MEASURE BY ACHIEVEMENT

Kirrawee High School
New South Wales

Student Number: ....

Teacher: ....

2013

HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

Mathematics

General Instructions

- Reading Time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen. Black pen is preferred
- Only board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In Questions 11-16, show relevant mathematical reasoning and/or calculations

Total marks - 100

Section I

10 marks

- Attempt Questions 1-10
- Answer on the Multiple Choice answer sheet provided
- Allow about 15 minutes for this section

Section II

90 marks

- Attempt Questions 11-16
- Allow about 2 hours 45 minutes for this section

This paper MUST NOT be removed from the examination room

HSC COURSE MATHEMATICS

NUMBER:		TEACHER:	
TASK:	Assessment		

Topic Area Outcome	P1	P2	P3	P4	P5	P6
Basic Arithmetic & Algebra			√			√
Plane Geometry		√				√
Trigonometric Ratios		√				√
Linear Functions & Lines 1		√				√
Series		√				√
Introductory Calculus					√	√
Further Trigonometry		√				√
The Parabola		√	√	√		√

Topic Area Outcome	H1	H2	H3	H4	H5	H6	H7	H8	H9
Applications of calculus		√		√	√	√			√
Integration		√		√	√			√	√
Probability		√		√	√				√
Plane Geometry 2		√			√				√
Linear Functions and Lines 2		√			√	√			
Series Applications		√		√					√
Trigonometric Functions		√		√	√	√		√	√
Logarithmic and Exponential Functions		√	√		√	√	√	√	√
Applications of Calculus to the Physical World		√	√	√	√	√	√		√

Outcome	Description
H1	Seeks to apply mathematical techniques to problems in a wide range of practical contexts. (NOTE: this outcome will NOT form part of the formal assessment program).
H2	Constructs arguments to prove and justify results.
H3	Manipulates algebraic expressions involving logarithmic and exponential functions.
H4	Expresses practical problems in mathematical terms based on simple given models.
H5	Applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and series to solve problems.
H6	Uses the derivative to determine the features of the graph of a function.
H7	Uses the features of a graph to deduce information about the derivative.
H8	Uses techniques of integration to calculate areas and volumes/.
H9	Communicates using mathematical language, notation, diagrams and graphs.

Section I

10 marks
Attempt Questions 1 - 10
Allow about 15 minutes for this section
Answer on the multiple choice answer sheet provided.

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1. Evaluate correct to three significant figures $\sqrt{\dfrac{4^2+6^2}{383-12^2}}$

(A) 0.218 (B) 0.466 (C) 4.012 (D) 4.019

2. Simplify $\dfrac{9n^2-64}{3n-8}$

(A) $3n-8$ (B) $3n+8$ (C) $3n^2-8$ (D) n^2-64

3. Rationalise the denominator, give your answer in simplest surd form $\dfrac{1}{2\sqrt{3}-1}$

(A) $\dfrac{2\sqrt{3}-1}{6}$ (B) $\dfrac{2\sqrt{3}+1}{6}$ (C) $\dfrac{2\sqrt{3}-1}{11}$ (D) $\dfrac{2\sqrt{3}+1}{11}$

4. What is the exact value of $\cos\dfrac{5\pi}{6}$?

(A) $\dfrac{-1}{2}$ (B) $\dfrac{1}{\sqrt{2}}$ (C) $\dfrac{-\sqrt{3}}{2}$ (D) $\dfrac{\sqrt{3}}{2}$

5. Find the primitive of $5+\dfrac{2}{x}$

(A) $5x+C$ (B) $2lnx+C$ (C) $5+2lnx+C$ (D) $5x+2lnx+C$

6. Factorise x^3-27

(A) $(x-3)^3$
(B) $(x-3)(x^2+3x+9)$
(C) $(x+3)(x^2-3x+9)$
(D) $(x-3)(x^2+6x+9)$

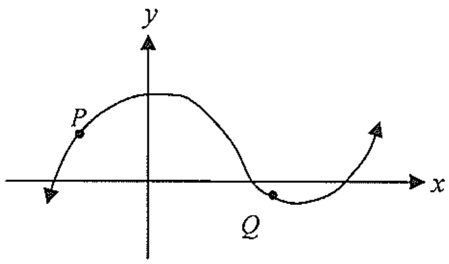
7. A parabola has the origin as the vertex and focus $(4,0)$. What is its equation?

(A) $x^2 = 16y$ (B) $x^2 = -16y$ (C) $y^2 = 16x$ (D) $y^2 = -16x$

8. What is the gradient of the tangent to $y = 2x^2 - 5x - 7$ at $x = 2$?

(A) -9 (B) -2 (C) 3 (D) 8

9. For the curve $y = f(x)$, which is correct?



(A) $f'(x) > 0$ at P and $f'(x) < 0$ at Q
(B) $f'(x) < 0$ at P and $f'(x) < 0$ at Q
(C) $f'(x) > 0$ at P and $f'(x) > 0$ at Q
(D) $f'(x) < 0$ at P and $f'(x) > 0$ at Q

10. Simplify $\cos\theta + \sin\theta \tan\theta$

(A) $\operatorname{cosec}\theta$ (B) $\cot\theta$ (C) $\sec\theta$ (D) 1

Section II

90 marks
Attempt Questions 11 - 16
Allow about 2 hours 45 minutes for this section.
Answer each question in a SEPARATE answer booklet.
Extra answer booklets are available.
Show relevant mathematical reasoning and/or calculations

Question 11 (15 marks) Use the Question 11 answer booklet. **Marks**

(a) Differentiate $\ln(3x-1)$ with respect to x **2**

(b) Solve $|2x-1| \leq 3$ **2**

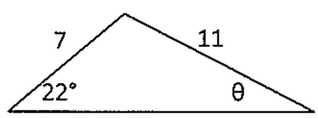
(c) Find $\int \sqrt{3x-2}\, dx$ **2**

(d) Evaluate $\int_0^{\frac{\pi}{12}} \sec^2 3x\, dx$ **3**

(e) $x^2 - 7x + 12 < 0$ **2**

(f) In a geometric series the fifth term is 162 and the second term is 6. Find the first term and the common ratio of this series. **2**

(g) Find the value of θ in the diagram. Give your answer to the nearest degree. **2**



NOT TO SCALE

Question 12 (15 marks) Use the Question 12 answer booklet. **Marks**

(a) Differentiate with respect to x :

(i) $x^2 e^x$ **2**

(ii) $\dfrac{\sin x}{x}$ **2**

(iii) $(1+\tan x)^5$ **2**

(b) Evaluate $\displaystyle\int_e^{e^4} \frac{7}{x}\, dx$ **2**

(c) The gradient of the curve is given by $\dfrac{dy}{dx} = 3x^2 + 5$. The curve passes through the point $(-1, 4)$. What is the equation of the curve? **2**

(d) Carol is training for a fun run by running every week for 32 weeks. She runs 2 km in the first week and after that runs 500 m more than the previous week, until she reaches 16 km in a week. She then continues to run 16 km each week.

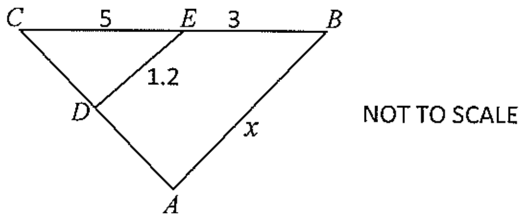
(i) How far does Carol run in the eighth week? **1**

(ii) What is the total distance that Carol runs in 32 weeks? **2**

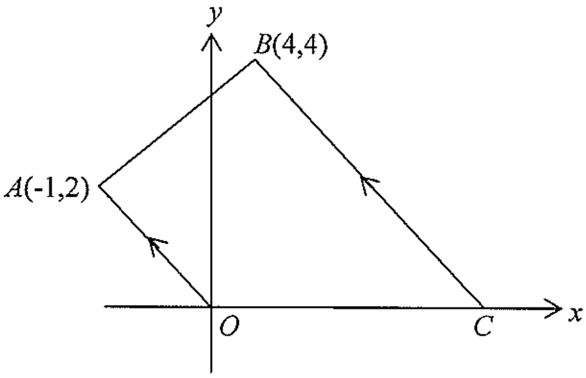
(e) Sketch the curve $y = 3\cos 2x$ for $0 \le x \le \pi$ **2**

Question 13 (15 marks) Use the Question 13 answer booklet. **Marks**

(a) In the diagram, AB is parallel to DE, CE is 5 cm, EB is 3 cm and DE is 1.2 cm. Find the length AB. **2**



(b) In the diagram $OABC$ is a trapezium with $OA \| CB$. The coordinates of O, A and B are $(0,0)$ $(-1,2)$ and $(4,4)$ respectively.



(i) Calculate the length of OA. **1**

(ii) Write down the gradient of the line OA. **1**

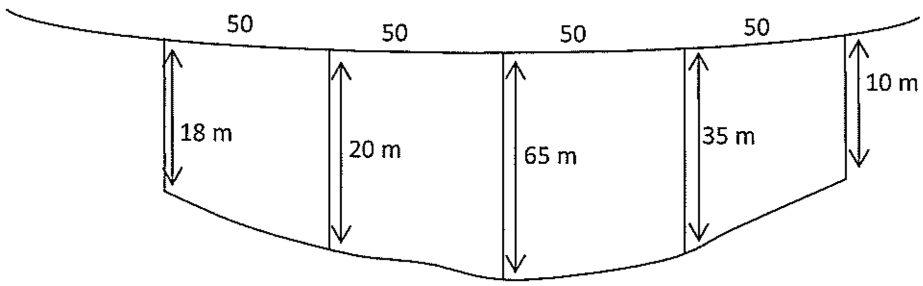
(iii) What is the size of $\angle AOC$ (to the nearest degree) **1**

(iv) Find the equation of the line BC and hence find the coordinates of C. **2**

(v) Show that the perpendicular distance from O to the line BC is $\dfrac{12\sqrt{5}}{5}$ **2**

(vi) Hence or otherwise, calculate the area of the trapezium $OABC$. **2**

(c)



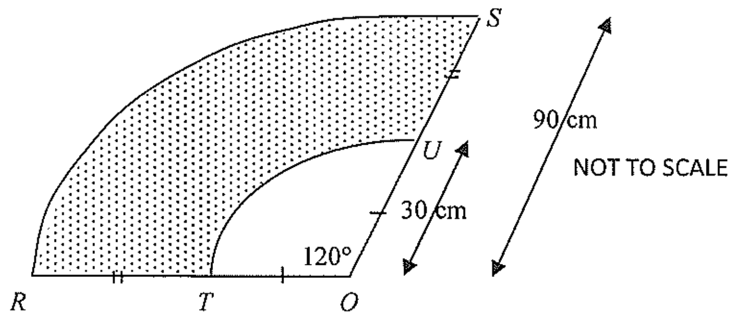
At a certain location a river is 200 metres wide. At this location the depth of the river in metres, has been measured at 50 metre intervals. The cross-section is shown above.

(i) Use Simpson's rule with five depth measurements to calculate the approximate area of the cross-section. **3**

(ii) The river flows at 0.3 metres per second. Calculate the approximate volume of water flowing through the cross section in 20 seconds. **1**

QUESTION 14 (15 marks) Use the Question 14 answer booklet. **Marks**

(a)



Karen's car windscreen wiper traces out the area *RSUT* where *RS* and *UT* are arcs of the circles centre *O*, radii 90 cm and 30 cm respectively as shown in the diagram.

(i) Find the length of the arc *RS* (correct to one decimal place) **2**

(ii) Calculate the shaded area *RSUT* (correct to one decimal place) **3**

(b) A function $f(x)$ is defined by $f(x)=(x+1)(x^2-16)$

(i) Find all solutions of $f(x)=0$ **2**

(ii) Find the coordinates of the turning points of the graph $y=f(x)$ and determine their nature **3**

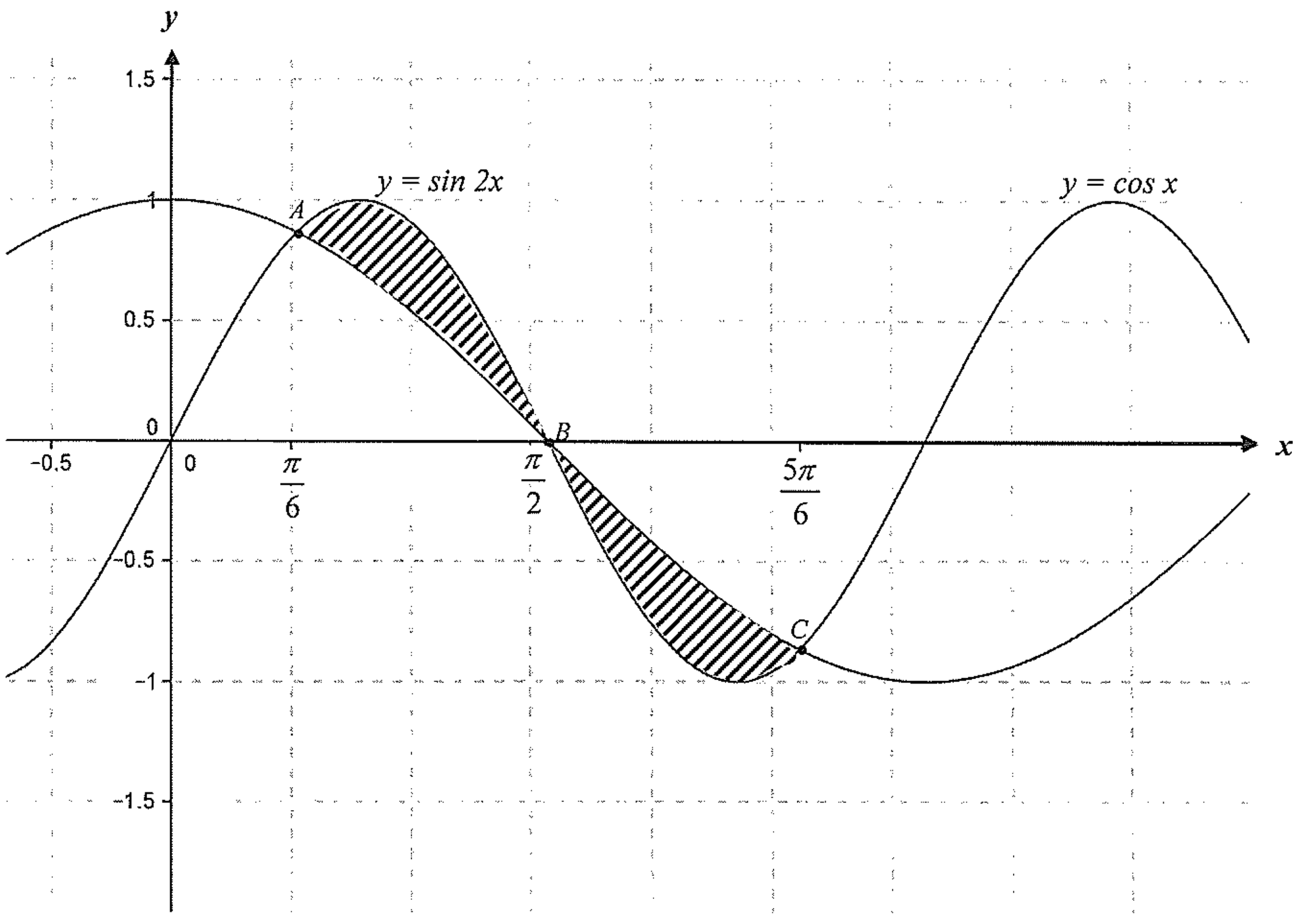
(iii) Hence sketch the graph $y=f(x)$, showing the turning points and the points where the curve meets the x-axis **2**

(iv) For what values of x is the graph of $y=f(x)$ concave up? **1**

(c) Solve $\log_2(5x-8)=5$ **2**

QUESTION 15 (15 marks) Use the Question 15 answer booklet. **Marks**

(a)



The diagram shows the graph of $y = \sin 2x$ and $y = \cos x$. The first three points of intersection to the right of the y-axis and labelled *A, B,* and *C.* These points have x values of $A = \frac{\pi}{6}$, $B = \frac{\pi}{2}$, $C = \frac{5\pi}{6}$

(i) Find the area of the shaded region in the diagram **3**

(ii) State giving reasons why this area is not equivalent to $\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \sin 2x - \cos x \, dx$ **1**

(c) A particle moves in a straight line so that its displacement, in metres is given by $x = \dfrac{t-3}{t+3}$ where t is measured in seconds

(i) What is the displacement when $t = 0$? **1**

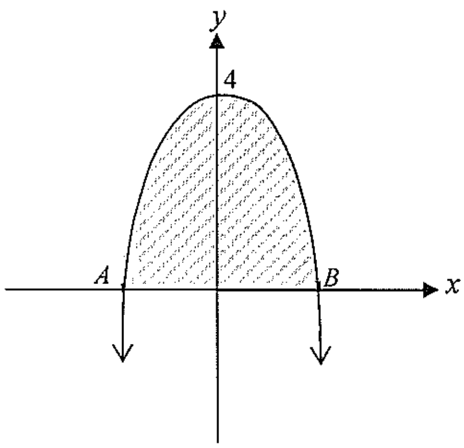
(ii) Show that $x = 1 - \dfrac{6}{t+3}$ **2**

(iii) Hence find expressions for the velocity and the acceleration in terms of t. **2**

(iv) Is the particle ever at rest? Give reasons for your answer. **1**

(v) What is the acceleration of the particle when $t = 2$? **1**

(d) The shaded region lying between the curve $y = 4 - x^2$ and the x axis is rotated about the x axis.

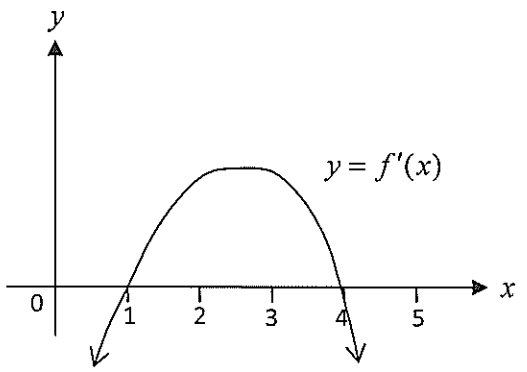


(i) Show that the coordinates of A and B are $(-2, 0)$ and $(2, 0)$ respectively. **1**

(ii) Hence or otherwise find the volume of the solid formed when $y = 4 - x^2$ is rotated about the x axis between $x = -2$ and $x = 2$ (in exact form). **3**

QUESTION 16 (15 marks) Use the Question 16 answer booklet. **Marks**

(a) The diagram shows the graph of the gradient function of the curve $y = f(x)$.



For what values of x does $f(x)$ have a local maximum? Justify your answer. **2**

(b) The population of a certain insect is growing exponentially according to $N = 260e^{kt}$ where t is the time in weeks after the insects are first counted. At the end of 6 weeks the insect population has doubled.

(i) Show that the value of $k = \dfrac{\ln 2}{6}$ **1**

(ii) How many insects are there when $t = 24$? **1**

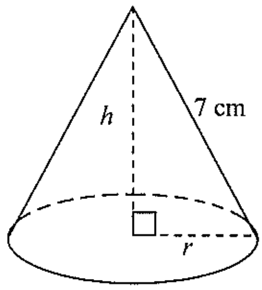
(iii) How long does it take for the number of insects to have a population more than 1000? **2**

(c) Dianne retires with a lump sum of \$285 000. The money is invested in a fund which pays interest at the end of each month at a rate of 3% per annum, and Dianne receives a fixed monthly payment of \$$M$ from the fund.

(i) Show that $A_n = 285000 \times 1.0025^n - \$M\left(1 + 1.0025 + \ldots\ldots\ldots\ldots + 1.0025^{n-1}\right)$ **1**

(ii) Dianne chooses the value of M so that there will be nothing left in the fund at the end of the 20^{th} year (240 payments). Find the value of M. **3**

(d) A right circular cone has a slant height 7 cm.



(i) Show that the volume can be expressed as $V = \frac{\pi}{3}\left(49h - h^3\right)$ where h is the perpendicular height. **2**

(ii) Hence find the maximum possible volume of the cone (in exact form). **3**

End of examination

Student Number: ....

Teacher: ....

Kirrawee High School

Mathematics

HSC Trial Examination 2013

Section I

10 marks

Allow about 15 minutes for this section

Attempt Questions 1 – 10

Select the alternative A, B, C, or D that best answers the question.
Fill in the response oval completely.

Sample: 2 + 4 = A 2 B 6 C 8 D 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ~~●~~ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

correct

A ~~●~~ B ~~●~~ C ○ D ○

1. A ○ B ○ C ○ D ○

2. A ○ B ○ C ○ D ○

3. A ○ B ○ C ○ D ○

4. A ○ B ○ C ○ D ○

5. A ○ B ○ C ○ D ○

6. A ○ B ○ C ○ D ○

7. A ○ B ○ C ○ D ○

8. A ○ B ○ C ○ D ○

9. A ○ B ○ C ○ D ○

10. A ○ B ○ C ○ D ○

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\quad n \neq -1;\quad x \neq 0,\ \text{if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x,\quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\quad a \neq 0$$

$$\int \sec ax\,\tan ax\,dx = \frac{1}{a}\sec ax,\quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\quad a > 0,\ -a < x < a$$

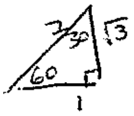
$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right),\quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x,\quad x > 0$

Trial Solutions 2013.

1. B   2. $\frac{\cancel{(3n-8)}(3n+8)}{\cancel{(3n-8)}}$ (B)   3. $\frac{1}{2\sqrt{3}-1} \times \frac{2\sqrt{3}+1}{2\sqrt{3}+1} = \frac{2\sqrt{3}+1}{12-1} = \frac{2\sqrt{3}+1}{11}$ (D)

4. 

$-\frac{\sqrt{3}}{2}$ (C)   5) $5x + 2\ln x + C$ (D)   6. (B)   7 (C)   8. (C)

9. (A)   10. (C).

11) a) $y = \ln(3x-1)$ (Setting Out.)

$y' = \frac{3}{3x-1}$.

b) $2x - 1 \leq 3$ or $2x - 1 \geq -3$.

$+1 \quad +1$

$2x \leq 4$ $2x \geq -2$

$x \leq 2$ ($\frac{1}{2}$) $x \geq -1$ ($\frac{1}{2}$)

$\therefore -1 \leq x \leq 2$.

c) $\int (3x-2)^{\frac{1}{2}}\,dx$

$= \frac{(3x-2)^{\frac{3}{2}}}{\frac{3}{2} \times 3} + C$

$= \frac{2}{9}(3x-2)^{\frac{3}{2}} + C$

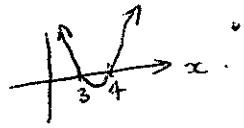
d) $\left[\frac{1}{3}\tan 3x\right]_0^{\frac{\pi}{12}}$

$= \frac{1}{3}\tan\frac{\pi}{4} - \frac{1}{3}\tan 0$

$= \frac{1}{3}$

e) $(x-3)(x-4) = 0$ $3 < x < 4$.

$x = 3$ or 4.



f) $T_5 \Rightarrow 162 = ar^4$

$T_2 \Rightarrow 6 = ar$

$\frac{\cancel{a}r^4}{\cancel{a}r} = r^3 = \frac{162}{6}$.

$r^3 = 27$ $r = 3$.

$a \times 3 = 6$

$a = 2$.

g) $\frac{\sin\theta}{7} = \frac{\sin 22}{11}$.

$\sin\theta = \frac{7 \times \sin 22}{11}$.

$\theta = 14°$

12) a) i) $u = x^2$ $v = e^x$ $\frac{dy}{dx} = 2xe^x + x^2e^x$

$u' = 2x$ $v' = e^x$

ii) $u = \sin x$ $v = x$. $\frac{dy}{dx} = \frac{x\cos x - \sin x}{x^2}$

$u' = \cos x$ $v' = 1$

12) iii) $y = (1+\tan x)^5$

$y' = 5(1+\tan x)^4 \times \sec^2 x$.

$= 5\sec^2 x(1+\tan x)^4$

b) $\int_e^{e^4} \frac{7}{x}\,dx = [7\ln x]_e^{e^4} = 7\ln e^4 - 7\ln e$

$= 28 - 7$

$= 21$

c) $\frac{dy}{dx} = 3x^2 + 5$ $(-1, 4)$

$y = \frac{3x^3}{3} + 5x + C$

$4 = (-1)^3 - 5 + C$.

$4 = -1 - 5 + C$

$C = 10$ $\therefore y = x^3 + 5x + 10$

d) i) $a = 2000\,m$ $d = 500\,m$

$T_8 = 2000 + 7 \times 500$

$= 2000 + 3500$

$= 5500\,m$

$= 5.5\,km$

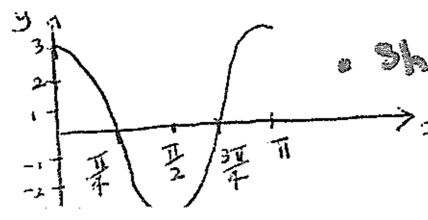
ii) $S_{29} = \frac{29}{2}\{4000 + 28 \times 500\}$

$= 261\,000\,m$

$= 261\,km$

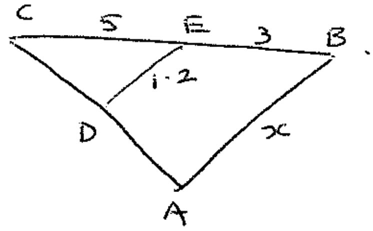
$S_{32} = 261\,km + 3 \times 16\,km$

$= 309\,km$.   (312).

e) 

• Shape   • Intercepts Amp + Period.

13. a)



$\frac{x}{1\cdot 2} = \frac{8}{5}$

$x = \frac{9\cdot 6}{5}$

$= \cancel{3\cdot 2}$ $1\cdot 92$.

b) i) $D = \sqrt{(-1-0)^2+(2-0)^2}$

$= \sqrt{1+4}$

$= \sqrt{5}$ units.

ii) $m = \frac{-2}{1}$

$= -2$.

$\angle AOC = \theta$.

iii) $\tan\theta = -2$.

$\theta = 116°34'$

$= 117°$.

iv). $y-4 = -2(x-4)$.

$y-4 = -2x+8$.

$2x+y-12=0$

when $y=0$ $\quad 2x-12=0$

$2x=12$

$x=6$.

$C=(6,0)$.

v) $2x+y-12=0$.

$(0,0)$

$D = \left|\frac{2\times0+1\times0-12}{\sqrt{2^2+1^2}}\right|$

$= \left|\frac{-12}{\sqrt{5}}\right| = \frac{12\sqrt{5}}{5}$.

vi) Dist $BC = \sqrt{(6-4)^2+(0-4)^2}$.

$= \sqrt{2^2+16}$

$= \sqrt{20}$

$= 2\sqrt{5}$

Area $= \frac{1}{2}\times\frac{12\sqrt{5}}{5}\times(2\sqrt{5}+\sqrt{5})$

$= \frac{1}{2}\times\frac{12\sqrt{5}}{5}\times 3\sqrt{5}$

$= \frac{180}{10} = 18$ unit^2.

c) i) $A \doteqdot \frac{100}{6}\left[18+4\times20+65\right] + \frac{100}{6}\left[65+4\times35+10\right]$

$\doteqdot \frac{100}{6}\times\left[163+215\right]$.

$\doteqdot \frac{100}{6}\times 378 \doteqdot 6300\ m^2$

ii) River Volume of Water $= 0.3\times 6300\times 20$

$= 37800\ m^3$.

14. a) i) $l = r\theta$

$= 90\times\frac{2\pi}{3}$

$= 30\times 2\pi$

$= 60\pi$

$= 188\cdot 5$ cm.

$\theta = 120°$. $\quad \pi = 180°$

$= \frac{120\times\pi}{180}$ $\quad \frac{\pi}{180} = 1°$

$= \frac{2\pi}{3}$.

ii). Area of large Arc $= \frac{1}{2}r^2\theta$

$= \frac{1}{2}\times 90^2\times\frac{2\pi}{3}$.

$= 8482.300$

Area of Small Arc

$= \frac{1}{2}\times 30^2\times\frac{2\pi}{3}$.

$= 942.4777961$.

$-$

Area $= 7,539\cdot 8$ cm^2.

b) $f(x) = (x+1)(x^2-16) \Rightarrow f(x) = (x+1)(x-4)(x+4)$.

i). $f(x) = 0 \rightarrow x = -1$ or $x = 4$ or $x = -4$.

ii). $f(x) = (x+1)(x^2-16)$

$u = x+1 \quad v = (x^2-16)$

$u' = 1 \quad v' = 2x$.

$f'(x) = x^2 - 16 + 2x(x+1)$.

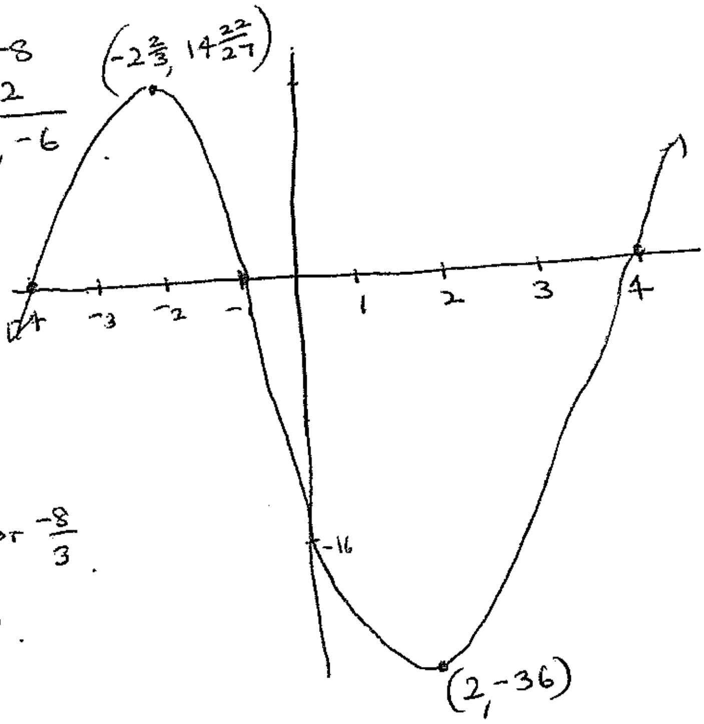
$= x^2 - 16 + 2x^2 + 2x$.

$= 3x^2 + 2x - 16$.

$\times\ -48$

$+\ 2$

$8, -6$



$f'(x) = 0$

$0 = 3x^2 - 6x + 8x - 16$.

$0 = 3x(x-2) + 8(x-2)$.

$0 = (3x+8)(x-2)$.

$\therefore x = 2$ or $3x + 8 = 0$

$3x = -8$.

$y = 3x - 12$. $\quad x = -2\frac{2}{3}$ or $\frac{-8}{3}$.

$= -36$. $\quad y = 14\frac{22}{27}$.

$f''(x) = 6x + 2$.

when $x = 2$ $f''(x) = 12 + 2$ $\therefore$ the pt $(2, -36)$

$= 14$ is a min t.p

as $f''(x) > 0$

when $x = -2\frac{2}{3}$ $f''(x) = -14$ $\therefore$ the pt $\left(-2\frac{2}{3}, 14\frac{22}{27}\right)$.

is a max t.p as $f''(x) < 0$

iv) Concave up $f''(x) > 0$.

$6x + 2 > 0$

$6x > -2$

$x > -\frac{1}{3}$.

c). $\log_2(5x - 8) = 5$.

$5x - 8 = 2^5$.

$5x - 8 = 32$.

$+8 \quad +8$

$5x = 40 \quad x = 8$.

$$A = 2\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin 2x - \cos x \, dx$$

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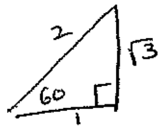


$$= 2\left[-\tfrac{1}{2}\cos 2x - \sin x\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$$= 2\left[\left(-\tfrac{1}{2}\cos 2\times\tfrac{\pi}{2} - \sin\tfrac{\pi}{2}\right) - \left(-\tfrac{1}{2}\cos\tfrac{\pi}{3} - \sin\tfrac{\pi}{6}\right)\right]$$

$$= 2\left[-\tfrac{1}{2}\times -1 - 1\right) - \left(-\tfrac{1}{2}\times\tfrac{1}{2} - \tfrac{1}{2}\right)\Big]$$

$$= 2\left[\left(\tfrac{1}{2} - 1\right) - \left(-\tfrac{1}{4} - \tfrac{1}{2}\right)\right]$$

$$= 2\left[-\tfrac{1}{2} + \tfrac{3}{4}\right]$$

$$= 2\times\tfrac{1}{4}$$

$$= \tfrac{1}{2}\ u^2$$

ii) as the area above and below the curve are exactly the same so $\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \sin 2x - \cos x \, dx = 0$.

Question 15.

$x = \dfrac{t-3}{t+3}$.

i) $t = 0 \quad x = \dfrac{-3}{3} = -1\text{m}$.

ii) $x = \dfrac{t+3}{t+3} - \dfrac{6}{t+3}$.

$= \dfrac{t+3-6}{t+3} = \dfrac{t-3}{t+3}$.

iii) $x = \dfrac{t-3}{t+3}$

$u = t-3 \quad v = t+3$

$u' = 1 \quad v' = 1$.

$$\dot{x} = \frac{t+3-(t-3)}{(t+3)^2} = \frac{\cancel{t}+3-\cancel{t}+3}{(t+3)^2} = \frac{6}{(t+3)^2}.$$

$\dot{x} = 6(t+3)^{-2}$.

$\ddot{x} = -12(t+3)^{-3}$

$= \dfrac{-12}{(t+3)^3}$

d) i) $V = \frac{1}{3}\pi r^2 h$

By Pythagoras $h^2 = 7^2 - r^2$

$h^2 = 49 - r^2$

$r^2 = 49 - h^2$

$= \frac{1}{3}\pi(49 - h^2)h$

$= \frac{\pi}{3}(49h - h^3)$

i) $V = \frac{49\pi}{3}h - \frac{\pi}{3}h^3$

$\frac{dV}{dh} = \frac{49\pi}{3} - \frac{3\pi h^2}{3}$

$0 = \frac{49\pi}{3} - \pi h^2$

$\pi h^2 = \frac{49\pi}{3}$

$h^2 = \frac{49}{3}$

$h = \frac{7}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{7\sqrt{3}}{3}$

$\frac{d^2V}{dh^2} = -\frac{6\pi h}{3}$

$= -2\pi h$

sub in $h = \frac{7\sqrt{3}}{3}$

$= -2\pi \times \frac{7\sqrt{3}}{3}$

$= -\frac{14\pi\sqrt{3}}{3}$

$\frac{d^2V}{dh^2} < 0$

so $h = \frac{7\sqrt{3}}{3}$ is the height of the maximum volume.

Maximum Volume is

$\therefore V = \frac{\pi}{3}\left(49 \times \frac{7\sqrt{3}}{3} - \left(\frac{7\sqrt{3}}{3}\right)^3\right)$.

$= \frac{\pi}{3}\left(\frac{343\sqrt{3}}{3} - \frac{343\sqrt{3}}{9}\right)$

$= \frac{\pi}{3}\left(\frac{1029\sqrt{3} - 343\sqrt{3}}{9}\right)$

$= \frac{\pi}{3}\left(\frac{686\sqrt{3}}{9}\right)$

$= \frac{686\sqrt{3}\pi}{27}$ cm^3

$= 138.25$ cm^3.

$\cancel{\dot{x} = \dfrac{6}{(t+3)^2}} \neq 0$ No. ~~it~~ approach~~es~~ zero ~~but~~ never ~~equals~~ it.

$$\cancel{\lim_{t \to \infty} \frac{6}{t^2+6t+9}} = \cancel{\frac{\frac{6}{t^2}}{\frac{t^2}{t^2} + \frac{6t}{t^2} + \frac{9}{t^2}}} = \cancel{\frac{0}{1}} = 0.$$

v) $\ddot{x} = \dfrac{-12}{5^3} = \dfrac{-12}{125}$ m/s^2 $= -0.096$ m/s^2

d) i) $y = 4 - x^2$. $\quad y = 0$

$0 = 4 - x^2$

$0 = (2-x)(2+x)$

$x = 2$ or $x = -2$.

$\therefore A = (-2, 0) \quad B = (2, 0)$.

ii) $V = 2\pi \displaystyle\int_0^2 y^2 \, dx$.

$y = 4 - x^2$

$y^2 = (4 - x^2)^2$

$= 16 - 8x^2 + x^4$

$= 2\pi \displaystyle\int_0^2 16 - 8x^2 + x^4 \, dx$

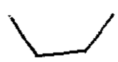
$= 2\pi \left[16x - \dfrac{8x^3}{3} + \dfrac{x^5}{5}\right]_0^2$.

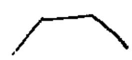
$= 2\pi \left[32 - \dfrac{64}{3} + \dfrac{32}{5}\right] - 0$

$= 2\pi \times \dfrac{256}{15} = \dfrac{512\pi}{15}$ u^3

16 a). T.P at $x = 1$ and $x = 4$.

$- \ 0 \ +$ $\quad + \ 0 \ -$





$\therefore x = 4$ is a local maximum as gradient is $+ \ 0 \ -$ which forms a max.

b) $N = 260 e^{kt}$

i) $520 = 260 e^{kt}$.

$2 = e^{kt}$.

$\ln 2 = \ln e^{k6}$

$\ln 2 = k6$.

$\dfrac{\ln 2}{6} = k$.

ii) $N = 260 \times e^{\frac{\ln 2}{6} \times 24}$.

$= 4160$.

iii) $1000 = 260 \times e^{\frac{\ln 2}{6} \times t}$.

$\ln \dfrac{1000}{260} = \dfrac{\ln 2}{6} \times t$.

$t = 11.66 \ldots$

$= 12$ weeks.

c)

3% p.a $= .03 \div 12$
$= .0025$ per month.

$$A_1 = 285000 \times 1.0025 - \$M.$$

$$A_2 = (285000 \times 1.0025 - \$M)1.0025 - \$M.$$
$$= 285000 \times 1.0025^2 - \$M \times 1.0025 - \$M.$$
$$= 285000 \times 1.0025^2 - \$M(1 + 1.0025).$$

$$\therefore A_n = 285000 \times 1.0025^n - \$M(1 + 1.0025 + \dots + 1.0025^{n-1})$$

$$\therefore A_{240} = 285000 \times 1.0025^{240} - \$M(1 + 1.0025 + \dots + 1.0025^{239})$$

$$A_{240} = 0.$$

$$\therefore \$M = \frac{285000 \times 1.0025^{240}}{1 + 1.0025 + \dots + 1.0025^{239}}.$$

$$\frac{a(r^n - 1)}{r - 1}.$$

$$= \frac{285000 \times 1.0025^{240}}{\frac{1.0025^{240} - 1}{.0025}}.$$

$$= 285000 \times 1.0025^{240} \times \frac{.0025}{(1.0025^{240} - 1)}.$$

$$= \$1580.60$$