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Student Number

2025

HSC TRIAL EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes.
- Working time – 3 hours.
- Write using a black pen.
- Write your Student Number at the top of every page.

Total Marks – 100

Section I – Pages 3 – 5

10 marks

Allow about 15 minutes for this section.

Sections II-IV – Pages 7 – 30

90 marks

- Attempt questions from questions 11 – 34.
- Answer all questions in the booklets provided.
- Allow about 2 hours and 45 minutes for this section.

This paper MUST NOT be removed from the examination room.

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Section I

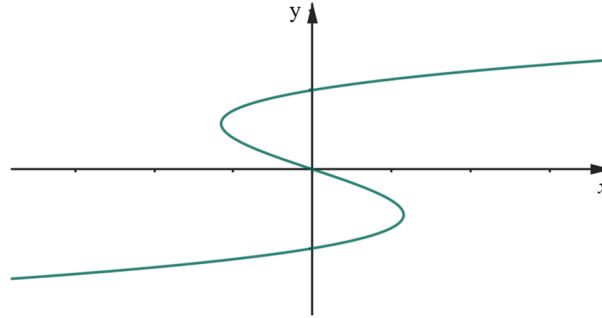
10 marks.

Attempt Questions 1–10.

Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for questions 1 – 10.

1. The graph of a relation is shown below.



What type of relationship does the graph represent?

- (A) One-to-one
 - (B) One-to-many
 - (C) Many-to-one
 - (D) Many-to-many
2. A function $f(x)$ is given such that:

$$\int_{-k}^k f(x) dx = 2 \int_0^k f(x) dx.$$

Which of the following could be a possible function for $f(x)$?

- (A) $f(x) = \sin 2x$
 - (B) $f(x) = \cos 2x$
 - (C) $f(x) = \tan 2x$
 - (D) $f(x) = \operatorname{cosec} 2x$
3. A die is rolled twice. Let events A and B be:

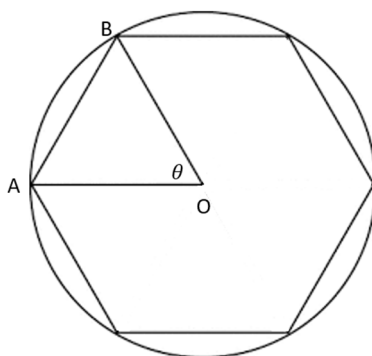
$$X = \{\text{The first roll is a '1'}\}$$

$$Y = \{\text{Sum of two rolls is '7'}\}$$

Which of the following statements is true?

- (A) Events X and Y are independent.
- (B) Events X and Y are mutually exclusive.
- (C) Event X is a subset of event Y .
- (D) The probabilities of events X and Y are different.

4. Ben scored 53 on a test where the scores were normally distributed with a mean of 41 and a standard deviation of 7.5. If the scores are adjusted to have a new mean of 44 and a new standard deviation of 6, what would Ben's adjusted score be, to the nearest whole number?
- (A) 52
(B) 53
(C) 54
(D) 55
5. Let $f(x) = x^3 - 3x^2 + 2$. In which domain is the function decreasing and concave down?
- (A) $(-\infty, 0)$
(B) $(0, 1)$
(C) $(1, 2)$
(D) $(2, \infty)$
6. A regular hexagon is inscribed inside a circle. Given the area of the $\triangle ABO = 14 \text{ cm}^2$, what is the perimeter of the hexagon to the nearest centimetre?



NOT TO SCALE

- (A) 43.9 cm
(B) 42.0 cm
(C) 34.1 cm
(D) 31.0 cm
7. If $f(x) = \sqrt{x}$ and $g(x) = \ln(x - 1)$, what is the domain of the composite function $f(g(x))$?
- (A) $[1, \infty)$
(B) $[1, e]$
(C) $[2, e]$
(D) $[2, \infty)$

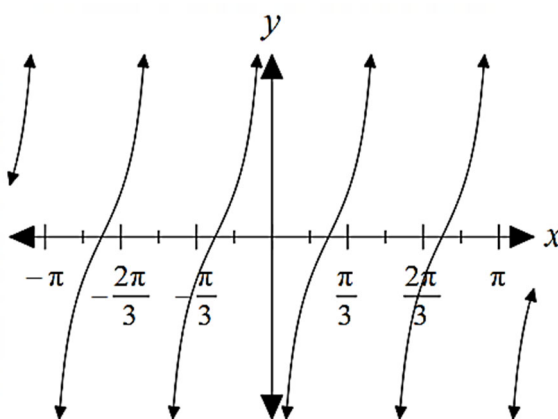
8. A probability density function is given below:

$$f(x) = \begin{cases} x - 1, & 1 \leq x \leq a \\ 0, & \text{else where} \end{cases}$$

Which one of the following, is the correct way to find the value of 'a'?

- (A) The larger root of $a^2 - 2a - 1 = 0$
- (B) The smaller root of $a^2 - 2a - 1 = 0$
- (C) The larger root of $a^2 + 2a - 1 = 0$
- (D) The smaller root of $a^2 + 2a - 1 = 0$

9. Which of the following functions best represents the graph below?



- (A) $y = \tan\left(4x + \frac{\pi}{2}\right)$
- (B) $y = \tan\left(4x + \frac{\pi}{4}\right)$
- (C) $y = \tan\left(2x + \frac{\pi}{2}\right)$
- (D) $y = \tan\left(2x + \frac{\pi}{4}\right)$

10. The function $f(x) = e^x$ is transformed by a horizontal translation of $\log_e 2$ units in the negative x direction followed by a horizontal dilation of scale factor 2. What is the new function $g(x)$?

- (A) $g(x) = 2e^{\frac{x}{2}}$
- (B) $g(x) = \frac{1}{2}e^{\frac{x}{2}}$
- (C) $g(x) = \frac{1}{2}e^{2x}$
- (D) $g(x) = 2e^{2x}$

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Mathematics Advanced

Section II Answer Booklet

30 marks.
Attempt Questions 11–19.

Instructions

- Write your student number at the top of each page.
 - Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
 - Your response should include relevant mathematical reasoning and/or calculations.
 - Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.
-

Question 11 (2 marks)**Marks**Evaluate x when $x = \log_{31} 32 \times \log_{30} 31 \times \log_{29} 30 \times \dots \times \log_2 3$ for $x \in \mathbb{Z}$.

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Question 12 (5 marks)**Marks**Given the equation of a circle is: $x^2 - 4x + y^2 + 2y - 4 = 0$.

a) Determine the centre and radius.

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b) The line $y = mx + 2$ is a tangent to the circle. Determine the values of m .

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Question 13 (4 marks)**Marks**

The ages of 26 million residents of Australia are normally distributed with mean of 45 years and standard deviation of 11 years.

- a) Calculate how many more people are older than 56 than are older than 67. 2

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- b) Determine the percentage, rounded to the nearest whole number, of individuals under 56 who are also younger than 34. 2

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Question 14 (2 marks)

Solve $3 \sin^2 x - \sqrt{3} \sin x = 0$ for $-\pi \leq x \leq \pi$ to 3 decimal places. 2

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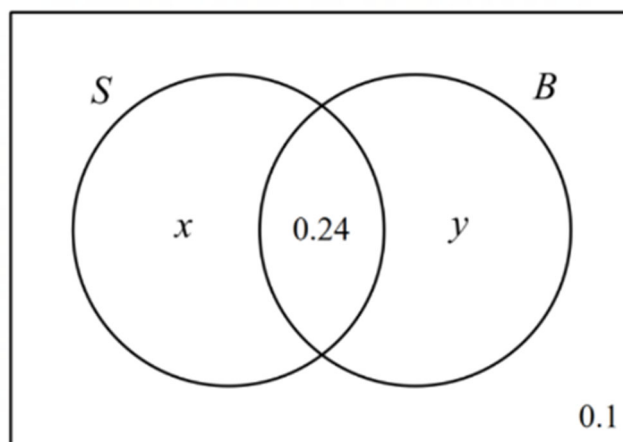
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Question 15 (3 marks)**Marks**

The subjects that a group of university students take is recorded. Out of the group, 24% of the students do both Statistics (S) and Business (B), while 10% do neither.



- a) It is known that the number of students in the group that take Statistics is twice the number of students that take Business. Find the values of x and y in the Venn diagram above. 2

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- b) If a student is selected at random, find the probability that they are not taking Statistics given that they are studying Business. 1

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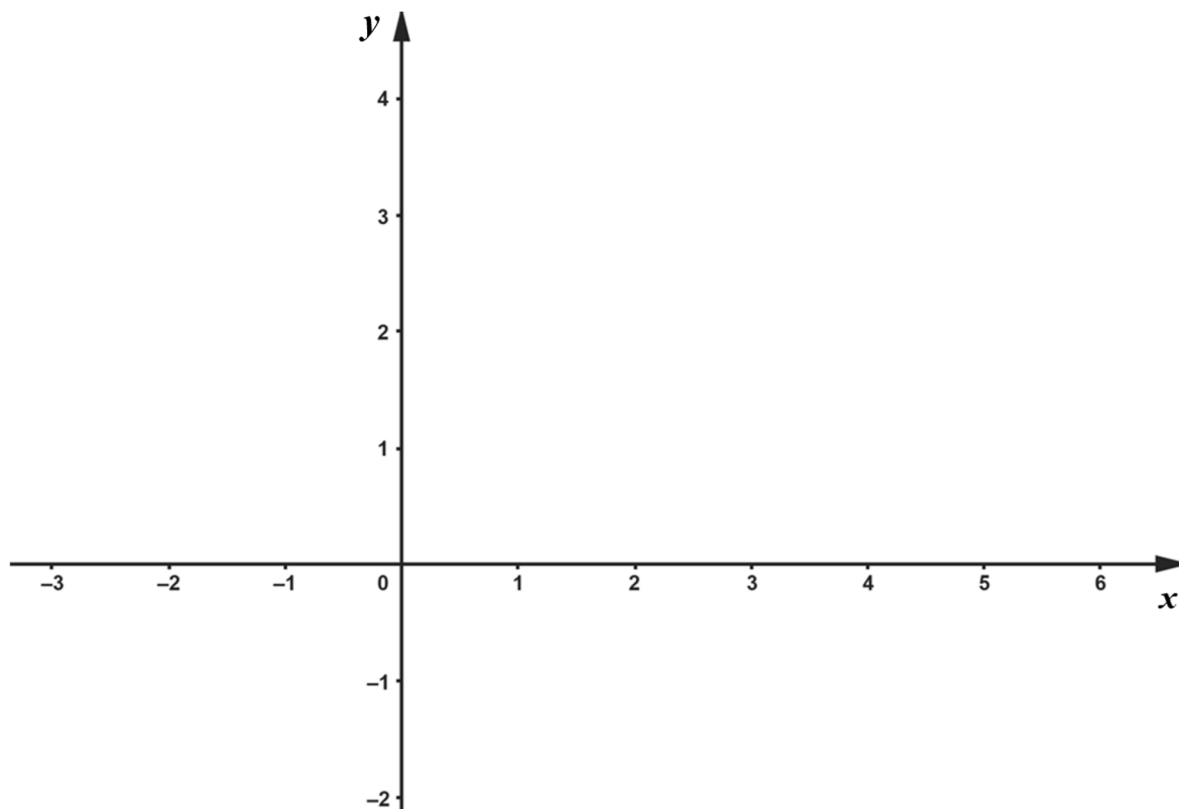
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Question 16 (3 marks)**Marks**

Consider the functions:

$$f(x) = |2x - 4| \text{ and } g(x) = |x + 1|$$

- a) On the same number plane, sketch the graphs of $f(x)$ and $g(x)$, showing all important features. 2



- b) Use your graph to solve the inequality: 1

$$|2x - 4| < |x + 1|$$

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Question 17 (3 marks)**Marks**The derivative of a function is given by: $f'(x) = e^x \sin x$.

- a) Differentiate and simplify $g(x) = e^x \sin x - e^x \cos x$. 1

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b) Hence or otherwise, find $f(x)$ if it is known that $f(\pi) = \frac{e^\pi}{2}$. 2

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Question 18 (4 marks)

Marks

Given the series $1\frac{1}{3} + 2\frac{1}{9} + 4\frac{1}{27} + 8\frac{1}{81} \dots$. Find the expression of the:

a) general term. 2

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b) sum. 2

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The decibel level of a sound is given by the formula:

$$L = 10 \log_{10} \left(\frac{I}{10^{-12}} \right)$$

Where L is the sound level in decibels, I is the intensity of the sound in watts per square meter.

- a) A chocolate making machine produces a sound with intensity of $2.5 \times 10^{-5} \text{ W/m}^2$.
Calculate the sound level of the machine to 4 decimal places.1

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- b) A nearby industrial coffee grinder produced a sound that is 20 decibels louder than the chocolate making machine. What is the percentage increase in sound intensity produced by the coffee grinder compared to the chocolate making machine? Leave your answer to the nearest whole number.3

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End of Section II

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If you use this space, clearly indicate which question(s) you are answering. Leave at least two lines between each question.

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Mathematics Advanced

Section III Answer Booklet

30 marks.

Attempt Questions 20–27.

Instructions

- Write your student number at the top of each page.
 - Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
 - Your response should include relevant mathematical reasoning and/or calculations.
 - Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.
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Question 20 (7 marks)**Marks**Let $f(x) = x^3 - 6x^2 + 9x + 1$.

a) Find any stationary points and determine their nature.

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b) Find any points of inflexion.

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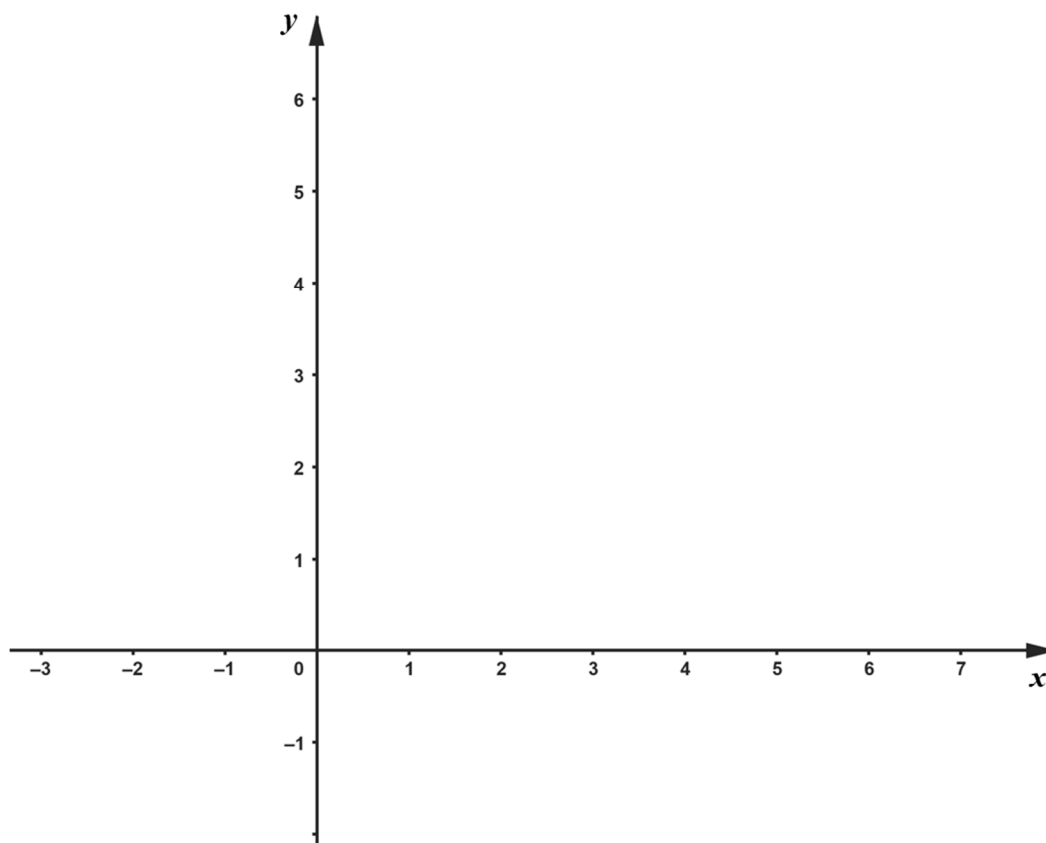
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c) Sketch the curve, showing all important features (you do not need to find the x intercept).

2

Question 21 (1 marks)**Marks**

The z scores on a test are normally distributed such that $P(-N < z < N) = p$ for some constants, $N > 0$ and $0 < p < 1$. What is the value of $P(z < N)$?

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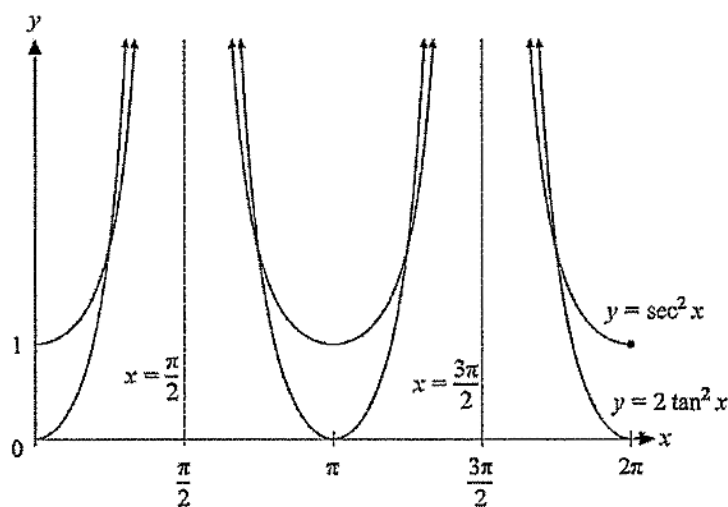
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Question 22 (3 marks)**Marks**

The diagram shows the graphs of the curves $y = \sec^2 x$ and $y = 2 \tan^2 x$ for $0 \leq x \leq 2\pi$.



- a) State the periods of each of the two curves.

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- b) Show that the x coordinates of the points of intersection of the two curves are solutions of the equation $\tan^2 x = 1$ and solve this equation for $\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$.

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Question 23 (2 marks)**Marks**

By using the trapezoidal rule with 4 subintervals, estimate the area under the curve for: 2

$$\int_0^4 \ln(x+1) dx.$$

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Question 24 (3 marks)**Marks**

Let the equation of a curve be $f(x) = \frac{x}{e^{x+1}}$. Find the equation of the normal to the curve at the point where $x = 1$ in general form. 3

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Question 25 (3 marks)

Marks

A car travels at 40 km/hr on a circular curve whose radius is 500 m.

- a) Find the distance, l km, that the car travels in one minute.
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- b) Calculate the size of the angle θ through which the car turns in 2 minutes. Give your answer to the nearest degree.
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Question 26 (3 marks)

Marks

Differentiate $f(x) = \ln(\tan^2 x)$ and simplify your answers in terms of $\sin x$ and $\cos x$.

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A continuous random variable, X has the following probability density function.

$$f(x) = \begin{cases} e^{-\frac{1}{2}x}, & \text{for } 0 \leq x \leq k \\ 0, & \text{for all other values of } x \end{cases}$$

a) Find the exact value of k .

2

b) Write the cumulative distribution function $F(x)$ of X for $-\infty < x < \infty$.

2

c) Write the simplest exact form of the median of X .

1

- d) Let X be the duration, in seconds, it takes for a student card to be registered by the school scanner. In words, interpret the value of the median in the context of this situation. 1

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- e) Given that $3P(X < c) = 2P(X > c)$, evaluate c to 3 decimal places. 2

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End of Section III

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Mathematics Advanced

Section IV Answer Booklet

30 marks.

Attempt Questions 28–34.

Instructions

- Write your student number at the top of each page.
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Marks

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Marks

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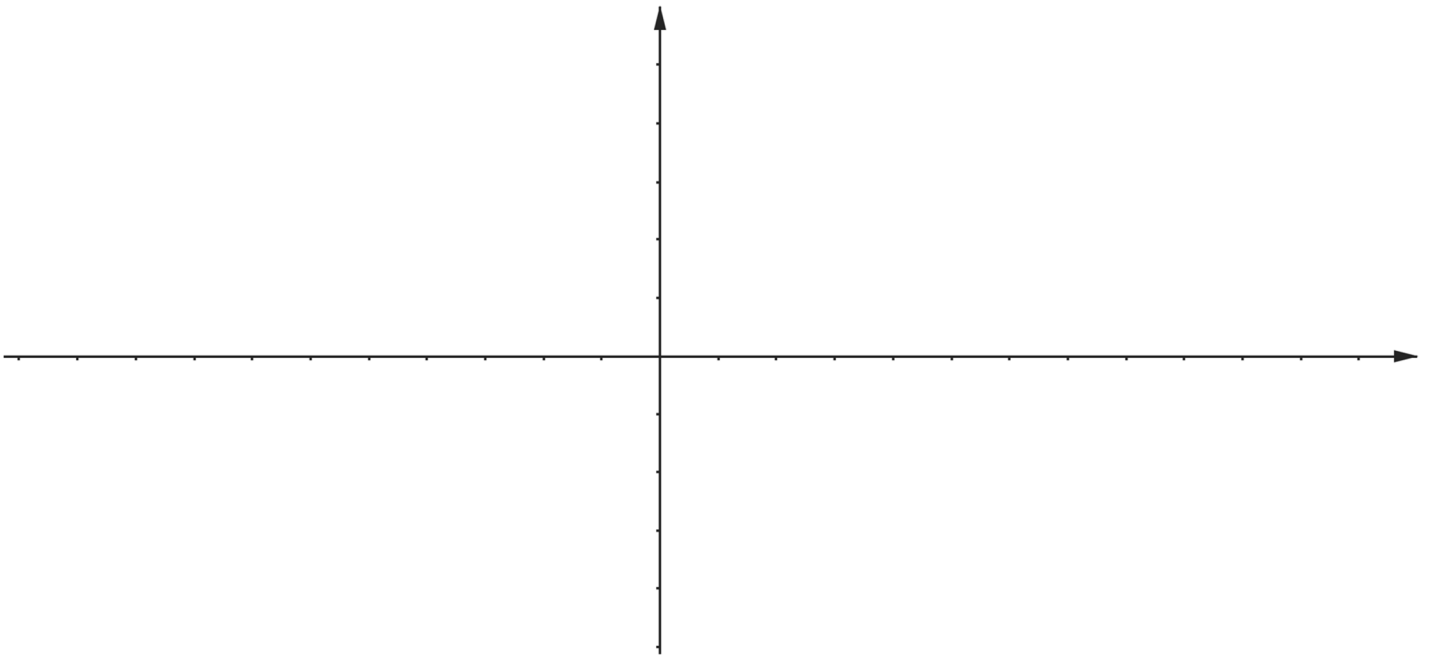
Marks

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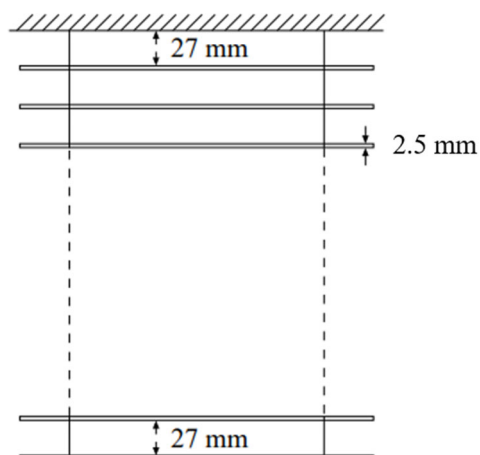
Question 31 (2 marks)**Marks**

Sketch the graph: $y = -2\cos(2\pi x)$ for $-1 \leq x \leq 1.5$ and show all important features. 2

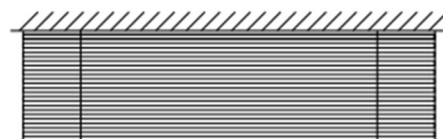
**Question 32** (5 marks)**Marks**

A venetian blind consists of twenty slats, each 2.5mm thick.

When the blind is down, the gap between the top slat and the top of the blind is 27mm and the gap between adjacent slats is also 27mm, as shown in the diagram below.



When the blind is up, all the slats are stacked at the top with no gaps, as shown in the diagram below.



a) Show that when the blind is raised, the bottom slat rises 540 mm. 2

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b) How far does the 19th slat rise? 1

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c) Explain briefly why the distances the slats rise form an arithmetic sequence. 1

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d) Find the sum of all the distances that the slats rise when the blind is raised. 1

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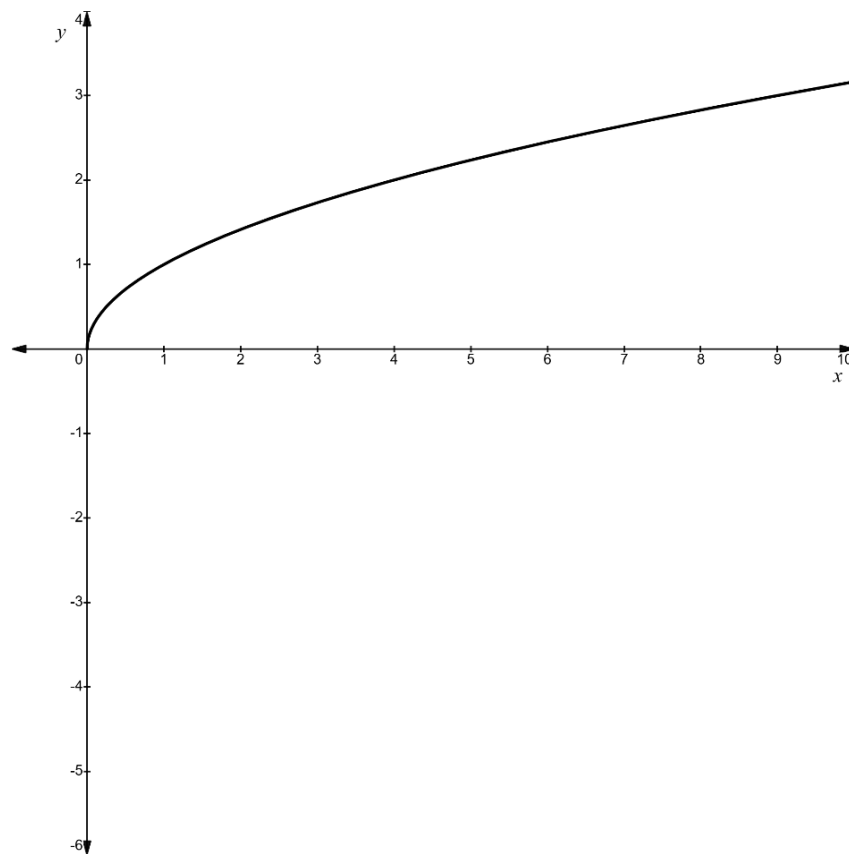
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Question 33 (7 marks)**Marks**

The graph of $f(x) = \sqrt{x}$ is given below.



a) On the same set of axes, sketch the graph of $g(x) = -2\sqrt{x-1} + 3$. 2

b) Describe the sequence of transformations applied to $f(x)$ to obtain $g(x)$. 2

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- c) Find the area enclosed between the curve $g(x)$, the line $y = 4 - x$ and the x -axis.

Considering only the region that lies above the x -axis.

3

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Question 34 (6 marks)**Marks**

The hours of daylight in Sydney can be modelled by:

$$y(t) = 2.24 \sin\left(\frac{\pi t}{6} + 1.34\right) + 12.19,$$

where y , the number of hours of daylight and t , is the months of the year.

(i.e.: $t = 1$ represents January 1st; ignoring leap years)

- a) In this model, what is the maximum number of daylight hours in one day? 1

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- b) Find the month that has the longest day in terms of number of hours of daylight. 2

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- c) A scientist needs 13 hours of continuous daylight for an experiment. Between which dates should they be looking to run the experiment? 3

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End of Examination

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If you use this space, clearly indicate which question(s) you are answering. Leave at least two lines between each question.

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Section IV extra writing space.

If you use this space, clearly indicate which question(s) you are answering. Leave at least two lines between each question.

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- 1) B 2) B 3) A 4) C 5) B
6) C 7) D 8) A 9) C 10) A

Section I

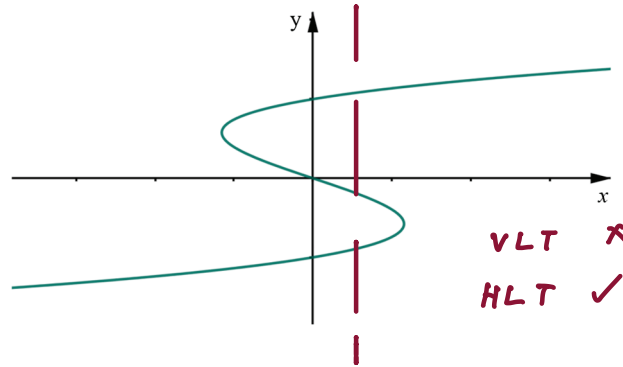
10 marks.

Attempt Questions 1–10.

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(B) One-to-many
(C) Many-to-one
(D) Many-to-many
2. A function $f(x)$ is given such that:

$$\int_{-k}^k f(x) dx = 2 \int_0^k f(x) dx.$$

Which of the following could be a possible function for $f(x)$?

- (A) $f(x) = \sin 2x$
(B) $f(x) = \cos 2x$
(C) $f(x) = \tan 2x$
(D) $f(x) = \operatorname{cosec} 2x$



$2 \times \int_0^k$ must be even func

3. A die is rolled twice. Let events A and B be:

$$X = \{\text{The first roll is a '1'}\}$$

$$Y = \{\text{Sum of two rolls is '7'}\}$$

Which of the following statements is true?

- (A) Events X and Y are independent.
(B) Events X and Y are mutually exclusive.
(C) Event X is a subset of event Y .
(D) The probabilities of events X and Y are different.

	1	2	3	4	5	6
1	1,1	1,2	1,3	1,4	1,5	1,6
2					7	
3				7		
4			7			
5		7				
6	7					

4. Ben scored 53 on a test where the scores were normally distributed with a mean of 41 and a standard deviation of 7.5. If the scores are adjusted to have a new mean of 44 and a new standard deviation of 6, what would Ben's adjusted score be, to the nearest whole number?

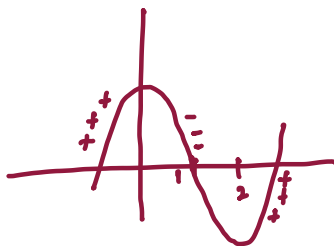
- (A) 52
(B) 53
(C) 54
(D) 55

$$z = \frac{53 - 41}{7.5} = 1.6$$

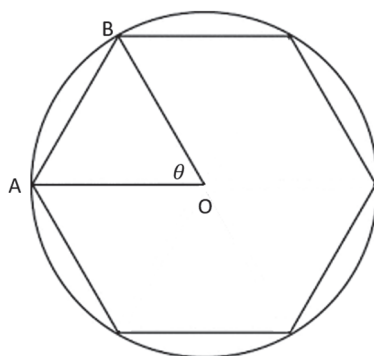
$$1.6 = \frac{x - 44}{6} = 53.6$$

5. Let $f(x) = x^3 - 3x^2 + 2$. In which domain is the function decreasing and concave down?

- (A) $(-\infty, 0)$
(B) $(0, 1)$
(C) $(1, 2)$
(D) $(2, \infty)$



6. A regular hexagon is inscribed inside a circle. Given the area of the $\triangle ABO = 14 \text{ cm}^2$, what is the perimeter of the hexagon to the nearest centimetre?



NOT TO SCALE

- (A) 43.9 cm
(B) 42.0 cm
(C) 34.1 cm
(D) 31.0 cm

$$\begin{aligned} A &= \frac{1}{2} r^2 \sin \theta \\ 14 &= \frac{1}{2} r^2 \sin \frac{\pi}{3} \\ 14 &= \frac{1}{2} \times \frac{\sqrt{3}}{2} r^2 \\ r &= 5.6 \text{ (} r > 0 \text{)} \\ p &= 6 \times 5.6 \dots \\ &= 34.1 \end{aligned}$$

7. If $f(x) = \sqrt{x}$ and $g(x) = \ln(x - 1)$, what is the domain of the composite function $f(g(x))$?

- (A) $[1, \infty)$
(B) $[1, e]$
(C) $[2, e]$
(D) $[2, \infty)$

$$\begin{aligned} f(x) &= \sqrt{x} & D: x &\geq 0 \\ g(x) &= \ln(x-1) & D: x &> 1 \\ f(g(x)) &= \sqrt{\ln(x-1)} \\ \text{when } x &= 2, \quad \sqrt{\ln 1} = 0 \\ \therefore D: x &> 2 \end{aligned}$$

8. A probability density function is given below:

$$f(x) = \begin{cases} x-1, & 1 \leq x \leq a \\ 0, & \text{else where} \end{cases}$$

$$\int_1^a (x-1) dx$$

$$1 = \int_1^a \left(\frac{x^2}{2} - x \right) dx$$

$$1 = \left(\frac{a^2}{2} - a \right) - \left(\frac{1}{2} - 1 \right)$$

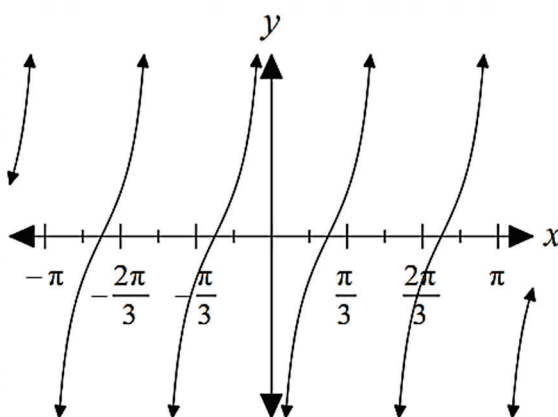
$$0 = \frac{a^2}{2} - a - \frac{1}{2}$$

$$0 = a^2 - 2a - 1$$

Which one of the following, is the correct way to find the value of 'a'?

- (A) The larger root of $a^2 - 2a - 1 = 0$
- (B) The smaller root of $a^2 - 2a - 1 = 0$
- (C) The larger root of $a^2 + 2a - 1 = 0$
- (D) The smaller root of $a^2 + 2a - 1 = 0$

9. Which of the following functions best represents the graph below?



$$p = \frac{\pi}{2}$$

$$n = \frac{\pi}{\frac{\pi}{2}} = 2$$

- (A) $y = \tan\left(4x + \frac{\pi}{2}\right)$
- (B) $y = \tan\left(4x + \frac{\pi}{4}\right)$
- (C) $y = \tan\left(2x + \frac{\pi}{2}\right)$
- (D) $y = \tan\left(2x + \frac{\pi}{4}\right)$

$$y = \tan\left(2\left(x + \frac{\pi}{4}\right)\right)$$

$$= \tan\left(2x + \frac{\pi}{2}\right)$$

10. The function $f(x) = e^x$ is transformed by a (horizontal translation of $\log_e 2$ units in the negative x direction) followed by a (horizontal dilation of scale factor 2.) What is the new function $g(x)$?

- (A) $g(x) = 2e^{\frac{x}{2}}$
- (B) $g(x) = \frac{1}{2}e^{\frac{x}{2}}$
- (C) $g(x) = \frac{1}{2}e^{2x}$
- (D) $g(x) = 2e^{2x}$

①

$$y = e^{x + \ln 2}$$

$$= e^x \cdot e^{\ln 2}$$

$$= 2e^x$$

②

$$y = 2e^{\frac{x}{2}}$$

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Mathematics Advanced

Section II Answer Booklet

30 marks.
Attempt Questions 11–19.

Instructions

- Write your student number at the top of each page.
 - Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
 - Your response should include relevant mathematical reasoning and/or calculations.
 - Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.
-

Question 11 (2 marks)

Marks

Evaluate x when $x = \log_{31} 32 \times \log_{30} 31 \times \log_{29} 30 \times \dots \times \log_2 3$ for $x \in \mathbb{Z}$.

2

$$\frac{\log 32}{\log 31} \times \frac{\log 31}{\log 30} \times \frac{\log 30}{\log 29} \times \dots \times \frac{\log 4}{\log 3} \times \frac{\log 3}{\log 2}$$

$$\therefore \frac{\log 32}{\log 2} = \frac{\log 2^5}{\log 2^1} = 5 \quad \text{to simplify}$$

(1) (1)

to realise the pattern

Done well!

Question 12 (5 marks)

Marks

Given the equation of a circle is: $x^2 - 4x + y^2 + 2y - 4 = 0$.

a) Determine the centre and radius.

2

$$(x-2)^2 + (y+1)^2 = 4 + 2^2 + 1^2$$

$$= 9$$

Done well!

$$(cc) = (2, -1) \quad r = 3$$

(1) (1)

b) The line $y = mx + 2$ is a tangent to the circle. Determine the values of m .

3

For tangent $\Delta = 0$ sub $y = mx + 2 \rightarrow (y+1)$

$$(x-2)^2 + (mx+2+1)^2 = 9$$

$$x^2 - 4x + 4 + m^2x^2 + 6mx + 9 = 9$$

$$x^2 + m^2x^2 - 4x + 6mx + 4 + 9 - 9 = 0$$

$$x^2(1+m^2) - x(6m-4) + 4 = 0$$

$$\rightarrow (6m-4)^2 - 4(m^2+1)(4) = 0 \quad \text{--- (1)}$$

$$36m^2 - 48m + 16 - 16m^2 - 16 = 0$$

$$20m^2 - 48m = 0$$

$$* 4m(5m - 12) = 0$$

$$4m = 0 \quad \text{or} \quad 5m - 12 = 0$$

$$m = 0 \quad m = \frac{12}{5}$$

Both solutions (1)

* mistake - m^2x^2 - many students wrote mx^2
- factorising $20m^2 - 48m = 0$. students didn't write $4m$

Question 13 (4 marks)

① - correct probability using calc or table

Marks

① - correct answer in 'million'

The ages of 26 million residents of Australia are normally distributed with mean of 45 years and standard deviation of 11 years.

a) Calculate how many more people are older than 56 than are older than 67.

2

Well done!

-1σ	-2σ	-1σ	$\bar{x}$	1σ	2σ	3σ	μ_1	$x_1 = 56$	$z_1 = 1$
12	23	34	45	56	67	78	μ_2	$x_2 = 67$	$z_2 = 2$

$M_2 = 13.5\% \times 26 \times 10^6$
 $= 3.51 \times 10^6$

$M_3 \quad P(x > 67) = 1 - 0.9772 = 0.0228$
 $P(x > 56) = 1 - 0.8413 = 0.1587$
 $\therefore (0.1587 - 0.0228) \times 26 \times 10^6$

$= 0.9772 - 0.8413$
 $= 0.1359 \times 26 \times 10^6$
 $= 3.533400 \times 10^6$

b) Determine the percentage, rounded to the nearest whole number, of individuals under 56 who are also younger than 34.

2

Well done!

-1σ	-2σ	-1σ	$\bar{x}$	1σ	2σ	3σ	M_2	$P(x < 34 x < 56)$
12	23	34	45	56	67	78		$= \frac{P(x < 34 \cap x < 56)}{P(x < 56)}$

$M_1 \quad \frac{P(34)}{P(56)} = \frac{16\%}{84\%}$
 $= 19\%$

$= \frac{0.1587}{0.8413}$
 $= 19\%$

- understanding its CP $\uparrow$
 writing the probability correctly — ①
 - correct rounded answer — ①

Question 14 (2 marks)

Solve $3 \sin^2 x - \sqrt{3} \sin x = 0$ for $-\pi \leq x \leq \pi$ to 3 decimal places.

2

Mistakes!

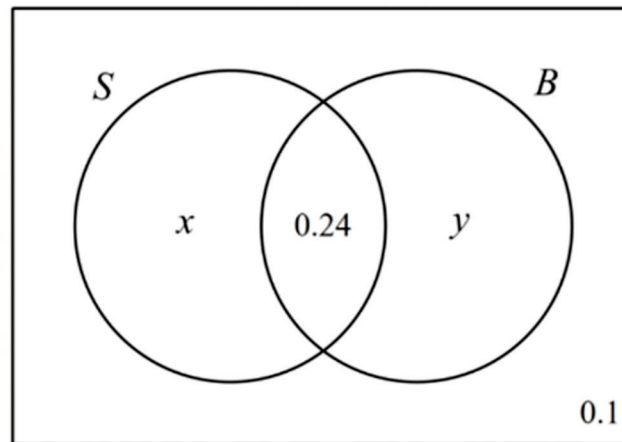
$\sin x (3 \sin x - \sqrt{3}) = 0$
 $\sin x = 0$
 $\therefore x = -\pi, 0, \pi$ — ①
 $3 \sin x = \sqrt{3}$
 $x = \sin^{-1} \frac{\sqrt{3}}{3}$
 $= 0.615, 2.5262$ — ①

- missed all solutions for $\sin x = 0$
 - gave answers in degree
 - gave answers in wrong quadrants and NOT 3 d.p

Question 15 (3 marks)

Marks

The subjects that a group of university students take is recorded. Out of the group, 24% of the students do both Statistics (S) and Business (B), while 10% do neither.



- a) It is known that the number of students in the group that take Statistics is twice the number of students that take Business. Find the values of x and y in the Venn diagram above. 2

$$x + y + 0.24 + 0.1 = 1 \quad \text{or} \quad x + y + 0.24 = 0.9 \quad \text{Well done!}$$

$$\therefore x + y = 0.66 \quad \text{--- i}$$

$$S = 2B \quad \text{1 mark for } x$$

$$x + 0.24 = 2(y + 0.24) \quad \text{1 mark for } y$$

$$x = 2y + 0.24 \quad \text{--- ii}$$

$$\text{ii} \rightarrow \text{i}$$

$$2y + 0.24 + y = 0.66$$

$$y = 0.14 \quad y \rightarrow \text{i}, \therefore x = 0.52$$

- b) If a student is selected at random, find the probability that they are not taking Statistics given that they are studying Business. 1

$$P(\bar{S}|B) = \frac{P(\bar{S} \cap B)}{P(B)}$$

$$= \frac{0.14}{0.24 + 0.14}$$

$$= \frac{7}{19} \quad \text{or} \quad 0.3684 \quad \text{Well done!}$$

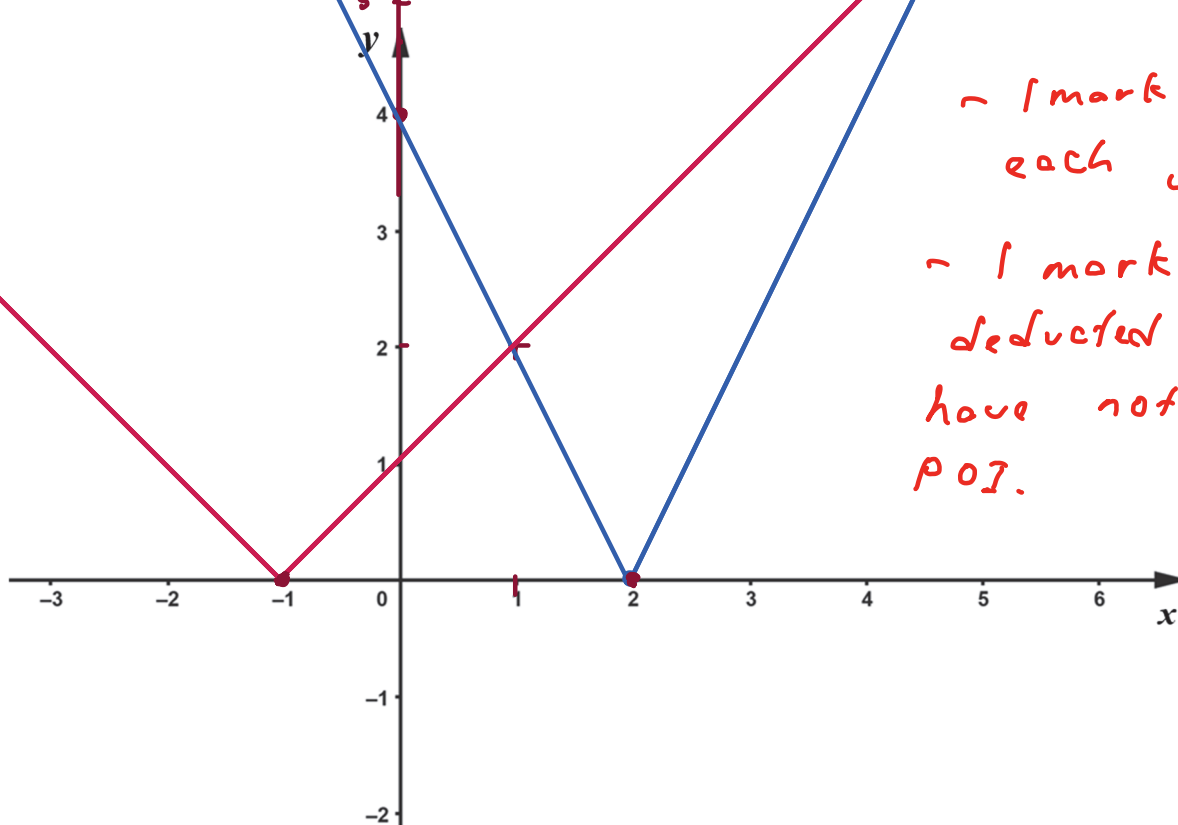
Question 16 (3 marks)

Marks

Consider the functions:

$f(x) = |2x - 4|$ and $g(x) = |x + 1|$

- a) On the same number plane, sketch the graphs of $f(x)$ and $g(x)$, showing all important features.



- b) Use your graph to solve the inequality:

$$|2x - 4| < |x + 1|$$

more was awarded based on your graph!

Question 17 (3 marks)

Marks

The derivative of a function is given by: $f'(x) = e^x \sin x$.

- a) Differentiate and simplify $g(x) = e^x \sin x - e^x \cos x$.

$$\begin{aligned} g'(x) &= \sin x \cdot e^x + e^x \cdot \cos x - [\cos x \cdot e^x + e^x (-\sin x)] \\ &= e^x \sin x + e^x \cos x - e^x \cos x + e^x \sin x \\ &= 2e^x \sin x \quad \text{--- } \textcircled{1} \end{aligned}$$

some students made mistake with the negative sign.

b) Hence or otherwise, find $f(x)$ if it is known that $f(\pi) = \frac{e^\pi}{2}$.

2

$$\begin{aligned}\int f'(x) dx &= \int e^x \sin x dx \\ &= \frac{1}{2} \int 2e^x \sin x dx \\ f(x) &= \frac{1}{2} [e^x \sin x - e^x \cos x] + C \\ f(\pi) &= \frac{1}{2} (e^\pi \sin \pi - e^\pi \cos \pi) + C \\ \frac{e^\pi}{2} &= \frac{1}{2} (0 + e^\pi) + C \\ C &= 0 \\ \therefore f(x) &= \frac{1}{2} (e^x \sin x - e^x \cos x)\end{aligned}$$

Question 18 (4 marks)

Marks

Given the series $1\frac{1}{3} + 2\frac{1}{9} + 4\frac{1}{27} + 8\frac{1}{81} \dots$. Find the expression of the:

a) general term.

2

$$\begin{aligned}\text{GP1} &= 1 + 2 + 4 + 8 \dots; a_1 = 1, r_1 = 2 \\ \text{GP2} &= \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} \dots; a_2 = \frac{1}{3}, r_2 = \frac{1}{3}\end{aligned}$$

} ① understanding
2 separate GPs

$$\begin{aligned}T_n &= a_1 r_1^{n-1} + a_2 r_2^{n-1} \\ &= 1(2)^{n-1} + \frac{1}{3}\left(\frac{1}{3}\right)^{n-1} \\ &= 2^{n-1} + \frac{1}{3^n}\end{aligned}$$

} ①

b) sum.

2

$$\begin{aligned}S_n &= \frac{a_1(r^n - 1)}{r - 1} + \frac{a_2(1 - r^n)}{1 - r} \\ &= \frac{1(2^n - 1)}{2 - 1} + \frac{\frac{1}{3}(1 - \frac{1}{3}^n)}{1 - \frac{1}{3}} \\ &= 2^n - 1 + \frac{1}{2}(1 - \frac{1}{3}^n) \\ &= 2^n - 1 + \frac{1}{2} - \frac{1}{2}\left(\frac{1}{3}^n\right) \quad \text{--- ①} \\ &= 2^n - \frac{1}{2}\left(\frac{1}{3}^n\right) - \frac{1}{2} \quad \text{or} \quad \frac{2^{n+1} - 3^n - 1}{2}\end{aligned}$$

Have to simplify to get the 2nd mark!

①

Question 19 (4 marks)

Marks

The decibel level of a sound is given by the formula:

$$L = 10 \log_{10} \left(\frac{I}{10^{-12}} \right)$$

Where L is the sound level in decibels, I is the intensity of the sound in watts per square meter.

- a) A chocolate making machine produces a sound with intensity of $2.5 \times 10^{-5} \text{ W/m}^2$.

Calculate the sound level of the machine to 4 decimal places.

1

$$L = 10 \log_{10} \left(\frac{2.5 \times 10^{-5}}{10^{-12}} \right) = 73.9794 \text{ dB}$$

Well done!

- b) A nearby industrial coffee grinder produced a sound that is 20 decibels louder than the chocolate making machine. What is the percentage increase in sound intensity produced by the coffee grinder compared to the chocolate making machine? Leave your answer to the nearest whole number.

3

Coffee = 93.9794 (ie 20 dB louder than choc machine)

$$\begin{aligned} I &= ? & 93.9794 &= 10 \log_{10} \frac{I}{10^{-12}} \\ & & 93.9794 &= \log_{10} \frac{I}{10^{-12}} \\ \log_2 8 &= 3 & \frac{10}{10} & 93.9794 &= \frac{I}{10^{-12}} \\ 2^3 &= 8 & I &= 2.5 \times 10^{-3} \end{aligned}$$

$$\begin{aligned} \% \text{ increase} &= \frac{2.5 \times 10^{-3} - 2.5 \times 10^{-5}}{2.5 \times 10^{-5}} \times 100\% \\ &= 9900\% \end{aligned}$$

Many students may have thought 9900% as too high and lost the last mark for NOT writing 9900% and maybe why left their answer as 99%

End of Section II

Question 20 (7 marks)**Marks**Let $f(x) = x^3 - 6x^2 + 9x + 1$.

a) Find any stationary points and determine their nature.

3

$$\begin{aligned}
 f'(x) &= 3x^2 - 12x + 9 \quad ; \quad f''(x) = 6x - 12 \\
 &= 3(x-3)(x-1) \quad \textcircled{1} \quad f''(3) = 18 - 12 = 6 > 0 \quad \textcircled{1} \quad \text{Test} = f'(x) \text{ or } f''(x) \\
 f(3) &= 1, \quad f(1) = 5 \quad ; \quad f''(1) = 6 - 12 = -6 < 0 \quad \text{must show numbers} \\
 \therefore \text{min @ } (3, 1) \quad \text{max @ } (1, 5) \quad \textcircled{1}
 \end{aligned}$$

Overall, students did well.

Must show numbers (calculation) when testing derivatives.

Students can use either the first or the second derivative to test for the nature here.

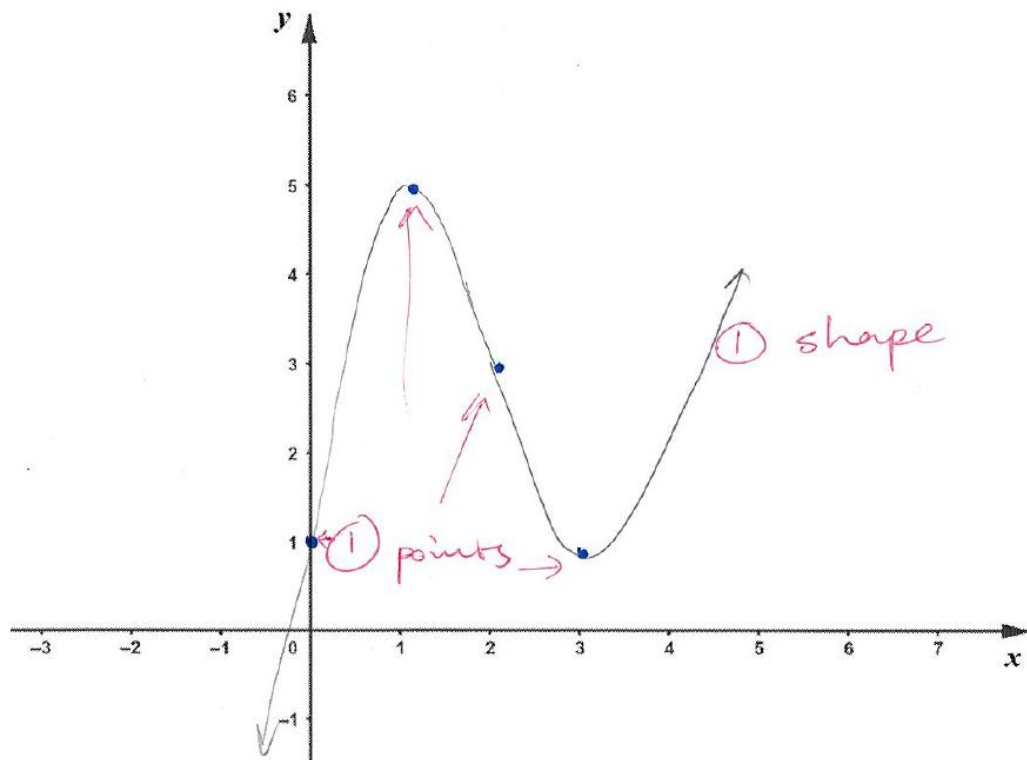
b) Find any points of inflexion.

2

1 mark for: $f''(x) = 6x - 12 = 0, \underline{x = 2}$. P.O.I is $\underline{(2, 3)}$.1 mark for testing: $f''(1) = -6 < 0$ and $f''(3) = 6 > 0$.When testing, the final value must be shown. i.e. $f''(3) > 0$ is insufficient.

Students must test the second derivative for the concavity.

c) Sketch the curve, showing all important features (you do not need to find the x intercept).



Overall, students did well.

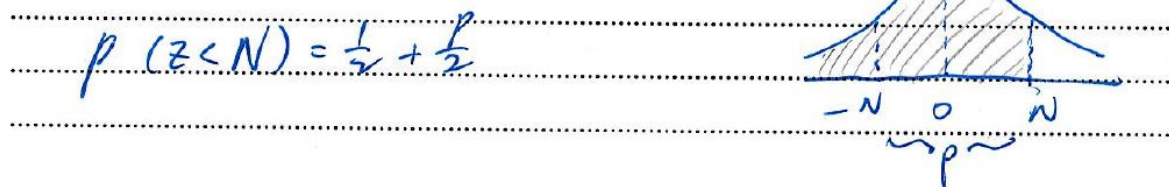
A few showed a 'cubic root' like P.O.I.

Question 21 (1 marks)

Marks

The z scores on a test are normally distributed such that $P(-N < z < N) = p$ for some constants, $N > 0$ and $0 < p < 1$. What is the value of $P(z < N)$?

1

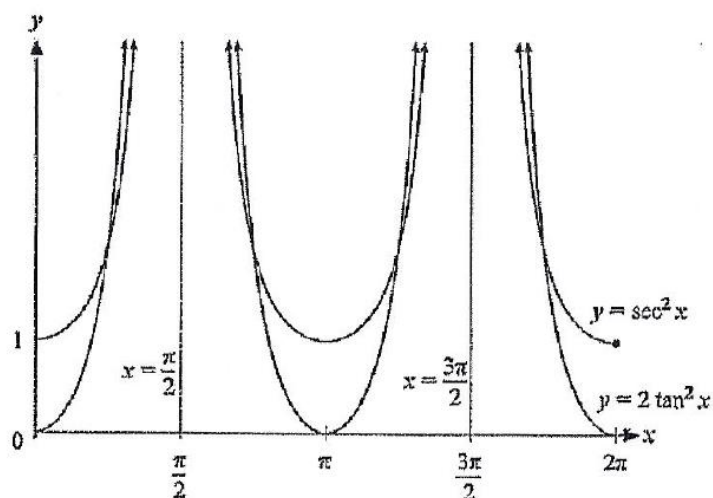


Overall, students did well.

Question 22 (3 marks)

Marks

The diagram shows the graphs of the curves $y = \sec^2 x$ and $y = 2 \tan^2 x$ for $0 \leq x \leq 2\pi$.



- a) State the periods of each of the two curves.

1

For both $y = \sec^2 x$ & $y = 2 \tan^2 x$, the period is π .

Overall, students did well.

- b) Show that the x coordinates of the points of intersection of the two curves are solutions of the equation $\tan^2 x = 1$ and solve this equation for $\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$.

2

$$2 \tan^2 x = \sec^2 x$$

$$2 \tan^2 x = 1 + \tan^2 x$$

$$\tan^2 x = 1$$

$$\tan x = \pm 1$$

$$x = \tan^{-1}(1) \quad \text{OR} \quad x = \tan^{-1}(-1),$$

$$x = \frac{\pi}{4}, \frac{5\pi}{4} \dots \quad x = -\frac{\pi}{4}, \frac{3\pi}{4} \dots$$

$$\therefore x = \frac{3\pi}{4} \text{ or } \frac{5\pi}{4}$$

- A few students missed out $x = \frac{3\pi}{4}$ as a solution. Pay attention to the given graph and the domain as there are two points of intersections, which indicating two solutions.

- Some students converted $\tan x$ into $\sin x$ and $\cos x$. Be careful when introducing a new denominator also when given $\tan^2 x$ and $\sec^2 x$, students need to recognise the use of the identities.

Question 23 (2 marks)

Marks

By using the trapezoidal rule with 4 subintervals, estimate the area under the curve for: 2

$$\int_0^4 \ln(x+1) dx.$$

x	0	1	2	3	4
$\ln(x+1)$	0	$\ln 2$	$\ln 3$	$\ln 4$	$\ln 5$

$$A \approx \frac{1}{2} \left(\frac{4-0}{4} \right) (0 + \ln 5 + 2(\ln 2 + \ln 3 + \ln 4))$$

$$= \frac{1}{2} (7.965 \dots) = \frac{1}{2} \ln 2880$$

$$= 3.98 \text{ (2 d.p.)}$$

Overall, students did well.

Common mistake is using 5 subintervals instead of 4.

Question 24 (3 marks)

Marks

Let the equation of a curve be $f(x) = \frac{x}{e^x+1}$. Find the equation of the normal to the curve at the point where $x = 1$ in general form.

3

$$f'(x) = \frac{(e^x+1) - e^x x}{(e^x+1)^2} = \frac{e^x(1-x)+1}{(e^x+1)^2} \quad \therefore f'(1) = \frac{1}{(e+1)^2} \quad (1)$$

$$\therefore m_{\text{normal}} = -(e+1)^2 \quad (1) \quad f(1) = \frac{1}{e+1} \quad (2)$$

$$y - \frac{1}{e+1} = -(e+1)^2(x-1) \quad (3)$$

$$(e+1)y - 1 = -(e+1)^3 x + (e+1)^2$$

$$(e+1)^3 x + (e+1)y - (1 + (e+1)^2) = 0 \quad // \quad (1)$$

Alternatively, $y = mx + c$
 $\frac{1}{e+1} = -(e+1)^2(1) + c$

$$c = \frac{1}{e+1} + (e+1)^2$$

$$\therefore y = -(e+1)^2 x + \frac{1}{e+1} + (e+1)^2$$

$$(e+1)^3 x + (e+1)y - 1 - (e+1)^3 = 0 \quad //$$

- GENERAL FORM! A few lost one mark due to leaving their expression with fractional coefficients.

Question 25 (3 marks)**Marks**

A car travels at 40 km/hr on a circular curve whose radius is 500 m.

- a) Find the distance, l km, that the car travels in one minute.

1

$$l = 1 \text{ min} \times \frac{40 \text{ km}}{60 \text{ min}} = \frac{2}{3} \text{ km}$$

- b) Calculate the size of the angle θ through which the car turns in 2 minutes. Give your answer to the nearest degree.

2

$$\theta = \frac{l}{r} = \frac{\frac{2}{3} \times 2 \text{ km}}{0.5 \text{ km}} = \frac{8}{3}^\circ \quad (1)$$

$$\theta = \frac{8}{3} \times \frac{180}{\pi} = 152.7887 \dots$$

$$= 153^\circ \text{ (nearest degree)} \quad (1)$$

Overall, this question was done well.

Question 26 (3 marks)**Marks**

Differentiate $f(x) = \ln(\tan^2 x)$ and simplify your answers in terms of $\sin x$ and $\cos x$.

3

$$\ln(\tan^2 x) = \ln\left(\frac{\sin^2 x}{\cos^2 x}\right)$$

$$= \ln \sin^2 x - \ln \cos^2 x$$

$$= \frac{2 \cos x}{\sin x} + \frac{2 \sin x}{\cos x}$$

$$= \frac{2 \cos^2 x + 2 \sin^2 x}{\sin x \cos x}$$

$$= \frac{2(\cos^2 x + \sin^2 x)}{\sin x \cos x}$$

$$= \frac{2}{\sin x \cos x}$$

1 mark for differentiation.

1 mark for using any trig identities.

1 mark for simplifying the final answer.

Question 27 (8 marks)

Marks

A continuous random variable, X has the following probability density function.

$$f(x) = \begin{cases} e^{-\frac{1}{2}x}, & \text{for } 0 \leq x \leq k \\ 0, & \text{for all other values of } x \end{cases}$$

a) Find the exact value of k .

2

$$\begin{aligned} \int_0^k e^{-\frac{1}{2}x} dx &= 1 \quad (1) \\ -2 \left[e^{-\frac{1}{2}x} \right]_0^k &= 1 \\ e^{-\frac{1}{2}k} - 1 &= -\frac{1}{2} \\ e^{-\frac{1}{2}k} &= \frac{1}{2} \\ -\ln 2 &= -\frac{1}{2}k \\ 2 \ln 2 &= k \\ \therefore k &= \ln 4 \quad (1) \end{aligned}$$

Students did well.

b) Write the cumulative distribution function $F(x)$ of X for $-\infty < x < \infty$.

2

$$\begin{aligned} \lim_{x \rightarrow \infty} \int_0^x e^{-\frac{1}{2}x} dx &= -2(e^{-\frac{1}{2}x} - 1) \quad \text{or} \quad 2(1 - e^{-\frac{1}{2}x}) \quad (1) \\ &= -2e^{-\frac{1}{2}x} + 2 \quad \quad \quad 2 - 2e^{-\frac{1}{2}x} \\ \therefore F(x) &= \begin{cases} 0, & x < 0 \\ -2(1 - e^{-\frac{1}{2}x}) & 0 \leq x \leq \ln 4 \\ 1, & x \geq \ln 4 \end{cases} \quad (1) \end{aligned}$$

Students did well.

c) Write the simplest exact form of the median of X .

1

$$F(x) = 0.5$$

$$-2(1 - e^{-\frac{x}{2}}) = \frac{1}{2}$$

$$\frac{3}{4} = e^{-\frac{x}{2}}$$

$$x = 2 \ln \frac{4}{3} //$$

①

d) Let X be the duration, in seconds, it takes for a student card to be registered by the school scanner. In words, interpret the value of the median in the context of this situation.

1

$$2 \ln \frac{4}{3} \approx 0.575 \text{ (3 d.p.)}$$

①

- median time

population

- values not
not compulsory

- Answers may vary, however, students need to mention the keyword 'median' not 'average' and show their understanding of cumulative probability distribution.

e.g. 'It will take no more than 0.575 seconds for 50% the students to get their cards registered by the school scanner.'

e) Given that $3P(X < c) = 2P(X > c)$, evaluate c to 3 decimal places.

2

$$3P(X < c) = 2(1 - P(X < c))$$

$$5P(X < c) = 2$$

$$P(X < c) = 0.4$$

(1)

$$2(1 - e^{-\frac{x}{2}}) = \frac{2}{5}$$

$$1 - e^{-\frac{x}{2}} = \frac{1}{5}$$

$$\frac{4}{5} = e^{-\frac{x}{2}}$$

$$-\frac{x}{2} = \ln \frac{4}{5}$$

$$x = -2 \ln \frac{4}{5}$$

$$\approx 0.446 \text{ (3 d.p.)} \quad (1)$$

Overall, students did well.

1 mark for showing $P(x < c) = 0.4$ or 40%.

1 mark for the final answer.

Question 28 (2 marks)**Marks**Evaluate $\int_{-1}^1 \frac{-6x^2}{(x^3+2)^2} dx$.

2

$$\int_{-1}^1 \frac{-2(3x^2)}{(x^3+2)^2} dx = \frac{-2}{-1} \left[(x^3+2)^{-1} \right]_{-1}^1 \quad \textcircled{1} \text{ integral}$$

$$= 2 \left((1^3+2)^{-1} - ((-1)^3+2)^{-1} \right)$$

$$= -\frac{4}{3} \quad \textcircled{1} \text{ Substitution + final solution}$$

Lots of students used substitution method which is not part of the advanced course. Students need to work on their algebra!!!

Question 29 (3 marks)**Marks**Solve the equation $1 + 4 + 7 + \dots + x = 925$.

3

$$a=1, d=3, S_n=925$$

$$S_n = \frac{n}{2} (2a + (n-1)d)$$

$$925 = \frac{n}{2} (2(1) + (n-1)(3)) \quad \textcircled{1}$$

$$1850 = n(2 + 3n - 3)$$

$$1850 = n(3n - 1)$$

$$3n^2 - n - 1850 = 0$$

$$n = \frac{1 \pm \sqrt{1^2 - 4(3)(-1850)}}{2(3)}$$

$$= 25, -\frac{74}{3}$$

$$\therefore n = 25, n \in \mathbb{Z}^+ \quad \textcircled{1}$$

$$\therefore x = a + (n-1)d$$

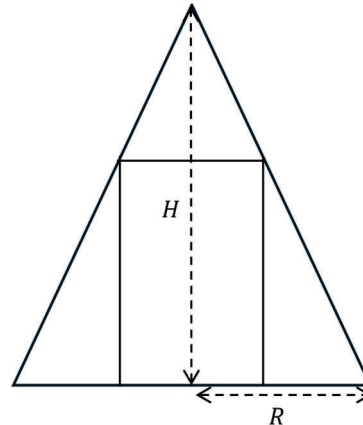
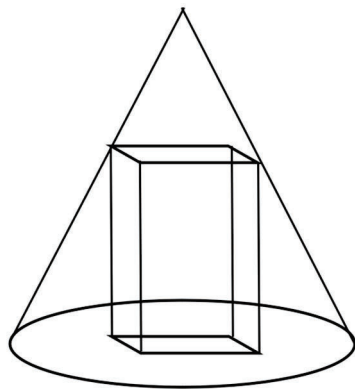
$$= 1 + (25-1)(3)$$

$$= 73 \quad \textcircled{1}$$

Question done well. A few students forgot to work out x .

Question 30 (5 marks)**Marks**

A right circular cone has height, H , and a base of radius r . A square prism is inscribed in the cone so that the base of the prism is on the cone's base. The prism has height, h , and a square base of side length x .



- a) By using similar triangles, show that $x = 2R \left(1 - \frac{h}{H}\right)$.

1

$$\frac{H-h}{H} = \frac{\frac{1}{2}x}{R}$$

$$x = 2R \left(\frac{H-h}{H}\right)$$

$$x = 2R \left(1 - \frac{h}{H}\right)$$

Do NOT need any

geometric reasoning.

Be careful that the correct ratio is used.

- b) Determine the height of the prism, in terms of H , that will maximise the volume of the prism.

4

$$V = x^2 h$$

$$= \left(2R \left(1 - \frac{h}{H}\right)\right)^2 h$$

$$= 4R^2 h \left(1 - \frac{h}{H}\right)^2$$

$$= 4R^2 h \left(1 - \frac{2h}{H} + \frac{h^2}{H^2}\right)$$

$$= 4R^2 h - \frac{8R^2 h^2}{H} + \frac{4R^2 h^3}{H^2}$$

$$\frac{dV}{dh} = 4R^2 - \frac{16R^2 h}{H} + \frac{12R^2 h^2}{H^2}$$

for stat. pts $\frac{dV}{dh} = 0$

$$4R^2 \left(1 - \frac{4h}{H} + \frac{3h^2}{H^2}\right) = 0$$

$$4R^2 \left(1 - \frac{h}{H}\right) \left(1 - \frac{3h}{H}\right) = 0$$

$$\therefore 1 - \frac{h}{H} = 0 \quad \text{or} \quad 1 - \frac{3h}{H} = 0$$

$$h = H$$

$$h = \frac{H}{3}$$

$$\frac{d^2V}{dh^2} = \frac{-16R^2}{H} + \frac{24R^2h}{H^2}$$

$$\text{at } h = H$$

$$\frac{d^2V}{dh^2} = \frac{-16R^2}{H} + \frac{24R^2H}{H^2}$$

$$= \frac{8R^2}{H} > 0$$

$\therefore$ concave up

$$\text{at } h = \frac{H}{3}$$

$$\frac{d^2V}{dh^2} = \frac{-16R^2}{H} + \frac{24R^2\left(\frac{H}{3}\right)}{H^2}$$

$$= \frac{-16R^2}{H} + \frac{8R^2}{H}$$

$$= \frac{-8R^2}{H} < 0$$

$\therefore$ concave down

$\therefore h = \frac{H}{3}$ is a maximum.

① testing for maximum

if using $\frac{dV}{dh}$ table,

students cannot use

just $+/-$, students

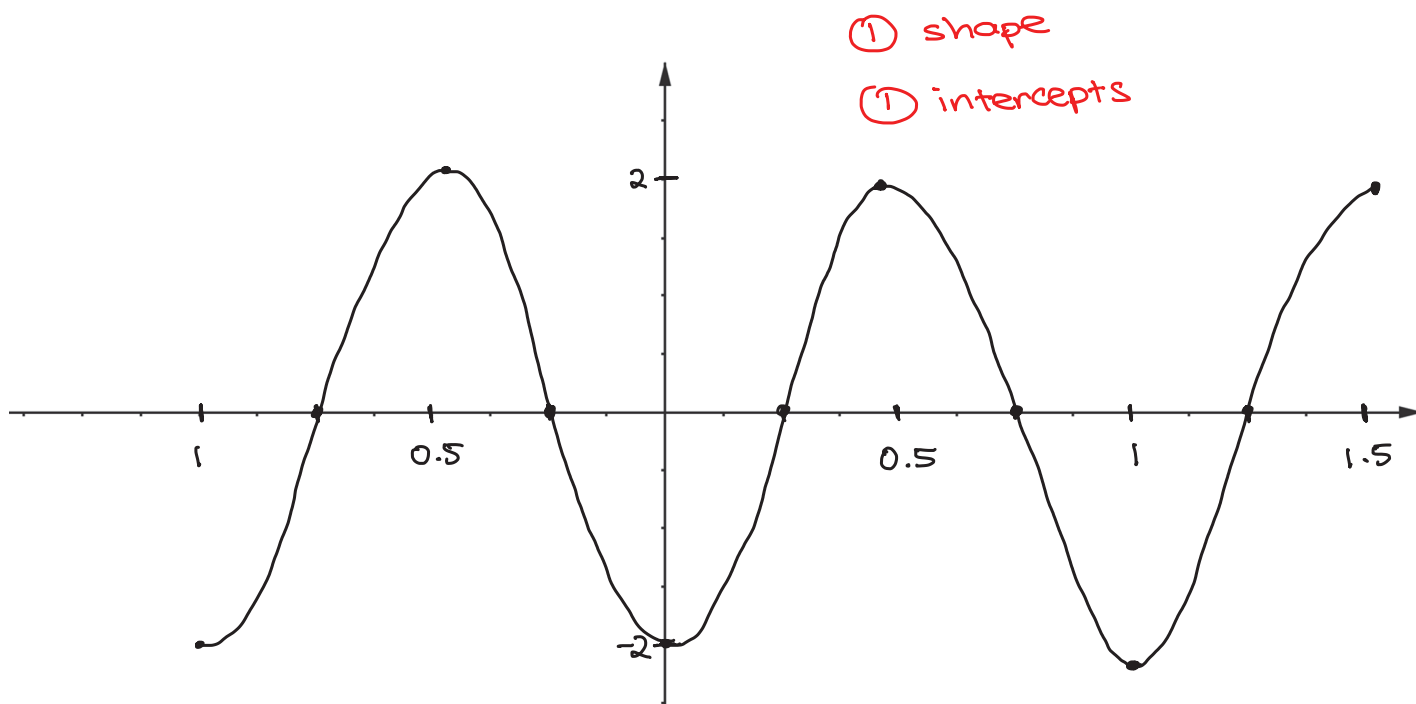
need to write some

value in terms of

R and H

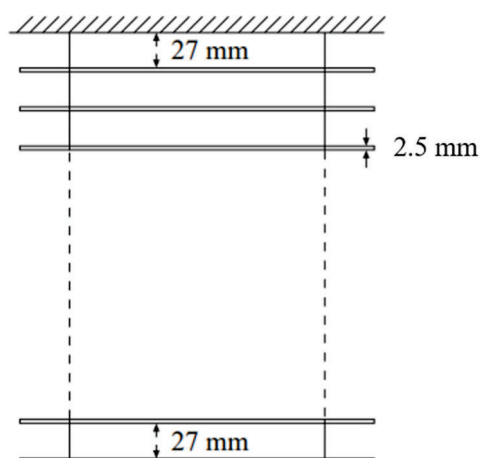
Question 31 (2 marks)**Marks**

Sketch the graph: $y = -2\cos(2\pi x)$ for $-1 \leq x \leq 1.5$ and show all important features. 2

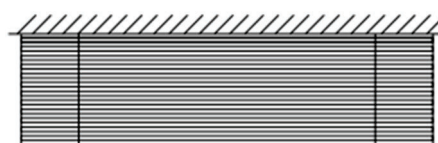
**Question 32** (5 marks)**Marks**

A venetian blind consists of twenty slats, each 2.5mm thick.

When the blind is down, the gap between the top slat and the top of the blind is 27mm and the gap between adjacent slats is also 27mm, as shown in the diagram below.



When the blind is up, all the slats are stacked at the top with no gaps, as shown in the diagram below.



a) Show that when the blind is raised, the bottom slat rises 540 mm.

2

When blind is down: $20(27 + 2.5) = 590$

When blind is up: $20(2.5) = 50$

$\therefore$ raised by $590 - 50 = 540 \text{ mm}$

① explanation

① equation

b) How far does the 19th slat rise?

1

$19 \times (27 + 2.5) - 19 \times 2.5 = 513 \text{ mm}$ ①

c) Explain briefly why the distances the slats rise form an arithmetic sequence.

1

1st slat: 27 arithmetic sequence

2nd slat: 2×27 with $a = 27$, $d = 27$

3rd slat: 3×27 increasing by 27 each

$\vdots$

time

20th slat: 20×27 show somewhere that d is 27.

d) Find the sum of all the distances that the slats rise when the blind is raised.

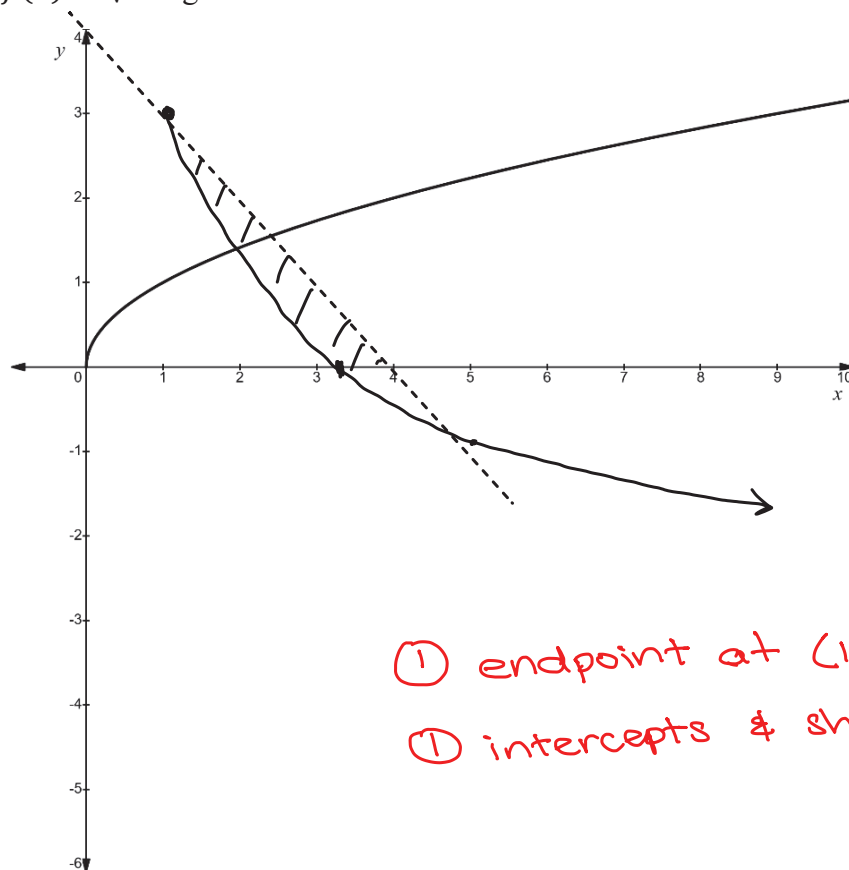
1

$$\begin{aligned} S_n &= \frac{n}{2} (2a + (n-1)d) \\ &= \frac{20}{2} (2(27) + (20-1)(27)) \\ &= 5670 \text{ mm} \quad \text{①} \end{aligned}$$

Question 33 (7 marks)

Marks

The graph of $f(x) = \sqrt{x}$ is given below.



a) On the same set of axes, sketch the graph of $g(x) = -2\sqrt{x-1} + 3$. 2

b) Describe the sequence of transformations applied to $f(x)$ to obtain $g(x)$. 2

translate 1 unit to the right
vertical dilation by 2 / horizontal dilation by $\frac{1}{2}$
reflect along x-axis
shift up by 3 units
② all correct
① incorrect order / reasoning
③ 2 or more thing incorrect.

Students need to be careful of the order the transform occur.

c) Find the area enclosed between the curve $g(x)$, the line $y = 4 - x$ and the x -axis.

Considering only the region that lies above the x -axis.

3

$$\begin{aligned}
 A &= \text{large triangle} - \text{area under the curve} \\
 &= \frac{1}{2} \times 3 \times 3 - \int_1^{\frac{13}{4}} (-2\sqrt{x-1} + 3) dx \quad (1) \\
 &= \frac{9}{2} - \left[-\frac{4}{3} (x-1)^{\frac{3}{2}} + 3x \right]_1^{\frac{13}{4}} \quad (1) \\
 &= \frac{9}{2} - \left(-\frac{4}{3} \left(\frac{13}{4} - 1 \right)^{\frac{3}{2}} + 3 \left(\frac{13}{4} \right) - \left(-\frac{4}{3} (1-1)^{\frac{3}{2}} + 3(1) \right) \right) \\
 &= \frac{9}{4} \quad (1)
 \end{aligned}$$

$$\begin{aligned}
 \text{OR} \\
 A &= \int_0^3 4 - y - \left(\frac{(y-3)^2}{4} + 1 \right) dy \quad (1) \\
 &= \int_0^3 3 - y - \frac{(y-3)^2}{4} dy \\
 &= \left[3y - \frac{y^2}{2} - \frac{(y-3)^3}{12} \right]_0^3 \quad (1) \\
 &= 3(3) - \frac{(3)^2}{2} - \frac{(3-3)^3}{12} - \left(3(0) - \frac{0^2}{2} - \frac{(0-3)^3}{12} \right) \\
 &= \frac{9}{4} \quad (1)
 \end{aligned}$$

Make sure the correct area is calculated.

Question 34 (6 marks)**Marks**

The hours of daylight in Sydney can be modelled by:

$$y(t) = 2.24 \sin\left(\frac{\pi t}{6} + 1.34\right) + 12.19,$$

where y , the number of hours of daylight and t , is the months of the year.

(i.e.: $t = 1$ represents January 1st; ignoring leap years)

- a) In this model, what is the maximum number of daylight hours in one day?

1

.....

- b) Find the month that has the longest day in terms of number of hours of daylight.

2

.....
 max when $y(t) = 14.43$

 $2.24 \sin\left(\frac{\pi t}{6} + 1.34\right) + 12.19 = 14.43$

 $\sin\left(\frac{\pi t}{6} + 1.34\right) = 1$ ①

 $\frac{\pi t}{6} + 1.34 = \frac{5\pi}{2}$

 $\frac{\pi t}{6} = 6.51398...$ students were able to
 get the first mark but
 $t = 12.4407...$ calculated t incorrectly.

 $\therefore$ longest day in december ①

- c) A scientist needs 13 hours of continuous daylight for an experiment. Between which dates should they be looking to run the experiment?

3

.....
 $13 = 2.24 \sin\left(\frac{\pi t}{6} + 1.34\right) + 12.19$

 $\sin\left(\frac{\pi t}{6} + 1.34\right) = 0.3616...$ ①

 $\frac{\pi t}{6} + 1.34 = 2.77160... , 6.6531...$

 $t = 2.734... , 10.1474...$ ①

 $= 20^{\text{th}} \text{ Feb} , 4^{\text{th}} \text{ Oct}$

 $\therefore$ between 5th Oct until 20th Feb ①

 Many students did the wrong time period.

End of Examination