Killara HIGH SCHOOL
MATHEMATICS DEPARTMENT

Centre Number

Student Number

2022 YEAR 12

Mathematics Advanced

Trial Examination

Date: Exam Block (08/08/2022

General Instructions:

- Reading time – 10 minutes
- Working time – 3 hours
- Write using blue or black pen
- NESA approved calculators may be used
- Show relevant mathematical reasoning and/or calculations
- No white-out may be used

Total Marks: 100

Section I - 10 marks (pages 3–7)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II - 90 marks (pages 8–42)

- Attempt Questions 11–30
- Allow about 2 hours and 45 minutes for this section

Q	Marks
MC	/10
11	/2
12	/2
13	/3
14	/3
15	/2
16	/3
17	/3
18	/4
19	/6
20	/7
21	/4
22	/5
23	/3
24	/5
25	/3
26	/13
27	/6
28	/6
29	/6
30	/4
Total	**/100**

This question paper must not be removed from the examination room.

This assessment task constitutes 30% of the course.

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Section I

10 marks
Allow about 15 minutes for this section

Use the multiple-choice sheet for Question 1–10

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1 If $A = \{1, 3, 5, 7, 9\}$ and $B = \{2, 3, 5, 7\}$ what is $A \cup B$?

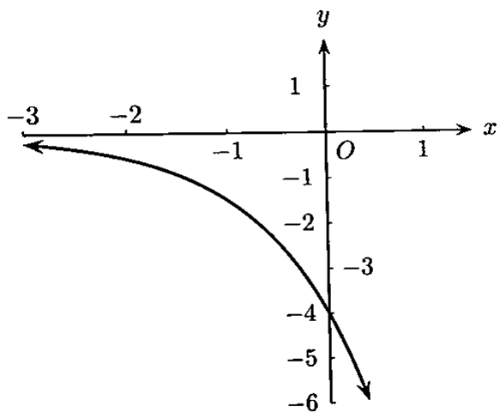
(A) $\{3, 5, 7\}$

(B) $\{2, 3, 5, 7\}$

(C) $\{6, 10, 4\}$

(D) $\{1, 2, 3, 5, 7, 9\}$

2 Which of the following could be the equation of the curve below?



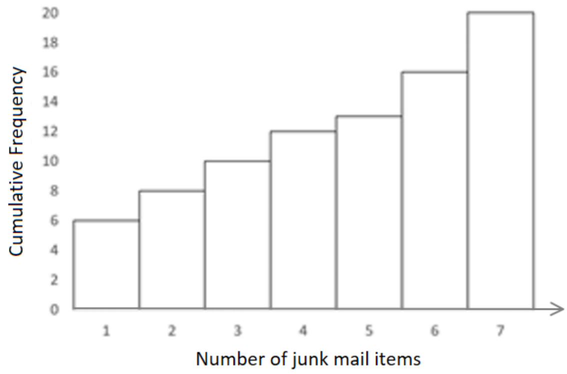
(A) $y = -4e^{-x}$

(B) $y = 3 - e^{x}$

(C) $y = -4e^{x}$

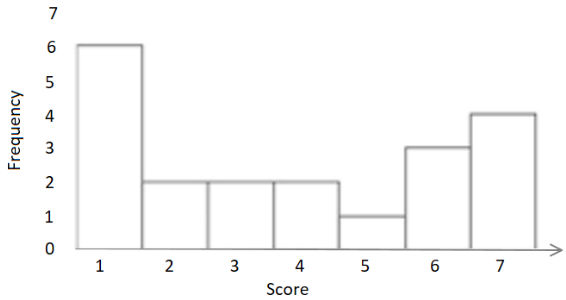
(D) $y = -4 - e^{x}$

3 The cumulative frequency histogram shows data collected in a survey on the number of junk mail items that people received daily in their inbox.

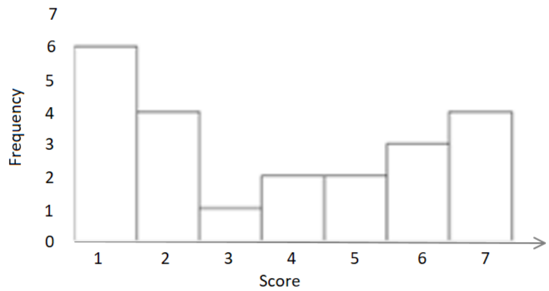


Which of the following shows the frequency histogram for the data given above?

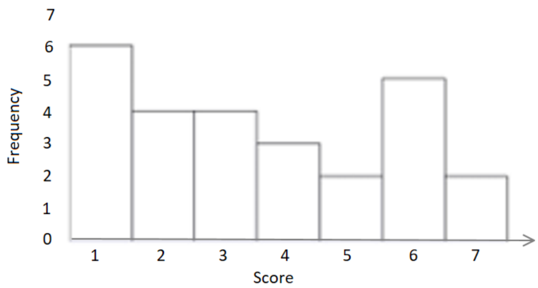
(A)



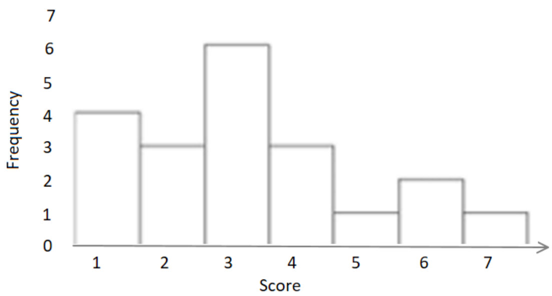
(B)



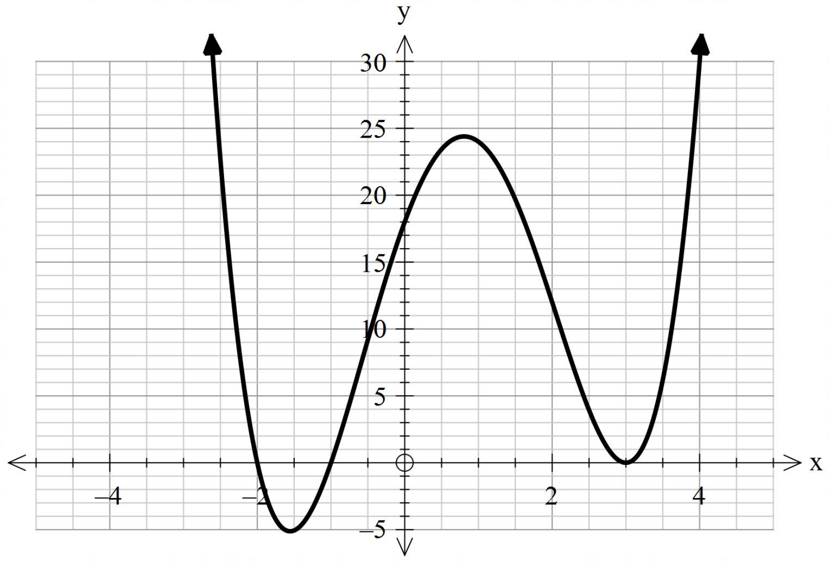
(C)



(D)



4 Which of the following equations represent the polynomial sketched below?



(A) $y = (x+2)(x-3)^2(x+1)$

(B) $y = (x-2)(x+3)^2(x-1)$

(C) $y = (x+2)(x-3)(3-x)(x+1)$

(D) $y = -(x+2)(x-3)^2(x-1)$

5 Differentiate $y = \dfrac{\tan x}{x}$ with respect to x.

(A) $\dfrac{dy}{dx} = \dfrac{\sec x^2 - \tan x}{x^2}$

(B) $\dfrac{dy}{dx} = \dfrac{\tan x - \sec^2 x}{x^2}$

(C) $\dfrac{dy}{dx} = \dfrac{\sec^2 x - \tan x}{x}$

(D) $\dfrac{dy}{dx} = \dfrac{x\sec^2 x - \tan x}{x^2}$

6 The table below shows the velocity, V of a particle given as a function of time, t.

t (s)	0	12	15	18	21	24
V (ms^{-1})	25	22	24	37	25	123

The distance in metres covered by the particle from $t = 12$ to $t = 21$ seconds, calculated using the trapezoidal rule is:

(A) 162

(B) 253.5

(C) 475.5

(D) 546

7 Given that f is a differentiable function for all real values of x,

then the derivative of $f(\tan 3x)$ is:

(A) $f'(\tan 3)$

(B) $3\sec^2 3x\, f'(x)$

(C) $3\sec^2 3x\, f'(\tan 3x)$

(D) $9\sec^2(3x)\, f'(\tan 3x)$

8 $f(x) = \begin{cases} 2a - x, & (-a, a) \\ 3x - 2a, & [a, \infty) \end{cases}$

Then, which of the following is NOT true?

(A) $f(x)$ is continuous for all $x < a$.

(B) $f(x)$ is continuous for all $x = a$.

(C) $f(x)$ is differentiable for all $x > a$.

(D) $f(x)$ is not differentiable for all $x = a$.

9 For a polynomial $f(x) = ax^3 + bx^2 + cx$, where a is a negative real number, and both b and c are real numbers. Also,

$$f(p) = 0$$
$$f(q) = 0$$
$$f'(t) = 0$$
$$f'(u) = 0$$

where $p < t < q < u < 0$.

Then, what can be said about the curve $f(x)$ at $x = t$?

(A) $f(x)$ has a maximum.

(B) $f(x)$ has a minimum.

(C) $f(x)$ has a horizontal point of inflection.

(D) The nature of the point at t can not be known without additional information.

10 If $\int_0^3 f(x)dx = 5$ then $\int_0^3 (2f(x) + 4)dx$ is equal to

(A) 5

(B) 10

(C) 14

(D) 22

End of Section I

Examination continues in the next booklet

Killara HIGH SCHOOL
MATHEMATICS DEPARTMENT

Centre Number

Student Number

2022 **YEAR 12**

Mathematics Advanced

Section II Answer Booklet 1

Section II

90 marks
Attempt Questions 11–30
Allow about 2 hours and 45 minutes for this section

Booklet 1 — Attempt Questions 11–25 (55 marks)
Booklet 2 — Attempt Questions 26–30 (35 marks)

Instructions:

- Write your Centre Number and student Number at the top of this page
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on page 28. If you use this space, clearly indicate which question you are answering
- No white-out may be used

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Section II

In Questions 11 – 30, your response should include relevant mathematical reasoning and/or calculations.

Question 11 (2 marks)

Solve the equation $|2x - 1| = 5$. **2**

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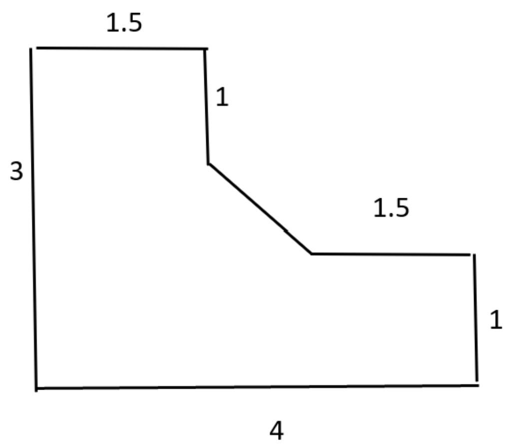
Question 12 (2 marks)

Find the area of the following shape. Measurements are given in centimetres. **2**



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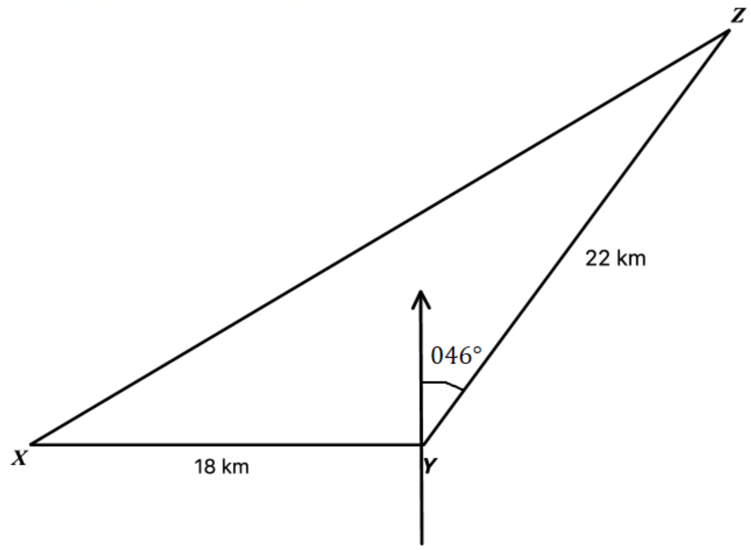
Question 13 (3 marks)



In the diagram X, Y and Z represent the locations of three towns. The town Y is due east of X, and the bearing of Z from Y is $046°$.

(a) Find the distance XZ correct to one decimal place. **2**

(b) What is the bearing of Y from Z. **1**

Question 14 (3 marks)

The curve $y = ax^2 + bx + 2$ has a minimum turning point at $(1, -1)$. **3**
Evaluate a and b.

Question 15 (2 marks)

Prove the following **2**

$$\sqrt{\frac{1 - \sin x}{1 + \sin x}} = \sec x - \tan x$$

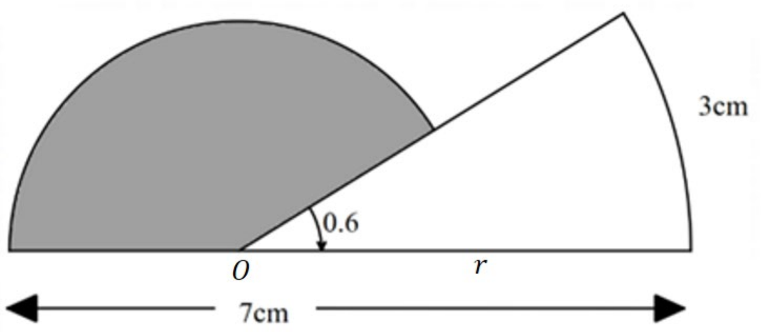
Question 16 (3 marks)

Find the equation of the tangent to the curve $y = \ln\sqrt{2x}, x > 0$, at the point where $x = \dfrac{e}{2}$. **3**

End of Question 16

Question 17 (3 marks)

In the diagram below, r is the radius of the unshaded sector and it subtends an angle of 0.6 radians at the origin O. **3**
Calculate the area of the shaded sector as shown below.



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End of Question 17

Question 18 (4 marks)

Joanne is planning a visit to Andes mountains, which has an altitude of 2400 m.

Altitude sickness is a serious condition that affects people who visit locations that are more than 2400 m above sea level. One in five people visiting these high regions suffer from altitude sickness.

This table shows the symptoms of altitude sickness, and the probability of suffering from it and displaying the symptom.

Symptom	Probability of the symptom
Headache	0.9
Shortness of breath	0.75
Nausea	0.3

(a) Explain why it is possible for the probabilities of symptoms to add to more than 1.0? **1**

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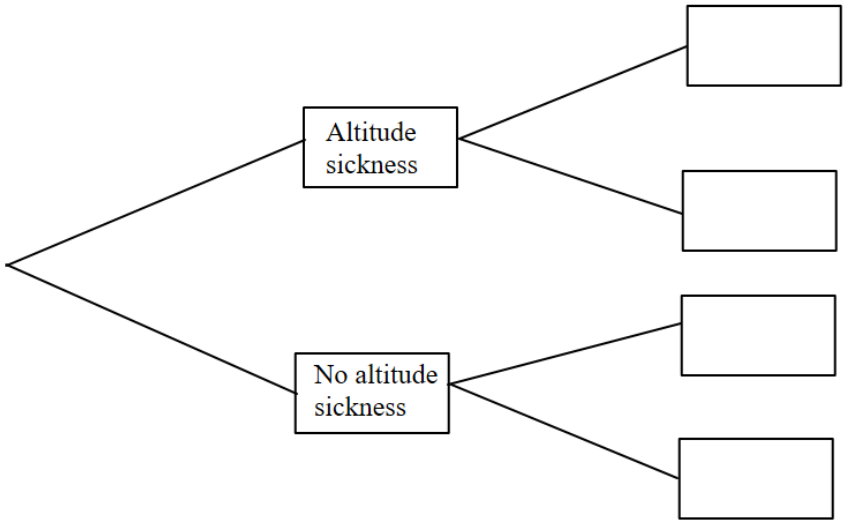
Question 18 continues on page 16

Question 18 (continued)

(b) The tree diagram below is completed partially. **3**

By completing the labels and probabilities on the branches of the tree diagram below, calculate the probability that Joanne will get altitude sickness and suffer from nausea when she visits Andes.



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End of Question 18

Question 19 (6 marks)

A company is investigating the effectiveness of its new weight loss diet. They surveyed 14 people who use their diet and recorded the time they have been on the diet and their weight loss over this period.

Weeks on the diet	1	5	6	3	6	9	4	2	7	10	2	5	9	11
Weight loss (kg)	0.5	6	7	3.5	6.5	7.5	3	4	6	9	1.5	4	7	8.5

(a) Pearson's coefficient for this data is $r = 0.923$. What does this value say about the strength and direction of the correlation between the number of weeks on the diet and the weight loss? **1**

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(b) The least-squares regression line is $y = 1 + mx$, where: **1**

x is the number of weeks on the diet
y is the weight loss, in kilograms
m is the gradient of the line

Using technology, or otherwise, find the value of m.

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(c) Interpret the value of m found in part (b) in the context of the data provided. **1**

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(d) The scatter plot shows the data collected from the 14 people on the diet. **2**

Draw the least-squares regression line found in part (b).

(e) Explain why this least-squares regression line would NOT be useful in predicting the weight loss of a person who had been on the diet for 18 weeks. **1**

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End of Question 19

Question 20 (7 marks)

A global pharmaceutical company transfers large amount of data files every evening from offices around the world to its head office. Once the file is received at the data warehouse, the time T taken to process the file is normally distributed with mean of 90 minutes and standard deviation of 15 minutes.

(a) An evening is selected at random. What is the probability that it takes more than two hours to process the file? **2**

(b) Sophia has sent through an important file from one of the offices to the warehouse. What is the probability that it will be processed in less than 50 minutes? **3**
(You may use the Standard normal distribution table provided)

Question 20 continues on page 20

Question 20 (continued)

(c) Due to the severe staff shortage and increased amount of data, the processing time has increased, and the new processing time Y is given by $Y = 1.1\,T + 40$. **1**

Calculate

(i) the new Mean processing time.

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(ii) standard deviation of the new processing time. **1**

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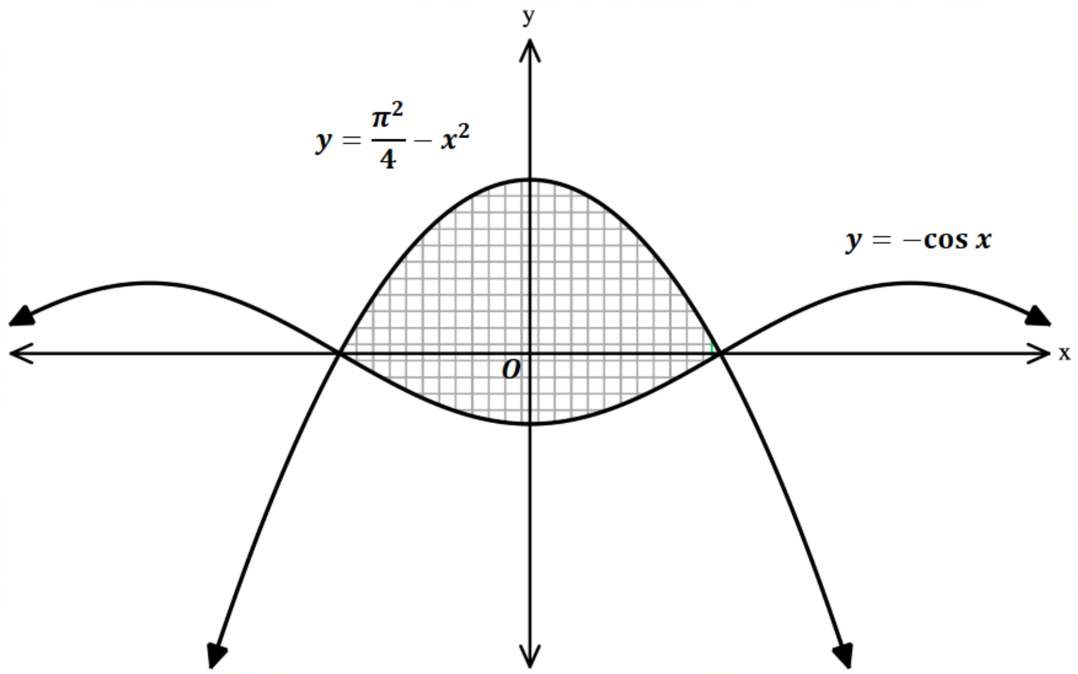
End of Question 20

Question 21 (4 marks)



The diagram shows a section of the graphs of $y = -\cos x$ and $y = \frac{\pi^2}{4} - x^2$.

(a) Show that $x = \frac{\pi}{2}$ is a solution to the equation **1**

$$\frac{\pi^2}{4} - x^2 = -\cos x$$

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Question 21 continues on page 22

Question 21 (continued)

(b) Find the exact value of the shaded region bounded by the curves $y = -\cos x$ and $y = \dfrac{\pi^2}{4} - x^2$. **3**

End of Question 21

Question 22 (5 marks)

Lori's bakery specialises in birthday cakes. The probability distribution of the number of birthday cakes sold in a day, X, is given below.

x	0	1	2	3	4
$P(X = x)$	0.1	0.2	0.25	0.35	0.1

(a) Calculate the mean number of birthday cakes sold in a day. **1**

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(b) On Monday, Lori makes four birthday cakes. If each birthday cake costs \$20 to make and sells for \$50, what is the expected profit or loss on that day? **2**

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(b) On Tuesday, Lori makes three birthday cakes. Let the random variable Y denote the number of birthday cakes not sold on that day. Obtain the probability distribution of Y. **2**

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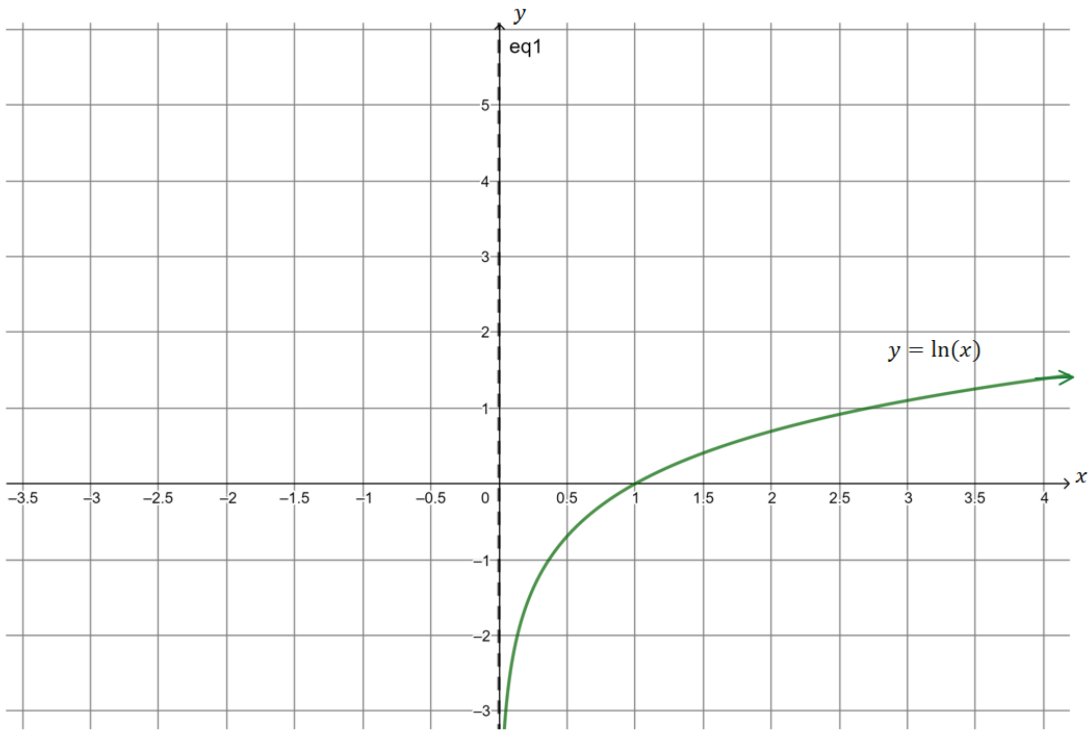
Question 23 (3 marks)

The sketch below shows the graph of $y = \ln(x)$. On the same set of axes, sketch the graph of **3**

$$y = 3\ln\left(\frac{2}{3}x + 1\right) + 2$$



Question 24 (5 marks)

A particle starts to move from the origin with a velocity $2\ \mathrm{m/s}$ along the x- axis.
Its acceleration at any time t seconds is given by

$$a = \frac{6}{2t+3}.$$

(a) Show that the velocity, v, of the particle at time t is: **2**

$$v = 3\ln\left(\frac{2}{3}t + 1\right) + 2 \quad \mathrm{m/s}$$

Question 24 (continued)

(b) Find the time taken by the particle to reach a velocity 4 m/s. **1**

(c) Does the particle ever return to the origin? Give reasons. **2**

End of Question 24

Question 25 (3 marks)

Evaluate **3**

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^2 x \, dx$$

End of Question 25

Proceed to Booklet 2 for Questions 26 – 30

Extra writing space

If you use this space, clearly indicate which question you are answering.

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Killara HIGH SCHOOL
MATHEMATICS DEPARTMENT

Centre Number

Student Number

2022 **YEAR 12**

Mathematics Advanced

Section II Answer Booklet 2

Booklet 2 – Attempt Questions 26 – 30 (35 marks)

Instructions:

- Write your Centre Number and student Number at the top of this page
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on page 43. If you use this space, clearly indicate which question you are answering
- No white-out may be used

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Question 26 (12 marks)

A1 Patel is a shop that sells mobile phones. The number of smartphones X that the shop sells in a day is found to be a numerical valued random phenomenon, with a probability function specified by the probability density function $f(x)$ given by

$$f(x) = \begin{cases} Ax, \text{for } 0 \leq x < 8 \\ A(20 - x) \quad \text{for } 8 \leq x \leq 20 \\ 0, \quad \text{otherwise} \end{cases}$$

(a) Find the value of A. **2**

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(b) What is the probability that more than 10 mobile phones will be sold tomorrow? **1**

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Question 26 continues on page 33

Question 26 (continued)

(c) The cumulative distribution function of $f(x)$ is represented by $F(x)$. Find $F(x)$ **3**

(d) Show that the median number of mobile phones that can be sold in a day is 9.8. **2**

Question 26 continues on page 34

Question 26 (continued)

(e) It is given that the first and third quartiles of the distribution are $Q_1 = 7.2$ and $Q_3 = 12.8$. Hence, Comment on the skewness of the distribution for the number of mobile phones sold per day. **2**

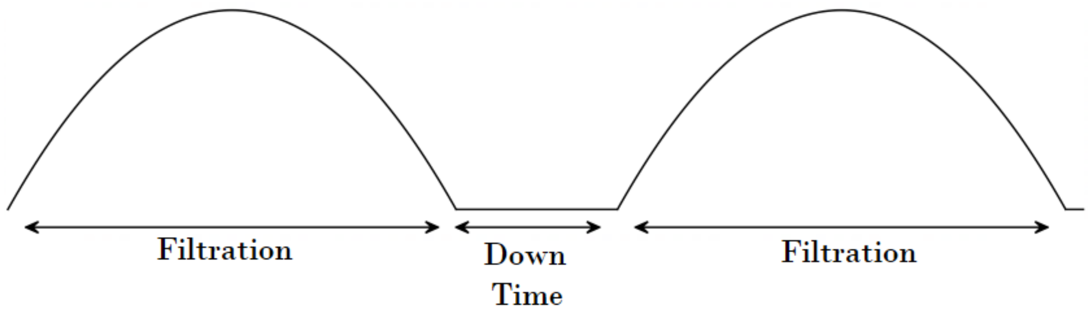
(f) In the last six months, at least 8 mobile phones were sold each day. The business can thrive in the longer term only if between 12 to 15 mobile phones are sold per day. **3**

What is the probability of the business surviving, given that at least 8 mobile phones can be sold per day?

End of Question 26

Question 27 (6 marks)

A variable speed pool pump is an efficient way of reducing electricity cost of maintaining a pool. The diagram below shows that the pump works in sequences of filtration followed by a brief downtime.



The flow rate at which a particular brand of pool pump filters water in summer, in litres per minute is given by

$$\frac{dV}{dt} = \left|275 \sin \frac{\pi}{240} t\right|$$

After each filtration cycle, the pump goes to sleep for 2 hours, and the sequence continues.

(a) Jasmine installed the pump on Saturday, and the first filtration cycle starts at 7:50 pm that evening. What is the first time after 7:50 pm at which the flow rate is zero? **2**

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Question 27 continues on page 36

Question 27 (continued)

(b) What is the volume of water that is cleaned 3 hours after the pump was turned on? **2**

(c) After how many hours can Jasmine expect her 120000 Litre pool to be completely clean? **2**

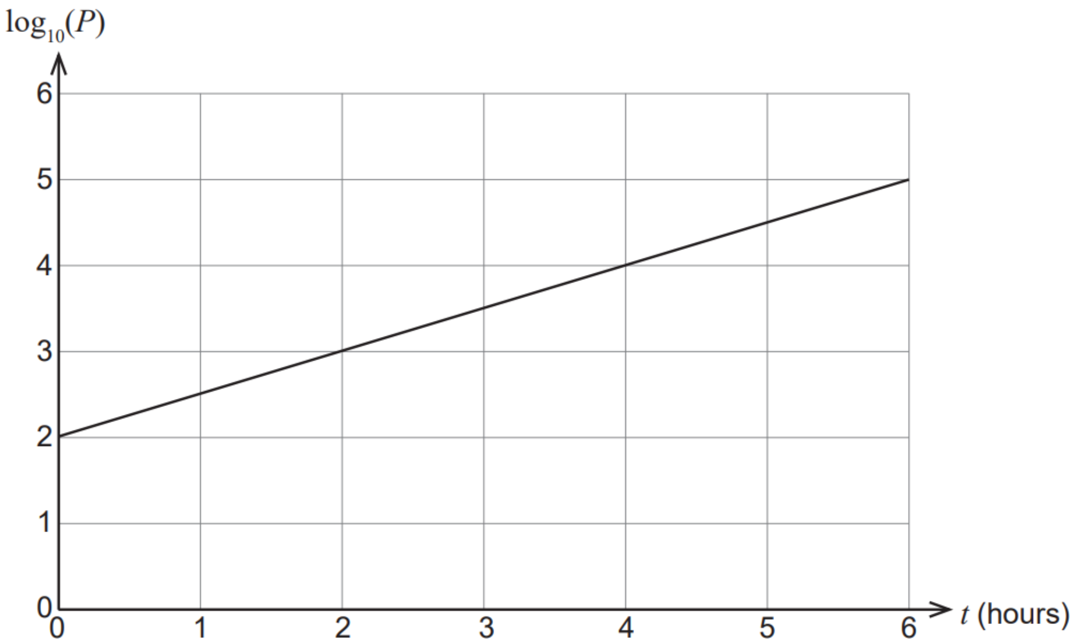
End of Question 27

Question 28(7 marks)

A marine biologist is studying the effect of temperature on the growth of a certain type of bacteria that are beneficial to coral reef reproductivity.

Sample A is a population of bacteria incubated at a temperature of 25°C under controlled laboratory conditions and the size of the population is measured at hourly intervals for six hours.

The logarithm of the population size appears to lie on a straight line when plotted against time (measured in hours), and the line of best fit is as shown below.



(a) Based on the graph above, what is the size of the bacteria population after two hours? **1**

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Question 28 continues on page 38

Question 28 (continued)

Sample B is another population of 100 of the same bacteria incubated at a higher temperature of 35°C, and was found to grow to a size of 1450 after 4 hours. The population P of bacteria after t hours can be expressed as

$$P = 100(10)^{kt}$$

(b) Using this model, determine how long will it take the population to reach the size of 5000? Give your answer to the nearest minute. **3**

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(c) By considering the rate at which the two cultures are growing, describe the effect of temperature on the population growth of this type of bacteria. (You must give supporting evidence and calculations) **2**

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End of Question 28

Question 29 (6 marks)

Wen has enrolled herself into a course on an online learning platform. Every time Wen completes a revision assignment, a score that represents the percentage of the course that she has understood is displayed.
The percentage score P, after x revision assignments is given by

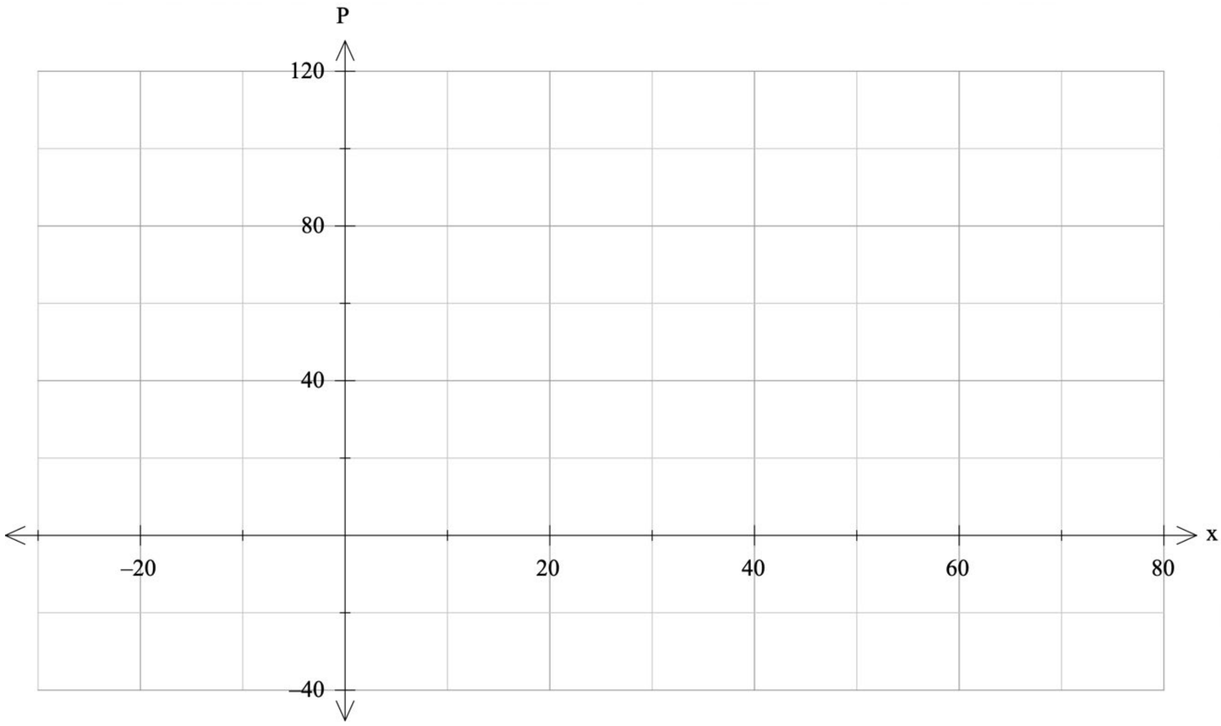
$$P = -\frac{x^2}{320}(x - 60),$$

(a) By determining the nature of stationary points, sketch the graph of **4**

$$P = -\frac{x^2}{320}(x - 60),$$

showing major features.
(Please see the graph template on page 40)

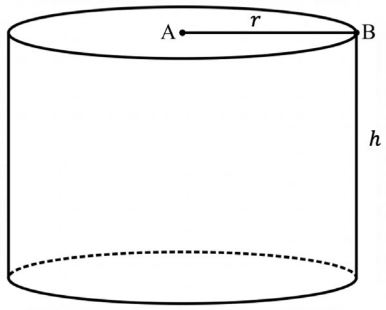
Question 29 (continued)



(b) Find the value of x, for which the percentage of the course P, being understood increases at the greatest rate. **2**

End of Question 29

Question 30 (4 marks)



A can is in the shape of a closed cylinder with height h cm and radius r cm, as shown in the diagram. The volume of the can is $500\ \text{cm}^3$.

Volume of the cylinder, $V = \pi r^2 \text{h}$
Surface area of the closed cylinder, $S = 2\pi rh + 2\pi r^2$
(DO NOT PROVE THIS)

Find the radius of the cylinder that will minimise the surface area, $S\ \text{cm}^2$, which is the amount of metal used to make the cylinder. **4**

(Extra working space for Question 30)

End of Examination

Extra writing space

If you use this space, clearly indicate which question you are answering.

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Killara HIGH SCHOOL
MATHEMATICS DEPARTMENT

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Mathematics Advanced

Section 1 - Multiple Choice Answer Sheet

Use this multiple-choice sheet for questions 1 – 10. Detach this sheet.

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9
A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

If you change your A ● B ⊗ C ○ D ○ mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

correct

A ⊗ B ⊗ C ○ D ○

1.	A ○	B ○	C ○	D ○
2.	A ○	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ○
5.	A ○	B ○	C ○	D ○
6.	A ○	B ○	C ○	D ○
7.	A ○	B ○	C ○	D ○
8.	A ○	B ○	C ○	D ○
9.	A ○	B ○	C ○	D ○
10.	A ○	B ○	C ○	D ○

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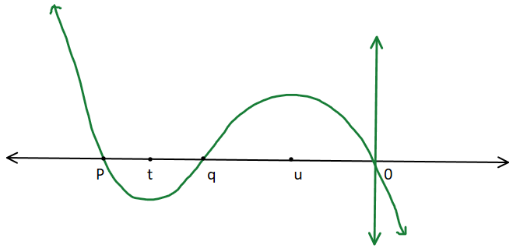
Table: The Standard Normal Distribution

The table below provides some of the probabilities of for the standard normal distribution.

$$Z \sim N(0,1) \qquad \phi(z) = P(Z \le z) = \int_{-\infty}^{z} \phi(t)dt$$

z	**0.00**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900
3.1	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929
3.2	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950
3.3	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965
3.4	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976
3.5	0.99977	0.99978	0.99978	0.99979	0.99980	0.99981	0.99981	0.99982	0.99983	0.99983

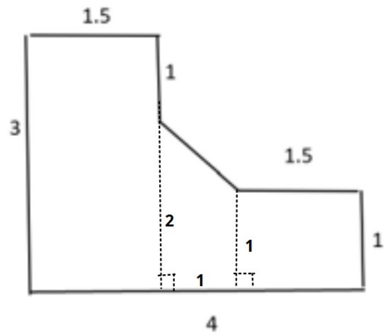
KHS 2022 Mathematics Advanced trial Marking scheme

1.	D	6.	$\frac{3}{2}[22+25+2(24+37)]$ $= 253.5$ B
2.	C	7.	C $\frac{d}{dx}\left(f(tan3x)\right) = f'(tan3x) \times 3\sec^2 3x$
3.	A	8.	D $Lim_{x\to a^-}\left(f'(x)\right) = -1$and $Lim_{x\to a^+}\left(f'(x)\right) = 3$ Hence, not differentiable at x = a.
4.	A	9.	B $f(x) = x(ax^2+bx+c)$ 
5.	D	10.	D $\int_0^3 (2f(x)+4)dx$ $= 2\int_0^3 f(x)dx + \int_0^3 4dx$ $= 2\times 5 + 4\times 3$ $= 22$

Question 11

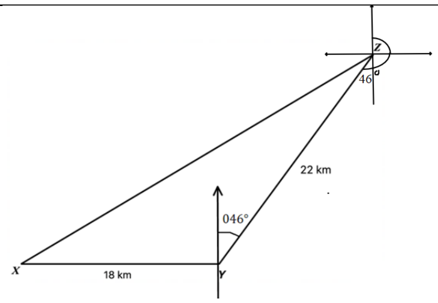
11	$\lvert 2x-1\rvert = 5$ $2x-1=5$ $1-2x=5$ $2x=6$ $-2x=4$ $x=3$ $x=-2$	2 marks: Correctly solves both equations 1 mark: correctly solves one of the equations	

Question 12

11	 Area = $1.5\times 3+\frac{1}{2}(2+1)+1.5\times 1$ $= 7.5\ cm^2$	2 marks: Correctly calculates the area 1 mark: minor error	

Question 13

13 (a)	$\angle XYZ = 90+46 = 136°$ XZ= $\sqrt{18^2+22^2-2\times 18\times 22\cos 136°}$ $= 35.9602..$ $\approx 36.0\ km$	1 mark: correctly calculates $\angle XYZ$ 1 mark: correct solution from correct	

		application of cosine rule	
13 (b)	 Bearing = $180 + 46 = 226°N$	1 mark: gives the correct bearing	

Question 14

14	$y = ax^2 + bx + 2$ $\frac{dy}{dx} = 2ax + b$ At $x = 1, \frac{dy}{dx} = 0$ $2a + b = 0$ ………….1 $(1, -1)$ is appoint on $y = ax^2 + bx + 2$ $-1 = a(1^2) + b(1) + 2$ Thus, $a + b = -3$ …………….2 1 − 2, $a = 3$ Then, $b = -3 - 3 = -6$	3 marks: correctly sets the conditions to generate equations. Correctly evaluates a and b, explaining each of the steps 2 marks: correctly sets the conditions to generate the equations and solves with minor error. 1 mark: Writes one of the equations between a and b, and makes some progress.	

Question 15

15	$\sqrt{\frac{1 - \sin x}{1 + \sin x}}$ $= \sqrt{\frac{(1 - \sin x)^2}{(1 + \sin x)(1 - \sin x)}}$ $= \sqrt{\frac{(1 - \sin x)^2}{1 - \sin^2 x}}$ $= \sqrt{\frac{(1 - \sin x)^2}{\cos^2 x}}$ $= \sqrt{\left(\frac{1 - \sin x}{\cos x}\right)^2}$ $= \sqrt{(\sec x - \tan x)^2}$ or $\frac{1 - \sin x}{\cos x}$ $= \sec x - \tan x$	2 marks: correct proof with full working Award 1 mark: 1 mark: multiplies with $1 - \sin x$ to make the denominator a single term.	

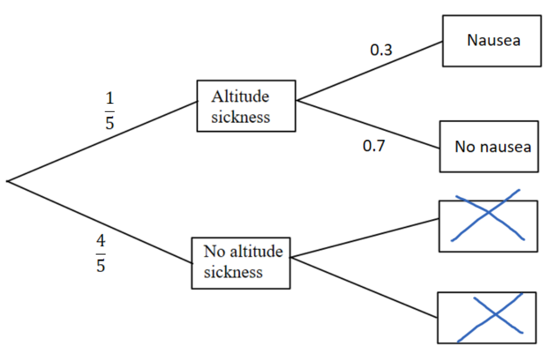
	Another method: RHS to LHS (Students must show all lines of working back to front)		

16	$y = \ln\sqrt{2x} = \frac{1}{2}\ln(2x)$ When $x = \frac{e}{2},\ \ y = \frac{1}{2}$ $\left(\frac{e}{2}, \frac{1}{2}\right)$ $y' = \frac{1}{2} \times \frac{2}{2x} = \frac{1}{2x}$ $x = \frac{e}{2},\quad y' = \frac{1}{e}$ Equation of the tangent is $x - ey = k$ Substitute $\left(\frac{e}{2}, \frac{1}{2}\right)$ Thus, $k = 0$ Equation of the tangent is $x - ey = 0$	1 mark: finds the point where $x = \frac{e}{2}$ 1 mark: correctly differentiates and finds the gradient of the tangent 1 mark: finds the equation of the tangent using their point and gradient.	

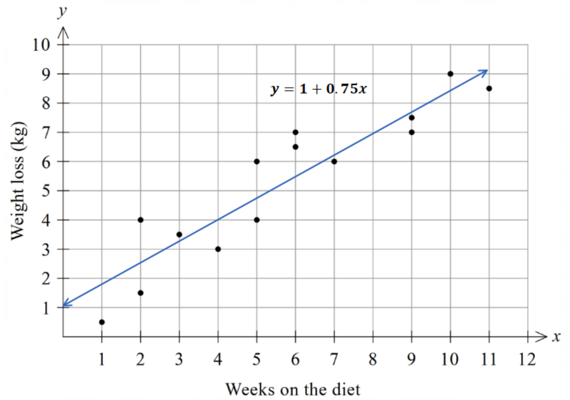
Question 17

17	$l = r\theta$ $3 = r \times 0.6$ $r = \frac{3}{0.6} = 5$ cm Area of the shaded sector Radius of the sector = $7 - 5 = 2$ cm Angle at the centre = $\pi - 0.6$ radians $A = \frac{1}{2} \times 2^2 \times (\pi - 0.6)$ $= 2\pi - 1.2$ cm^2	1 mark: correctly calculates r 2 marks: gives the correct angle subtended at the centre and gives the correct area Award 1 mark: gives the correct angle subtended at the centre and substitutes into the correct area formula	

Question 18

18 (a)	It is possible for a person to suffer from more than one of the symptoms at the same time.	1 mark: correct reasoning	
18 (b)	 P(altitude sickness with nausea) $= \frac{1}{5} \times 0.3 = \frac{3}{50}$	2 marks: correct probabilities and labels 1 mark: Minor error in the tree diagram 1 mark: calculates the correct probability from their tree diagram	

Question 19

19 (a)	R=0.923 This represents a **strong positive** correlation between the duration on diet and weight loss.	1 mark: correct reasoning	
19 (b)	Using calculator, $y = 1 + 0.75\,x$ m = 0.75	1 mark: Correct value of m given	
19 (c)	For every week on the diet, the weight loss is 0.75 kg. or a person on the diet loses weight at the rate of 0.75 kg/week	1 mark: rate of change is explained very clearly	
19 (d)		2 marks: correct trendline given (check gradient 0.75 Points to check: (0,1) and (4,3) 1 mark: A line is drawn	
19 (e)	The least-square regression line is based on the data for being on the diet for 1 to 12 weeks. 18 is outside the range of this data, and the result may not be reliable when extrapolating.	1 mark: must explain that this is an extrapolation	

Question 20

(a)	$T \sim N(90, 15)$ For Time 2 hours, $P(X > 120) = P\left(\frac{X-90}{15} > \frac{120-90}{15}\right)$ That is $P(z > 2) = 0.025 = 2.5\%$	1 mark: calculates the correct z value for the score of 120 1 mark: calculates the correct probability	
(b)	$P(x < 50)$ $= P\left(\frac{X-90}{15} < \frac{50-90}{15}\right)$ $= P\left(z < -\frac{8}{3}\right)$ $= P\left(z > \frac{8}{3}\right)$ $= 1 - P\left(z < \frac{8}{3}\right)$ $= 1 - 0.99617$ $= 0.00383$	1 mark: determines that the required probability can be got by $P\left(z < -\frac{8}{3}\right)$ 1 mark: correctly states that the probability is $1 - P\left(z < \frac{8}{3}\right)$ 1 mark: correctly uses the table to get the correct probability	

(c)(i)	$E(Y) = E(1.1T + 40)$ $= 1.1\,E(T) + 40$ $= 1.1 \times 90 + 40$ $= 139$ minutes	1 mark: correctly calculates the new mean	
(b)(ii)	$Var(Y) = E(1.1T + 40)$ $= 1.1^2\,V(T)$ $= 1.1^2 \times 15^2$ $\sigma_Y = 1.1 \times 15 = 16.5$ minutes	1 mark: correctly evaluates the new SD	

Question 21

20(a)	$x = \frac{\pi}{2}$ Substitute into the simultaneous equation $-\cos x = \frac{\pi^2}{4} - x^2$ $0 = \frac{\pi^2}{4} - \left(\frac{\pi}{2}\right)^2$ $0 = 0$	1 mark: correctly substitutes and verifies the solution satisfies the simultaneous equation	
(b)	$y = -\cos x$ and $y = \frac{\pi^2}{4} - x^2$ are both even functions. $\text{Area} = 2 \times \int_0^{\frac{\pi}{2}} \left(\frac{\pi^2}{4} - x^2 - -\cos x\right) dx$ $= 2 \times \int_0^{\frac{\pi}{2}} \left(\frac{\pi^2}{4} - x^2 - -\cos x\right) dx$ $= 2 \times \int_0^{\frac{\pi}{2}} \left(\frac{\pi^2}{4} - x^2 + \cos x\right) dx$ $= 2 \times \left[\frac{\pi^2 x}{4} - \frac{x^3}{3} + \sin x\right]_0^{\frac{\pi}{2}}$ $= 2 \times \left[\frac{\pi^2 x}{4} - \frac{x^3}{3} + \sin x\right]_0^{\frac{\pi}{2}}$ $= 2 \times \left[\frac{\pi^3}{8} - \frac{\pi^3}{3 \times 8} + \sin\frac{\pi}{2} - 0\right]$ $= 2 \times \left[\frac{2\pi^3}{24} + 1\right]$ $= \frac{\pi^3}{6} + 2\ \text{unit}^2$	1 mark: explains the function is even and gives the correct integrand to find the area. 2 marks: correctly integrates and gives the correct area, including the units Award only 1 mark: Minor error (must integrate correctly to get this mark)	

Question 22

(a)	$E(X) = 0 \times 0.1 + 1 \times 0.2 + 2 \times 0.25 + 3 \times 0.35 + 4 \times 0.1 = 2.15$	1 mark: Evaluates mean correctly	
(b)	The cost of making 4 birthday cakes = 20×4 = \$80 Expected income = \$50 $\times\ 2.15 = \$107.50$ Expected profit = $107.50 - 80 = \$27.50$	2 marks: correctly calculates the expected profit with full working 1 mark: Either calculates the correct expected income or the correct expected income	
(c)	$Y = 0$, when all the cakes are sold.		

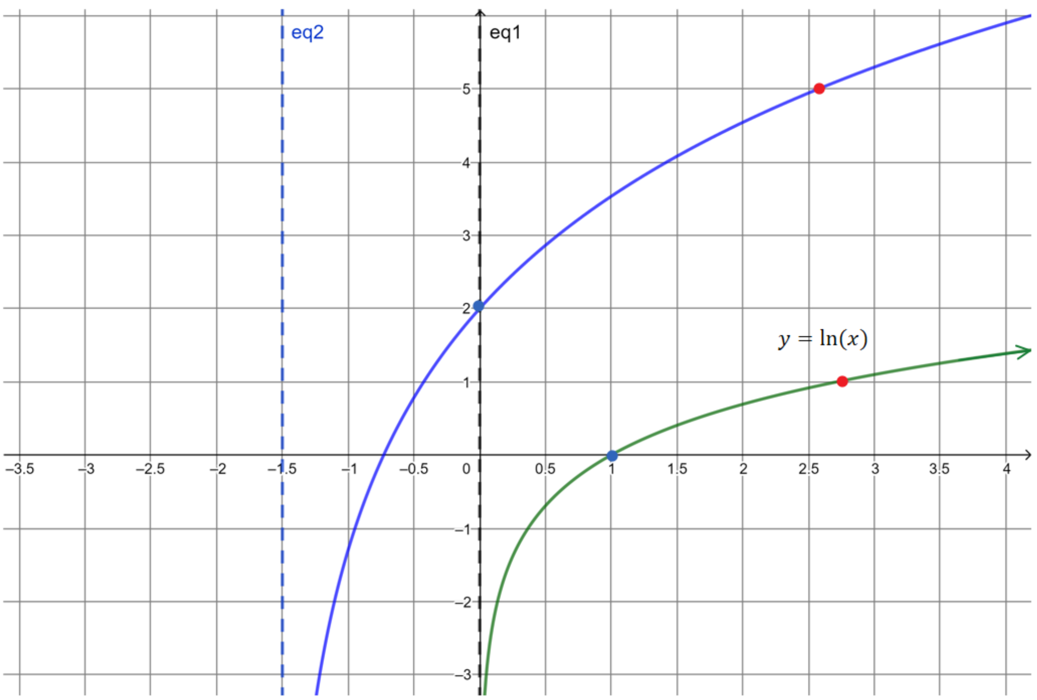
<table>
<tr>
<td></td>
<td>This means, the number of cakes sold are 3 or 4.

$P(Y=0)=P(X=3)+P(X=4)$
$=0.35+0.1=0.45$

<table>
<tr><td>y</td><td>0</td><td>1</td><td>2</td><td>3</td></tr>
<tr><td>$P(Y=y)$</td><td>0.45</td><td>0.25</td><td>0.2</td><td>0.1</td></tr>
</table>
</td>
<td>1 marks: correctly calculates P(Y=0) correctly with working

1 mark: correctly calculates all probabilities and hence, the Probability Distribution table (relates the value of Y to the values of X.)</td>
<td></td>
</tr>
</table>

Question 23

<table>
<tr>
<td>

vertical asymptote $x=-\frac{3}{2}$
$y=3\ln\left(\frac{2}{3}x+1\right)+2$
$\frac{y-2}{3}=\ln\left(\frac{x+\frac{3}{2}}{\frac{3}{2}}\right)$
Horizontal dilation of $\frac{3}{2}$; Vertical dilation of 3;
Horizontal translation of $\frac{3}{2}$ to left; translate up by 2 units

$(1,0)\mapsto\left(\frac{3}{2},0\right)\mapsto(0,0)\mapsto(0,2)$</td>
<td>1 mark: correct asymptote

1 mark: (1,0) transformed correctly to (0,2)

1 mark: correct shape and (e,1) is transformed correctly to approximately (2.6,5)

Not done well. The intent of this question is to transform $y = ln\, x$. To get the full mark, students need to show evidence of transformation of another point other</td>
</tr>
</table>

$(e,1) \mapsto \left(\frac{3}{2}e,3\right) \mapsto \left(\frac{3}{2}e-\frac{3}{2},3\right) \mapsto \left(\frac{3}{2}e-\frac{3}{2},5\right)$ $\frac{3}{2}e-\frac{3}{2} \approx 2.6$ 1 mark: correct vertical asymptote $x=-\frac{3}{2}$ 1 mark: Correct transformation of (1,0) 1 mark: correct transformation of (e,1) and correct shape	than the y-intercept. Plotting points and joining them will not constitute analysis of functions and their properties.

Question 24

(a)	$a=\frac{6}{2t+3}$ $v=\int \frac{6}{2t+3}dt$ $=3\int \frac{2}{2t+3}dt$ $=3\ln\lvert 2t+3\rvert + C$ $t \geq 0$ Hence, $v=3\ln(2t+3)+C$ When t=0, $v=2$ $2=3\ln 3 + C$ Thus, $C=2-3\ln 3$ $v=3\ln(2t+3)+2-3\ln 3$ 1 mark $v=3\ln\left(\frac{2t+3}{3}\right)+2$ $v=3\ln\left(\frac{2}{3}t+1\right)+2\ m/s$	2 marks: correct answer 1 mark: correctly finds the expression $v=3\ln(2t+3)+2-3\ln 3$	This is a 'show that' question where you need to show all lines of verbal and numerical working. Many students resorted to fudging to arrive at the answer. Generally, well done. Students who went from $=3\int \frac{2}{2t+3}dt$ To $=3\ln\lvert 2t+3\rvert + C$ Were awarded only one mark. All lines of derivation need to be shown.
(b)	$4=3\ln\left(\frac{2}{3}t+1\right)+2$ $3\ln\left(\frac{2}{3}t+1\right)=2$ $\ln\left(\frac{2}{3}t+1\right)=\frac{2}{3}$ $\frac{2}{3}t+1=e^{\frac{2}{3}}$ $t=\frac{3}{2}\left(e^{\frac{2}{3}}-1\right) \approx 1.42$ seconds	1 mark: correctly substitutes $v=4$, and solves for t in exact form	Well done. You need to give exact value (and then round the number if you wish to)
(c)	For $t \geq 0$, The acceleration is always positive. $a=\frac{6}{2t+3} \geq 0$ Initially, the velocity (2m/s) is positive, and acceleration continues to be positive, hence the particle continues to speed up (move away) from its initial position of $x=0$. Thus, the particle never returns to the origin (Students may refer to the graph in Q23, where the velocity is always positive ($v \geq 2$) but must mention a>0 and x = 0 initially	2 marks: gives the correct answer 1 mark: makes a correct statement about v or a.	To get the full marks, apart from discussing the nature of v and a, you must connect it with the displacement (describing the motion).

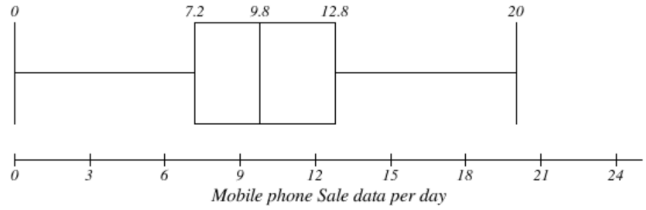
Question 25

25	$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^2 x\,dx$ $y = \tan x$ is an odd function Thus, $\tan^2(-x) = (-\tan x)^2 = \tan^2 x$ $\therefore y = \tan^2 x$ is an even function. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^2 x\,dx = 2 \times \int_{0}^{\frac{\pi}{4}} \tan^2 x\,dx$ $= 2 \times \int_{0}^{\frac{\pi}{4}} (sec^{\,2} x - 1)dx$ $= 2 \times [\tan x - x]_0^{\frac{\pi}{4}}$ $= 2 \times \left[\left(\tan\frac{\pi}{4} - \frac{\pi}{4}\right) - (\tan 0 - 0)\right]$ $= 2\left(1 - \frac{\pi}{4}\right)$	1 mark: identifies the function as even 1 mark: expresses $\tan^2 x$ as $\sec^2 x - 1$ 1 mark: correct evaluation	Very poorly done. This is a typical question you need to learn how to do. Many students expressed $\tan^2 x = \frac{Sin^2(x)}{\cos^2 x}$. This is not a form suitable for integration. The maximum mark they can get is 1 unless your expression has become too easy.

Book 2
Question 26

(a)	We know that $\int_{-\infty}^{\infty} f(x)dx = 1$ $\int_0^8 Axdx + \int_8^{20} A(20-x)dx = 1$ $A\left\{\left(\frac{x^2}{2}\right)_0^8 + \left(20x - \frac{x^2}{2}\right)_8^{20} = 1\right\}$	2 marks: correctly evaluates A 1 mark: demonstrates the understanding that $\int_{-\infty}^{\infty} f(x)dx = 1$	Poorly done. There were two types misconception with this question. * Students considering only one of the two parts and equating the

	$104A = 1$ $A = \frac{1}{104}$	And attempts to evaluate A	probability to 1. This would mean that the probability of the other part is zero. But the question clearly stated the probability is zero only for $x \in (-\infty, 0)$ and $x \in (20, \infty)$ * or each section equating to one. This would mean that the total probability is 2 which is erroneous.
(b)	$P(10 \le x \le 20)$ $= \frac{1}{104}\int_{10}^{20} (20 - x)dx$ $= \frac{1}{104}\left(20x - \frac{x^2}{2}\right)_{10}^{20}$ $= \frac{1}{104}\left[\left(20x - \frac{x^2}{2}\right)_{10}^{20}\right]$ $= \frac{1}{104}(400 - 200 - 200 + 50)$ $= \frac{25}{52}$ Or $P(10 \le x \le 20)$ $= 1 - P(x \le 10)$ $= 1 - \frac{1}{104}\int_0^8 xdx - \frac{1}{104}\int_8^{10} 20 - xdx$ $= 1 - \frac{1}{104}\left[\left(\frac{x^2}{2}\right)_0^8 + \left(20x - \frac{x^2}{2}\right)_8^{10}\right]$ $= 1 - \frac{1}{104}[32 + (200 - 50) - (160 - 32)]$ $= \frac{25}{52}$	1 mark: Correctly calculates the probability giving full working	P(more than 10) refers to the area under the pdf. Substituting into the pdf $x = 10$ will not give you the required probability. (This is a continuous random variable, so P(x=10) = 0. Poorly done. (Please note: when you are probability values greater than 1, you know something has gone wrong.
(c)	$F(x) = \int_a^x f(t)dt$ At $x = 8$, the cumulative frequency is $\frac{4}{13}$. $F(x) = \begin{cases} \frac{x^2}{208}, \text{for } 0 \le x < 8 \\ \frac{1}{104}\left(20x - \frac{x^2}{2} - 128\right) + \frac{4}{13} \quad \text{for } 8 \le x \le 20 \\ 1, \quad x > 20 \end{cases}$	3 marks: one mark for each part. CDF for each domain has to be shown clearly	Poorly done. Many students tried getting the total area under the curve. This is cumulative probability. So, we

			need $\frac{1}{104}\left(20x-\frac{x^2}{2}-128\right)+\frac{\mathbf{4}}{\mathbf{13}}$
(d)	Median by integration: $P(0 \le x < 8) = F(8)$ $= \frac{8^2}{208} = \frac{4}{13}$ We need to find $X = x$ such that $\frac{4}{13} + \int_8^x f(x)dx = \frac{1}{2}$ $\frac{4}{13} + \frac{1}{104}\left(20x - \frac{x^2}{2}\right)_8^x = \frac{1}{2}$ $\frac{1}{104}\left(20x - \frac{x^2}{2}\right)_8^x = \frac{5}{26}$ $20x - \frac{x^2}{2} - 128 = 20$ $x^2 - 40x + 296 = 0$ $x = \frac{40 \pm \sqrt{1600-1184}}{2}$ $x = 9.8\,, 30$ Reject $x = 30$ as domain is [8,20] Median number of phones expected to be sold = 9.8 **Method 2** Give full mark, if they do the following: Using CDF, for median, $\frac{1}{104}\left(20x - \frac{x^2}{2} - 128\right) + \frac{4}{13} = 0.5$ Let $x = 9.8$ $\frac{1}{104}\left(20 \times 9.8 - \frac{9.8^2}{2} - 128\right) + \frac{4}{13} = 0.5$ $0.4998076 \ldots . \approx 0.5$ Hence, the median number of mobile phones sold is 9.8	1 mark: writes the correct expression for finding the median $\frac{4}{13} + \int_8^x f(x)dx = \frac{1}{2}$ 1 mark: correctly calculates the value of median Note: 1 mark: Sets F(x)=0.5 1 mark: verifies by substitution or solves	'Show that' question – again you need to show all working. If you had the correct CDF and then verifying 9.8 to get a probability of 0.5 was awarded marks. However, substituting into pdf was not awarded marks as median is associated with CDF.
(e)	Lowest score = 0 Highest 20 Five number summary: 0, 7.2, 9.8, 12.8, 20  From the shape of the plot, slightly positively skewed. $Q_2 - Q_1 < Q_3 - Q_2$ $(9.8 - 7.2 = 2.6) < (12.8 - 9.8 = 3)$	2 marks: correct solution 1 mark: Gives the five number summary and makes a comment (A demonstration using a box-and-whisker or calculation must be given to get full marks)	Mostly well done. Misconception: From IQR, you cannot make statements about the shape of the distribution. Responses without accurate numerical and verbal explanation was awarded only 1 mark.

(f)	Conditional Probability $= \frac{P[(12 \le x \le 15) \cap (x \ge 8)]}{P(x \ge 8)}$ (This line is required) $= \frac{P[(12 \le x \le 15)]}{P(x \ge 8)}$ (This line is required) $= \frac{F(15) - F(12)}{1 - \frac{4}{13}}$ $= \frac{\frac{1}{104}\left[\left(20 \times 15 - \frac{15^2}{2}\right) - \left(20 \times 12 - \frac{12^2}{2}\right)\right]}{\frac{9}{13}}$ $= \frac{\frac{3}{16}}{\frac{9}{13}} = \frac{13}{48}$ *Alternately,* $\frac{\frac{1}{104}\int_{10}^{20}(20 - x)dx}{1 - \frac{1}{104}\int_0^8 xdx}$	1 mark: correct expression for conditional probability 1 mark: correctly calculates $P(x \ge 8)$ 1 mark: correctly calculates the conditional probability	Poorly done.

Question 27

27(a)	$\frac{dV}{dt} = \left\lvert 275 \sin \frac{\pi}{240} t \right\rvert$ $\frac{dV}{dt} = 0$ $\sin \frac{\pi}{240} t = 0$ $\frac{\pi}{240} t = \pi$ $\therefore t = 240$ minutes $= 4$ hours Time = 7:50 pm + 4 h = 11:50 pm Saturday	2 marks: sets $\frac{dV}{dt} = 0$, correctly solves and gives the correct time. 1 mark: sets $\frac{dV}{dt} = 0$, error in solving the equation and calculates time correctly	
27(b)	$\frac{\mathrm{dV}}{\mathrm{dt}} = 275 \sin \frac{\pi}{240} t$ Volume of water that is filtered is $V = \int 275 \sin \frac{\pi t}{240} dt$ $= -275 \times \frac{240}{\pi}\left[\cos \frac{\pi t}{240}\right] + C$ $t = 0, V = 0$ Hence, $C = \frac{275 \times 240}{\pi}$ Thus, $V = \frac{275 \times 240}{\pi} - 275 \times \frac{240}{\pi}\left[\cos \frac{\pi t}{240}\right]$ $V = \frac{66000}{\pi} - \frac{66000}{\pi}\left[\cos \frac{\pi t}{240}\right]$ $t = 3\ hours = 180\ minutes$ $V = \frac{66000}{\pi} - \frac{66000}{\pi}\left[\cos \frac{180\pi}{240}\right]$ $= \frac{66000}{\pi} - \frac{66000}{\pi}\left[-\frac{1}{\sqrt{2}}\right]$ $= \frac{66000}{\pi}\left[1 + \frac{1}{\sqrt{2}}\right]$ litres	2 marks: correct answer 1 mark: Minor error (Award 1 mark, if t = 3 is used and then calculations are correct)	

	$=35863.67\ldots$ $\approx 35863\ L$ or $V=\int_0^{180} 275\sin\frac{\pi t}{240}\,dt$ $=-275\times\frac{240}{\pi}\left[\cos\frac{\pi t}{240}\right]_0^{180}$ $=275\times\frac{240}{\pi}\left[\cos\frac{\pi t}{240}\right]_{180}^{0}$ $=\frac{66000}{\pi}\left[1+\frac{1}{\sqrt{2}}\right]$ litres $=35863.67\ldots$ $\approx 35863\ L$		
27(c)	From 25(b), $V=\frac{66000}{\pi}-\frac{66000}{\pi}\left[\cos\frac{\pi t}{240}\right]$ Hence, in one filtration cycle of 4 hours, the pump cleans $2\times\frac{66000}{\pi}$ Litres of water $\approx 42016.9\ L$ Thus after 2 full cycles of filtration, $120000-2\left(\frac{132000}{\pi}\right)=35966.19\ L$ need to be filtered. Using the solution from 27(b), it takes 3 hours to filter $35863\ L\approx 35966$.. Thus total time taken to clean the pool, including the down time is $2(4+2)+3=15$ hours (Exact calculation shown below, which is unnecessary as the difference 100 L is negligible n a 120000 L pool) **Time taken for 35966.19L** From 19(c), $V=\frac{66000}{\pi}-\frac{66000}{\pi}\left[\cos\frac{\pi t}{240}\right]$ $35966.19=\frac{66000}{\pi}-\frac{66000}{\pi}\left[\cos\frac{\pi t}{240}\right]$ $\frac{66000}{\pi}\left[\cos\frac{\pi t}{240}\right]=-14957.73$ $\left[\cos\frac{\pi t}{240}\right]=-0.7119\ldots$	*states the volume filtered in 2 cycles *calculates the rest of water that needs to be filtered * gives reasoning to use the result from 19(c) to give the total time to clean. 2 mark: gives the correct answer from correct working 1 mark: At least two of the three processes are demonstrated.	

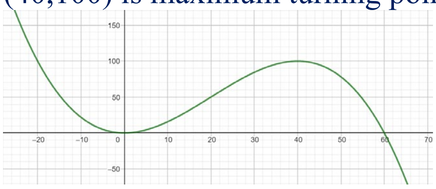
	$\frac{\pi t}{240} = 2.3631\ldots$ $t = \frac{2.3631.. \times 240}{\pi} = 180.529..$ $t = 3.0088 \approx 3h\ 0\ \text{min}\ 31\ sec$		

Question 28

(a)	From the graph, When t = 2, $\log_{10} P = 3$ $\therefore P = 10^3 = 1000$ bacteria	1 mark: identifies the correct value of $\log_{10} P$ And evaluates P.	
(b)	For temperature of 35°C, $$P = 100(10)^{kt}$$ t = 4, P = 1450 Hence, $1450 = 100\ (10)^{4k}$ $10^{4k} = 14.5$ $4k = \log_{10} 14.5$ $\text{k} = \frac{1}{4}(\log_{10} 14.5) \approx 0.2903$ $(4\ \text{sig. fig.})\qquad (*)$ Hence, $P = 100(10)^{kt}$ $\frac{P}{100} = (10)^{kt}$ $\therefore\ \ t = \frac{1}{k}\log_{10}\left(\frac{P}{100}\right)$ Thus, $\frac{t}{4} = \frac{4}{(\log_{10} 14.5)} \times \log_{10}\left(\frac{P}{100}\right)$ $t = 4 \times \frac{\log_{10}\left(\frac{P}{100}\right)}{(\log_{10} 14.5)}$ P = 5000 $t = 4 \times \frac{\log_{10}\left(\frac{5000}{100}\right)}{(\log_{10} 14.5)}$ $t = 5.8516\ldots. \approx$ 5h 51 minutes	1 mark: substitutes and calculates the value of k correctly 3 marks: • correctly substitutes to the expression for P and Correctly makes t subject of the formula and evaluates t (2 marks here) Significant progress 1 mark • Correctly rounds t to the nearest minute	
(c)	From the graph, at temperature of 25 degrees, the gradient of $\log_{10} P$ is $\frac{1}{2}$. From (b), at temperature of 35 degrees, $P = 100\ (10)^{\frac{t}{4}(\log_{10} 14.5)}$ $P = (10)^{\frac{t}{4}(\log_{10} 14.5)+2}$ Hence, $\log_{10} P = \frac{t}{4}(\log_{10} 14.5) + 2$ The gradient of $\log_{10} P$ is $\text{k} = \frac{1}{4}(\log_{10} 14.5)$ $\approx 0.2903\quad (4\ \text{sig. fig.})$	* identifies the gradient in model 1 to be ½ * In model 2. Calculates the gradient of $\log_{10} P$ as $\text{k} = \frac{1}{4}(\log_{10} 14.5) \approx 0.2903$ *compares the gradients and gives the correct effect of temperature on the bacteria population	

	Comparing gradients, $$\frac{1}{2} > \frac{1}{4}(\log_{10} 14.5) \approx 0.29$$ Hence, the bacterial population grows faster at lower temperatures. Or mentions that $\frac{dP}{dt} = kP$ and compares the two k values at 25 and 35 degrees.	2 marks: All of them correct 1 mark: At least one of the gradients given correctly.	

Question 29

(a)	$P = -\frac{x^2}{320}(x-60)$ $P = -\frac{x^3}{320} + \frac{3x^2}{16}$ $\frac{dP}{dx} = -\frac{3x^2}{320} + \frac{6x}{16}$ Stationary points $\frac{dP}{dx} = 0$ $\frac{3x^2}{320} = \frac{6x}{16}$ $x = 0$ and $x = 40$ $\frac{d^2P}{dx^2} = -\frac{6x}{320} + \frac{3}{8}$ $x = 0, \frac{d^2P}{dx^2} = \frac{3}{8}.$ (0,0) is minimum turning point. $x = 40, \frac{d^2P}{dx^2} = -\frac{3}{8}.$ (40,100) is maximum turning point. 	1 mark: finds the stationary points 1 mark: determines the nature of stationary points 1 mark: correct shape 1 mark: correct intercepts and turning points	
(b)	$\frac{d^2P}{dx^2} = -\frac{6x}{320} + \frac{3}{8}$ For point of inflection, $\frac{d^2P}{dx^2} = -\frac{6x}{320} + \frac{3}{8} = 0$ $\frac{3x}{160} = \frac{3}{8}$ $x = 20$ From the diagram in (a), the concavity changes at x = 20. Thus, x= 20 gives the point of inflection. Using the diagram, the gradient is the greatest here.	1 mark: finds the possible point of inflection. 1 mark: uses the graph or proves the point of inflection, and explains the gradient is the greatest	

	Greatest rate when $x = 20$		

Question 30

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<td>30</td>
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We need to find the radius of the cylinder that make the surface area minimum.
That is S needs to be a function of r.

$v = \pi r^2 h$

$500 = \pi r^2 h$
$h = \dfrac{500}{\pi r^2}$
Surface area

$S = 2\pi r h + 2\pi r^2$

$S = 2\pi r\left(\dfrac{500}{\pi r^2}\right) + 2\pi r^2$

$S = \dfrac{1000}{r} + 2\pi r^2$

For stationary points, $\dfrac{dS}{dr} = 0$

$\dfrac{dS}{dr} = \dfrac{-1000}{r^2} + 4\pi r = 0$
$4\pi r^3 = 1000$
$r^3 = \dfrac{1000}{4\pi}$
$r = \sqrt[3]{\dfrac{250}{\pi}} \approx 4.30 \ldots..$

Testing for maxima/minima.

<table>
<tr><td>r</td><td>4</td><td>$\sqrt[3]{\dfrac{250}{\pi}}$</td><td>5</td></tr>
<tr><td>$\dfrac{ds}{dr}$</td><td>-12.23</td><td>0</td><td>22.83</td></tr>
</table>

Hence, S is minimum when
$$r = \sqrt[3]{\frac{250}{\pi}} \approx 4.30 \text{ cm}$$
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1 mark: expresses h in terms of r. Proves that
$h = \dfrac{500}{\pi r^2}$

1 mark: Derives S as a function of r
$S = \dfrac{1000}{r} + 2\pi r^2$

1 mark: correctly differentiates and sets $\frac{dS}{dr} = 0$ to find the stationary point.

1 mark: determines the nature of stationary point to give the value of r that optimises S.
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