Student Name: ____________________________________

Teacher: ____________________________________

Kirrawee High School
New South Wales

2014
HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes.
- Working time – 3 hours.
- Write using blue or black pen.
- Only board-approved calculators may be used.
- Start each question in a new booklet.
- A table of standard integrals is provided at the back of the paper
- All necessary working should be shown.

Total Marks – 100

Section I Pages 3-5

10 marks

- Answer Questions 1 – 10
- Allow about 15 minutes for this section

Section II Pages 6-11

90 marks

- Attempt Questions 11 – 16
- Allow about 2 hours 45 minutes for this section

NAME:

NUMBER:

TEACHER:

HSC COURSE MATHEMATICS

TASK: Trial Examination

Topic Area Outcome	H1	H2	H3	H4	H5	H6	H7	H8	H9
Applications of Calculus		√		√	√	√			√
Integration		√		√	√			√	√
Probability		√		√	√				√
Plane Geometry 2		√			√				√
Linear Functions and Lines 2		√			√	√			
Series Applications		√		√					√
Trigonometric Functions		√		√	√	√		√	√
Logarithmic and Exponential Functions		√	√		√	√	√	√	√
Applications of Calculus to the Physical World		√	√	√	√	√	√		√

Outcome	**Description**
H1	Seeks to apply mathematical techniques to problems in a wide range of practical contexts. (NOTE: This outcome will NOT form part of the formal assessment program.)
H2	Constructs arguments to prove and justify results.
H3	Manipulates algebraic expressions involving logarithmic and exponential functions.
H4	Expresses practical problems in mathematical terms based on simple given models.
H5	Applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and series to solve problems.
H6	Uses the derivative to determine the features of the graph of a function.
H7	Use the features of a graph to deduce information about the derivative.
H8	Uses techniques of integration to calculate areas and volumes.
H9	Communicates using mathematical language, notation, diagrams and graphs.

Section I
Multiple Choice
10 Marks

Answer on the answer sheet provided.

1. Which of the following is equal to $\dfrac{1}{2\sqrt{5}-\sqrt{3}}$?

(A) $\dfrac{2\sqrt{5}-\sqrt{3}}{7}$ (B) $\dfrac{2\sqrt{5}+\sqrt{3}}{7}$ (C) $\dfrac{2\sqrt{5}-\sqrt{3}}{17}$ (D) $\dfrac{2\sqrt{5}+\sqrt{3}}{17}$

2. The quadratic equation $x^2+3x-1=0$ has roots α and β.

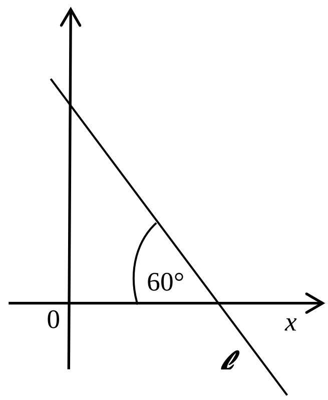
What is the value of $\alpha\beta+(\alpha+\beta)$?

(A) 4 (B) 2 (C) -4 (D) -2

3. $\displaystyle\int \frac{1}{2x}\,dx = ?$

(A) $\dfrac{1}{2}\ln x+c$ (B) $\ln x+c$ (C) $2\ln x+c$ (D) $2\ln 2x+c$

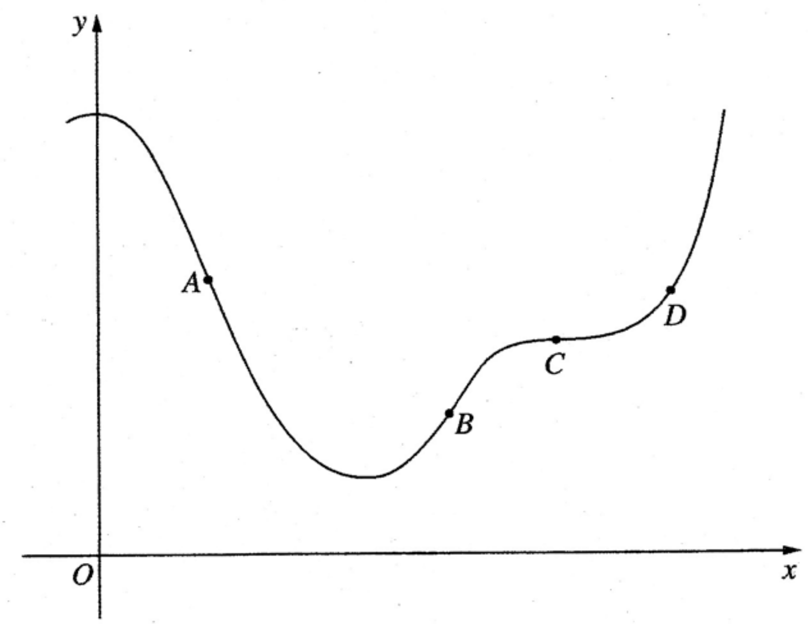
4. The diagram shows the line ℓ



What is the slope of the line ℓ?

(A) $\sqrt{3}$ (B) $-\sqrt{3}$ (C) $\dfrac{1}{\sqrt{3}}$ (D) $-\dfrac{1}{\sqrt{3}}$

5. The diagram shows points A, B C and D on the graph $y = f(x)$



At which point is $f'(x) > 0$ and $f''(x) = 0$?

(A) A (B) B (C) C (D) D

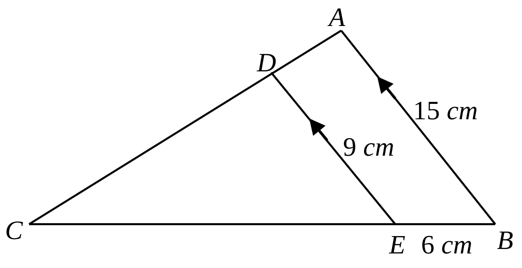
6. Which inequality defines the domain of the function $f(x) = \dfrac{1}{\sqrt{x+3}}$?

(A) $x > -3$ (B) $x \geq -3$ (C) $x < -3$ (D) $x \leq -3$

7. What is the perpendicular distance of the point $(-3,1)$ from the line $3x - 2y = 4$?

(A) $\dfrac{7}{\sqrt{5}}$ (B) $\dfrac{7}{\sqrt{13}}$ (C) $\dfrac{15}{\sqrt{15}}$ (D) $\dfrac{15}{\sqrt{13}}$

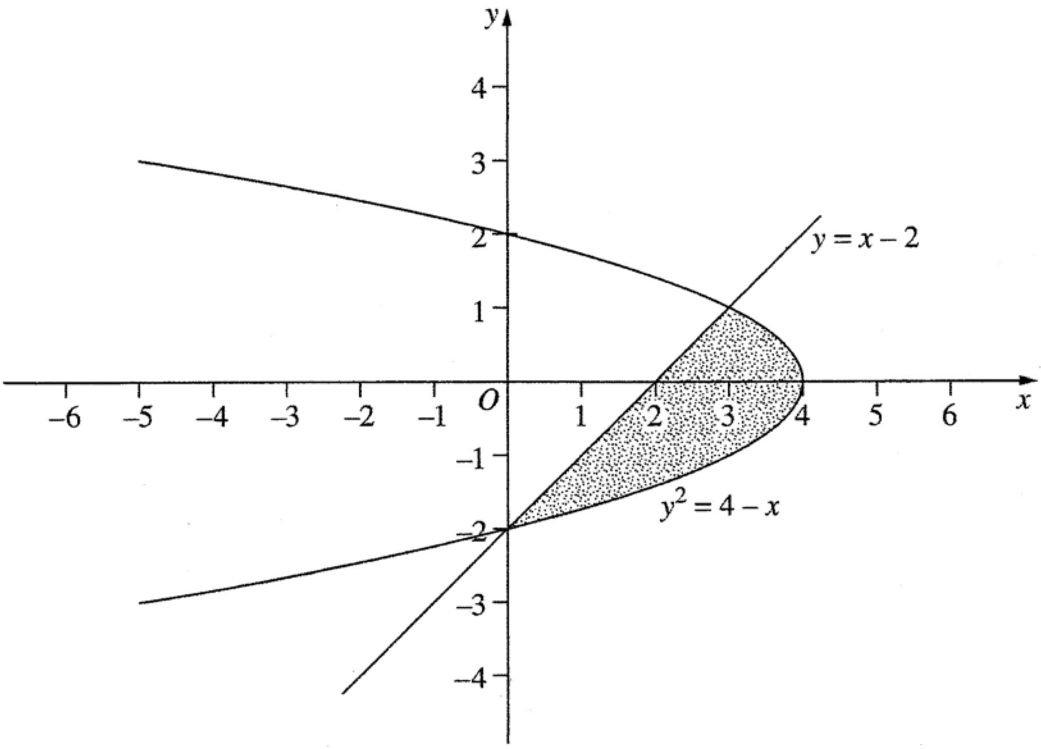
8. In the diagram below ABC is a triangle and $AB \parallel DE$



Given that $AB = 15\ cm$, $DE = 9\ cm$ and $BE = 6\ cm$, what is the length of BC?

(A) 3.6 cm (B) 6 cm (C) 9 cm (D) 15 cm

9. The diagram shows the region enclosed by $y = x - 2$ and $y^2 = 4 - x$



Which of the following pairs of inequalities describes the shaded region in the diagram?

(A) $y^2 \le 4 - x$ and $y \ge x - 2$
(B) $y^2 \le 4 - x$ and $y \le x - 2$
(C) $y^2 \ge 4 - x$ and $y \ge x - 2$
(D) $y^2 \ge 4 - x$ and $y \le x - 2$

10. A particle is moving along the x axis. The displacement of the particle at t seconds is x metres.

At a certain time, $\dot{x} = -3\ ms^{-1}$ and $\ddot{x} = 2\ ms^{-2}$

Which statement best describes the motion of the particle at that time?

(A) The particle is moving to the right with increasing speed
(B) The particle is moving to the left with increasing speed
(C) The particle is moving to the right with decreasing speed
(D) The particle is moving to the left with decreasing speed

Section II

90 Marks
Attempt Questions 11-16
Allow about 2 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available. In Questions 11-16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 **(15 marks)** **Use a SEPARATE writing booklet** **Marks**

(a) Simplify $\dfrac{4}{x^2 - x} - \dfrac{x-1}{x^2}$ **2**

(b) (i) Draw the graphs of $y = |x|$ and $y = x + 4$ on the same set of axes **2**

(ii) Hence, or otherwise, solve the equation $|x| = x + 4$ **1**

(c) For the series $e^x + e^{2x} + e^{3x} + ...$

(i) Is the series arithmetic or geometric? Explain why. **2**

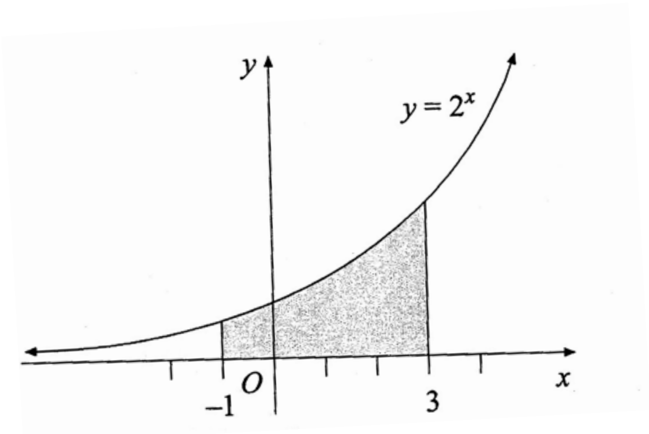
(ii) Write an expression for the sum of the first 10 terms. **1**

(d) Consider the function $y = 2^x$

x	-1	0	1	2	3
2^x					

(i) Copy and complete the above table in your Writing Booklet. **1**

(ii) Using Simpson's rule with these five function values, find an estimate for the area shaded in the diagram below. **2**



(e) Find the exact solution of $5^x = 4$ **2**

(f) What are the solutions to $2\cos x = -\sqrt{3}$ for $-\pi \le x \le \pi$ **2**

Question 12 (15 marks) Use a SEPARATE writing booklet

(a) Differentiate:

(i) $x^2 \log_e x$ **2**

(ii) $\dfrac{e^{3x}}{\cos x}$ **2**

(b) Find:

(i) $\int \sec^2 6x \, dx$ **1**

(ii) $\displaystyle\int_2^3 \frac{6x^2}{x^3 - 2} \, dx$ **3**

(c) Consider the curve $y = 12x - x^3$

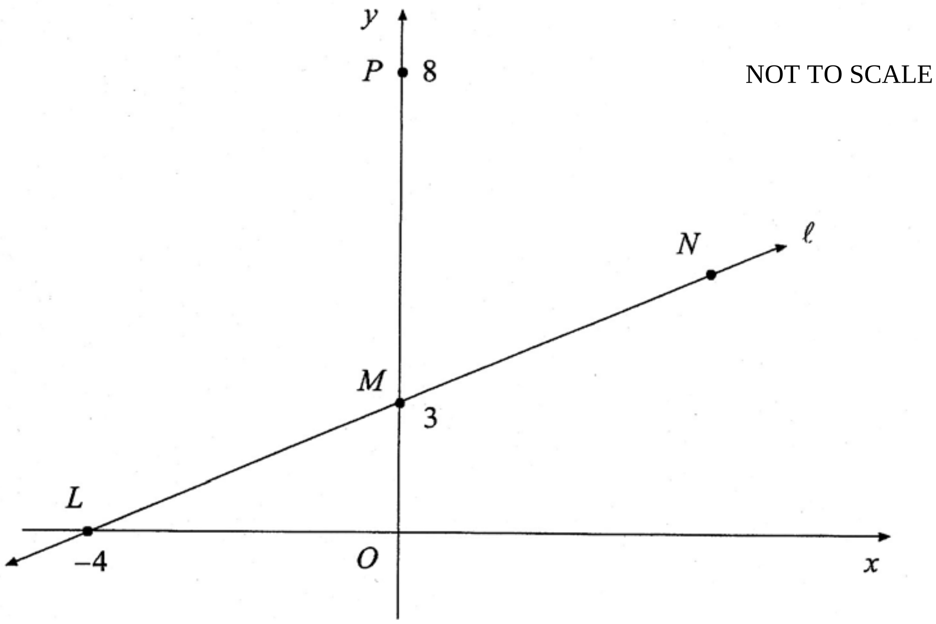
(i) Find where $\dfrac{d^2y}{dx^2} = 0$ **3**

(ii) Sketch the curve showing all important features **4**

Marks

Question 13 **(15 marks)** **Use a SEPARATE writing booklet**

(a)



The line ℓ cuts the x axis at $L(-4,0)$ and the y axis at $M(0,3)$ as shown.
N is a point on the line ℓ, and P is the point $(0,8)$.

Copy the diagram into your writing booklet.

(i) Find the equation of the line ℓ. **2**

(ii) Show that the point (16, 15) lies on the line ℓ. **1**

(iii) By considering the lengths of ML and MP, show the ΔLMP is isosceles. **2**

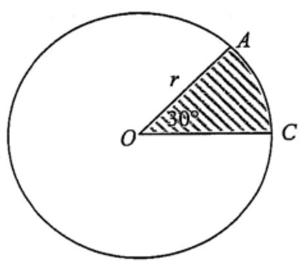
(iv) Calculate the gradient of the line PL. **1**

(v) M is the midpoint of the interval LN. Find the coordinates of the point N. **2**

(vi) Show that $\angle NPL$ is a right angle. **2**

(vii) Find the equation of the circle that passes through the points N, P, and L. **2**

(b)



(i) Find the radius of the circle if the area of the shaded sector is 12π cm^2. **1**

(ii) Hence find the exact length of major arc AC. **2**

Marks

Question 14 (15 marks) Use a SEPARATE writing booklet

(a) Consider the parabola with equation $x^2 = 12(y-1)$

(i) Find the coordinates of the vertex of the parabola **1**

(ii) Find the coordinates of the focus of the parabola **2**

(b) After a week of rain the local dam starts to fill until, at 10am Sunday the dam overflows. At this point the height (*H*) of the river starts to change at the rate of

$$\left(1-\frac{t}{20}\right) \text{ metres per hour.}$$

Initially the height of the river is 5 metres.

(i) Show that the height of the river is given by the formula **1**

$$H = -\frac{t^2}{40} + t + 5$$

(ii) Find the maximum height of the river during this flood. **2**

(iii) A bridge crossing this river will be blocked once the height of the river reaches 12.5 metres. **3**

At what times and days will the bridge be blocked and then re-opened?

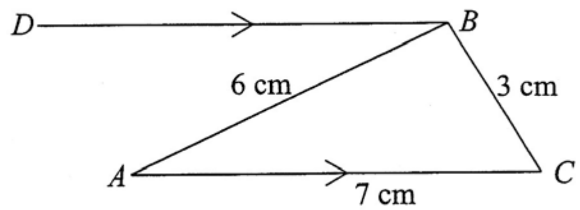
(c) (i) Sketch the graph of $y = e^x$ over the domain $0 \le x \le 3$ **2**

(ii) The area under the curve $y = e^x$ in part (i) is rotated about the *x* axis. Find the volume generated. Write your answer in exact form and correct to 3 significant figures. **4**

Marks

Question 15 **(15 marks)** **Use a SEPARATE writing booklet**

(a)



Not to scale

In the diagram, AC is parallel to DB, AB is 6 cm, BC is 3 cm and AC is 7 cm.

(i) Use the cosine rule to find the size of $\angle ACB$, to the nearest degree. **3**

(ii) Hence find the size of $\angle DBC$, giving reasons for your answer. **1**

(b) The graphs $y = x^2 - 3x - 5$ and $y = x + k$ have only one point of intersection, P.

(i) Show that the x coordinate of P satisfies $x^2 - 4x - 5 - k = 0$ **1**

(ii) Find the value of k. **2**

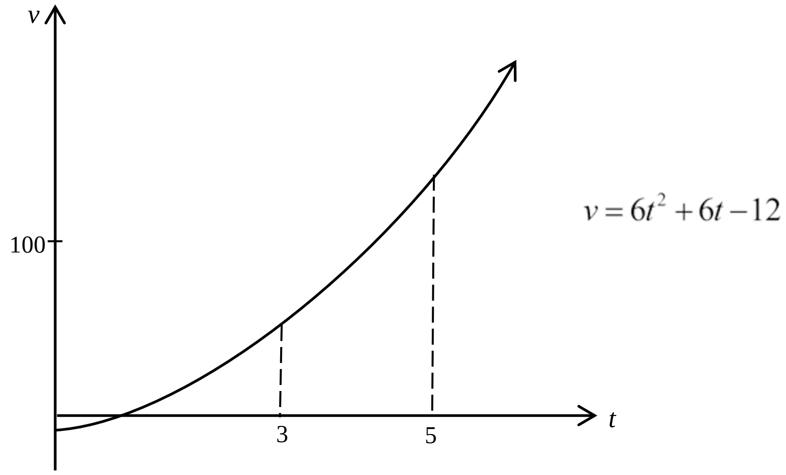
(iii) Find the coordinates of P. **2**

(c) (i) Sketch the curve $y = 4\cos\dfrac{x}{2}$ over the domain $-2\pi \le x \le 2\pi$ **2**

(ii) Find the equation of the tangent at the point where $x = \dfrac{\pi}{3}$ **4**

Question 16 **(15 marks)** **Use a SEPARATE writing booklet** **Marks**

(a) An object moves in a straight line so that at any time $t \geq 0$, its velocity v is given by $v = 6t^2 + 6t - 12$



Find (i) when its velocity is zero **1**

(ii) its acceleration at this time **2**

(iii) the distance it travels between when $t = 3$ and $t = 5$ **2**

(b) In November 1946, 18 koalas were introduced to Kangaroo Island. **5**
By November 2006, the number of koalas had increased to 5000.

Assume that the number N of koalas is increasing exponentially and satisfies an equation of the form $N = N_0 e^{kt}$, where N_0 and k are constants and t is measured in years from November 1946.

Find the values of N_0 and k, and predict the number of koalas that will be present on Kangaroo Island in November 2014.

(c) Felicity receives a money box on the day she's born. Her parents decide that each month, on the 1st of the month, they will deposit money into her money box and give her this money on her 21st birthday.

The 1st month they deposit \$10 into the money box.
The 2nd month they deposit \$20, the 3rd month they deposit \$30.
Each month they deposit \$10 more into the money box than they did the month before.

(i) How many times over the 21 years will Felicity's parents deposit money into her money box? **1**

(ii) How much will be deposited into the money box in the month of her 21st birthday? **2**

(iii) How much will Felicity receive from her parents on her 21st birthday? **2**

End of paper

TRIAL HSC – MATHEMATICS – ANSWERS – 2014

MULTIPLE CHOICE

1. D
2. C
3. A
4. B
5. B
6. A
7. D
8. D
9. B
10. B

QUESTION 11

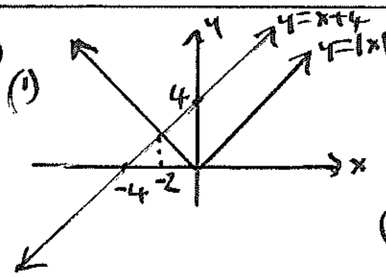
(a) $\dfrac{4}{x^2-x} - \dfrac{x-1}{x^2}$ (2)

$= \dfrac{4x-(x-1)^2}{x^2(x-1)}$

$= \dfrac{4x-x^2+2x-1}{x^2(x-1)}$

$= \dfrac{6x-x^2-1}{x^2(x-1)}$

(b) (i)



(2)

(ii) $|x| = x+4$ (1)

$x = -2$

(c) $e^x + e^{2x} + e^{3x} + \ldots$

(i) $\dfrac{T_2}{T_1} = \dfrac{e^{2x}}{e^x} = e^x$

$\dfrac{T_3}{T_2} = \dfrac{e^{3x}}{e^{2x}} = e^x$ (2)

$\therefore$ Geometric $r = e^x$

(ii) $S_n = \dfrac{a(r^n-1)}{r-1}$

$S_{10} = \dfrac{e^x(e^{10x}-1)}{e^x-1}$ (1)

(d) (i)

x	-1	0	1	2	3
2^x	$\frac{1}{2}$	1	2	4	8

(1)

(ii) $A = \frac{1}{3}\left(\frac{1}{2}+2+4(1)\right) + \frac{1}{3}\left(2+8+4(4)\right)$

$= 10\frac{5}{6}$ units^2 (2)

(e) $5^x = 4$

$\log 5^x = \log 4$

$x \log 5 = \log 4$ (2)

$x = \dfrac{\log 4}{\log 5}$

(f). $2\cos x = -\sqrt{3}$

$\cos x = -\dfrac{\sqrt{3}}{2}$ (2nd, 3rd)

$x = \pi - \frac{\pi}{6}, \ \pi + \frac{\pi}{6} \ -2\pi$

$= \dfrac{5\pi}{6}, \ -\dfrac{5\pi}{6}$ (2)

QUESTION 12

(a)(i) $y = x^2 \log_e x$

$y' = x^2\left(\frac{1}{x}\right) + \log_e x\,(2x)$

$= x + 2x\log_e x$ (2)

(ii) $y = \dfrac{e^{3x}}{\cos x}$

$y' = \dfrac{\cos x(3e^{3x}) - e^{3x}(-\sin x)}{(\cos x)^2}$

$= \dfrac{e^{3x}(3\cos x + \sin x)}{\cos^2 x}$ (2)

(b)(i) $\int \sec^2 6x \, dx$ (1)

$= \frac{1}{6}\tan 6x + C$

(ii) $\displaystyle\int_2^3 \frac{6x^2}{x^3-2}\,dx$

$= \Big[2\log_e(x^3-2)\Big]_2^3$

$= 2\log_e 25 - \frac{1}{2}\log_e 6$

$= 2\log_e \frac{25}{6}$ (3)

(c)(i) $y = 12x - x^3$

$y' = 12 - 3x^2$

$y'' = -6x$ (3)

$y'' = 0, \quad -6x = 0$

$x = 0$

(ii) Stat points $y' = 0$

$12 - 3x^2 = 0$

$3(4-x^2) = 0$

$x = \pm 2$

when $x = 2$
$y = 16$
$y'' = -12$ ∩

when $x = -2$
$y = -16$
$y'' = 12$ ∪

Max t.pt $(2, 16)$

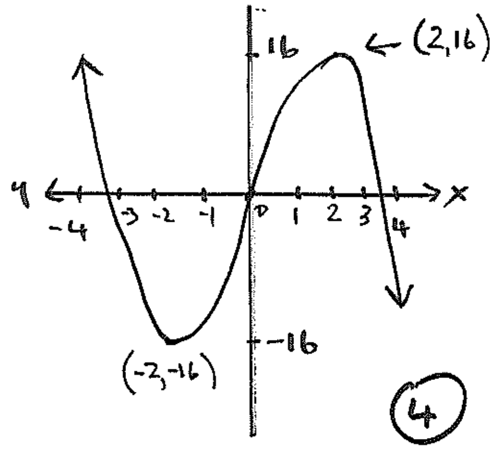
min t.pt $(-2, -16)$

Solve $y = 0$

$12x - x^3 = 0$

$x(12-x^2) = 0$

$x = 0, \pm\sqrt{12}$



(4)

Test $(0,0)$ for pt. of inflexion

x	-2	0	2
y''	12	0	-12

concavity changes

$\therefore (0,0)$ is pt. inflexion

TRIAL HSC — MATHEMATICS — ANSWERS — 2014

QUESTION 13

(a)(i) grad $= \frac{3}{4}$ (2)

y int $= 3$

Eqn : $y = \frac{3}{4}x + 3$ or

$3x - 4y + 12 = 0$

(ii) subst $x = 16, y = 15$

$3(16) - 4(15) + 12$

$= 48 - 60 + 12$

$= 0.$ (1)

(16,15) satisfies eqn

$\therefore$ lies on line

(iii) $ML = \sqrt{3^2 + 4^2} = 5$ (2)

$MP = 8 - 3 = 5$

$ML = MP$

$\therefore \triangle LMP$ is isosceles

(iv) Grad PL $= \frac{8-0}{0--4}$

$= 2$ (1)

(v) $N = (4,6)$ (2)

(vi) $N(4,6)$ $P(0,8)$

grad NP $= \frac{8-6}{0-4}$ (1)

$= \frac{2}{-4} = -\frac{1}{2} = m_1$

grad PL $= 2 = m_2$

$m_1 \times m_2 = -\frac{1}{2} \times 2$ (1)

$= -1$

$\therefore NP \perp PL$

$\therefore \angle NPL = 90°$

(vii) radius $= 5$ (2)

centre $= (0,3)$

Eqn : $x^2 + (y-3)^2 = 25$

(b)(i) $\frac{1}{2}r^2\theta = 12\pi$

$\frac{1}{2}r^2\frac{\pi}{6} = 12\pi$

$r^2 = 144$

$r = 12$ cm (1)

(ii) $l = r\theta$ (major arc)

$= \frac{11\pi}{6} \times 12$

$= 22\pi$ cm (2)

QUESTION 14

(a) $x^2 = 12(y-1)$

(i) vertex $= (0,1)$ (1)

(ii) $4a = 12$

$a = 3$ (focal length)

Focus $= (0,4)$ (2)

(b)(i) $\frac{dH}{dt} = 1 - \frac{t}{20}$

$H = t - \frac{t^2}{40} + c$

sub. $t=0$, $H=5$

$5 = c$ (1)

$\therefore H = \frac{-t^2}{40} + t + 5$

(ii) max height when $\frac{dH}{dt} = 0$

$1 - \frac{t}{20} = 0$

$t = 20$ (1)

max height $= \frac{-20^2}{40} + 20 + 5$

$= -10 + 20 + 5$

$= 15$ m (1)

(iii) when $H = 12.5$

$12.5 = \frac{-t^2}{40} + t + 5$

$500 = -t^2 + 40t + 200$

$t^2 - 40t + 300 = 0$

$(t-30)(t-10) = 0$

$t = 30, 10$ hours

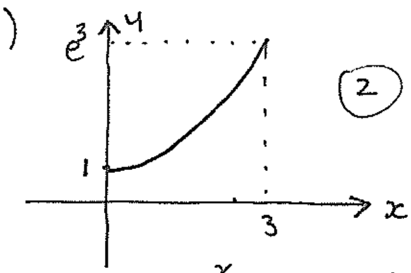
10am Sunday + 30h

= 4pm Monday

10am Sunday + 10h

= 8pm Sunday (3)

(c)(i)



(2)

$y = e^x$, $0 \le x \le 3$

(ii) $V = \pi\int_a^b y^2\,dx$

$= \pi\int_0^3 e^{2x}\,dx$ (1)

$= \pi\left[\frac{e^{2x}}{2}\right]_0^3$ (1)

$= \frac{\pi}{2}(e^6 - 1)$ (1)

$= 6.32$ (3 sig. fig) (1)

NB Only award correct rounding in this question.

Trial HSC - Mathematics - Answers - 2014

QUESTION 15

(a)(i) $\cos C = \dfrac{a^2+b^2-c^2}{2ab}$

$= \dfrac{3^2+7^2-6^2}{2(3)(7)}$ 3

$= \dfrac{11}{21}$

$C = 58^\circ$ (nearest degree)

(ii). $\angle DBC = 180^\circ - 58^\circ$

$= 122^\circ$ 1

cointerior angles in parallel lines $DB \parallel AC$

(b)(i) Solve simultaneously

$y = x^2 - 3x - 5$ ①

$y = x + k$ ②

sub ① in ②

$x^2 - 3x - 5 = x + k$

$x^2 - 4x - 5 - k = 0$ 1

(ii) only one root

so $b^2 - 4ac = 0$

$16 + 4(5+k) = 0$

$16 + 20 + 4k = 0$

$4k = -36$

$k = -9$ 2

(iii) $x^2 - 4x - 5 + 9 = 0$

$x^2 - 4x + 4 = 0$

$(x-2)^2 = 0$

$x = 2$

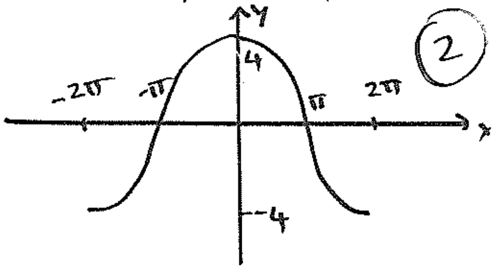
$y = -7$ 2

$P(2, -7)$

(c)(i) $y = 4\cos\frac{x}{2}$

period $= 4\pi$

amp $= 4$



2

(ii). $y = 4\cos\frac{x}{2}$

$y' = -4\sin\frac{x}{2} \cdot \frac{1}{2}$

$= -2\sin\frac{x}{2}$

when $x = \frac{\pi}{3}$ 4

$y = 4\cos\pi/6$

$= \frac{4\sqrt{3}}{2} = 2\sqrt{3}$

$y' = -2\sin\pi/6$

$= -2(\frac{1}{2}) = -1$

Eqn of tan

$y - 2\sqrt{3} = -1(x - \pi/3)$

$y - 2\sqrt{3} = -x + \pi/3$

QUESTION 16

(a) $v = 6t^2 + 6t - 12$

(i) $v = 0$

$6t^2 + 6t - 12 = 0$

$t^2 + t - 2 = 0$

$(t-1)(t+2) = 0$

$t \neq -2 \quad \therefore t = 1$

2

(ii) $a = v'$

$= 12t + 6$

when $t = 1$ 2

$a = 12(1) + 6$

$= 18$

(iii) $d = \int_3^5 (6t^2 + 6t - 12)\, dt$

$= \left[2t^3 + 3t^2 - 12t\right]_3^5$

$= \left[2(125) + 3(25) - 12(5)\right]$

$- \left[2(27) + 3(9) - 12(3)\right]$

$= 265 - 45$ 2

$= 220$ units

(b) $N = N_0 e^{kt}$

1946 : $t = 0$ $N = 18$

2006 : $t = 60$ $N = 5000$

$\therefore N_0 = 18$

$5000 = 18e^{60k}$

$e^{60k} = \frac{5000}{18}$

$60k = \ln\frac{5000}{18}$

$k = \dfrac{\ln\frac{5000}{18}}{60}$

$= 0.093780\ldots$

In 2014, $t = 68$

$N = 18e^{\frac{68}{60}\ln\frac{5000}{18}}$

$= 10587.57105$ 5

$= 10587$ approx.

(c)(i) $21 \times 12 = 252$ 1

(ii) $10 + 20 + 30 + \ldots$

$T_{252} = a + 251d$

$= 10 + 251 \times 10$ 1

$= \$2520$

(iii) $S_{252} = \frac{n}{2}(a + l)$

$= \frac{252}{2}[10 + 2520]$

$= \$318\,780$

2