Killara High School

Student Number

2025 Year 12 Trial Examination

Mathematics Advanced

11/08/2025

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using blue or black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- No white-out may be used

Total Marks: 100

Section I - 10 marks (page 3 – 8)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II - 90 marks (page 12 - 42)

- Attempt Questions 11 - 31
- Allow about 2 hours and 45 minutes for this section

This question paper must not be removed from the examination room

This assessment task constitutes 30% of the course.

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1. Which value of x satisfies both of these equations?

$$y = 2x - 3$$

$$y = 4x + 1$$

A. $x = -2$

B. $x = -1$

C. $x = 1$

D. $x = 2$

2. What is the exact value of

$$\int_1^4 \frac{1}{9x}\,dx$$

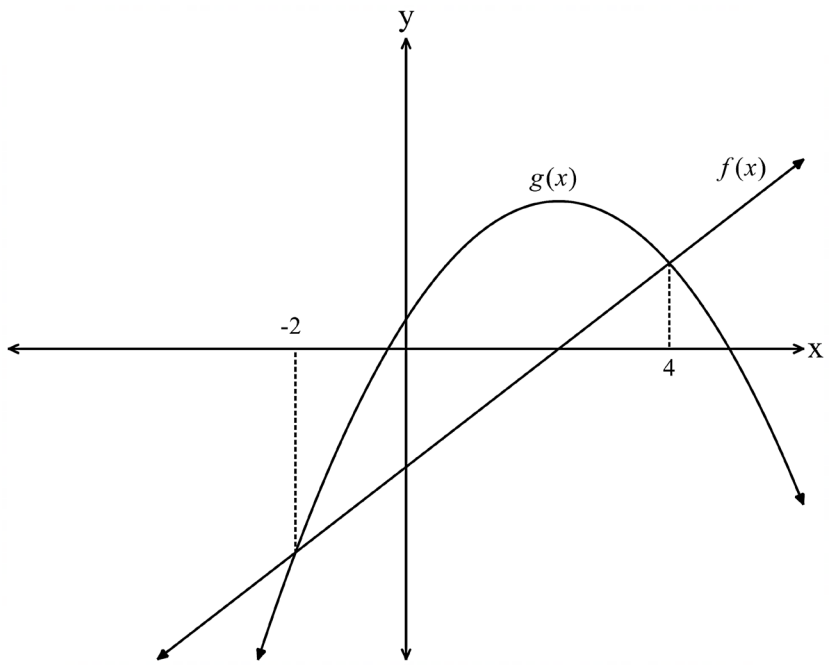
A. $\frac{1}{3}\ln 3$

B. $\frac{1}{3}\ln 4$

C. $\frac{1}{9}\ln 4$

D. $\frac{1}{9}\ln 3$

3. Consider $f(x)$ and $g(x)$ on the diagram below.



Solve the following inequatilty:

$$g(x) \geq f(x)$$

A. $x \in (-\infty, -2]$

B. $x \in [-2, 4]$

C. $x \in [4, \infty)$

D. $x \in (-\infty, -2] \cup [4, \infty)$

4. Consider the following geometric sequence

$$81, a, b, 3$$

Find the value of $b - a$

A. -18

B. 18

C. -26

D. 26

5. Suppose that

$$\int_3^{10} f(x)\ dx = C \quad \text{and} \quad \int_7^{10} f(x)\ dx = D$$

Determine the value of

$$\int_3^{7} f(x)\ dx$$

A. $C + D$

B. $C + D - 3$

C. $C - D$

D. $D - C$

6. The discrete random variable X has the following probability distribution.

x	0	1	2	3
$P(X = x)$	a	$3a$	$5a$	$7a$

Calculate $E(X)$.

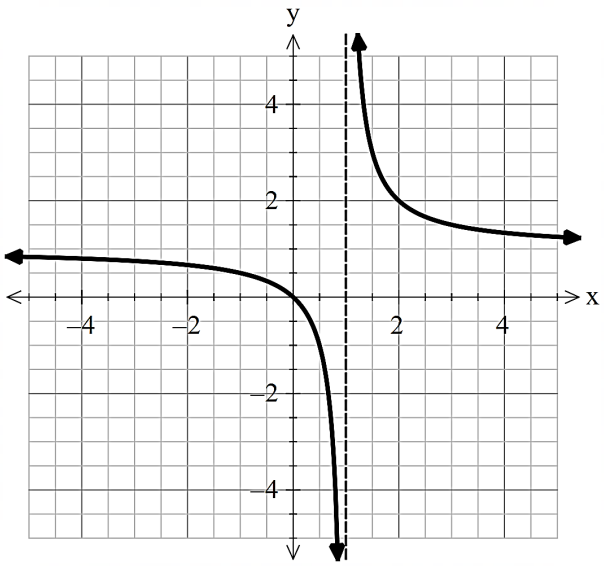
A. $\frac{1}{16}$

B. 1

C. $\frac{35}{16}$

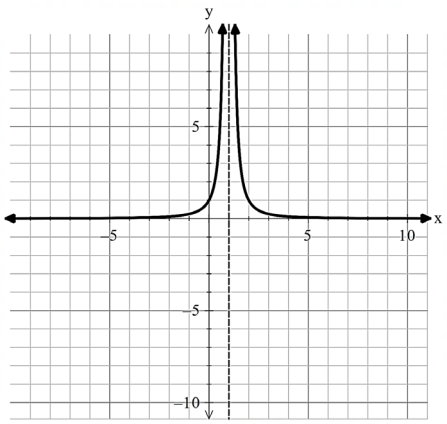
D. $\frac{17}{8}$

7. The following functions represents $f(x)$.

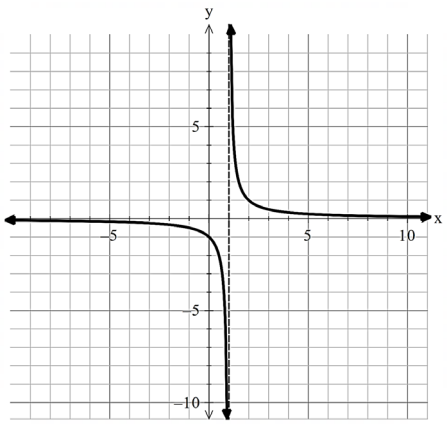


Which option below represents $f'(x)$?

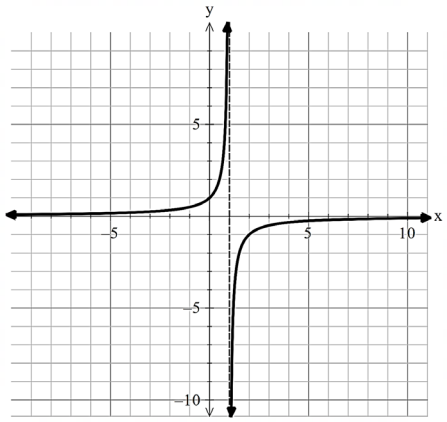
A.



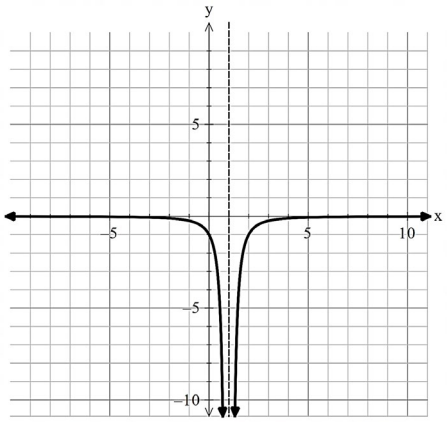
B.



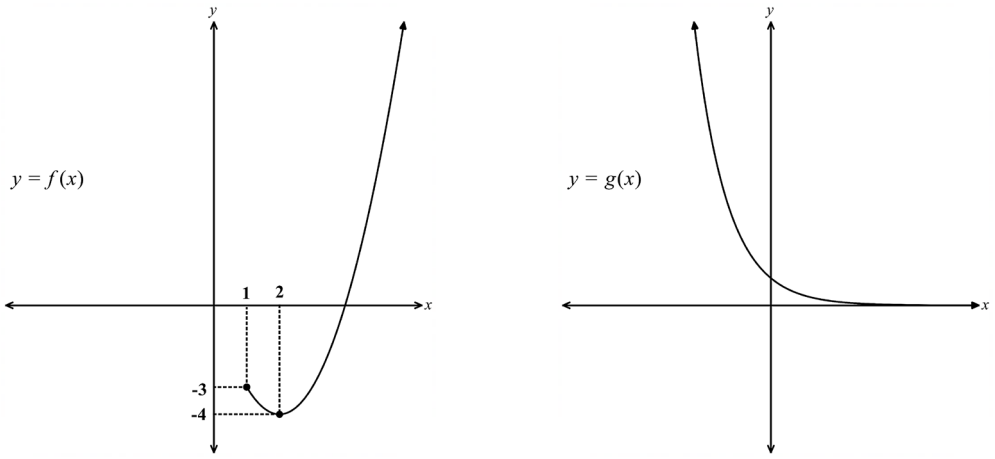
C.



D.



8. The graph of $f(x) = x^2 - 4x$ for $x \geq 1$ and $g(x) = e^{-x}$ are shown.



Determine the range of the composite function $g\big(f(x)\big)$.

A. $(0, e^3)$

B. $(0, e^3]$

C. $(0, e^4)$

D. $(0, e^4]$

9. Let $y = f(x)$ be a differentiable function for all real x, such that

- $f'(3) = 0$
- $f'(x) < 0$ when $x < 3$ and when $x > 3$

When $x = 3$, the graph of $f(x)$ has a

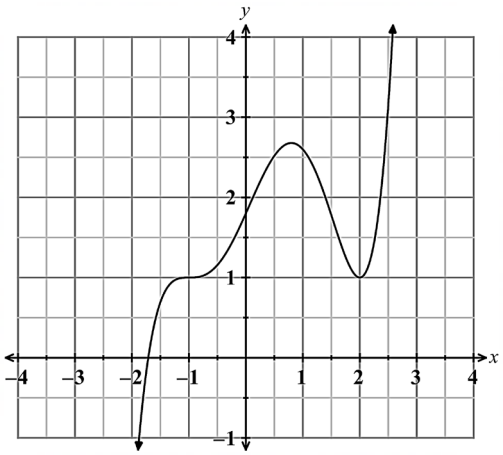
A. Local minimum

B. Local maximum

C. Point of inflection

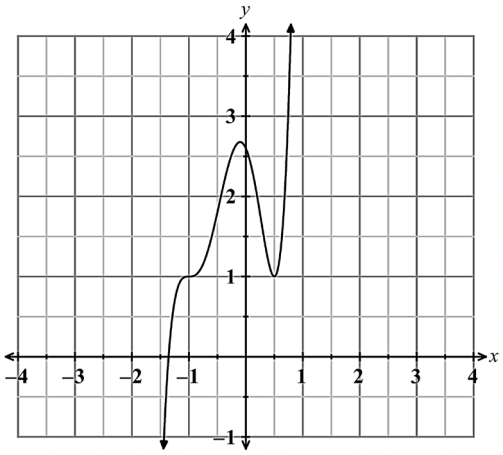
D. Gradient of 3

10. The graph of $y = f(x)$ is shown below.

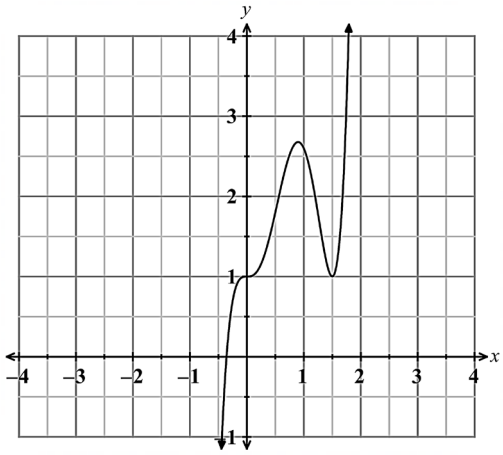


Which of the following options best represent the graph of $y = f(2x + 1)$?

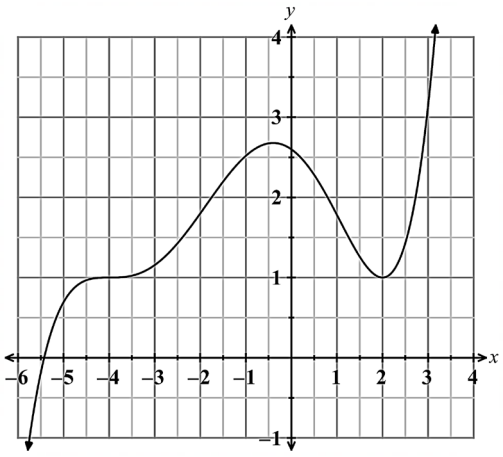
A.



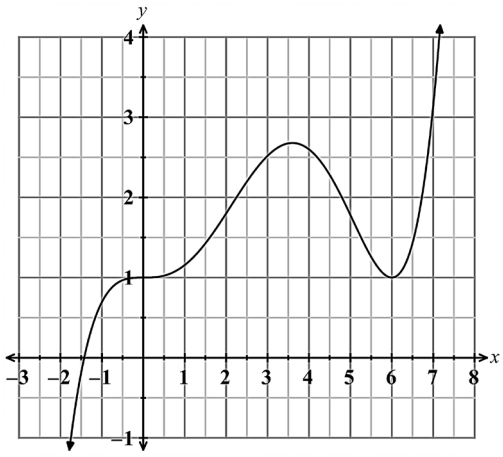
B.



C.



D.



End of Section I

Killara High School

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2025 Year 12 Trial Examination

Mathematics Advanced

Section II Answer Booklet 1

11/08/2025

Section II

90 Marks
Attempt Questions 11 - 31
Allow about 2 hours and 45 minutes for this section.

Booklet I – Attempt Questions 11 – 25 (55 marks)
Booklet II – Attempt Questions 26 – 31 (35 marks)

Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on page 29 - 30
- If you use this space, clearly indicate which question you are answering.

Q	Marks
11	/3
12	/3
13	/3
14	/3
15	/4
16	/6
17	/2
18	/3
19	/4
20	/6
21	/4
22	/4
23	/2
24	/3
25	/5
Total	/55

Section II

In Questions 11 - 31, your response should include relevant mathematical reasoning and/or calculations.

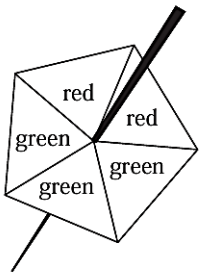
Question 11 (3 marks)

Find the sum of the terms in the geometric series. **3**

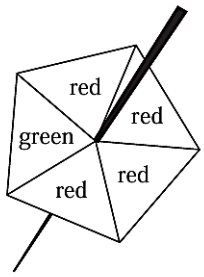
$$2 + 6 + 18 + 54 + \cdots + 118098$$

Question 12 (3 marks)

Harry has two fair 5-sided spinners.



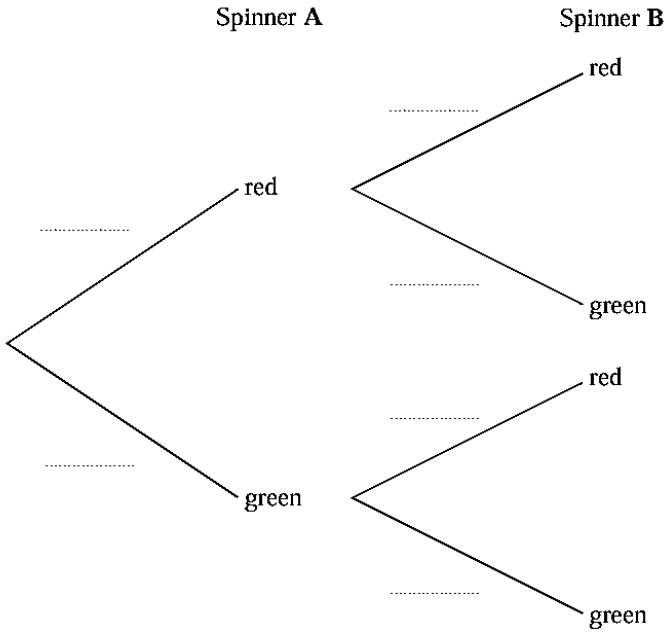
Spinner **A**



Spinner **B**

Harry is going to spin each spinner once.

(a) Complete the probability tree diagram below by labelling the probabilities on each branch. **2**

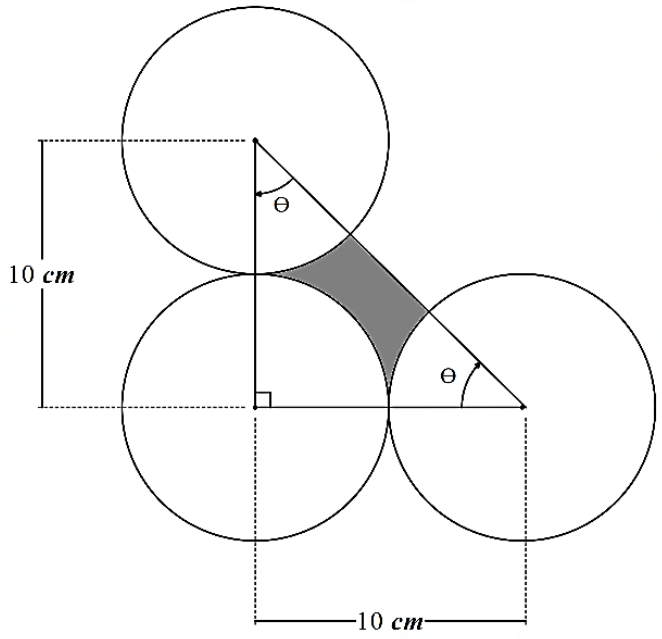


(b) Calculate the probability that at least one of the spinners will land on green. **1**

Question 13 (3 marks)

In the diagram below, the circles are equal in size, and the triangle is a right-angled isosceles triangle. **3**

Calculate the area of the shaded section. Leave your answer as an exact value.



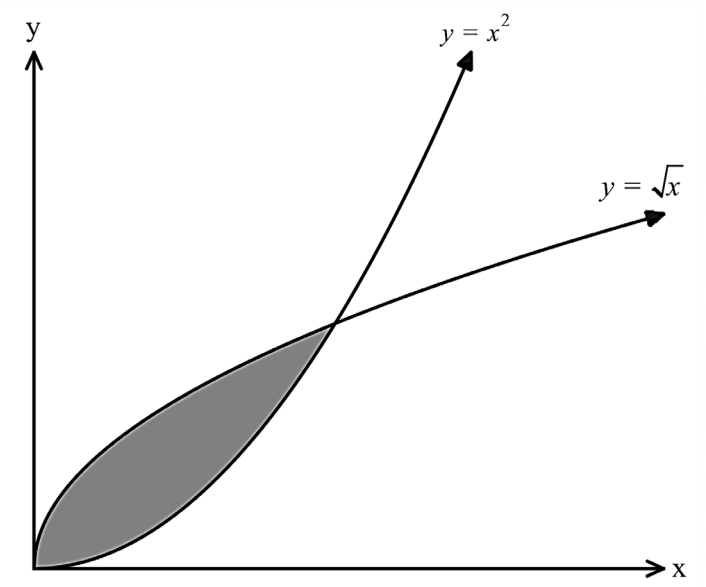
Question 14 (3 marks)

Solve the following equation for $\theta \in [0, \pi]$. **3**

$$2 \sin\theta \cos\theta = \sin\theta$$

Question 15 (4 marks)

The graph of $y = x^2$ and $y = \sqrt{x}$ are shown below for $x \geq 0$.



(a) Find the points of intersection between the two graphs. **2**

(b) Hence, find the area of the shaded region. **2**

Question 16 (6 marks)

A mum deposited \$100 into her son's saving account on his first birthday. She deposits \$125 on his second birthday. \$150 on his third birthday, and so on.

(a) Calculate the amount of money she will deposit into her son's account on his 15^{th} birthday. **2**

(b) On which birthday will the mum deposit \$700? **2**

(c) Hence, calculate the total amount she will have deposited over 25 years. **2**

Question 17 (2 marks)

Prove the following identities. **2**

$$(1 - \tan x)^2 + (1 + \tan x)^2 = 2\sec^2 x$$

Question 18 (3 marks)

Solve for a: **3**

$$\ln(a) - \ln 3 = \ln(a+2) - \ln(a-2)$$

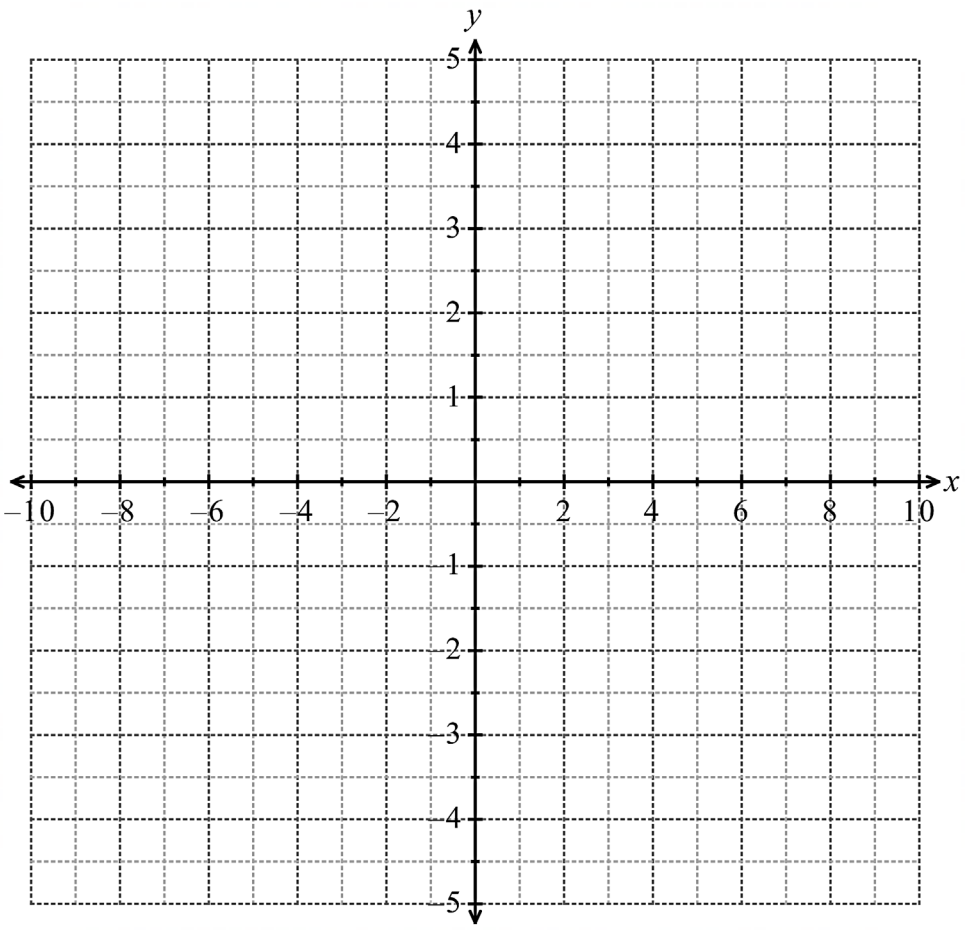
Question 19 (4 marks)

Given the two functions: **4**

$$f(x) = \frac{1}{x^2 + 2} - 3$$

$$g(x) = \sqrt{x + 2}$$

Sketch $y = f\big(g(x)\big)$ over its natural domain.



Question 20 (6 marks)

The probability density function, f, of a random variable X is given by

$$f(x) = \begin{cases} kx^2(2-x), & 0 \leq x \leq 2 \\ 0, & elsewhere \end{cases}$$

(a) Show that $k = \frac{3}{4}$. **2**

Question 20 continues on page 22

Question 20 (continued)

(b) Hence, show that $P(X \geq 1.5) = \frac{67}{256}$ **2**

(c) Show that the mode of the probability distribution function is $x = \frac{4}{3}$. **2**

End of Question 20

Question 21 (4 marks)

The results of a large group of students in a Mathematics competition are represented by a random variable X and are normally distributed with a mean of 70 and a standard deviation of 10.

(a) Determine the $z-$ score for a student who scored 66. **1**

(b) Students who scored in the top 2.5% of the group receive a monetary prize? **1**

What is the minimum mark required to receive the prize?

(c) Students who gained a mark with a $z-$ score range $0 \leq z \leq 1$ received a credit certificate. **2**

Those students with a mark with a $z-$ score range of $1 \leq z \leq 2$ received a distinction certificate.

If 816 credit certificates were awarded, how many distinction certificates were issued?

Question 22 (4 marks)

(a) Show that **2**

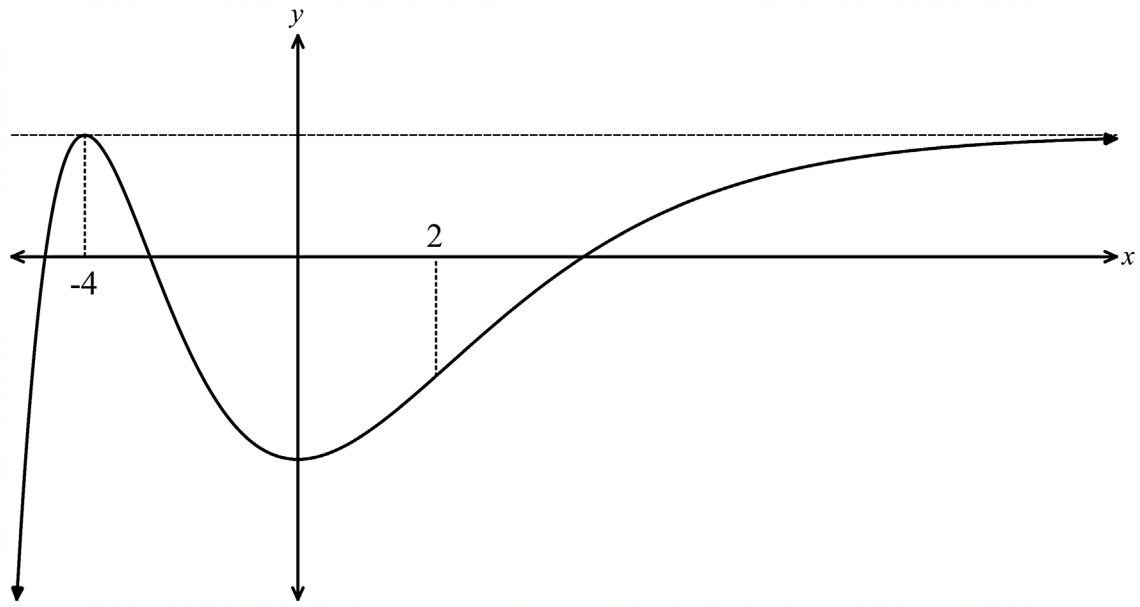
$$\frac{d}{dx}[x^2 \ln(x)] = 2x \ln(x) + x$$

(b) Hence show that **2**

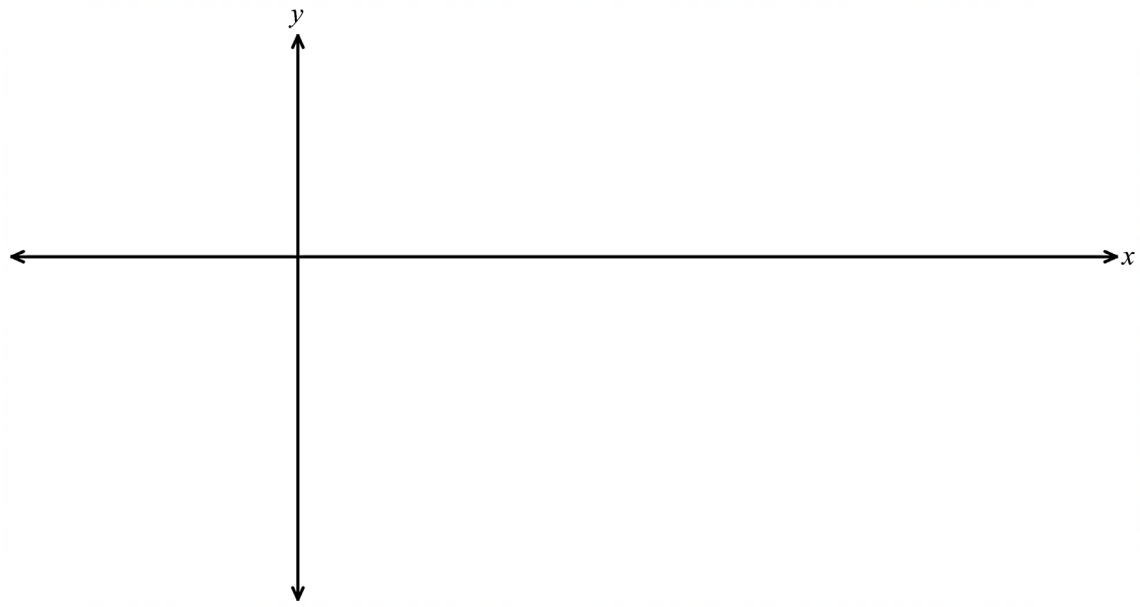
$$\int x \ln(x)\, dx = \frac{x^2 \ln(x)}{2} - \frac{x^2}{4} + C$$

Question 23 (2 marks)

The diagram shows the graph of a function $y = f(x)$. **2**



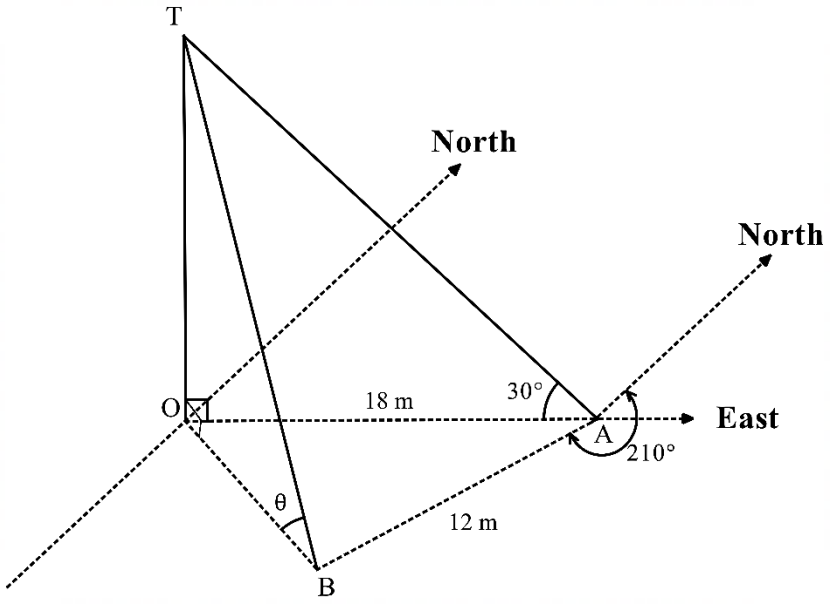
Sketch the graph of $y = f'(x)$



Question 24 (3 marks)

A flagpole, OT, lies on flat level ground as shown on the diagram below. **3**

- Person A is standing 18 metres due east of the tower with an angle of elevation of 30°.
- Person B is standing 12 metres from Person A on a bearing of 210°.



Find the angle of elevation θ from B to T to the nearest degree.

Question 25 (5 marks)

The velocity of a particle moving in a straight line at the end of t seconds is given by

$$v(t) = 3t^2 - 15t + 18, \quad \text{where } 0 \leq t \leq 4$$

(a) Show that the particle is stationary at $t = 2$ and $t = 3$. **2**

Question 25 continues on page 28

Question 25 (continued)

(b) Hence, or otherwise find the total distance travelled by the particle between $t = 1$ to $t = 3$. **3**

End of Booklet 1

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2025 **Year 12 Trial Examination**

Mathematics Advanced

Section II Answer Booklet 2

11/08/2025

Q	Marks
26	/4
27	/6
28	/6
29	/5
30	/6
31	/8
Total	/35

Booklet 2 – Attempt Questions 26 - 31 (35 marks)

Instructions

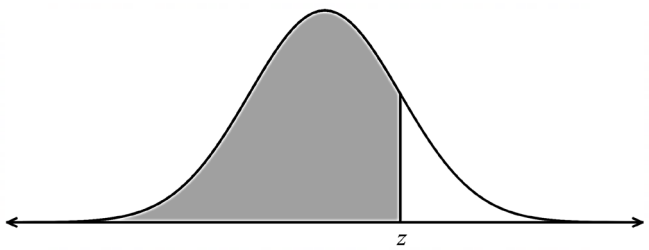
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on page 43 - 44.
- If you use this space, clearly indicate which question you are answering.

Question 26 (4 marks)

A random variable is normally distributed with a mean of 0 and standard deviation of 1. The table gives the probability that this random variable lies below z for some positive values of z.

z	-2	-1.25	-0.5	0.25	1	1.75	2.5	3.25
Probability	0.0228	0.1057	0.3085	0.5987	0.8413	0.9599	0.9938	0.9994

The probability values given in the table above are represented by the shaded area in the diagram below.



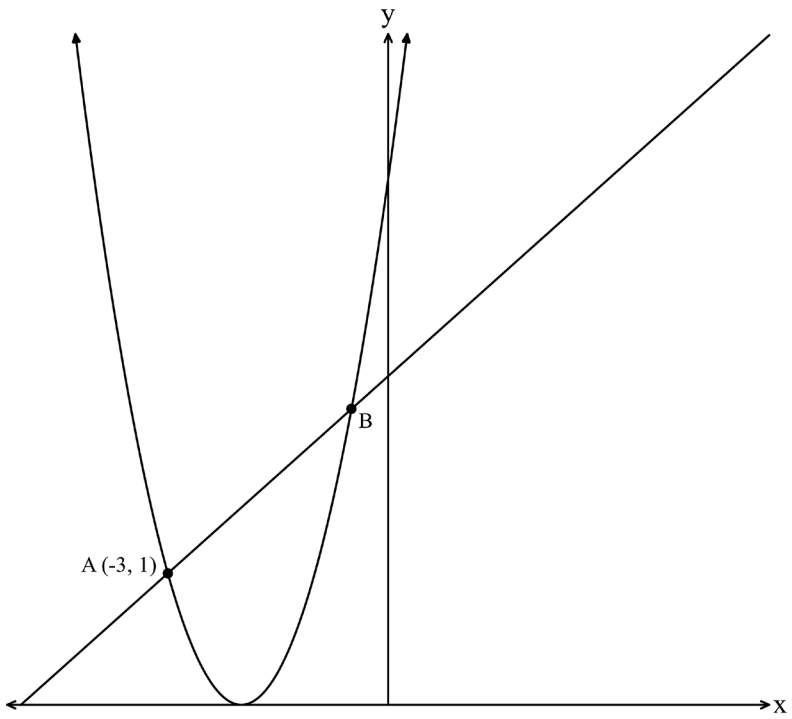
Let X be a normally distributed random variable with a mean of 72 and a standard deviation of 8.

(a) Calculate the probability that $62 < X < 92$. **2**

(b) Calculate the probability that $X < 62$ given that $X < 92$ to the nearest 4 decimal places. **2**

Question 27 (6 marks)

The normal to the curve $y = (x + 2)^2$ at the point $A(-3,1)$ meets the curve again at B as shown in the diagram below.



(a) Find the equation of the normal **2**

Question 27 continue on page 33

Question 27 (continued)

(b) Show that the coordinate of B is given by $\left(-\frac{1}{2}, 2\frac{1}{4}\right)$ **2**

(c) Hence, find the angle between the line AB and the tangent at B. **2**

End of Question 27

Question 28 (6 marks)

Consider the following function:

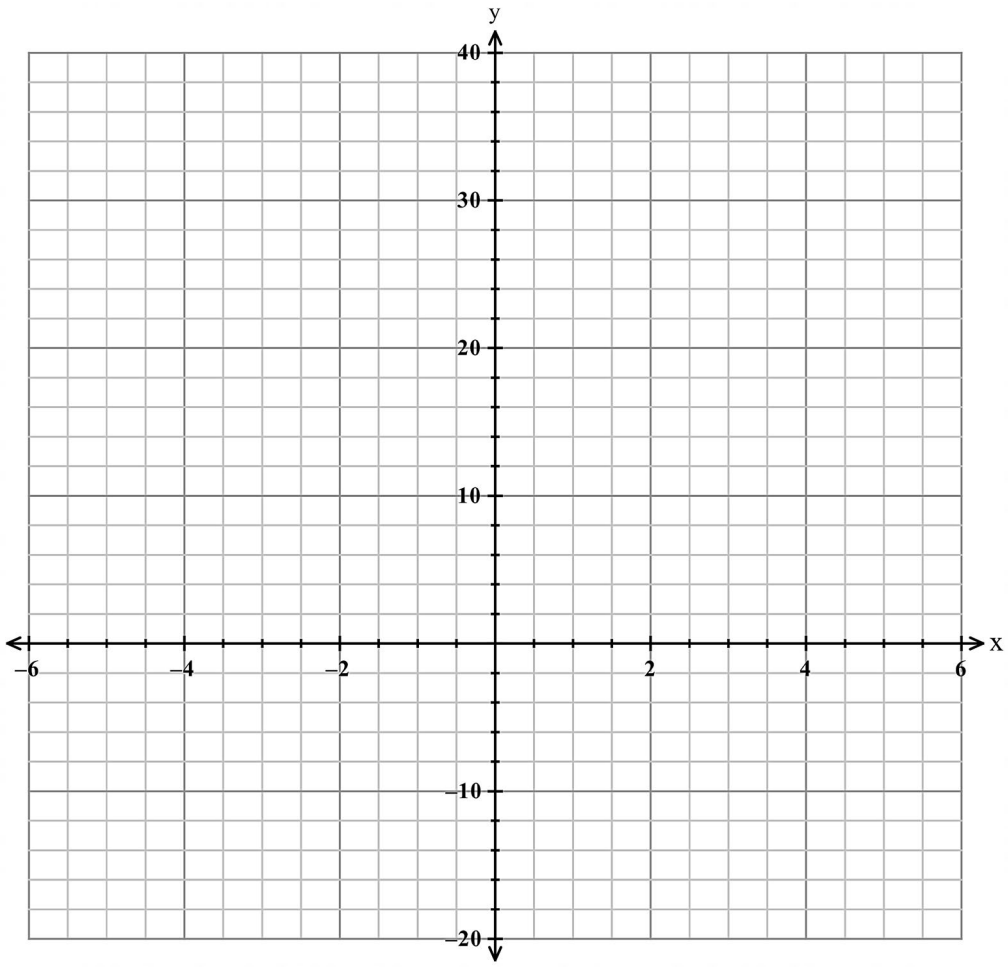
$$f(x) = (x + 3)(x - 2)(x - 5).$$

(a) Show that the turning points of $f(x)$ is $(-1, 36)$ and $\left(\frac{11}{3}, -\frac{400}{27}\right)$. **3**

Question 28 continue on page 35

Question 28 (continued)

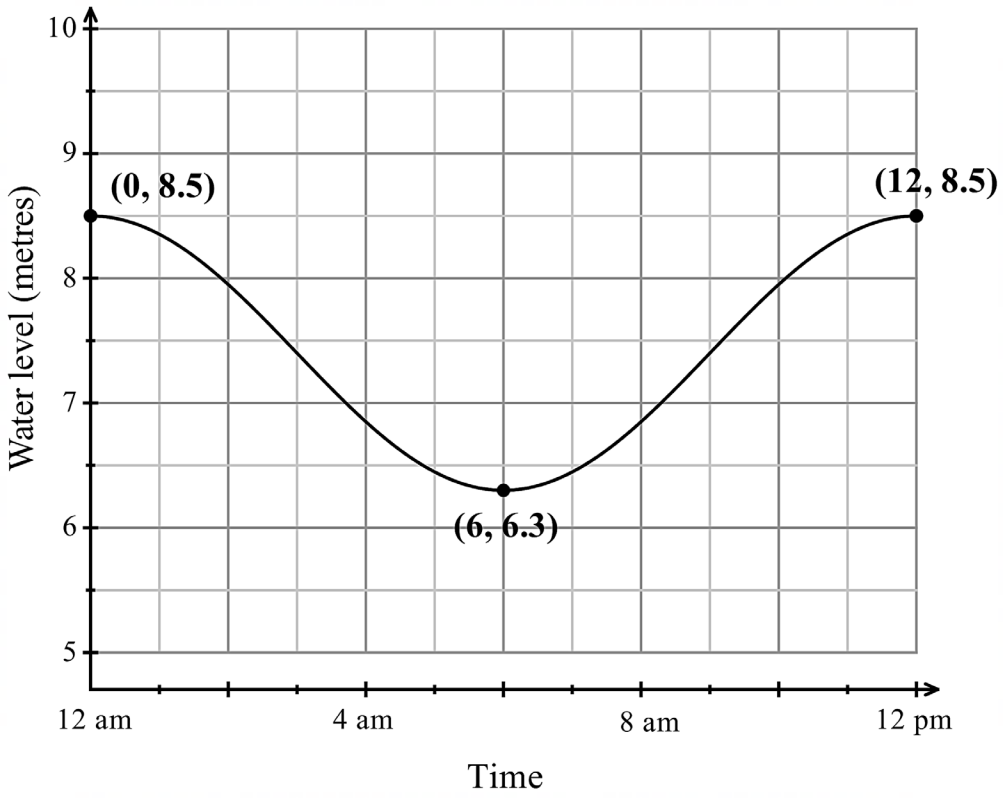
(b) Hence, sketch $f(x)$ and label its turning points and intercepts. **3**



End of Question 28

Question 29 (5 marks)

The graph shows the water level under a bridge over a 12-hour period.



(a) Show that the equation of the curve that models the water level as a function of time after 12: 00 am is given by: **2**

$$w(t) = 1.1\cos\left(\frac{\pi}{6}t\right) + 7.4$$

Question 29 continue on page 37

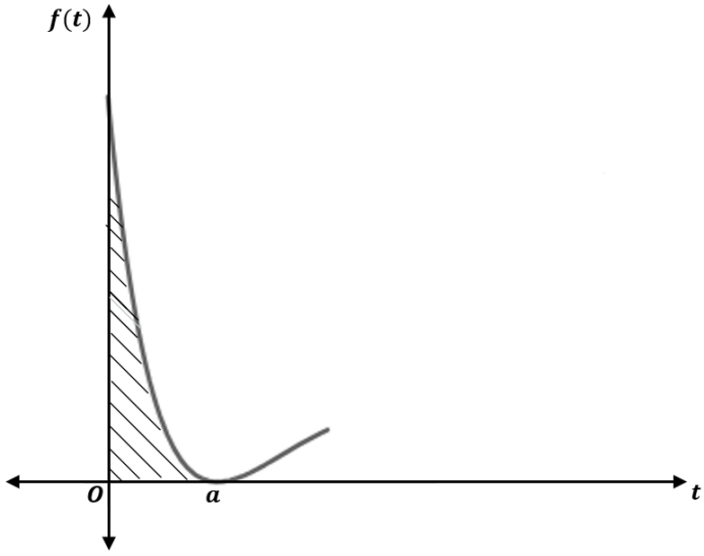
Question 29 (continued)

(b) Calculate how long in the 12-hour period shown that the rate of change of water level is increasing at more than 0.55 metres per hour? Correct your answer to the nearest minute. **3**

End of Question 29

Question 30 (6 marks)

Part of the graph of the function $f(t) = (t-1)^2 e^{-t}, 0 \leq t < \infty$ is shown below.



(a) Complete the graph of $f(t) = (t-1)^2 e^{-t}, 0 \leq t < \infty$. **4**

Showing clearly the exact coordinates of all stationary points.

Question 30 continue on page 39

Question 30 (continued)

(b) The function $f(t)$ is transformed to $g(t) = 2f(t) + 5$. **2**

It is given that the shaded area shown above, bounded by the curve $f(t) = (t-1)^2 e^{-t}, 0 \leq t \leq a$ and the x-axis is $1 - \frac{4}{e}$ unit^2. *(Do not prove this).*

Therefore evaluate the following integral

$$\int_0^a g(t)\ dt$$

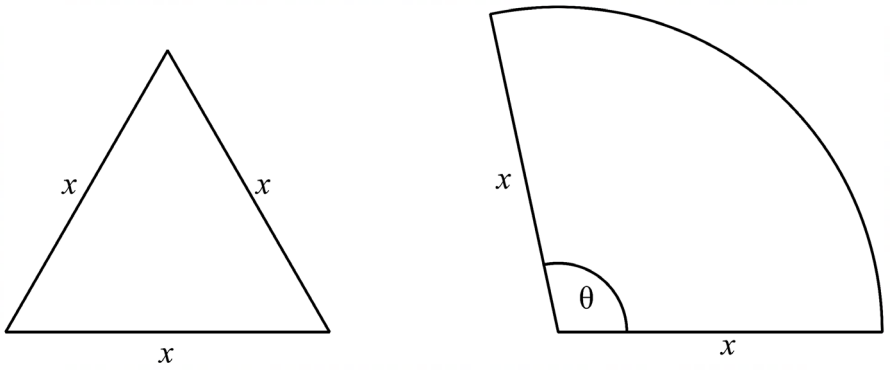
End of Question 30

Question 31 (8 marks)

A wire total length 60 cm is to be cut into two pieces.

The first piece is bent to form an equilateral triangle of side length x cm.

The second piece is bent to form a circular sector of radius x cm with an angle of θ radians at the centre.



(a) Show that **2**

$$x\theta = 60 - 5x$$

Question 31 continue on page 41

Question 31 (continued)

(b) The total area of the two shapes is $\boldsymbol{A}\ cm^2$. **3**

Hence, or otherwise, show that

$$A = \frac{1}{4}\left(\sqrt{3} - 10\right)x^2 + 30x$$

Question 31 continue on page 42

Question 31 (continued)

(c) Hence, find the maximum total area of both shapes to two decimal places. **3**

End of Examination

2025 Trial HSC Mathematics Advanced
Marking Guidelines

Section I

Multiple-choice Answer Key

Question	Answer
1	A
2	C
3	B
4	A
5	C
6	D
7	D
8	D
9	C
10	A

Section II

Question 11

Criteria	Marks
In addition, correctly calculates the sum of the series	3
In addition, identifies that the series contains 11 terms	2
Identifies a and r, either explicitly or by substituting into a formula	1

Sample answer:

$a = 2 \qquad r = 3$

$T_n = 2 \times 3^{n-1} = 118098$

$3^{n-1} = 59049$

$n - 1 = log_3 59049$

$n = log_3 59049 + 1$

$n = 11$

$\therefore S_{11} = \dfrac{2(3^{11} - 1)}{3 - 2} = 177146$

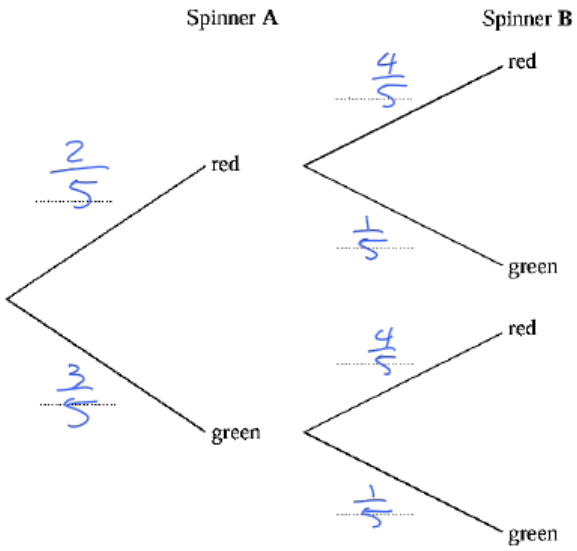
Markers' feedback:

- Some students identified that $n = 11$ without any working. I presume they used a calculator to generate terms and simply counted them. While this is not a scalable method, a mark was still awarded.
- Some students tried using formulas for arithmetic series instead of geometric series – it is strongly recommended to read the question carefully.

Question 12 (a)

Criteria	Marks
Correctly labelled tree with spinner B conditional probabilities	2
Correctly labelled tree with spinner A probabilities	1

Sample answer:



Question 12 (b)

Criteria	Marks
Calculate probability of no greens	1

Sample answer:

$$1 - P(Both\ red) = 1 - \left(\frac{2}{5} \times \frac{4}{5}\right)$$

$$= \frac{17}{25}$$

<u>Markers' feedback:</u>

- Most students labelled the tree correctly
- Some students didn't notice that spinner B was different to spinner A, and for spinner B wrote the same probabilities that they had used for spinner A
- One student wrote total probabilities instead of conditional probabilities for spinner B – this didn't earn a mark.
- Most students used the complement $P(\overline{GG}) = 1 - P(RR)$. Some calculated directly by adding the various cases together. Both approaches earned a mark.

Question 13

Criteria	Marks
In addition, correctly combines calculated values above to arrive at shaded area.	3
In addition, correctly calculates area of sectors.	2
Correctly calculates area of triangle	1

Sample answer:

$\theta = \frac{\pi}{4}$ $\quad$ r = 5 cm

$$\boldsymbol{Area\ of\ triangle} = \frac{1}{2} \times 10 \times 10 = 50\ cm^2$$

$$\boldsymbol{Sector} = \frac{\pi \times 5^2}{4} + 2 \times \left(\frac{1}{2} \times \frac{\pi}{4} \times 5^2\right)$$

$$= \frac{25\pi}{4} + \frac{25\pi}{4}$$

$$= \frac{25\pi}{2}$$

$$\therefore \boldsymbol{Shaded\ area} = 50 - \frac{25\pi}{2}$$

Markers' feedback:

- Most students calculated the area of the triangle correctly.
- Many students calculated area of sectors correctly.
- Some realised that the sectors make up a semi-circle and just used $A = \frac{1}{2}\pi r^2$. This was fine.
- Quite a few students sought that $100 - 25\pi = 75\pi$ — this is not fine.
- Some students wrote their answer is terms of without realising that $\theta = \frac{\pi}{4}$. This lost the mark.
- Some students got mixed up between degrees and radians — the formula $A = \frac{1}{2}\theta r^2$ only works if is in radians.

Question 14

Criteria	Marks
Find solution $\theta = \pi$	3
Find solution $\theta = 0$	2
Find solution $\theta = \frac{\pi}{3}$	1

Sample answer:

$2\,sin\,\theta\,cos\,\theta - sin\,\theta = 0$

$sin\,\theta\,(2\,cos\,\theta - 1) = 0$

$sin\,\theta = 0$

$\theta = 0, \pi$

$2\,cos\,\theta - 1 = 0$

$cos\,\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}$

Markers' feedback:

- Many students divided the equation through by without acknowledging that this is only allowed if $sin\theta \neq 0$. As a result, they lost two of the solutions.
- Some students did "guess and check" and only found one solution. They got 1 mark for that solution.
- Some students listed values outside the domain – these solutions were disregarded.
- Some students provided solutions in degrees rather than radians – while technically incorrect (the domain was clearly specified in radians), they were still awarded a mark.

Question 15 (a)

Criteria	Marks
Identified both point's x coordinates	2
Missed one point	1

Sample answer:

$x^2 = \sqrt{x}$
$x^4 = x$
$x^4 - x = 0$
$x(x^3 - 1) = 0$

$\therefore x = 0, 1$
$\therefore y = 0, 1$

Question 15 (b)

Criteria	Marks
Correct integration with complete substitution and correct answer	2
Mistake in bounds or only one mistake in integration	1

Sample answer:

$$\int_0^1 \sqrt{x} - x^2 \, dx$$
$$= \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^3}{3}\right]_0^1$$
$$= \frac{1}{3}$$

Markers' feedback:

Question was well done, though many students missed the (0,0) intercept.

Question 16 (a)

Criteria	Marks
Correctly identified a and d and found T15	2
Mistake in a or d leading to incorrect final result from correct Term calculation	1

Sample answer:

$100, 125, 150, \ldots$

$a = 100$ and $d = 25$

$T_{15} = 100 + 14 \times 25$
$= 450$

Question 16 (b)

Criteria	Marks
Correct formula with correct answer	2
Correct formula with arithmetic mistake	1

Sample answer:

$T_n = 700$
$100 + 25(n - 1) = 700$
$100 + 25n - 25 = 700$
$25n = 625$
$n = 25$

Question 16 (c)

Criteria	**Marks**
Correct summation formula with correct answer	2
Correct formula with arithmetic mistake	1

Sample answer:

$$S_{25} = \frac{25}{2}(100 + 700)$$
$$= 10000$$

Markers' feedback:

Done well

Question 17

Criteria	**Marks**
Use of expansion and the sec^2 result	2
Use of either correct expansion or sec^2 result	1

Sample answer:

$$LHS = 1 - 2\tan(x) + \tan^2(x) + 1 + 2\tan(x) + \tan^2(x)$$
$$= 2 + 2\tan^2(x)$$
$$= 2(1 + \tan^2(x))$$
$$= 2\sec^2(x)$$
$$= RHS$$

Markers' feedback:

Mostly well done, few mistakes

Question 18

Criteria	**Marks**
Correct single answer with correct working	3
Two answers correctly found or only one arithmetic mistake leading to two answers and removing one	2
Arithmetic mistake leads to one answer so no final removal	1

Sample answer:

$\ln\left(\frac{a}{3}\right) = \ln\left(\frac{a+2}{a-2}\right)$
$\frac{a}{3} = \frac{a+2}{a-2}$
$a^2 - 2a = 3a + 6$
$(a-6)(a+1) = 0$
$\therefore a = 6 \text{ or } -1$
$\therefore a \text{ can only be } 6 \text{ because } a \geq 0$

Markers' feedback:

Poorly completed, most students struggled with log laws and index laws. Students got both answers, about half correctly removed the extra answer.

Question 19

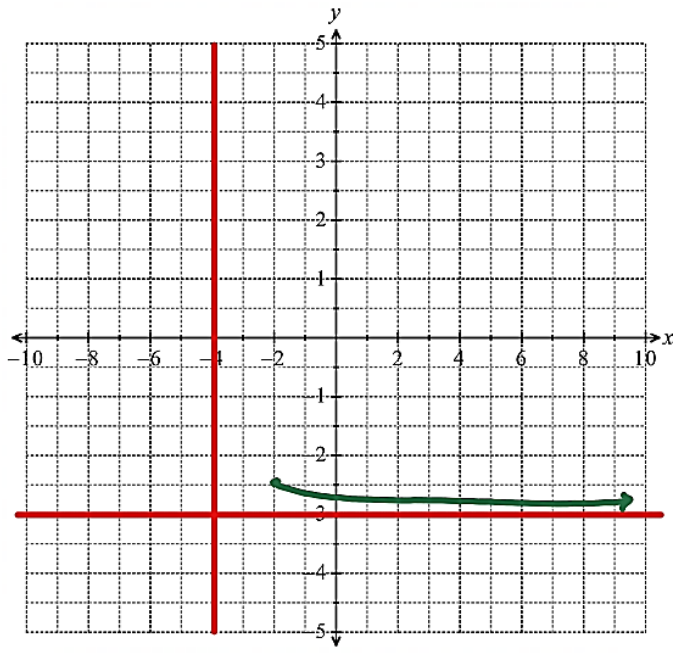
Criteria	**Marks**
Correct domain, asymptotic behaviour, intercepts and general shape	4
Missed a specific limitation of the graph, like its asymptotic nature or general shape	3
If incorrect domain, most scored was 2, had some asymptotic behaviour and intercepts correct	2
Had only one facet of the curve correct	1

Sample answer:

$$f\big(g(x)\big) = \frac{1}{x+4} - 3$$

Domain:

$x \geq -2$



Markers' feedback:

Very poorly done, most students did not account for the limited domain of the function despite it being hinted at in the question.

Question 20 (a)

Criteria	Marks
If the integration was conducted correctly and k was solved to equal $\frac{3}{4}$	2
One mark for letting the integral equal 1 and taking k common (and having dx)	1

Sample answer:

$$k\int_0^2 x^2(2-x)\,dx = 1$$

$$k\left[\frac{2x^3}{3} - \frac{x^4}{4}\right]_0^2 = 1$$

$$\frac{4k}{3} = 1$$

$$k = \frac{3}{4}$$

Markers Feedback: Most students did not have a problem with integration; however many either did not get the limit correct or did not let the integral value equal to 1.

Question 20 (b)

Criteria	Marks
Solving correctly for $\frac{67}{256}$ or showing that 1 – p[X<5] was giving the same value was given a mark.	2
A mark was not given for the integration as it was already given in part a). However, having the integral from 1.5 to 2 was given one mark.	1

Sample answer:

$$\frac{3}{4}\int_{1.5}^2 x^2(2-x)\,dx$$

$$= \frac{3}{4}\left[\frac{2x^3}{3} - \frac{x^4}{4}\right]_{1.5}^2$$

$$= \frac{67}{256}$$

Markers Feedback: Some students did not recognize that the limit had to be between 1.5 to 2. This led to many students not solving correctly even though the result was given. Many students chose to take the 1 – p(b) value which also led to many calculation errors. However, most students did achieve the 2 marks.

Question 20 (c)

Criteria	Marks
Showing $P''(x) = 3 - \frac{9}{2}x$ and solving for -3 or showing that the Nature of the points around the value determined a maximum point to give the mode was also acceptable.	**2**
Showing that $P'(x) = 0$ and stating that $x = 0$ or $\frac{4}{3}$	**1**

Sample answer:

$$P(x) = \frac{3}{4}x^2(2 - x)$$

$$P'(x) = 3x - \frac{9}{4}x^2$$

Then $P'(x) = 0$

$$3x\left(1 - \frac{3}{4}x\right) = 0$$

$$x = 0 \; or \frac{4}{3}$$

$$P''(x) = 3 - \frac{9}{2}x$$

Then $P''\left(\frac{4}{3}\right) = -3$

So $x = \frac{4}{3}$ *is the mode of the PDF.*

Markers' feedback: Most students knew how to take the derivative; however, many did not take the second derivative. This led to many losing a mark.

Question 21 (a)

Criteria	Marks
Substitute correctly to achieve a value of -0.4	**1**

Sample answer:

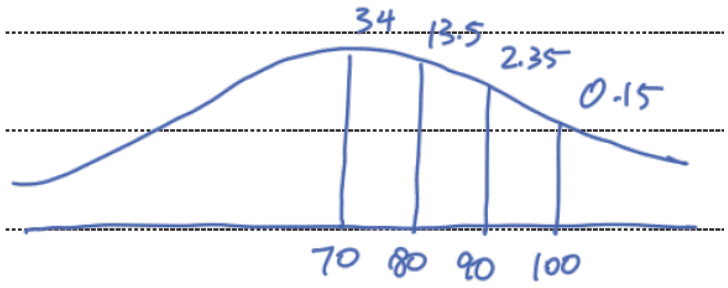
$$\frac{66-70}{10} = -0.4$$

Markers' feedback: Many achieved a positive value (they did not get the mark).

Question 21 (b)

Criteria	Marks
A breakdown to show how to get the 2.5% value.	**1**

Sample answer:



Markers feedback: Most students failed to show either a diagram or some working out to achieve the 2.5%

$\therefore$ The minimum mark is 90

Question 21 (c)

Criteria	Marks
The second mark was for multiplying by 13.5.	**2**
Unitory Method to show the value of 1% to equal 24.	**1**

Sample answer:

$816 = 34\%$

$24 = 1\%$

$\therefore 24 \times 13.5$

$= 324$ distinction certificates awarded

Markers feedback: This question was done well.

Question 22 (a)

Criteria	Marks
The second mark was putting it into the product rule. Solving correctly.	**2**
One mark for u, u', v and v'	**1**

Sample answer:

$u = x^2 \qquad v = ln(x)$

$u' = 2x \qquad v' = \frac{1}{x}$

$\frac{dy}{dx} = 2x\, ln(x) + x$

Markers feedback: This question was done well. Working out should be shown. Do not assume that the answer only is acceptable.

Question 22 (b)

Criteria	Marks
Letting the entire function equal x^2 ln(x) and solving to for the RHS. Adding the C.	**2**
Taking the 2 common and showing that the x^2 will be halved.	**1**

Sample answer:

$\int 2x\, ln(x) + x\ dx = x^2\, ln(x)$

$2\int x\, ln(x)\ dx + \int x\, dx = x^2\, ln(x)$

$2\int x\, ln(x)\, dx + \frac{x^2}{2} = x^2\, ln(x)$

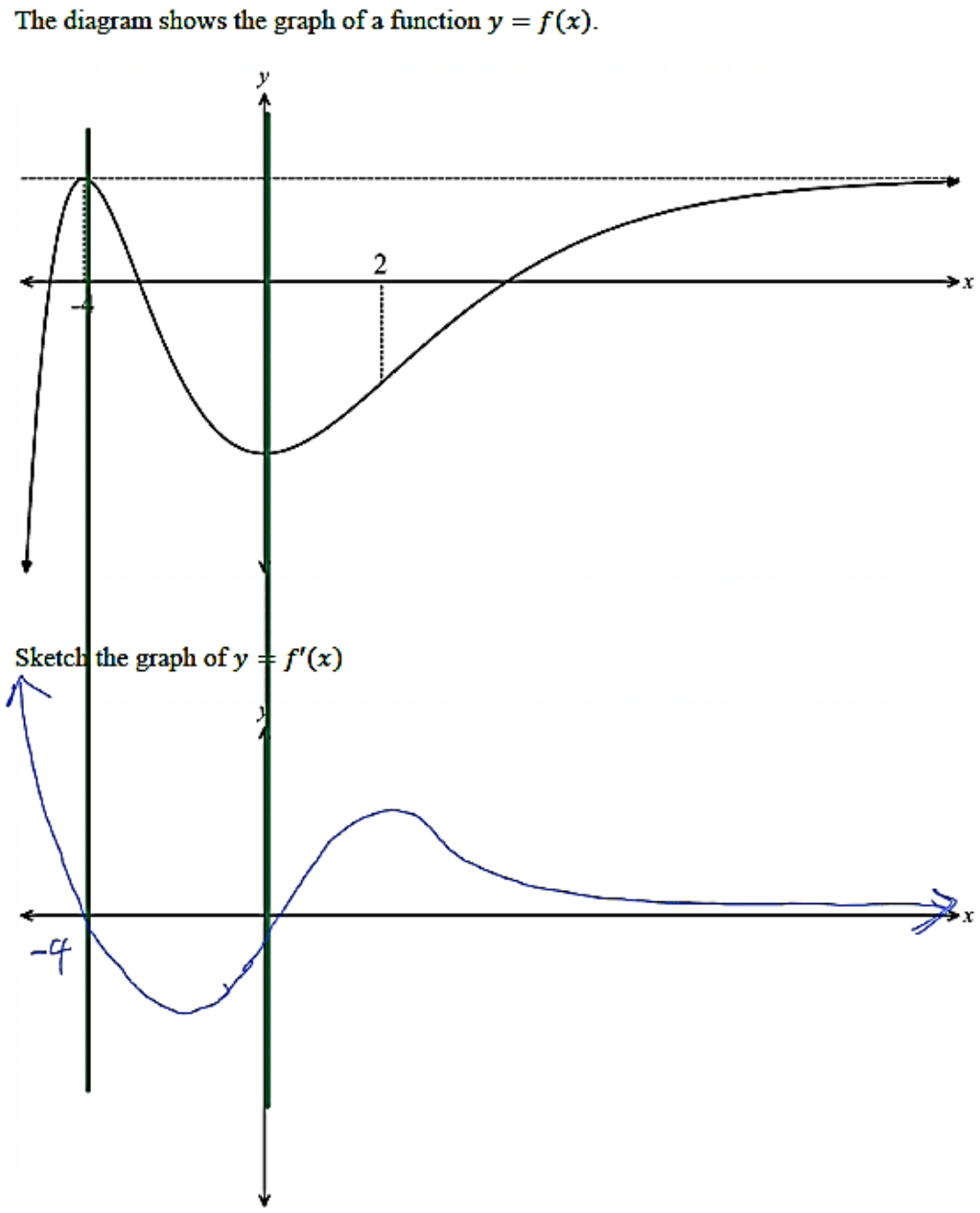
$\int x\, ln(x)\ dx = \frac{x^2\, ln(x)}{2} - \frac{x^2}{4} + C$

Markers' feedback: This was not done particularly well. Many students fumbled around the already given formula and did not get to the main integral.

Question 23

Criteria	Marks
Going through approximately the origin and then tending to the x – axis.	**2**
Starting from the positive and going through -4.	**1**

Sample answer:



<u>Markers' feedback:</u> This question was not done particularly well, many just copied the question and drew that on the second graph. Most understood that it went through x = -4.

Question 24

Criteria	Marks
Calculating $\theta = 63$ degrees.	**3**
Calculating OB correctly. (leave in exact form)	**2**
One mark for calculating OT correctly.	**1**

Sample answer:

$$tan\,30° = \frac{OT}{18}$$

$$OT = 18 \times tan\,30°$$

$$OT = \frac{18}{\sqrt{3}}$$

$$OB = \sqrt{18^2 + 12^2 - 2(18)(12)\,cos\,60°}$$

$$= \sqrt{252}$$

$$tan\,\theta = \frac{OT}{OB}$$

$$\theta = tan^{-1}\left(\frac{OT}{OB}\right)$$

$$= tan^{-1}\left(\frac{\frac{18}{\sqrt{3}}}{\sqrt{252}}\right)$$

$$= 63°(nearest\ degree)$$

Markers' feedback:

Most students achieved one mark for calculating the value of OT. Many did not correctly use OB to substitute to find the value of theta.

Question 25 (a)

Criteria	Marks
Provides correct steps	2
Let $v(t) = 0$, attempted to find the values of t. Or attempted to use substitution of the given values of t to show $v = 0$	1

Sample answer:

$v(t) = 3t^2 - 15t + 18$
$= t^2 - 5t + 6$
$(t-2)(t-3) = 0$
$\therefore t = 2, 3$

Question 25 (b)

Criteria	Marks
Provides correct answers	3
Attempted to find two definite integrals using the correct bounds, but made minor arithmetic errors	2
Recognises the change of direction at $t = 2$ by either drawing a graph of $v(t)$ or equivalent merit Or. If a student did not identify the positive and negative areas under the velocity function, they integrated from $t = 1\ to\ t = 3$, performed correct calculation subsequently.	1

Sample answer:

$$\int_1^2 3t^2 - 15t + 18\ dt + \left|\int_2^3 3t^2 - 15t + 18\ dt\right|$$
$$= \frac{5}{2} + \left|-\frac{1}{2}\right|$$
$= 3$ m

Markers' feedback:

Majority of the students were able to connect the definition of stationary velocity to $v(t) = 0$ and show part a) correctly. Very few students were able to recognise the change of the direction and the different sign of the areas under the velocity graph in order to calculate the distance travelled correctly. Further practice in this type of question is required.

Question 26 (a)

Criteria	Marks
Provides correct answer	2
Correctly finds the two corresponding z-scores	1

Sample answer:

$$z = \frac{62 - 72}{8} = -1.25$$
$$z = \frac{92 - 72}{8} = 2.5$$

$$P(62 < X < 92) = P(-1.25 < X < 92)$$
$$= 0.9938 - 0.1057$$
$$= 0.8881$$

Question 26 (b)

Criteria	Marks
Provides correct answer	2
Provides correct conditional probability formulae in relation to the question	1

Sample answer:

$$P(X < 62 | X < 92)$$
$$= \frac{P(X < 62 \cap X < 92)}{P(X < 92)}$$
$$= \frac{P(z < -1.25 \cap z < 2.5)}{P(z < 2.5)}$$
$$= \frac{0.1057}{0.9938}$$
$$= 0.10635$$
$$= 0.1064 \text{ (4 d. p)}$$

Markers' feedback:

Mostly done well in part a. Part b) was done poorly. Many students recognized the conditional probability in relation to this question but unable to identify the correct probability of each in the formula.

Question 27 (a)

Criteria	Marks
Provides correct linear equation of the normal	2
Provides correct gradient of the normal	1

Sample answer:

$y = (x+2)^2$
$y' = 2(x+2)$
$\therefore y'(2) = -2$
$\therefore m_{normal} = \frac{1}{2}$
$\therefore y - 1 = \frac{1}{2}(x+3)$
$\therefore y = \frac{1}{2}x + \frac{5}{2}$

Question 27 (b)

Criteria	Marks
Finds the second point of intersection (both x- and y- value) using simultaneous equation	2
Uses simultaneous equations to solve for the x-coordinates of the points of intersection, but made minor errors	1

Sample answer:

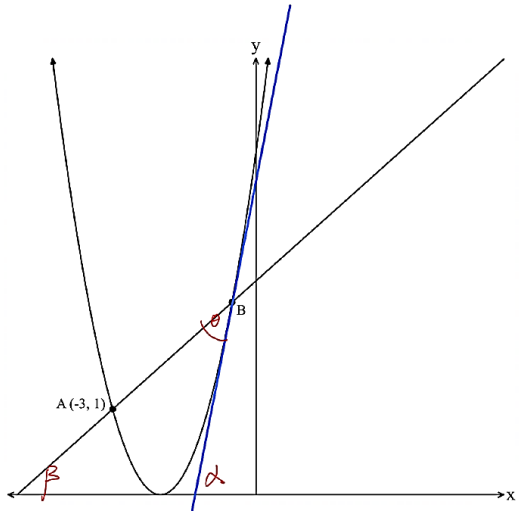
$\frac{1}{2}x + \frac{5}{2} = (x+2)^2$
$\frac{x}{2} + \frac{5}{2} = x^2 + 4x + 4$
$2x^2 + 7x + 3 = 0$
$(2x+1)(x+3) = 0$

$\therefore x = -\frac{1}{2}$ or -3
$\therefore y\left(-\frac{1}{2}\right) = 2\frac{1}{4}$
$\therefore B\left(-3, 2\frac{1}{4}\right)$

Question 27 (c)

Criteria	Marks
Provides the correct answer	2
Find the gradient of tangent B and converts it to angle of inclination	1

Sample answer:



$y' = 2(x + 2)$

$y'\left(\frac{1}{2}\right) = 3$ (gradient of the tangent at B)

$\tan\alpha = 3$

$\alpha = \tan^{-1}(3)$

$\alpha = 71.565$ ....

$\tan\beta = \frac{1}{2}$

$\beta = \tan^{-1}\left(\frac{1}{2}\right)$

$= 26.565$ ...

$\therefore \theta = \alpha - \beta$(exterior angle sum of triangle)

$= 71.56 \ldots - 26.565 \ldots$

$= 45$

Markers' feedback:

Part a) was done mostly correct. A lot of students made various mistakes in algebraic manipulation in solving the coordinates of points of intersections. Students who used given point to substitute were not awarded mark in part b. Part c) was done very poorly. Further revision on angle of inclination with geometrical application in angle relationship is required.

Question 28 (a)

Criteria	**Marks**
Provides the correct y-coordinates of the stationary points	3
Finds the correct x −values of the derivatives	2
Differentiates f(x) correctly	1

Sample answer:

$f(x) = x^3 - 4x^2 - 11x + 30$
$f'(x) = 3x^2 - 8x - 11$
$= (3x - 11)(x + 1)$

$\therefore x = \frac{11}{3}\ or - 1$

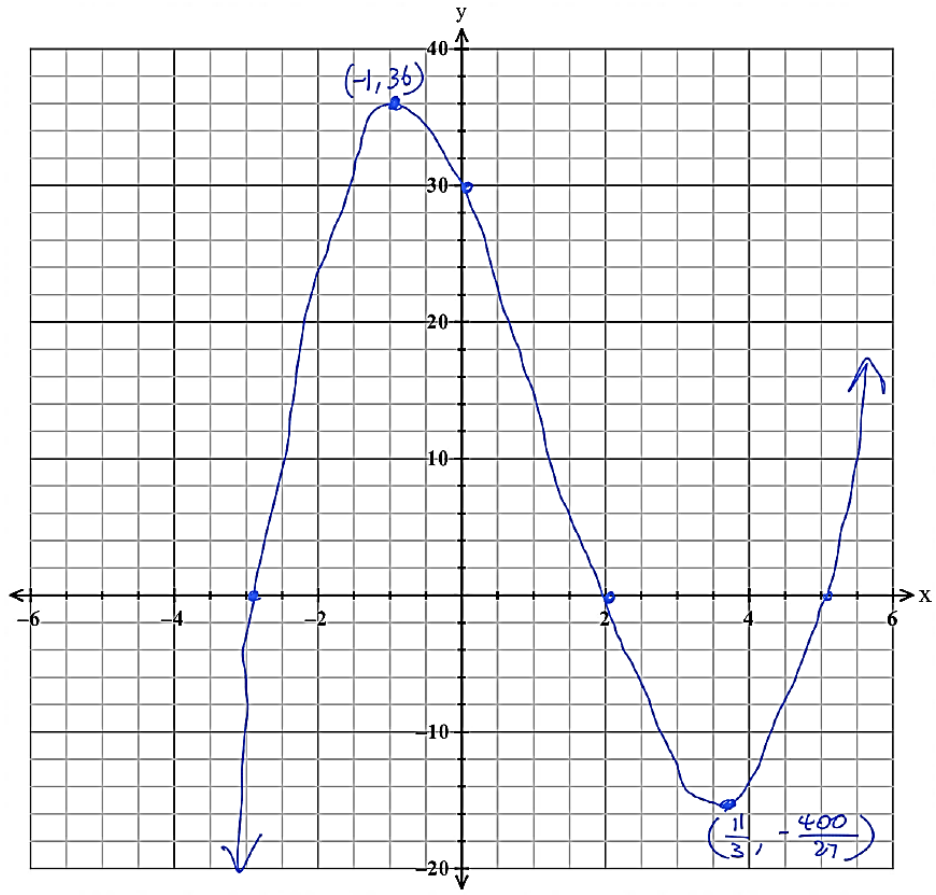
$\therefore f\left(\frac{11}{3}\right) = -\frac{400}{27}$ and $f(-1) = 36$

Question 28 (b)

Criteria	Marks
Provides the correct shape and all other key features	3
Provides correct shape, two of the y-intercept, x-intercepts and turning points	2
Provides correct shape of the polynomial	1

Sample answer:

$y-\text{intercept} = 30$
$x-\text{intercept} = -3, 2, 5$



Marker's feedback:
Students have done this question very well.

Question 29 (a)

Criteria	Marks
Explain all three features with reference to the amplitude, period and midline.	**2**
Explain two out of the three features with reference to the amplitude, period and midline.	**1**

Sample answer:

$\boldsymbol{Midline}: \dfrac{8.5 + 6.3}{2} = 7.4$

$\boldsymbol{Amplitude}: 8.5 - 7.4 = 1.1$

$\boldsymbol{Period}: \dfrac{2\pi}{a} = 12$

$a = \dfrac{\pi}{6}$

Markers' feedback:

Many students did not explain all the features with references to the amplitude, period or midline, hence they did not receive all the marks.

Question 29 (b)

Criteria	Marks
Provide the correct solution	3
Correctly solve $\frac{11\pi}{60}\sin\left(\frac{\pi}{6}t\right) = 0.55$	2
Correctly find the derivative of $w(t)$	1

Sample answer:

$w(t) = 1.1\cos\left(\dfrac{\pi}{6}t\right) + 7.5$

$w'(t) = \dfrac{11\pi}{60}\sin\left(\dfrac{\pi}{6}t\right)$

$\sin\left(\dfrac{\pi}{6}t\right) = -\dfrac{3}{\pi}$

$\dfrac{\pi}{6}t = \pi + \sin^{-1}\left(\dfrac{3}{\pi}\right),\ 2\pi - \sin^{-1}\left(\dfrac{3}{\pi}\right)$

$t = 8.424418\ldots \quad or \quad 9.5755811\ldots$

$\therefore 9.5755811\ldots - 8.424418\ldots$

$= 1.151162\ldots hours$

$\therefore 1\ hour\ 9\ minutes\ (nearest\ minute)$

Marker's feedback:

Almost all students were able to differentiate $w(t)$ correctly but couldn't solve for the correct values of t. Students who did not solve for t correctly also did not consider All Station To Central. Another common error was that students converted their answers to degrees instead of working with radians.

Question 30 (a)

Criteria	Marks
Provide the correct solution	4
Correctly show the turning point or f(t) → 0 as t → ∞	3
Correctly find the turning points of f(t)	2
Correctly find the derivative of f(t)	1

Sample answer:

$u = (t-1)^2 \qquad v = e^{-t}$

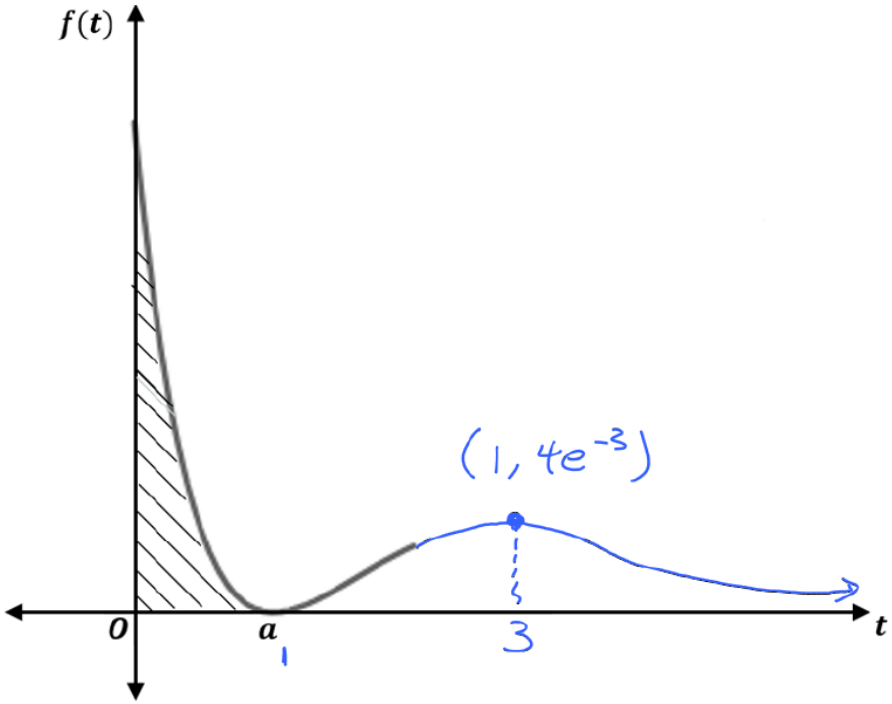
$u' = 2(t-1) \qquad v' = -e^{-t}$

$f'(t) = 2e^{-t}(t-1) - e^{-t}(t-1)^2$

$= e^{-t}(t-1)(3-t)$

$\therefore t = 1 \text{ or } t = 3$

$\therefore f(1) = 0 \text{ and } f(3) = 4e^{-3}$



Markers' feedback:

Most students recognised that they had to use the product rule to differentiate but did not manage to simplify and factorise correctly to find t = 1 or 3 for the turning points. Also many students did not complete the graph.

Question 30 (b)

Criteria	Marks
Provide the correct solution	2
Correctly use $\int f(t) = 1 - \frac{4}{e}$ or correctly integrate $\int_0^a 5 \ dt$	1

Sample answer:

$$\int_0^a 2f(t) + 5 \ dt$$

$$= 2\int_0^a f(t)dt + \int_0^a 5 \ dt$$

$$= 2\left(1 - \frac{4}{e}\right) + 5a$$

$$= 7 - \frac{8}{e} \ (\boldsymbol{because\ a = 1})$$

Marker's feedback:

This question was not done well by students. Many did not know how to apply $\int_0^a f(t)dt = 1 - \frac{4}{e}$ in their working.

Question 31 (a)

Criteria	Marks
Provide the correct solution	2
Find the correct expression for the perimeter of the triangle or the perimeter of sector.	1

Sample answer:

$3x + 2x + x\theta = 60$

$x\theta = 60 - 5x$

Marker's feedback:
This question was done well by most students.

Question 31 (b)

Criteria	Marks
Provide the correct solution	3
Find the correct simplified expression for both the area of triangle and the area of sector	2
Find the correct simplified expression for the area of triangle or area of the sector	1

Sample answer:

$$Area\ of\ triangle = \frac{1}{2} \times x^2 \times sin\left(\frac{\pi}{3}\right)$$

$$= \frac{x^2\sqrt{3}}{4}$$

$$Area\ of\ sector = \frac{1}{2} \times x^2 \times \theta$$

$$= \frac{1}{2} \times x^2 \times \frac{60 - 5x}{x}$$

$$= 30x - \frac{5x^2}{2}$$

$$Total\ area = \frac{x^2\sqrt{3}}{4} + 30x - \frac{5x^2}{2}$$

$$= \frac{x^2}{4}\left(\sqrt{3} - 10\right) + 30x$$

$$= \frac{1}{4}\left(\sqrt{3} - 10\right)x^2 + 30x$$

Markers' feedback:
This question was done well by most students. The most common error was not recognising each angle in the triangle was $60°\ or\ \frac{\pi}{3}$. Second most common error was algebraic error such that students were not able to simplify their expression to show the required expression for A.

Question 31 (c)

Criteria	Marks
Provide the correct solution	3
Correctly solve $A' = 0$	2
Correctly find A'	1

Sample answer:

$$A = \frac{x^2}{4}\left(\sqrt{3} - 10\right) + 30x$$

$$A' = \frac{1}{2}\left(\sqrt{3} - 10\right)x + 30$$

$$\frac{1}{2}\left(\sqrt{3} - 10\right)x + 30 = 0$$

$$x = -\frac{60}{\sqrt{3} - 10}$$

This is the maximum turning point because A' is a concave down parabola.

$$\therefore A\left(-\frac{60}{\sqrt{3} - 10}\right) \approx 108.85\ cm^2$$

Markers' feedback:

This question was done well by most students.