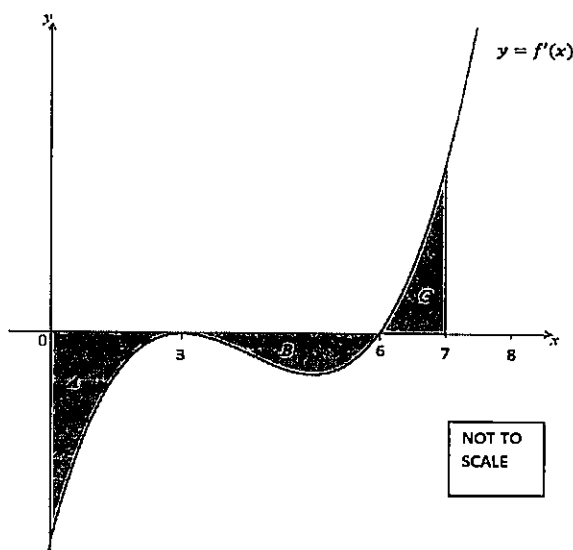
b) The flow rate of water first into and then out of a tank is given by

$$R = 3\pi t(12 - t) \text{ litres/second.}$$

- i. When does the water stop flowing into the tank? 1
 - ii. The tank was initially empty. Show that the volume of water in the tank is given by 2
- $$V = \pi t^2(18 - t)$$
- iii. Find the rate at which the water is flowing out of the tank when the tank is empty again. 1

Question 15 continues on the next page

c)



The regions A , B and C are enclosed by the curve $y = f'(x)$, the x -axis, the y -axis, and the line $y = 7$. The area of region A is $10u^2$, while the area of regions B and C are both $5u^2$.

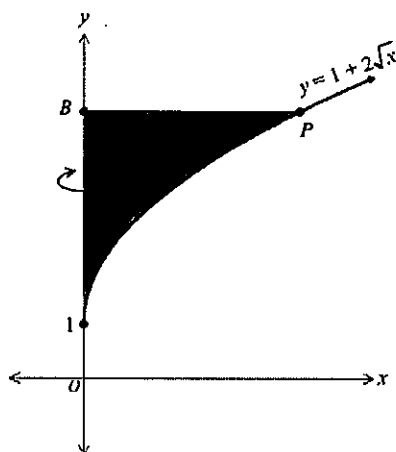
i. Given that $f(0) = 12$, find $f(7)$.

2

ii. Let $g(x) = (f(x))^2$. Given that $f'(7) = 6$, find the equation of the tangent to the graph $y = g(x)$ at $x = 7$.

2

d)



The shaded region in the diagram is bounded by the curve $y = 1 + 2\sqrt{x}$, the y -axis and the horizontal interval BP . The x -coordinate of P is 4.

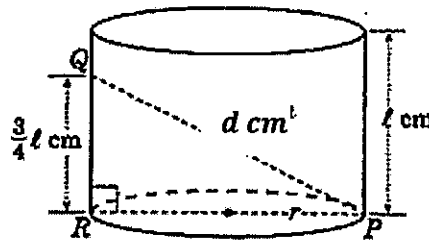
Calculate the exact volume of the solid of revolution formed when the shaded region is rotated about the y -axis.

3

PLEASE TURN OVER

Question 16 (15 marks) Start a new sheet of paper

a)



The diagram shows a hollow cylindrical tube of length l cm and radius r cm. P is a point on the circumference at one edge of the tube, at the very bottom of the rim of the tube. Q is a point on the opposite side of the tube, three-quarters of the way up the tube. RP is the diameter with R directly below Q . The length of PQ is d cm.

- i. Show that the volume of the tube is given by $V = \frac{\pi l}{4} (d^2 - \frac{9l^2}{16})$ 2
 - ii. If d remains constant, find l in terms of d that will give the maximum volume for the tube. 3
- b) A certain insect plague is following the law of natural growth. The insect population P satisfies the equation $P = P_0 e^{kt}$. Time t is measured in months and P_0 and k are constants.
- Initially there were 10 000 insects in the plague and after 8 months there were 40 000.
- i. Show that $P_0 = 10\,000$ and $k = \frac{1}{4} \ln 2$ 2
 - ii. After how many whole months would the population exceed 1 million? 2
- c) \$30 000 is borrowed at 9 % per annum reducible interest, calculated monthly. The loan is repaid in equal monthly instalments of \$ M over 5 years. Interest is charged before repayments are made every month.
- Let A_n be the amount owing after n monthly repayments.
- i. Show that $A_n = 30\,000 \times R^n - M \left(\frac{R^n - 1}{R - 1} \right)$ where $R = \frac{403}{400}$ 3
 - ii. Show that the monthly repayment is \$622.75 1
 - iii. What is the balance owing after the 24th payment? 2

End of Paper

Section I Multiple Choice (10 marks)

Attempt Question 1 – 10 (1 mark each)

Allow approximately 15 minutes for this section.

1. The graph of $y = e^{-x}$ is:

A. Monotonically increasing

B. Monotonically decreasing

C. Neither monotonically increasing nor monotonically decreasing

D. Cannot be determined

$$\frac{d}{dx} e^{-x} = -e^{-x} < 0$$

for all real x

$\therefore$ **B**

2. What is the 6th term of the series $10 - 15 + 22.5 - 33.75 \dots$

A. -75.9375

B. 75.9375

C. -113.90625

D. 113.90625

Geometric series, $r = -\frac{3}{2}$, $a = 10$.

$$\therefore T_6 = 10 \left(-\frac{3}{2}\right)^5$$

$$= -75.9375$$

$\therefore$ **A**

3. Evaluate $\int_{-2}^2 4x^3 - 4x \, dx$:

A. 0

B. 8

C. 16

D. 24

function is odd

$$\Rightarrow = 0$$

A

4. $\frac{3+\sqrt{2}}{3-\sqrt{2}} - 5$ is equivalent to:

A. $\frac{-21(2+\sqrt{2})}{7}$

B. $\frac{2(23+3\sqrt{2})}{7}$

C. $\frac{6(5\sqrt{2}-8)}{7}$

D. $\frac{6(\sqrt{2}-4)}{7}$

$$= \frac{(3+\sqrt{2})^2}{9-2} - 5 = \frac{11+6\sqrt{2}}{7} - 5 = \frac{6\sqrt{2}-24}{7} = \frac{6}{7}(\sqrt{2}-4) = \text{D}$$

5. The axis of symmetry for the parabola $y = -x^2 + 8x + 3$ is:

A. y - axis

B. $x = -4$

C. $x = 2$

D. $x = 4$

axis of symmetry at $-\frac{b}{2a} = \frac{8}{2} = 4$

$\therefore$ **D**

6. John has 10 marbles in a bag, 3 of which are green and the rest are yellow. He randomly draws 1 out of the bag, notes its colour and then puts it back before drawing another out of the bag. What is the probability that both marbles drawn are the same colour?

A. 0.42

B. 0.58

C. 0.7

D. 3

$$P(\text{same color}) = P(GG) + P(YY) = \left(\frac{3}{10}\right)^2 + \left(\frac{7}{10}\right)^2 = \frac{58}{100} \quad \therefore \boxed{B}$$

7. The parabola with vertex $(-\frac{3}{2}, 0)$, focal length $\frac{1}{2}$, and exists only on the negative side of the y-axis is given by:

A. $y^2 = 2(x + \frac{3}{2})$

B. $y^2 = -2(x + \frac{3}{2})$

C. $x^2 = 2(y + \frac{3}{2})$

D. $x^2 = -2(y + \frac{3}{2})$

$$x^2 = -4\left(\frac{1}{2}\right)\left(y + \frac{3}{2}\right) \quad \therefore \boxed{D}$$

$$= -2\left(y + \frac{3}{2}\right)$$

8. The quadrilateral formed by $(1, 2)$, $(3, 3)$, $(1, 7)$, $(-1, 3)$ is: (Give the most specific answer)

A. Parallelogram

B. Rhombus

C. Square

D. Kite

$(3, 3)$ and $(-1, 3)$ have same y-coordinate
 $(1, 2)$ and $(1, 7)$ have same x-coordinate
 $\therefore \boxed{D}$

9. For what values of m will the geometric series $1 + 2m^2 + 4m^4 + \dots$ have a limiting sum?

A. $m < \frac{1}{2}$

B. $-\frac{1}{2} < m < \frac{1}{2}$

C. $-\frac{1}{4} < m < \frac{1}{4}$

D. $-\frac{1}{\sqrt{2}} < m < \frac{1}{\sqrt{2}}$

$$-1 < 2m^2 < 1$$

$$-\frac{1}{\sqrt{2}} < m < \frac{1}{\sqrt{2}} \quad \therefore \boxed{D}$$

10. Suppose that the point $P(a, f(a))$ lies on the curve $y = f(x)$. If $f'(a) = 0$ and $f''(a) > 0$, which of the following statements describes the point P on the graph of $y = f(x)$?

A. P is a maximum turning point

B. P is a horizontal point of inflexion

C. P is a minimum turning point

D. P is a point of inflexion

$$f''(a) > 0$$

$\Rightarrow$ concave up

$\therefore$ minimum turning point

$\boxed{C}$

End of Section I

Suggested Solutions	Marks	Marker's Comments
a) i) $y = (x^2 + 9x - 3)$ $y' = 3(x^2 + 9x - 3) \cdot (2x + 9)$ — (1) ii) $y = \frac{x}{x^2 + 1}$ $y' = \frac{x^2 + 1 \cdot 1 - x(2x)}{(x^2 + 1)^2}$ $= \frac{1 - x^2}{(x^2 + 1)^2}$ — (1) iii. $y = x^2 \sqrt{x+1}$ $y' = (x+1)^{1/2} \cdot 2x + x^2 \cdot \frac{1}{2}(x+1)^{-1/2}$ — (1) $= \frac{2 \cdot 2x(x+1) + x^2}{2(\sqrt{x+1})}$ $= \frac{4x^2 + 4x + x^2}{2(\sqrt{x+1})} = \frac{5x^2 + 4x}{2(\sqrt{x+1})}$ — (1) OR. $y = x^2 (x+1)^{1/2}$ $y = (x^5 + x^4)^{1/2}$ $y' = \frac{1}{2}(x^5 + x^4)^{-1/2} \times (5x^4 + 4x^3)$ — (1) $= \frac{5x^4 + 4x^3}{2x^2 (x+1)^{1/2}}$ $= \frac{5x^2 + 4x}{2(\sqrt{x+1})}$ — (1)		simplify your answer.

Suggested Solutions	Marks	Marker's Comments
b) $f(x) = \sqrt{x}$ using 1st principles $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \times \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} \quad \text{--- (1)}$ $= \lim_{h \rightarrow 0} \frac{x+h - x}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h \rightarrow 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} \quad \text{--- (1)}$ $= \frac{1}{\sqrt{x} + \sqrt{x}}$ $= \frac{1}{2\sqrt{x}}$		
c) i) $x = ?$ when $t = 5$ $x = (-2)(3)$ $= -6m$ from origin --- (1) ii) $x = t^2 - 9t + 14$ --- (1) $v = 2t - 9$ iii) change direction when $v = 0$ $\therefore 2t = 9$ --- (1) $t = 4.5s$ iv) $v = 5$ --- (1) $5 = 2t - 9$ $-5 = 2t - 9$ $t = 7$ $t = 2$ $\therefore$ happens first when $t = 2s$ --- (1)		

Suggested Solutions

Marks

Marker's Comments

$$d) \quad 2x^2 + 4x - 9 = 0$$

$$i) \quad \alpha + \beta = -\frac{4}{2} = -2$$

$$ii) \quad \alpha\beta = \frac{-9}{2}$$

$$\begin{aligned} iii) \quad \alpha^3 + \beta^3 &= (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2) \\ &= (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta) \\ &= (\alpha + \beta)((\alpha + \beta)^2 - 2\alpha\beta - \alpha\beta) \\ &= -2 \times ((-2)^2 - (-\frac{9}{2})) \\ &= -35 \end{aligned}$$

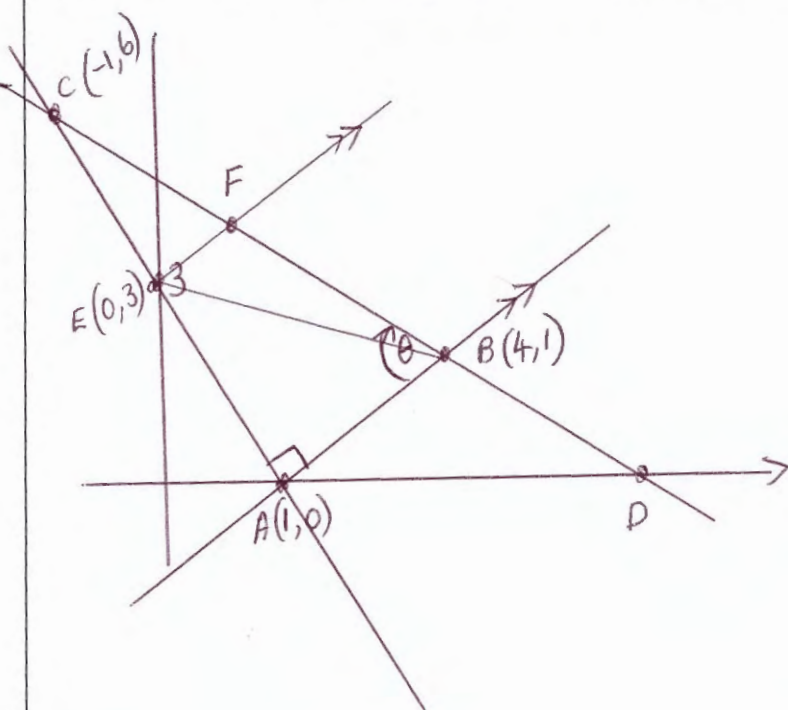
(1)

MATHEMATICS: Question 12 - page 1

Suggested Solutions

Marks

Marker's Comments



(i) $m_{AC} = -\frac{3}{1}$ & y intercept = 3
 $\therefore$ eqn of AC: $y = -3x + 3$
 $3x + y - 3 = 0$

(ii) $\tan \alpha = -\frac{3}{1} \therefore \alpha = 108^\circ$
 $\alpha = \text{Angle DAC}$

(iii) $m_{AB} = \frac{1-0}{4-1} \quad m_{AC} = -\frac{3}{1}$
 $= \frac{1}{3}$
 $m_{AB} \cdot m_{AC} = \frac{1}{3} \times -\frac{3}{1}$
 $= -1$
 $\therefore AB \perp AC$

Draw a diagram
 at least $\frac{1}{3}$ page
 in size.

②

①

②

MATHEMATICS: Question 2 - Page

Suggested Solutions	Marks	Marker's Comments
(iv) $d_{AB} = \sqrt{(4-1)^2 + (1-0)^2}$ $= \sqrt{9+1}$ $= \sqrt{10}$ $d_{AC} = \sqrt{(0-6)^2 + (1+1)^2}$ $= \sqrt{36+4}$ $= \sqrt{40}$ $= 2\sqrt{10}$	1	(2)
(v) In $\triangle ABC$ $\tan \theta = \frac{AC}{AB}$ $= \frac{2\sqrt{10}}{\sqrt{10}}$ $\theta = 63^\circ$	(1)	
(vi) $E = \left(\frac{1+(-1)}{2}, \frac{0+6}{2}\right)$ $E = (0, 3)$	(1)	
(vii) Area $\triangle CEB = \frac{1}{2} \times EC \times AB$ $= \frac{1}{2} \times \sqrt{(-1-0)^2 + (6-3)^2} \times \dots$ $= \frac{1}{2} \times \sqrt{10} \times \sqrt{10}$ $= \frac{10}{2}$ $= 5u^2$	1	(2)
Note! You could also use $A = \frac{1}{2} \times EC \times EB \times \sin \angle BEC.$		

MATHEMATICS: Question

Suggested Solutions	Marks	Marker's Comments
(viii) In $\triangle CEF$ and $\triangle CAB$ $\angle C$ is common $\angle CEF = \angle CAB$ (corresponding angles) $EF \parallel AB$ $\therefore \triangle CEF \sim \triangle CAB$ (equiangular).	1 1	finding equal angles (2) identifying triangles and giving reason for similarity.
(ix) $\frac{EF}{AB} = \frac{CE}{CA}$ (corresponding sides of similar triangles are in proportion (or same ratio)). $\frac{EF}{\sqrt{10}} = \frac{\sqrt{10}}{2\sqrt{10}}$ $EF = \frac{10}{2\sqrt{10}}$ $= \frac{\sqrt{10}}{2}$	1 1	(2)
(Alternatively, you could use $CE = \frac{1}{2}AC$ (a line drawn from one side of a triangle to another, parallel to the 3rd side, divides the other 2 sides in same ratio).		

MATHEMATICS: Question 13

Suggested Solutions

Marks

Marker's Comments

a) $\frac{d}{dx}(\sin^2 x) = 2 \sin x \cos x$
 $= \sin 2x$

1

accepted both answers

b) Solve $\sqrt{3} \sin^2 \theta = \sin \theta \cos \theta$ for $0 \leq \theta \leq 2\pi$

3

question says answer in radians

$\sqrt{3} \sin^2 \theta - \sin \theta \cos \theta = 0$

$\sin \theta (\sqrt{3} \sin \theta - \cos \theta) = 0$ ✓

$\sin \theta = 0$ or $\sqrt{3} \sin \theta - \cos \theta = 0$

$\theta = 0, \pi, 2\pi$ or $\sqrt{3} \sin \theta = \cos \theta$
 $\tan \theta = \frac{1}{\sqrt{3}}$

$\theta = \frac{\pi}{6}$ or $\frac{7\pi}{6}$

$\therefore \theta = 0^\circ, \frac{\pi}{6}, \pi, \frac{7\pi}{6}$ and 2π

Note: If students did only
 $\sqrt{3} \sin \theta = \cos \theta$
 got only 1 mark out of 3

1 mark for factorising

1 mark for solving
 $\sqrt{3} \sin \theta - \cos \theta = 0$

1 mark for final 5 answers

accepted
 0.52359 and
 3.665182

c) Solve $\log_2 x + \log_2 (x-3) = 2$

3

1 mark for simplifying logarithmic law

$\log_2 (x^2 - 3x) = \log_2 4$

$x^2 - 3x = 4$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x = 4$ or -1

but $x > 3$

$\therefore x = 4$ only

1 mark for factorising correctly

1 mark for accepting 4 only

MATHEMATICS: Question 13

Suggested Solutions	Marks	Marker's Comments
d) 10 birds $\begin{cases} 5 \text{ red} \\ 5 \text{ green} \end{cases}$ i) $p(\text{all red}) = \frac{5}{10} \times \frac{4}{9} \times \frac{3}{8} = \frac{1}{12}$ ii) $p(\text{at least one is green}) = 1 - p(\text{all red})$ $= 1 - \frac{1}{12} = \frac{11}{12}$	1 1	Getting $\frac{1}{12}$ Getting $\frac{11}{12}$
e) Find probability that expt finishes before the 4th row <u>Method 1</u> $\left. \begin{aligned} p(HH) &= \frac{1}{4} \\ p(TT) &= \frac{1}{4} \\ p(HTT) &= \frac{1}{8} \\ p(THT) &= \frac{1}{8} \end{aligned} \right\} \begin{aligned} &\text{add} \\ &\frac{3}{4} \end{aligned}$ <u>Method 2</u> $\begin{aligned} &1 - p(\text{not finishes}) \\ &= 1 - \left(\frac{1}{8} + \frac{1}{8} \right) \\ &= \frac{3}{4} \end{aligned}$ <u>Method 3</u> $\begin{aligned} P(\text{even}) &= P_2 + P_3 \\ &= \frac{\text{probability for second throw}}{4} + \frac{\text{probability for 3rd throw}}{8} + \frac{1}{8} \\ &= \frac{1}{4} + \frac{1}{4} + \frac{1}{8} + \frac{1}{8} \\ &= \frac{3}{4} \end{aligned}$	2	1 mark for correct working of probabilities 1 mark for getting the answer $\frac{3}{4}$.
f) (i) $S(n) = 3n^2 - 2n + 1$ and use $T_n = S_n - S_{n-1}$ to find $T_n = 6n - 5$ $\begin{aligned} S(n-1) &= 3(n-1)^2 + 2(n-1) + 1 \\ &= 3(n^2 - 2n + 1) - 2(n-1) + 1 \\ &= 3n^2 - 6n + 3 - 2n + 2 + 1 \\ &= 3n^2 - 8n + 6 \end{aligned}$ So $S_n - S_{n-1}$ gives	(2)	✓ 1 mark for finding S_{n-1} and ✓ mark for working $T_n = S_n - S_{n-1}$ to give $6n - 5$

Suggested Solutions

Marks

Marker's Comments

$$\therefore T_n = (3n^2 - 2n + 1) - (3n^2 - 8n + 6)$$

$$= 6n - 5$$

$$\therefore T_n = 6n - 5$$

$$(ii) S_n = C + T_1 + T_2 + \dots + T_n$$

Show it is an AP.

$$T_1 = 1$$

$$T_2 = 7$$

$$T_3 = 13$$

$$T_4 = 19 \dots$$

Method 1

$$S_2 = 3(2^2) - 2(2) + 1$$

$$= 9$$

From formula

$$S_2 = C + T_1 + T_2$$

$$9 = C + 1 + 7$$

$$\therefore C = 1$$

Method 2

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$= \frac{n}{2} [2 + (n-1)6]$$

$$= \frac{n}{2} [6n - 4]$$

$$= 3n^2 - 2n$$

$$\text{Compare with } S_n = 3n^2 - 2n + 1$$

$$3n^2 - 2n + C = 3n^2 - 2n + 1$$

$$\therefore C = 1$$

1 mark.

Show it is an AP

working

getting value of C.

MATHEMATICS: Question 14

Suggested Solutions

Marks

a) Area of the shaded Region.

3

A = Area of first quadrant of a circle

$$- \int_0^1 (x^2 - 3x + 2) dx$$

$$= \frac{\pi(2^2)}{4} - \left[\frac{x^3}{3} - \frac{3x^2}{2} + 2x \right]_0^1$$

$$= \pi - \left(\frac{1}{3} - \frac{3}{2} + 2 \right)$$

$$= \pi - \frac{5}{6}$$

Area is $\pi - \frac{5}{6}$ unit².

1 mark for finding area of quarter of circle as π

Correct limits and integrating $x^2 - 3x + 2$

Correct working to get answer $\pi - \frac{5}{6}$

b). Simpson's rule with 5 function values.

3

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	0	$\frac{\pi}{4\sqrt{2}}$	$\frac{\pi}{2}$	$\frac{3\pi}{4\sqrt{2}}$	0

$$\int_0^\pi x \sin x \cdot dx$$

$$I \approx \frac{\pi}{12} \left[(0 + 0) + 4 \left(\frac{\pi}{4\sqrt{2}} + \frac{3\pi}{4\sqrt{2}} \right) + 2 \left(\frac{\pi}{2} \right) \right]$$

$$= \frac{\pi}{12} \left[\frac{4\pi}{\sqrt{2}} + \pi \right]$$

$$= \frac{\pi}{12} \left[\frac{4\pi + \sqrt{2}\pi}{\sqrt{2}} \right]$$

$$= \frac{\pi^2}{12\sqrt{2}} (4 + \sqrt{2})$$

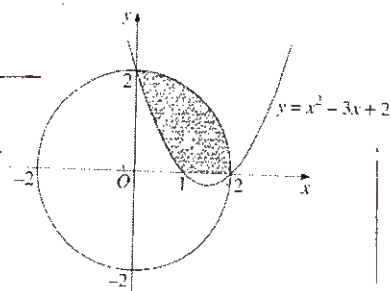
$$= \frac{\pi^2 (2\sqrt{2} + 1)}{12}$$

$$= 3.15$$

1 mark to get $\frac{\pi}{12} \left(\frac{1 \pm 3}{4} \right)$

correct use of formula.

Correct manipulations to get 3.15



Suggested Solutions	Marks	Marker's Comments
c) Find the length of BD and show interval EF is $6\sin^3\theta$ i) $\sin\theta = \frac{BD}{6}$ $\therefore BD = 6\sin\theta$ $\angle EBD = \theta$ also $\sin\theta = \frac{DE}{BD} = \frac{DE}{6\sin\theta}$ $\therefore DE = 6\sin^2\theta$ $\angle FDG = \theta$ $\sin\theta = \frac{EF}{DE} = \frac{EF}{6\sin^2\theta}$ $\therefore EF = 6\sin^3\theta$	2,	> show BD is $6\sin\theta$ Working to get to $6\sin^3\theta$
(ii) show that the limiting sum $BD + EF + GH + \dots$ is given by $6\sec\theta \tan\theta$ Series is $6\sin\theta + 6\sin^3\theta + 6\sin^5\theta + \dots$ $a = 6\sin\theta$ $r = \sin^2\theta$ Sum = $\frac{a}{1-r}$ $= \frac{6\sin\theta}{1-\sin^2\theta}$ $= \frac{6\sin\theta}{\cos^2\theta}$ $= 6 \frac{\sin\theta}{\cos\theta} \times \frac{1}{\cos\theta}$ $= 6 \tan\theta \sec\theta$	3	1 mark to getting the series and values of a and r 1 mark for correct formula and substitution Showing trig identities and leading to $6\tan\theta \sec\theta$ Note if students $\frac{6\sin\theta}{\cos^2\theta} = 6 \tan\theta \sec\theta$

Suggested Solutions

Marks

Marker's Comments

d). (i) 3 VG, 3 meat and 2 carb.

2.

$M_1, M_2, M_3, V_1, V_2, V_3, C_1, C_2$

M_1	///	///	///	///	///	///	///	///	///
M_2	///	///	///	///	///	///	///	///	///
M_3	///	///	///	///	///	///	///	///	///
V_1	///	///	///	///	///	///	///	///	///
V_2	///	///	///	///	///	///	///	///	///
V_3	///	///	///	///	///	///	///	///	///
C_1	///	///	///	///	///	///	///	///	///
C_2	///	///	///	///	///	///	///	///	///

56 possible outcome

(ii) $p(\text{same category})$ ie $p(VV) = \frac{6}{56}$
 $p(MM) = \frac{6}{56}$
 $p(CC) = \frac{2}{56}$

$\therefore p(\text{same category}) = \frac{14}{56} = \frac{1}{4}$ → 1 mark

(iii) Any wishes to order 2 different dishes at random. find $p(\text{ordered exactly the same dishes})$.

ways of picking V_1, V_2 and V_2, V_1 is 2

$\therefore \frac{2}{56} = \frac{1}{28}$ → 1 mark

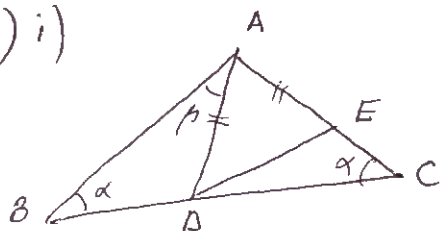
To get full marks one had to show $M_1, M_1, M_2, M_2 \dots$ etc does not exist

Suggested Solutions

Marks

Marker's Comments

a) i)



$$\angle DAC + \beta + \alpha + \alpha = 180 \text{ (angle sum of } \triangle ABC)$$

$$\therefore \angle DAC = 180 - 2\alpha - \beta \quad \text{--- (i)}$$

$$\text{ii) } \angle ADC = \alpha + \beta \text{ (exterior angle of } \triangle ABD)$$

$$\angle DAC + \angle AED + \angle ADE = 180 \text{ (angle sum of } \triangle ADE)$$

$$\left(\begin{array}{l} \angle ADE = \angle AED \text{ (equal angles opposite equal sides)} \\ 180 - 2\alpha - \beta + 2\angle ADE = 180 \end{array} \right.$$

$$2\angle ADE = 180 - 180 + 2\alpha + \beta$$

$$\angle ADE = \alpha + \frac{\beta}{2}$$

$$\angle ADE = \alpha + \frac{\beta}{2}$$

$$\angle EDC = \angle ADC - \angle ADE \text{ (angle subtraction)}$$

$$= \alpha + \beta - \left(\alpha + \frac{\beta}{2} \right)$$

$$= \frac{\beta}{2}$$

1 mark for $\angle EDC$

remaining 2 marks

for $\angle ADC$, $\angle ADE$

with reasoning

$$\text{b) } R = 3\pi t(12 - t)$$

$$\text{i) stop flowing when } R = 0$$

$$\therefore t = 0 \text{ or } 12 \quad t > 0$$

$$\text{but } t \text{ is } 12 \text{ s only} \quad \text{--- (1)}$$

$$\text{ii) Tank empty.}$$

$$\int R = 36\pi t - 3\pi t^2$$

$$V = 18\pi t^2 - \pi t^3 + C \quad \text{--- (1)}$$

Suggested Solutions	Marks	Marker's Comments
when $t=0$, $v=0$ $\therefore 0 = 18\pi(0)^2 - \pi(0)^3 + C$ (i) to evaluate value of C $\therefore C = 0$ So $v = \pi t^2(18 - t)$ iii) when $v=0$ $t=0$ or 18 when $t=18$ $R = 36\pi(18)(12 - 18)$ $= -324\pi \text{ L/s}$ (i) c) $A = \int_0^3 f'(x)$ given $f(0)=12$ $= [f(x)]_0^3$ $-10 = f(3) - f(0)$ $-10 = f(3) - 12$ $f(3) = 2$ $B = \int_3^6 f'(x) = [f(x)]_3^6$ $-5 = f(6) - f(3)$ $f(6) = -5 + 2$ $= -3$ $C = \int_6^7 f'(x) = [f(x)]_6^7$ $5 = f(7) - f(6)$ $f(7) = 5 + (-3)$ $= 2$ (i)		to find value of t in R and must have units. 1 mark to show an understanding how to get to $f(7)$

Suggested Solutions

Marks Awarded

Marker's Comments

ii) $g(x) = (f(x))^2$, $f'(7) = 6$, $f(7) = 2$

Eqn of tangent to $y = g(x)$ at $x = 7$

$$g(x) = (f(x))^2$$

$$g'(x) = 2f(x) \cdot f'(x)$$

when $x = 7$

$$g'(x) = 2(2) \times 6 = 24 \quad \text{--- (1)}$$

when $x = 7$ $y = (f(7))^2$ (cfe) !!

(1) $y = 4$

Eqn of tangent.

$$y - 4 = 24(x - 7)$$

(1) $y = 24x - 164$

must have correct y value
ie it has to be squared from (i)
only then a student could
get 2 marks for this part

d) $y = 1 + 2\sqrt{x}$ rotate abt y axis.

$$\therefore 2\sqrt{x} = y - 1$$

$$x = \left(\frac{y-1}{2}\right)^2 = \frac{(y-1)^2}{4}$$

when $x = 4$ at pt B

$$y = 1 + 2\sqrt{4} = 5.$$

$$V = \pi \int_1^5 \left(\frac{(y-1)^2}{4}\right)^2$$

$$= \frac{\pi}{16} \int_1^5 (y-1)^4$$

$$= \frac{\pi}{16} \left[\frac{(y-1)^5}{5} \right]_1^5 = \frac{\pi}{16 \times 5} (4^5 - 0^5)$$

$$= \frac{64\pi}{5} u^3$$

1 correct limit

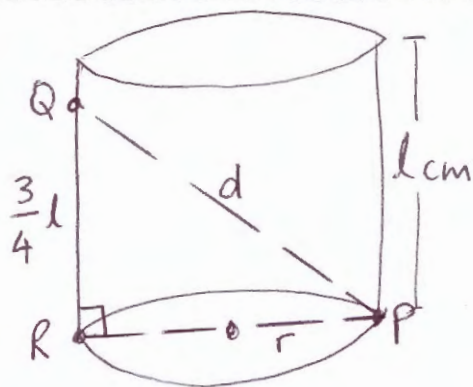
1 mark has π , and eqn squared.

correct value from
substitution.

Suggested Solutions

Marks

Marker's Comments



(i) $V = \pi r^2 l$

In $\triangle RPQ$

$(2r)^2 + \left(\frac{3}{4}l\right)^2 = d^2$ (by Pythagoras' Theorem)

$4r^2 + \frac{9}{16}l^2 = d^2$

$4r^2 = d^2 - \frac{9}{16}l^2$

$r^2 = \frac{d^2}{4} - \frac{9}{4 \times 16}l^2$

$\therefore V = \pi r^2 l$
 $= \pi \left(\frac{d^2}{4} - \frac{9}{4 \times 16}l^2 \right) l$
 $= \frac{\pi}{4} l \left(d^2 - \frac{9}{16}l^2 \right)$

(ii) Maximum volume occurs when $\frac{dV}{dl} = 0$

$V = \frac{\pi}{4} l d^2 - \frac{9\pi l^3}{4 \times 16}$

$\frac{dV}{dl} = \frac{\pi d^2}{4} - \frac{27\pi l^2}{64}$

(2)

— given answer

Suggested Solutions	Marks	Marker's Comments
Max when $\frac{dV}{dl} = 0$ ie $\frac{\pi d^2}{4} - \frac{27\pi l^2}{64} = 0$ $\frac{16\pi d^2}{64} - \frac{27\pi l^2}{64} = 0$ $16\pi d^2 - 27\pi l^2 = 0$ $27\pi l^2 = 16\pi d^2$ $l^2 = \frac{16\pi d^2}{27\pi l^2}$ $l = \frac{4d}{\sqrt{27}} \quad l > 0$ To check concavity $\frac{d^2V}{dl^2} = -\frac{54\pi}{64}$ Since $d > 0$ and $\frac{d^2V}{dl^2} < 0$ & $l > 0$ $\therefore$ concave down $\therefore$ max volume when $l = \frac{4d}{\sqrt{27}}$ (or $\frac{4d}{3\sqrt{3}}$)	1 1	(3) Needed to check concavity.

MATHEMATICS: Question

Suggested Solutions	Marks	Marker's Comments
(b) $P = P_0 e^{kt}$ Initially $t=0$ $P=10000$ $\therefore 10000 = P_0 e^{0k}$ $\underline{10000 = P_0}$ $\therefore P = 10000 e^{kt}$ Subst $t=8$ & $P=40000$ to find k. $40000 = 10000 e^{8k}$ $e^{8k} = \frac{40000}{10000}$ $e^{8k} = 4$ $\log_e 4 = 8k$ $k = \frac{\log_e 4}{8}$ $k = \frac{\ln 4}{8}$ $k = \frac{\ln 2^2}{8}$ $k = \frac{2 \ln 2}{8}$ $k = \frac{\ln 2}{4}$	1 2	needed to this.

Suggested Solutions

Marks

Marker's Comments

(ii) Population exceeds 1 million when

$$10000 e^{\frac{\ln 2}{4} t} > 1000000$$

$$\text{ie } e^{\frac{\ln 2}{4} t} > 100$$

$$\log_e e^{\frac{\ln 2}{4} t} > \log_e 100$$

$$\frac{\ln 2}{4} t > \log_e 100$$

$$\ln 2 t > 4 \ln 100$$

$$t > \frac{4 \ln 100}{\ln 2}$$

$$t > 26.16$$

∴ Popn exceeds 1000000 after 27 mths (nearest month).

(c) $P = 30000$ $9\% \text{ pa} = \frac{9}{12 \times 100}$
 $= \frac{3}{400}$
 5 years = 60 months.

Let $R = 1 + \frac{3}{400} = \frac{403}{400}$

Amount owing after 1 month

$$A_1 = 30000 \times R - M$$

$$A_2 = (30000 \times R - M) R - M$$

$$= 30000 R^2 - MR - M$$

(2)

[illegible]