MEASURE BY ACHIEVEMENT

Kirrawee High School
New South Wales

Student Number: ____________________

Teacher: ____________________

2015

HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes.
- Working time – 3 hours.
- Write using blue or black pen.
- Only board-approved calculators may be used.
- Start each question in a new booklet.
- A table of standard integrals is provided at the back of the paper
- All necessary working should be shown.
- Answer multiple choice on Multiple Choice answer sheet provided at back of this exam. This may be detached.

Total Marks – 100

Section I Pages 3-4

10 marks

- Answer Questions 1 – 10
- Allow about 15 minutes for this section

Section II Pages 5-10

90 marks

- Attempt Questions 11 – 16
- Allow about 2 hours 45 minutes for this section

NAME:

NUMBER:

TEACHER:

HSC COURSE MATHEMATICS

TASK: Trial Examination

Topic Area Outcome	H1	H2	H3	H4	H5	H6	H7	H8	H9
Applications of Calculus		√		√	√	√			√
Integration		√		√	√			√	√
Probability		√		√	√				√
Plane Geometry 2		√			√				√
Linear Functions and Lines 2		√			√	√			
Series Applications		√		√					√
Trigonometric Functions		√		√	√	√		√	√
Logarithmic and Exponential Functions		√	√		√	√	√	√	√
Applications of Calculus to the Physical World		√	√	√	√	√	√		√

Outcome	**Description**
H1	Seeks to apply mathematical techniques to problems in a wide range of practical contexts. (NOTE: This outcome will NOT form part of the formal assessment program.)
H2	Constructs arguments to prove and justify results.
H3	Manipulates algebraic expressions involving logarithmic and exponential functions.
H4	Expresses practical problems in mathematical terms based on simple given models.
H5	Applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and series to solve problems.
H6	Uses the derivative to determine the features of the graph of a function.
H7	Use the features of a graph to deduce information about the derivative.
H8	Uses techniques of integration to calculate areas and volumes.
H9	Communicates using mathematical language, notation, diagrams and graphs.

Section I

Multiple Choice
10 Marks

Answer on the answer sheet provided.

1. What is the value of $\sum_{n=1}^{4} n^2$?

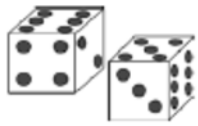
(A) 576 (B) 120 (C) 30 (D) 16

2. What is the equation of the parabola with vertex (4,2) and focus (3,2)?

(A) $(x-4)^2 = 4(y-2)$ (B) $(x-4)^2 = -4(y-2)$

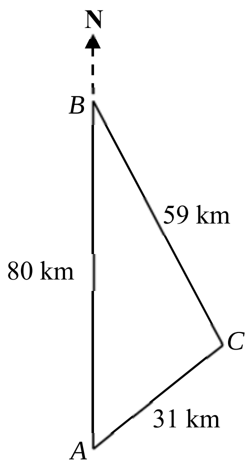
(C) $(y-2)^2 = 4(x-4)$ (D) $(y-2)^2 = -4(x-4)$

3. Two regular dice are thrown. What is the probability that the throw will not result in a double?



(A) $\frac{1}{6}$ (B) $\frac{5}{6}$ (C) $\frac{25}{36}$ (D) $\frac{35}{36}$

4. In the diagram below, Town *B* is 80 km due north of town *A* and 59 km from Town *C*. Town *A* is 31 km from Town *C*.



What is the bearing of Town *C* from Town *B*?

(A) 019° (B) 122° (C) 161° (D) 341°

5. What is the domain and range of the function $f(x) = |x-2| + 1$?

(A) Domain: All real x, Range : $y \geq 1$ (B) Domain: $x < 2$, Range : $y \geq 1$
(C) Domain: $x \geq 0$, Range: $y \geq 0$ (D) Domain: All real x, Range : $y \geq -1$

6. The period of the function $y = 3\cos\left(\frac{x}{3}\right)$ is :

(A) $\frac{\pi}{3}$ (B) π (C) 3π (D) 6π

7. The number of animals P on an island, at time t is given by $P=7000e^{-kt}$, where k is a positive constant. Over time the number of animals on the island is:

(A) Increasing exponentially (B) Decreasing exponentially
(C) Increasing at a constant rate (D) Decreasing at a constant rate

8. For what values of m will the geometric series $1 + 2m + 4m^2 + 8m^3 + ...$ have a limiting sum?

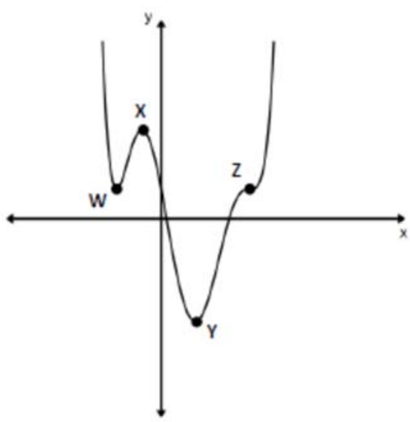
(A) $-1 \leq m \leq 1$ (B) $-\frac{1}{2} \leq m \leq \frac{1}{2}$
(C) $-\frac{1}{2} < m < \frac{1}{2}$ (D) $m < \frac{1}{2}$

9. If $\log_{10} 7 = a$, then $\log_{10}\left(\frac{1}{70}\right)$ *is* equal to :

(A) $-(1+a)$ (B) $(1+a)^{-1}$ (C) $\frac{a}{10}$ (D) $\frac{1}{10a}$

10. The diagram below shows the graph of $y = f(x)$. Points W,X,Y and Z are stationary points and Z is a point of inflexion on $f(x)$. Which of the points on $y = f(x)$ corresponds to the description: $y > 0$, $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} < 0$

(A) W
(B) X
(C) Y
(D) Z



Section II

90 Marks
Attempt Questions 11-16
Allow about 2 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available. In Questions 11-16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet **Marks**

(a) Find the value of $\dfrac{3.6\times 7.4}{\sqrt{5.6+2.5}}$ correct to two significant figures. **1**

(b) Factorise x^3-125 **1**

(c) If $\left(\sqrt{7}-3\right)\left(2\sqrt{7}+2\right)=p+q\sqrt{7}$, find p and q **2**

(d) Evaluate $\displaystyle\lim_{x\to 3}\frac{x^2+2x-15}{x-3}$ **2**

(e) Find the equation of the tangent to the curve $y=e^{2x}-3x$ at the point (0, 1). **2**

(f) An arithmetic series has a first term of 2 and a last term of 126. If there are 32 terms in the series, find the sum of the series. **2**

(g) Solve the following equation for k. **2**

$\log_x 54=\log_x k+2\log_x 3$

(h) Prove that $\dfrac{\sin A}{1+\cos A}+\dfrac{1+\cos A}{\sin A}=2\operatorname{cosec} A$ **3**

Question 12 **(15 marks)** **Use a SEPARATE writing booklet** **Marks**

(a) Differentiate with respect to x.

(i) $x^2 \sin 2x$ **2**

(ii) $\dfrac{x}{\log_e x}$ **2**

(b) Find the following indefinite integrals:

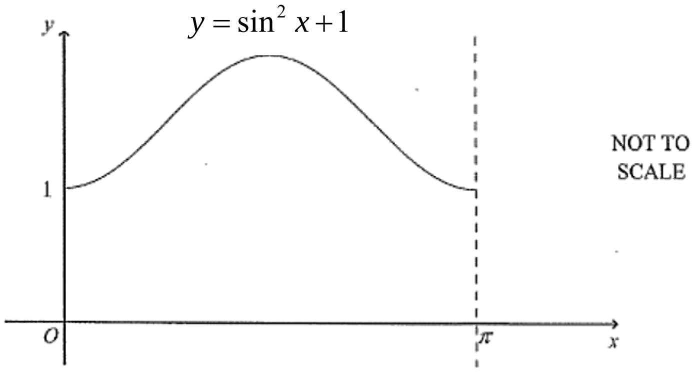
(i) $\int (2x+5)^3 \, dx$ **2**

(ii) $\int 4e^{3-2x} \, dx$ **2**

(c) If $\int_4^8 \frac{1}{x-2} \, dx = \log_e N$, find the value of N. **3**

(d) A stained-glass window can be modelled as the region bounded by the curve $y = \sin^2 x + 1$, the coordinate axes and $x = \pi$.

The graph of $y = \sin^2 x + 1$ for $0 \le x \le \pi$ is shown below.



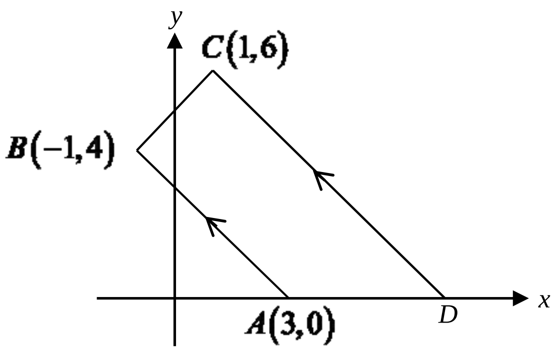
(i) Copy and complete the table with exact values in your writing booklet. **2**

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	1				

(ii) Use Simpson's Rule with 5 function values to calculate the approximate area of glass needed for the window, correct to 2 decimal places. **2**

Question 13 **(15 marks)** **Use a SEPARATE writing booklet** **Marks**

(a) In the diagram below, $ABCD$ is a trapezium with AB parallel to DC. The coordinates of A, B and C are $(3,0)$, $(-1,4)$ and $(1,6)$ respectively. D lies on the x-axis.



(i) Find the length of AB. **1**

(ii) Find the gradient of AB. **1**

(iii) Show that the equation of the line CD is $x+y-7=0$, and hence find the coordinates of D. **2**

(iv) Show that the perpendicular distance from A to the line CD is $2\sqrt{2}$ units. **2**

(v) Hence, or otherwise, find the area of the trapezium $ABCD$. **2**

(b) A box contains 4 blue and 9 green discs. Two discs are chosen from the box without replacement. Find the probability of selecting:

(i) two blue discs. **1**

(ii) at least one green disc. **1**

(iii) two discs of a different colour. **2**

(c) If $3x^2+7 \equiv A(x+2)^2+Bx+C$, find the values of A, B and C. **3**

Question 14 **(15 marks)** **Use a SEPARATE writing booklet** **Marks**

(a) A function is given by $f\left(x\right)=x^3-3x^2-9x+11$

(i) Find the coordinates of the stationary points of $f(x)$ and determine their nature. **3**

(ii) Hence, sketch the graph $y=f\left(x\right)$ showing all stationary points and the y intercept. **2**

(b) Solve $\cos 2\theta=\frac{1}{\sqrt{2}}$ in the domain $0\le\theta\le2\pi$ **2**

(c) A Year 12 Biology class tested to see how much bacteria was present in a variety of food samples at their school canteen.

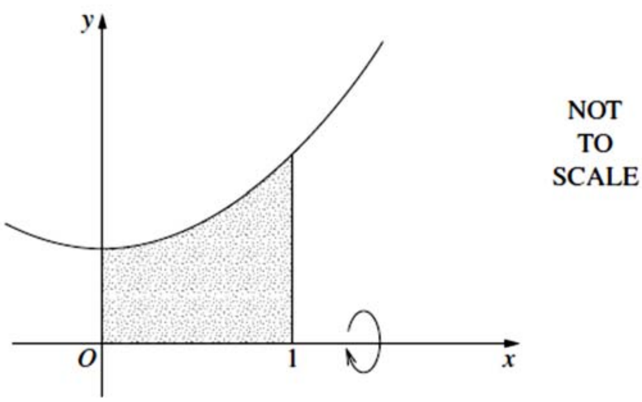
It is known that after t hours the number of bacteria (N) present in a particular type of food is given by the formula $N=Ae^{kt}$.

(i) If initially there were 20 000 bacteria present, calculate the value of A. **1**

(ii) After three hours, there were 45 000 bacteria present. Calculate the value of k, correct to 2 decimal places. **2**

(iii) How long would it take for the initial number of bacteria to triple in quantity, correct to the nearest hour? **2**

(d) The shaded region in the diagram below is bounded by the curve $y=x^2+1$, the x axis, and the lines $x=0$ and $x=1$.



Find the exact volume of the solid of revolution formed when the shaded region is rotated about the x axis. **3**

Question 15 **(15 marks)** **Use a SEPARATE writing booklet** **Marks**

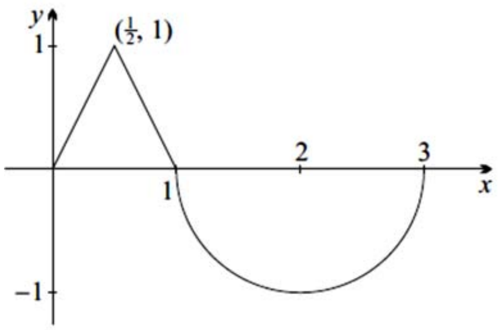
(a) If a, b and c are three consecutive terms of a geometric series, show that $\ln a$, $\ln b$ and $\ln c$ are three consecutive terms of an arithmetic series. **2**

(b) Let α and β be the roots of $3x^2 - 4x - 2 = 0$.

(i) Find the value of $\alpha\beta$ **1**

(ii) Find $\frac{5}{\alpha} + \frac{5}{\beta}$ **2**

(c)



The diagram above shows the function $y = g(x)$ with domain $0 \le x \le 3$. The arc is a semi-circle. Find $\int_0^3 g(x)\,dx$ **2**

(d) A particle moves in a straight line and at time t it has velocity v, where $v = 3t^2 - 2\sin 3t + 6$.

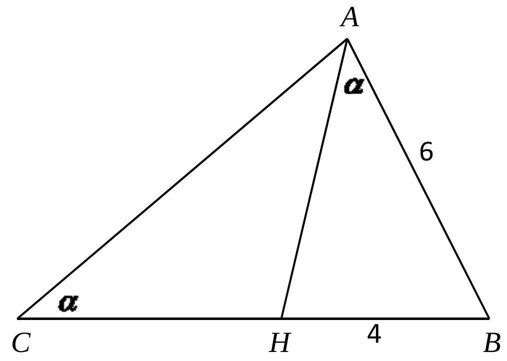
(i) Find an expression for the acceleration of the particle at time t. **1**

(ii) When $t = \frac{\pi}{3}$, show that the acceleration of the particle is $2\pi + 6$. **1**

(iii) When $t = 0$, the particle is at the origin. Find an expression for the displacement of the particle from the origin at time t. **2**

Question 15 (Continued) **Marks**

(e)



In the diagram above, $\angle BCA = \angle BAH = \alpha$, $AB = 6$ and $BH = 4$

(i) Show that ΔABC is similar to ΔHBA. **2**

(ii) Hence, or otherwise, find the length HC. **2**

Question 16 **(15 marks)** **Use a SEPARATE writing booklet** **Marks**

(a) Jessica plans to work for the next ten years.

During that time she wants to save \$50 000.

She decides to invest a fixed amount of money at the beginning of each month during this time.

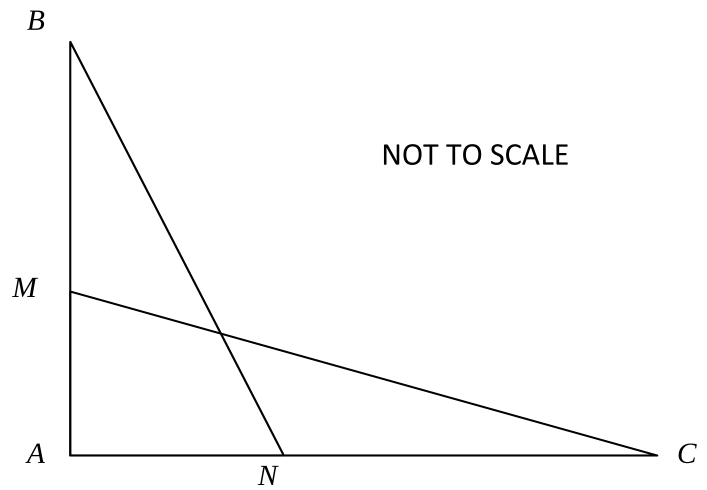
Interest is paid at a fixed rate of 6% p.a. **compounded monthly.**

(i) Let M be the monthly investment in dollars. **3**
Show that the total investment R after 10 years is given by
$R = M\left[(1.005)+(1.005)^2+(1.005)^3+....+(1.005)^{120}\right]$

(ii) Find the amount, M, which needs to be deposited at the beginning of each month, in order for Jessica to reach her goal. **2**

(b) Two equal length intervals AB and AC are drawn.

Points M and N are placed on the intervals with the condition $\angle ABN = \angle ACM$



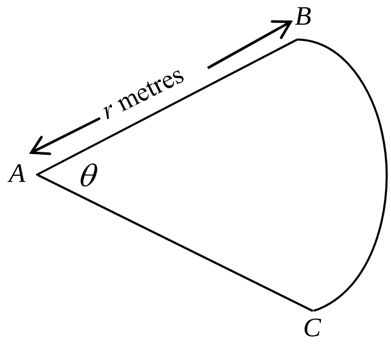
(i) Prove that $\Delta ABN \equiv \Delta ACM$ **2**

(ii) Hence prove that $BM = CN$ **2**

Question 16 (Continued)

Marks

(c) In the figure below, AB and AC are radii of length r metres of a circle with centre A.



NOT TO SCALE

The arc BC of the circle subtends an angle of θ at A.

The perimeter of the figure ABC is 12 metres.

(i) Show that the area Y (in square metres) of the sector ABC is given by: **2**

$$Y = \frac{72\theta}{(\theta + 2)^2}$$

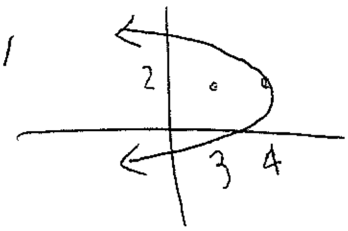
(ii) Hence, show that the maximum area of the sector is 9 square metres. **4**

End of paper

Year 12 2019 Trial HSC
Mathematics Solutions

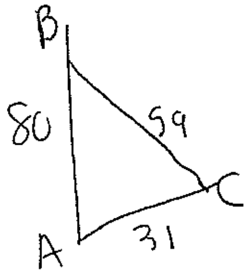
Section 1 MC

1, C

2, 

D

3, B

4, 

$\cos B = \dfrac{80^2+59^2-31^2}{2\times80\times59}$

$B = 19^\circ$

Bearing $= 180-19 = 161^\circ$

C

5, A

6, $\dfrac{2\pi}{\frac{1}{3}} = 6\pi$ D

7, B

8, $|r|<1$

$-1 < 2m < 1$

$-\frac{1}{2} < m < \frac{1}{2}$

C

9, $\log_{10}(7\times10)^{-1}$

$= -(\log_{10}7 + \log_{10}10)$

$= -(a+1)$

A

10, B

Section II   Q11

(a) 9.4

(b) $(x-5)(x^2+5x+25)$

(c) $\sqrt{7}(2\sqrt{7}+2)-3(2\sqrt{7}+2)$

$= 2\times 7+2\sqrt{7}-6\sqrt{7}-6$

$= 8-4\sqrt{7}$

$p=8, \quad q=-4$

(d) $\lim_{x\to 3} \frac{(x+5)\cancel{(x-3)}}{\cancel{x-3}}$

$=3+5$

$=8$

(e) $y=e^{2x}-3x$

$y'=2e^{2x}-3$

at $x=0$

$y'=2e^{0}-3$

$=-1$

Eqn

$y-1=-1(x)$

$y-1=-x$

$x+y-1=0$

2

(f) AP $a = 2$

$n = 32$

$l = 126$

$$S_{32} = \frac{32}{2}(2 + 126)$$

$$= 2048$$

(g) $\log_x 54 = \log_x k + 2\log_x 3$

$\log_x 54 = \log_x k + \log_x 3^2$

$\log_x 54 = \log_x 9k$

$9k = 54$

$k = 6$

(h) $\frac{\sin A}{1 + \cos A} + \frac{1 + \cos A}{\sin A}$

$$= \frac{\sin^2 A + 1 + 2\cos A + \cos^2 A}{(1 + \cos A)\sin A}$$

$$= \frac{\sin^2 A + \cos^2 A + 1 + 2\cos A}{\sin A(1 + \cos A)}$$

$$= \frac{1 + 1 + 2\cos A}{\sin A(1 + \cos A)}$$

$$= \frac{2 + 2\cos A}{\sin A(1 + \cos A)}$$

$$= \frac{2\underline{(1 + \cos A)}}{\sin A(1 + \cos A)} = \frac{2}{\sin A} = 2\operatorname{cosec} A$$

(3)

Q12

(a)(i) $\cancel{\dfrac{d}{dx}(x^2-3)^{-1}}$

$\cancel{= -(x^2-3)^{-2} \cdot 2x}$

$\cancel{= -\dfrac{2x}{(x^2-3)^2}}$

(i) $\dfrac{d}{dx}(x^2 \sin 2x)$

$= u'v + v'u$

$= 2x \sin 2x + 2\cos 2x \, x^2$

(ii) $\dfrac{d}{dx}\left(\dfrac{x}{\log_e x}\right)$

$= \dfrac{u'v - v'u}{v^2}$

$= \dfrac{\log_e x - \frac{1}{x} \cdot x}{(\log_e x)^2}$

$= \dfrac{\log_e x - 1}{(\log_e x)^2}$

(b)(i) $\int (2x+5)^3 \, dx$

$= \dfrac{1}{2}\dfrac{(2x+5)^4}{4} + c$

$= \dfrac{(2x+5)^4}{8} + c$

4

(b)(ii) $\int 4e^{3-2x}$

$= -2e^{3-2x} + C$

(c) $\int_4^8 \frac{1}{x-2}$

$= \left[\log_e(x-2)\right]_4^8$

$= \log_e 6 - \log_e 2$

$= \log_e \frac{6}{2}$

$= \log_e 3$, $N = 3$

(d) (i)

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	1	1.5	2	1.5	1

(ii) $h = \frac{b-a}{n}$

$= \frac{\pi}{4}$

$A = \frac{h}{3}\left(f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right)$

$= \frac{\pi}{12}\left(1 + (4 \times 1.5) + 2\right) + \frac{\pi}{12}\left(2 + (4 \times 1.5) + 1\right)$

$= \frac{9\pi}{12} + \frac{9\pi}{12}$

$= \frac{18\pi}{12} = 4.71$ units^2

5

Q13

13a) (i) $AB = \sqrt{4^2+4^2}$

$= \sqrt{32}$

$= 4\sqrt{2}$ units

(ii) $m = \dfrac{y_2-y_1}{x_2-x_1}$

$= -\dfrac{4}{4}$

$= -1$

(iii) $m=-1$ thru $C(1,6)$

$y-6 = -1(x-1)$

$y-6 = -x+1$

$x+y-7=0$

at D $y=0$

$x=7$

$D=(7,0)$

(iv) $x+y-7=0$ 3, 0

$d = \dfrac{|3-7|}{\sqrt{1+1}}$

$= \dfrac{4}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}}$

$= \dfrac{4\sqrt{2}}{2} = 2\sqrt{2}$ units

6

(v) $CD = (1,6),\ (7,0)$

$$d = \sqrt{6^2+6^2}$$
$$= \sqrt{72}$$
$$= 6\sqrt{2}$$

$$A = \frac{h}{2}(a+b)$$
$$= \frac{2\sqrt{2}}{2}\left(4\sqrt{2}+6\sqrt{2}\right)$$
$$= \sqrt{2} \times 10\sqrt{2}$$
$$= 20 \text{ units}^2$$

(b) (i) $P(2 \text{ blue}) = \frac{4}{13} \times \frac{3}{12}$
$$= \frac{1}{13}$$

(ii) $P(\text{at least one green}) = 1 - 2 \text{ blue}$
$$= 1 - \frac{1}{13}$$
$$= \frac{12}{13}$$

(iii) $P(\text{Blue, green}) + P(\text{green, blue})$
$$= \frac{4}{13} \times \frac{9}{12} + \frac{9}{13} \times \frac{4}{12}$$
$$= \frac{6}{13}$$

13 (c) $A(x+2)^2 + Bx + C$

$= A(x^2+4x+4) + Bx + C$

$= Ax^2 + (4A+B)x + 4A + C$

$A = 3$

$4A + B = 0$

$12 + B = 0$

$B = -12$

$4A + C = 7$

$12 + C = 7$

$C = -5$

$A = 3, \ B = -12, \ C = -5$

Q 14

(a) (i) $f(x) = x^3 - 3x^2 - 9x + 11$

$f'(x) = 3x^2 - 6x - 9$

stat points $f'(x) = 0$

$3x^2 - 6x - 9 = 0$

$3(x^2 - 2x - 3) = 0$

$(x-3)(x+1) = 0$

$x = 3$ or -1

$y = -16$ or 16

$(3, -16), (-1, 16)$ stat pts

Nature

$f''(x) = 6x - 6$

at $x = 3$, $f''(x) = 12$
$> 0 \therefore$ min

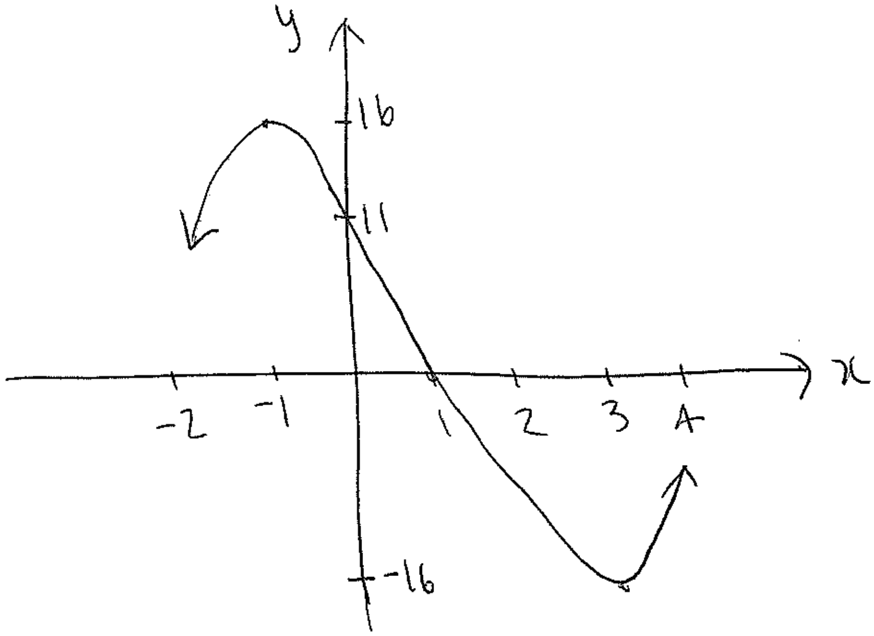
at $x = -1$, $f''(x) = -12$
$< 0 \therefore$ max

$(3, -16)$ minimum

$(-1, 16)$ maximum.

(ii) at $x = 0$

$f(x) = 11$



(b)

$\cos 2\theta = \frac{1}{\sqrt{2}} \quad 0 \le \theta \le 2\pi$

$2\theta = \frac{\pi}{4}, \ 2\pi - \frac{\pi}{4}, \ 2\pi + \frac{\pi}{4}, \ 4\pi - \frac{\pi}{4}$

$\theta = \frac{\pi}{8}, \ \frac{7\pi}{8}, \ \frac{9\pi}{8}, \ \frac{15\pi}{8}$

(c) (i) at $t = 0$, $N = 20000$

$20000 = Ae^0$

$A = 20000$

(ii)

$45000 = 20000e^{3k}$

$2.25 = e^{3k}$

$\ln 2.25 = 3k$

$k = \frac{\ln 2.25}{3} = 0.27$

(iii)

$3 = e^{0.27t}$

$\ln 3 = 0.27t$

$t = \frac{\ln 3}{0.27} = 4$ hours

10

(d)

$$y = x^2 + 1$$

$$y^2 = (x^2+1)^2$$

$$= x^4 + 2x^2 + 1$$

$$V = \pi \int_0^1 x^4 + 2x^2 + 1 \, dx$$

$$= \pi \left[\frac{x^5}{5} + \frac{2x^3}{3} + x \right]_0^1$$

$$= \pi \left(\left(\frac{1}{5} + \frac{2}{3} + 1 \right) - 0 \right)$$

$$= \frac{28\pi}{15} \text{ units}^3$$

Q15

(a) a, b, c GP

$\therefore \frac{b}{a} = \frac{c}{b}$

$\ln\left(\frac{b}{a}\right) = \ln\left(\frac{c}{b}\right)$

ie $\ln b - \ln a = \ln c - \ln b$

$\therefore$ common difference

$\therefore$ $\ln a, \ln b$ and $\ln c$ form ap

(b) (i) $3x^2 - 4x - 2 = 0$

$\alpha\beta = \frac{c}{a}$

$= -\frac{2}{3}$

(ii) $\frac{5}{\alpha} + \frac{5}{\beta}$

$= \frac{5\beta + 5\alpha}{\alpha\beta}$

$= \frac{5(\beta + \alpha)}{\alpha\beta}$

$\alpha + \beta = -\frac{b}{a}$

$= \frac{4}{3}$

$\frac{5(\beta+\alpha)}{\alpha\beta} = \frac{5 \times \frac{4}{3}}{-\frac{2}{3}} = -10$

(12)

(c) $\int_0^3 g(x)\,dx$

$= \text{triangle} - \text{semi circle}$

$= \frac{1}{2}bh - \frac{1}{2}\pi r^2$

$= \frac{1}{2}\times 1\times 1 - \frac{1}{2}\pi\times 1^2$

$= \frac{1}{2} - \frac{1}{2}\pi$

$= \frac{1}{2}(1-\pi)$

(d) (i) $v = 3t^2 - 2\sin 3t + 6$

$a = 6t - 6\cos 3t$

(ii) at $t = \frac{\pi}{3}$

$a = 6\times\frac{\pi}{3} - 6\cos\pi$

$= \frac{6\pi}{3} - (6\times -1)$

$= 2\pi + 6$

(iii) $v = 3t^2 - 2\sin 3t + 6$

$x = t^3 + \frac{2}{3}\cos 3t + 6t + c$

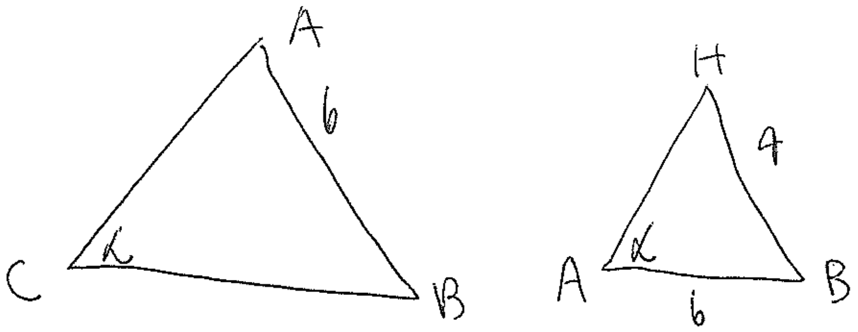
at $t=0$, $x=0$

$0 = 0 + \frac{2}{3} + 0 + c$

$c = -\frac{2}{3}$

$x = t^3 + \frac{2}{3}\cos 3t + 6t - \frac{2}{3}$

(e)



(i) In ΔABC and ΔHBA

$\angle BCA = \angle BAH = \alpha$ given

$\angle ABC = \angle HBA$ (common)

$\therefore$ equiangular

$\therefore$ triangles are similar

(ii) $\frac{CB}{6} = \frac{6}{4}$

$CB = 9$

$CH = 9 - 4$

$= 5$

Q16

(a)(i) $r = 6\% \div 12 = 0.5\%$

$n = 10 \times 12 = 120$

$$A_1 = P\left(1+\frac{r}{100}\right)^n$$

$$= m\left(1+\frac{0.5}{100}\right)^{120}$$

$$= m(1.005)^{120}$$

$$A_2 = m(1.005)^{119}$$

$$A_3 = m(1.005)^{118}$$

$$\vdots$$

$$A_{119} = m(1.005)^2$$

$$A_{120} = m(1.005)$$

$$R = m\left(1.005 + (1.005)^2 + (1.005)^3 + \dots (1.005)^{120}\right)$$

(ii) 4 $R = 50000$

$$50000 = m\left(1.005 + 1.005^2 \dots + 1.005^{120}\right)$$

GP, $a = 1.005$, $r = 1.005$, $n = 120$

$$50000 = m\left(1.005\,\frac{(1.005)^{120}-1}{1.005-1}\right)$$

$$50000 = m(164.6987)$$

$$m = \$303.58$$

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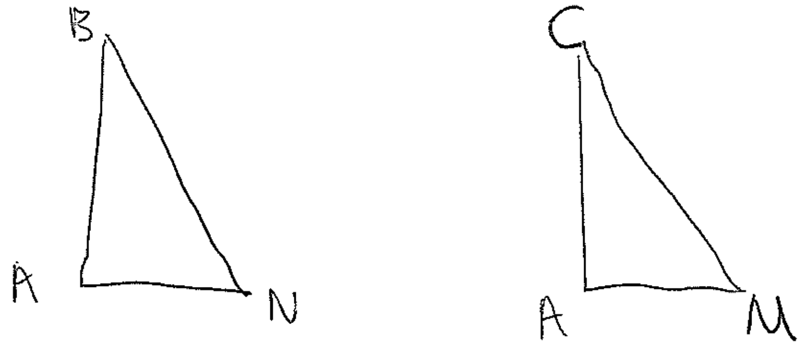
(b)



(i)

In $\triangle ABN$ and $\triangle ACM$

$\angle A$ is common

$AB = AC$ (given)

$\angle ABN = \angle ACM$ (given)

$\therefore \triangle ABN \equiv \triangle ACM$ (AAS)

(ii)

$AM = AN$ (matching sides of congruent triangles)

Also $AB = AC$ (given)

$\therefore AB - AM = AC - AN$

$\therefore BM = CN$

(c)(i) $P = 2r + r\theta$

$12 = 2r + r\theta$

$12 = r(2+\theta)$

$r = \dfrac{12}{2+\theta}$

Area Sector ABC

$= \dfrac{1}{2} r^2 \theta$

$= \dfrac{1}{2} \times \dfrac{144}{(2+\theta)^2} \theta$

$= \dfrac{72\theta}{(\theta+2)^2}$

(ii) Max

$Y' = 0$

$Y' = \dfrac{72(\theta+2)^2 - 2(\theta+2)72\theta}{(\theta+2)^4}$

$= \dfrac{72(\theta+2)(\theta+2-2\theta)}{(\theta+2)^4}$

$\dfrac{72(\theta+2)(2-\theta)}{(\theta+2)^4} = 0$

$\theta = 2$

θ	1	2	3
Y'	2.6 >0	0	-0.576 <0

$\therefore\ \theta = 2$ is a max

at $\theta = 2$

$$A = \frac{72\theta}{(\theta+2)^2}$$

$$= \frac{72 \times 2}{4^2}$$

$$= 9\ m^2$$