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Student Number

Teachers Name

KIRRAWEE HIGH SCHOOL

2017 YEAR 12
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes.
- Working time – 3 hours.
- Write using black pen.
- Only board-approved calculators may be used.
- All necessary working should be shown.
- A multiple choice answer sheet provided
- A formula sheet is provided

Total Marks – 100

Section I

10 marks

- Answer Questions 1 – 10 on the sheet provided

Section II

90 marks

- Attempt Questions 11 – 16

Question	Mark	Out of
Multiple Choice 1-10		10
Section II Q11		15
Q12		15
Q13		15
Q14		15
Q15		15
Q16		15
Total		100

NUMBER:

MATHEMATICS

TEACHER:

TASK: TRIAL HSC EXAMINATION

Question	Max Mark	Mark	Outcome Topic Area	H1	H2	H3	H4	H5	H6	H7	H8	H9	H10
1-10	10		Credit & Borrowing	✓		✓						✓	✓
11	15		Annuities & Loan Repayments	✓		✓			✓			✓	✓
12	15		Interpreting Sets of Data & Normal Distribution	✓	✓					✓		✓	✓
13	15		Sampling & Populations	✓	✓					✓	✓	✓	✓
14	15		Further Applications of Area & Volume				✓	✓					✓
15	15		Applications of Trigonometry				✓	✓					✓
16	15		Spherical Geometry				✓	✓					✓
			Multistage Events & Applications of Probability	✓	✓						✓	✓	✓
			Further Algebraic Skills			✓						✓	✓
			Modelling Linear Relationships			✓						✓	✓
			Modelling Non-Linear Relationships			✓						✓	✓
			Mathematics & Health	✓	✓	✓		✓		✓		✓	✓
Max	Mark												
100													

Outcome	Description
H1	Uses mathematics and statistics to evaluate and construct arguments in a range of familiar and unfamiliar contexts.
H2	Analyses representations of data in order to make inferences, predictions and conclusions.
H3	Makes predictions about situations based on mathematical models, including those involving cubic, hyperbolic or exponential functions.
H4	Analyses two-dimensional and three-dimensional models to solve practical problems, including those involving spheres and non-right-angled triangles.
H5	Interprets the results and measurements and calculations and makes judgements about reasonableness, including the degree of accuracy of measurements and calculations and the conversion of appropriate units.
H6	Makes informed decisions about financial situations, including annuities and loan repayments.
H7	Answers questions requiring statistical processes, including the use of the normal distribution, and the correlation of bivariate data.
H8	Solves problems involving counting techniques, multistage events and expectation.
H9	Chooses and uses appropriate technology to location and organise information from a range of contexts.
H10	Uses mathematical argument and reasoning to evaluate conclusions drawn from other sources, communicating a position clearly to others, and justifies a response.

Section I
Multiple Choice
10 Marks

Use the multiple choice answer sheet for Questions 1 – 10

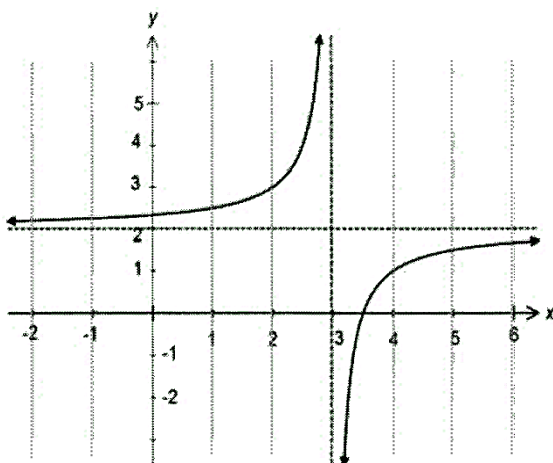
1 Solve the equation $3x - 1 = \frac{x + 3}{2}$.

- (A) $x = \frac{4}{5}$
- (B) $x = 1$
- (C) $x = 2$
- (D) $x = 7$

2 Given $\log_a 3 = 0.6$ and $\log_a 2 = 0.4$, find $\log_a 18$.

- (A) 1.8
- (B) 1.6
- (C) 3.0
- (D) 0.74

3 Part of the function $y = \frac{a}{x+b} + c$ is shown



The value of a , b and c respectively are:

- (A) 2, 3, 2
- (B) 2, -3, 2
- (C) -2, -3, 2
- (D) -2, 3, 2

4 For $f(x) = \begin{cases} x+1 & x \geq 3 \\ x^2 + 2x - 1 & -1 < x < 3 \\ 3^x & x \leq 1 \end{cases}$, find the value of $f(2) + f(-2) - f(6)$

(A) $\frac{1}{9}$

(B) $14\frac{1}{9}$

(C) 9

(D) $3\frac{8}{9}$

5 A circle has the equation $4x^2 - 4x + 4y^2 + 24y + 21 = 0$. What is the radius and centre?

(A) Centre $\left(\frac{1}{2}, -3\right)$ and radius of 2

(B) Centre $\left(\frac{1}{2}, 3\right)$ and radius of 2

(C) Centre $\left(\frac{1}{2}, -3\right)$ and radius of 4

(D) Centre $\left(\frac{1}{2}, 3\right)$ and radius of 4

6 An infinite geometric series has a first term of 12 and a limiting sum of 15. What is the common ratio?

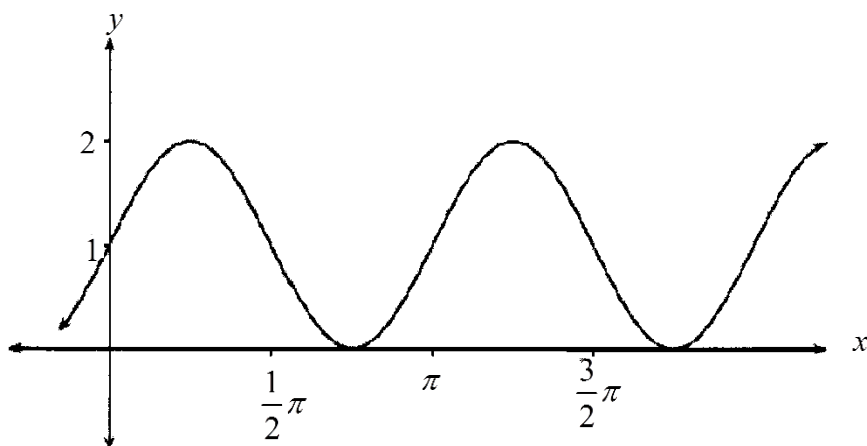
(A) $\frac{1}{5}$

(B) $\frac{1}{4}$

(C) $\frac{1}{3}$

(D) $\frac{1}{2}$

7



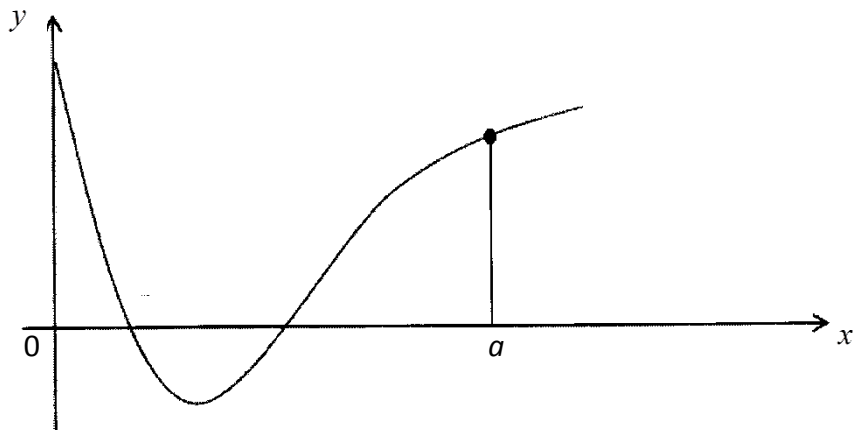
The equation of the graph sketched above could be:

- (A) $y = 1 + \sin 2x$
- (B) $y = 1 - \sin 2x$
- (C) $y = 1 + 2 \sin 2x$
- (D) $y = 1 - 2 \sin x$

8 What is the domain of the function $f(x) = \sqrt{6 - 2x}$?

- (A) All real x such that $x \leq 3$
- (B) All real x such that $x < 3$
- (C) All real x such that $x \geq 3$
- (D) All real x such that $x > 3$

- 9 The diagram shows the graph of $y = f(x)$



Which of the following statements is true?

- (A) $f'(a) > 0$ and $f''(a) < 0$
- (B) $f'(a) < 0$ and $f''(a) < 0$
- (C) $f'(a) > 0$ and $f''(a) > 0$
- (D) $f'(a) < 0$ and $f''(a) > 0$
- 10 Differentiate $\log_e \left(\frac{x-5}{x+5} \right)$

- (A) $\frac{25}{x^2 - 25}$
- (B) $\frac{10}{x^2 - 25}$
- (C) $\frac{1}{x-5} + \frac{1}{x+5}$
- (D) $\frac{1}{x+5} + \frac{1}{x-5}$

Section II

Question 11 (15 marks)

Marks

- (a) Factorise $2x^2 - 7x + 3$ 1
- (b) Express $\frac{5}{\sqrt{3}-1}$ with a rational denominator 1
- (c) Solve $|3x-1| < 2$ 2
- (d) Find the coordinates of the focus of the parabola $x^2 = 16(y-2)$ 2
- (e) Evaluate $\lim_{x \rightarrow 3} \frac{x^2 + 2x - 15}{x - 3}$ 2
- (f) The area of a sector of a circle of radius 6cm is 50cm^2 .
Find the length of the arc of the sector. 2
- (g) The exterior angle of a regular polygon is 8° . What is the interior angle sum of the polygon? 2
- (h) Solve $4\cos^2 \theta - 3 = 0$ for $0 \leq \theta \leq 2\pi$ 3

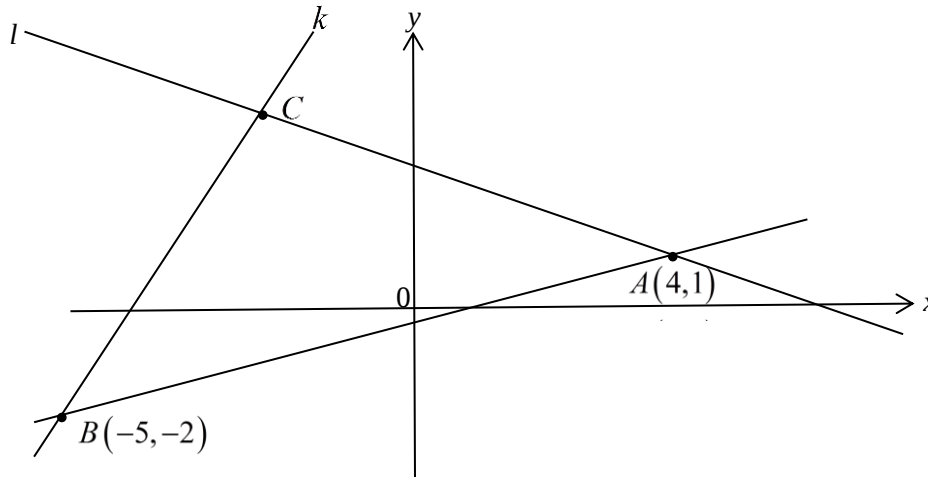
Question 12 (15 marks)

Marks

- (a) Differentiate $y = x^2 - x$ from first principles.

2

- (b) The diagram shows a triangle ABC . The point $A(4,1)$ lies on the l given by the equation $x + 2y = 6$ and the point $B(-5,-2)$ lies on the line k , given by the equation $y = 2x + 8$.



- (i) Show that the point C , which is the point of intersection of l and k has coordinates $(-2, 4)$. 1
- (ii) Find the gradient of the line joining A and B 1
- (iii) Show the equation of the line AB is $x - 3y - 1 = 0$ 1
- (iv) Find the perpendicular distance from the point C to line AB . 1
- (v) Hence, or otherwise, find the area of the triangle ABC . 2
- (c) Show that $\frac{2\cos^3 \theta - \cos \theta}{\sin \theta \cos^2 \theta - \sin^3 \theta} = \cot \theta$ 3
- (d) On the curve $y = \ln x + \frac{1}{x}$:
- (i) Show $\frac{dy}{dx} = \frac{x-1}{x^2}$ 1
- (ii) Find its stationary point and determine its nature. 1
- (iii) Find its point of inflexion. 2

Question 13 (15 marks)

Marks

(a) Differentiate with respect to x

(i) $e^{3x} \tan x$ 2

(ii) $\frac{\sin x}{5-x}$ 2

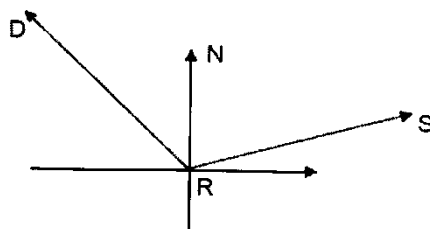
(b) Evaluate:

(i) $\int \frac{1}{1-2x} dx$ 2

(ii) $\int_0^{\pi} \sec^2 \frac{x}{3} dx$ 2

(c) Find the equation of the locus of the point $P(x, y)$ such that the distance from P to the point $A(2, 3)$ is twice the distance from P to the point $B(-1, 4)$. Write your answer in simplest form. 3

(d) Danny and Stella are pilots of two light planes which leave Rodeo Station at the same time. Danny flies on a bearing of 330°T at a speed of 180km/h and Stella flies on a bearing of 080°T at a speed of 240km/h . Copy the diagram into your answer page and mark the information on the diagram.

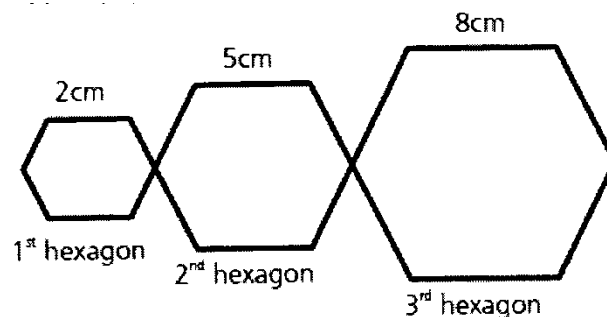


(i) How far apart are Danny and Stella after 2 hours? 2

(ii) What is the bearing of Stella from Danny after 2 hours (to the nearest degree)? 2

Question 14 (15 marks)**Marks**

- (a) Consider the curves $y = x^2$ and $y = 4x + 5$
- (i) Find any points of intersection. **1**
- (ii) Sketch the graphs of the two equations on the same set of axes. **2**
- (iii) Find the area of the region enclosed by these two equations. **2**
- (b) Find the value of m for which the equation $(m-1)x^2 + 3x - 3 = 0$ has one root twice the other. **3**
- (c) Show that $3x^2 + 4x + 5$ is positive definite. **2**
- (d) Melanie is using wire to construct a geometrical design which consists of n regular hexagons with sides 2cm , 5cm , 8cm and so on going up by the same amount each hexagon. The diagram below shows the first 3 hexagons of her design.



- (i) Find the perimeter of the n th hexagon. **1**
- (ii) Show that the total length of the wire is $L = 9n^2 + 3n$. **2**
- (iii) If the total length of the wire is 6 metres, find the number of hexagons that Melanie has constructed. **2**

Question 15 (15 marks)

Marks

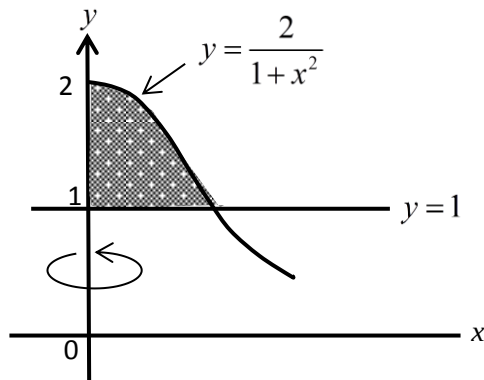
- (a) A cone has a radius of 2cm and slant height of 12cm . The curved part of the cone unwrapped to form a sector.

(i) What is the exact size of the angle subtended in this sector in radians? 2

(ii) Calculate the area of the segment formed from this angle. 1

- (b) The normal of the parabola $x^2 = 16y$ at $(-4, 1)$ cuts the parabola again at Q . Find the coordinates of Q . 3

- (c) The shaded region on the graph below is rotated about the y axis.



- (i) Show that the volume of the solid formed is given by 2

$$V = \pi \int_1^2 \left(\frac{2}{y} - 1 \right) dy$$

- (ii) Hence find its volume. 1

- (d) A cylindrical container closed at both ends is made from a thin sheet of plastic. The surface area of the cylinder is $600\pi \text{ cm}^2$.

- (i) Show the height of the cylinder is given by $h = \frac{300}{r} - r$, where r is the radius. 2

- (ii) Find an expression for the volume V in terms of r . 1

- (iii) Find the height of the container if the volume is to be a maximum. 3

Question 16 (15 marks)**Marks**

- (a) Solve the following equation for x :

2

$$e^{2x} + 3e^x - 10 = 0$$

- (b) Find the exact value of the gradient of the tangent to the curve $y = x \cos x$ at the point where $x = \frac{\pi}{3}$

2

- (c) Michael invest \$50 000 in an account which earns 8% interest, compounded annually. He intends to withdraw \$ M at the end of each year, immediately after the interest has been paid. He wishes to be able to do this for exactly 20 years, so that the account will then be empty.

- (i) How much money does he have in the account immediately after he has made his first withdrawal?

1

- (ii) Show that the amount of money in the account immediately after his 20th

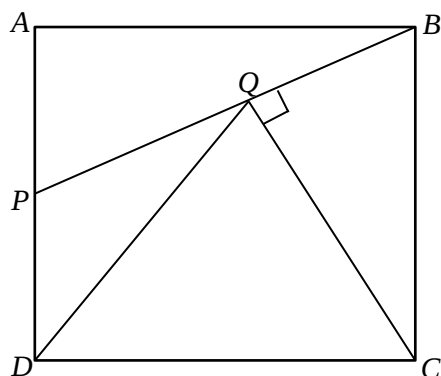
3

withdrawal is $A_{20} = 50000(1.08)^{20} - M \left[\frac{1.08^{20} - 1}{0.08} \right]$

- (iii) Calculate the value of M which leaves his account empty after the 20th withdrawal.

2

- (d)



NOT TO SCALE

$ABCD$ is a square of side length 2 units. P is a midpoint of AD . CQ is drawn perpendicular to PB and $\angle APB = x^\circ$.

- (i) Prove that $\angle APB = \angle QBC$

1

- (ii) Hence, or otherwise, show that $QC = \frac{4}{\sqrt{5}}$ units

2

- (iii) Show that $QD = CD$

2

End of Examination

Year 12 Mathematics 2017

Trial HSC Solutions

Multiple Choice

- 1, B
- 2, B
- 3, C
- 4, A
- 5, A
- 6, D
- 7, A
- 8, D
- 9, A
- 10, B

Question 11

a, $(2x-1)(x-3)$

b, $\frac{5}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{5\sqrt{3}+5}{2}$

c, $|3x-1| < 2$

$$3x-1 < 2$$

$$3x < 3$$

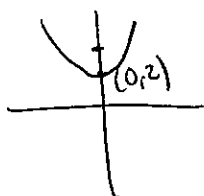
$$1-3x < 2$$

$$-3x < 1$$

$$x > -\frac{1}{3}$$

$$-\frac{1}{3} < x < 1$$

d,



$$x^2 = 4a(y-k)$$

Vertex (0, 2)

$$a=4$$

Focus (0, 6)

e, $\lim_{x \rightarrow 3} \frac{(x+5)(x-3)}{x-3} = 8$

f, $A = \frac{1}{2} r^2 \theta$

$$90 = \frac{1}{2} \times 6^2 \times \theta$$

$$\theta = \frac{25}{9}$$

$$l = r\theta$$

$$= 6 \times \frac{25}{9}$$

$$= \frac{50}{3} \text{ cm}$$

g, $n = \frac{360}{8} = 45$

$$S = 43 \times 180$$

$$= 7740^\circ$$

h, $4 \cos^2 \theta = 3$

$$\cos^2 \theta = \frac{3}{4}$$

$$\cos \theta = \pm \frac{\sqrt{3}}{2}$$

$$0 \leq \theta \leq 360$$

$$\theta = 30^\circ, 150^\circ, 210^\circ, 330^\circ$$

$$0 \leq \theta \leq 2\pi$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

Question 12

a, $f(x) = x^2 - x$

$$f(x+h) = (x+h)^2 - (x+h)$$
$$= x^2 + 2xh + h^2 - x - h$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - x - h - (x^2 - x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2xh + h^2 - h}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h(2x + h - 1)}{h}$$

$$= 2x - 1$$

b, (i) $x + 2y = 6$ (1)
 $y = 2x + 8$ (2)

Sub C (-2, 4) into (1)
 $-2 + 8 = 6$ True

Sub C into (2)
 $4 = 2(-2) + 8$ True

So C is POI

(iii) $m = \frac{1+2}{4+5} = \frac{1}{3}$

(iii) $m = \frac{1}{3}$ pt (4, 1)

$$y - 1 = \frac{1}{3}(x - 4)$$

$$3y - 3 = x - 4$$

$$x - 3y - 1 = 0$$

(iv) (-2, 4) $x - 3y - 1 = 0$

$$d = \frac{|-2 \times 1 - 3 \times 4 - 1|}{\sqrt{1+9}}$$

$$= \frac{15}{\sqrt{10}} \text{ units}$$

(v) $AB = \sqrt{9^2 + 3^2}$

$$= \sqrt{90}$$

$$= 3\sqrt{10}$$

$$A = \frac{1}{2}bh$$

$$= \frac{1}{2} \times \frac{15}{\sqrt{10}} \times 3\sqrt{10}$$

$$= \frac{45}{2} \text{ units}^2$$

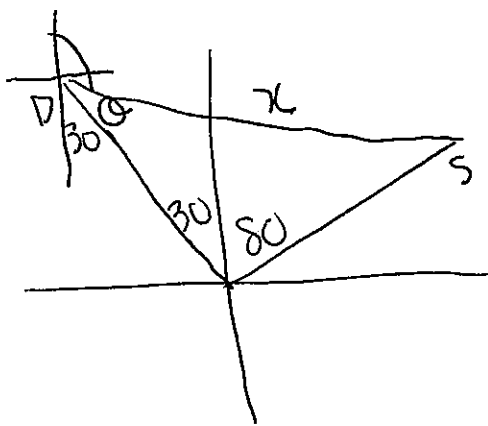
C, LHS = $\frac{\cos \theta (2\cos^2 \theta - 1)}{\sin \theta (\cos^2 \theta - \sin^2 \theta)}$

$$= \frac{\cos \theta (2\cos^2 \theta - 1)}{\sin \theta (\cos^2 \theta - (1 - \cos^2 \theta))}$$

$$= \frac{\cos \theta (2\cos^2 \theta - 1)}{\sin \theta (2\cos^2 \theta - 1)}$$

$$= \cot \theta$$

d,



(1) after 2 hrs

$$D = 360$$

$$S = 480$$

$$x^2 = 360^2 + 480^2 - 2 \times 360 \times 480 \cos 110^\circ$$

$$x = 691.5 \text{ km}$$

$$(1') \cos \theta = \frac{360^2 + 691.5^2 - 480^2}{2 \times 360 \times 691.5}$$

$$\theta = 40^\circ 43'$$

$$\text{Bearing} = 180 - 40^\circ 43' - 30^\circ = 109^\circ$$

Question 13

a, (1) $y = e^{3x} \tan x$

$$u = e^{3x}$$

$$v = \tan x$$

$$u' = 3e^{3x}$$

$$v' = \sec^2 x$$

$$y' = e^{3x} \sec^2 x + 3e^{3x} \tan x$$

(11) $y = \frac{\sin x}{5-x}$

$$u = \sin x$$

$$v = 5-x$$

$$u' = \cos x$$

$$v' = -1$$

$$y' = \frac{(5-x) \cos x - \sin x}{(5-x)^2}$$

b, (1) $\int \frac{1}{1-2x} dx$

$$= -\frac{1}{2} \int \frac{-2}{1-2x} dx$$

$$= -\frac{1}{2} \ln(1-2x) + C$$

(11) $\int_0^\pi \sec^2 \frac{x}{3} dx$

$$= \left[3 \tan \frac{x}{3} \right]_0^\pi$$

$$= 3 \tan \frac{\pi}{3} - 3 \tan 0 = 3\sqrt{3}$$

c, $PA = 2PB$

$$(x-2)^2 + (y-3)^2 = 4(x+1)^2 + 4(y-4)^2$$

$$x^2 - 4x + 4 + y^2 - 6y + 9$$

$$= 4x^2 + 8x + 4 + 4y^2 - 32y + 64$$

$$3x^2 + 12x + 3y^2 - 26y + 55 = 0$$

d, $y = \ln x + x^{-1}$

(1) $y' = \frac{1}{x} - x^{-2}$

$$= \frac{1}{x} - \frac{1}{x^2}$$

$$= \frac{x-1}{x^2}$$

(11) stat pt $y' = 0$

$$\frac{x-1}{x^2} = 0$$

$$x=1, y=1$$

$$(1,1)$$

$$y'' = -x^{-2} + 2x^{-3}$$

$$\text{at } x=1$$

$$y'' = -1 + 2$$

$$= 1 \quad \checkmark \quad (1,1) \text{ is min} \quad (3)$$

iii) For inflexion $y'' = 0$
 Δ change of concav

$$-\frac{1}{x^2} + \frac{2}{x^3} = 0$$

$$\frac{2-x}{x^3} = 0$$

$x = 2$ possible

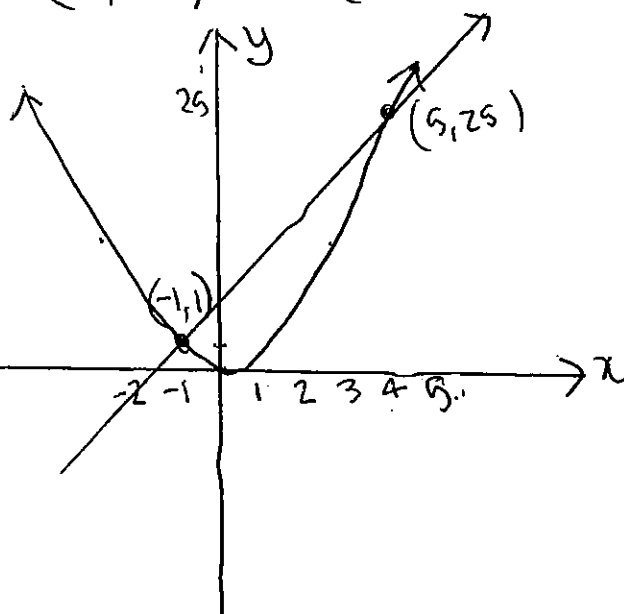
x	1	2	3
y''	1	0	$-\frac{1}{27}$

$\therefore (2, \ln 2 + \frac{1}{2})$
 is POI.

Question 14.

a, $y = x^2$ (1)
 $y = 4x + 5$ (2)

(1) $x^2 = 4x + 5$
 $x^2 - 4x - 5 = 0$
 $(x-5)(x+1) = 0$
 $x = 5$ or $x = -1$
 $y = 25$ or $y = 1$
 $(5, 25)$ Δ $(-1, 1)$



$$A = \int_{-1}^5 4x + 5 - x^2$$

$$= \left[2x^2 + 5x - \frac{x^3}{3} \right]_{-1}^5$$

$$= 36 \text{ units}^2$$

b, $(m-1)x^2 + 3x - 3 = 0$

$$x = 2B$$

$$3B = \frac{-3}{m-1}, \quad 2B^2 = \frac{-3}{m-1}$$

$$2B^2 = 3B$$

$$B(2B-3) = 0$$

$$B = \frac{3}{2}$$

$$\therefore \frac{a}{2} = \frac{-3}{m-1}$$

$$m-1 = \frac{-2}{3}$$

$$m = \frac{1}{3}$$

c, $3x^2 + 4x + 5$
 $a > 0 \quad \Delta < 0$
 for pos def

$$a = 3$$

$$\therefore > 0$$

$$\Delta = 16 - 60$$

$$< 0$$

$\therefore$ pos def

$$d, \quad T_n = a + (n-1)d$$

$$= 2 + (n-1)3$$

$$= 3n-1$$

$$P = 6 \times T_n$$

$$= 6(3n-1)$$

$$= 18n-6$$

$$(ii) \quad L = 6 \times S_n$$

$$= 6 \times \frac{n}{2} (4 + (n-1)3)$$

$$= 3n(4+3n-3)$$

$$= 3n+9n^2$$

$$(iii) \quad 9n^2 + 3n - 600 = 0$$

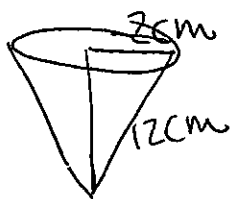
$$3n^2 + n - 200 = 0$$

$$(3n+25)(n-8) = 0$$

$$n=8$$

Question 15

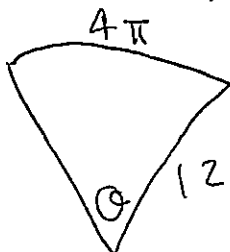
(a) (i)



$$C = 2\pi r$$

$$r = 2$$

$$= 4\pi \text{ cm}$$



$$L = r\theta$$

$$4\pi = 12\theta$$

$$\theta = \frac{\pi}{3}$$

$$(ii) \quad A = \frac{1}{2} r^2 (\theta - \sin \theta)$$

$$= \frac{1}{2} \times 12^2 \left(\frac{\pi}{3} - \sin \frac{\pi}{3} \right)$$

$$= \frac{72\pi}{3} - \frac{72\sqrt{3}}{2}$$

$$= 24\pi - 36\sqrt{3} \text{ units}^2$$

$$b, \quad x^2 = 16y$$

$$y = \frac{x^2}{16}$$

$$y' = \frac{2x}{16}$$

$$= \frac{x}{8}$$

$$\text{at } x = -4$$

$$y' = -\frac{1}{2}$$

$$\text{normal } m = 2 \quad (-4, 1)$$

$$y-1 = 2(x+4)$$

$$y-1 = 2x+8$$

$$y = 2x+9$$

$$\text{Solve } x^2 = 16y \quad (1)$$

$$y = 2x+9 \quad (2)$$

$$\text{Sub (2) into (1)}$$

$$x^2 = 16(2x+9)$$

$$x^2 = 32x+144$$

$$x^2 - 32x - 144 = 0$$

$$(x+4)(x-36) = 0$$

$$x = 36$$

$$y = (2 \times 36 + 9)$$

$$(36, 81)$$

$$C_1(1) \quad y = \frac{2}{1+x^2}$$

$$y + yx^2 = 2$$

$$yx^2 = 2 - y$$

$$x^2 = \frac{2-y}{y}$$

$$= \frac{2}{y} - 1$$

$$V = \pi \int x^2 dy$$

$$= \pi \int_1^2 \left(\frac{2}{y} - 1 \right) dy$$

$$\begin{aligned} (11) \quad V &= \pi \left[2 \ln y - y \right]_1^2 \\ &= \pi (2 \ln 2 - 2 - 2 \ln 1 + 1) \\ &= \pi (2 \ln 2 - 1) \text{ units}^3 \end{aligned}$$

$$d, (1) \quad SA = 2\pi r^2 + 2\pi r h$$

$$600\pi = 2\pi r^2 + 2\pi r h$$

$$600\pi - 2\pi r^2 = 2\pi r h$$

$$h = \frac{600\pi - 2\pi r^2}{2\pi r}$$

$$= \frac{2\pi(300 - r^2)}{2\pi r}$$

$$= \frac{300}{r} - r$$

$$\begin{aligned} (11) \quad V &= \pi r^2 h \\ &= \pi r^2 \left(\frac{300}{r} - r \right) \\ &= 300\pi r - \pi r^3 \end{aligned}$$

$$(111) \quad \frac{dV}{dr} = 300\pi - 3\pi r^2$$

$$\max \frac{dV}{dr} = 0$$

$$3\pi r^2 = 300\pi$$

$$r^2 = 100$$

$$r = \pm 10$$

but r is positive

$$\therefore r = 10$$

$$\frac{d^2V}{dr^2} = -6\pi r$$

$$\text{at } r = 10$$

$$\frac{d^2V}{dr^2} = -60\pi$$

$$< 0$$

$\therefore$ maximum $\wedge$

$$h = \frac{300}{10} - 10$$

$$= 20 \text{ cm}$$

Question 16

$$a, \quad e^{2x} + 3e^x - 10 = 0$$

$$\text{let } e^x = u$$

$$u^2 + 3u - 10 = 0$$

$$(u+5)(u-2) = 0$$

$$u = -5 \text{ or } 2$$

$$e^x = -5$$

no soln

$$e^x = 2$$

$$x = \ln 2$$

(b)

$$b, y = x \cos x$$

$$u = x \quad v = \cos x$$

$$u' = 1 \quad v' = -\sin x$$

$$y' = -x \sin x + \cos x$$

$$\text{at } x = \frac{\pi}{3}$$

$$m = -\frac{\pi}{3} \times \frac{\sqrt{3}}{2} + \frac{1}{2}$$

$$= -\frac{\sqrt{3}\pi}{6} + \frac{1}{2}$$

$$= \frac{3 - \sqrt{3}\pi}{6}$$

$$C_1(i) A_1 = (50000 \times 1.08) - m$$

$$= 54000 - m$$

$$(ii) A_2 = A_1 \times 1.08 - m$$

$$= 50000 \times 1.08^2 - 1.08m - m$$

$$A_3 = A_2 \times 1.08 - m$$

$$= 50000 \times 1.08^3 - 1.08^2 m$$

$$- 1.08m - m$$

$$= 50000 \times 1.08^3 - m(1 + 1.08 + 1.08^2)$$

$$A_{20} = 50000 \times 1.08^{20} - m(1 + 1.08 + \dots + 1.08^{19})$$

$$\uparrow$$

$$C.P \quad u=1$$

$$r=1.08$$

$$= 50000 \times 1.08^{20} - m \left(\frac{1(1.08^{20} - 1)}{1.08 - 1} \right)$$

$$= 50000(1.08)^{20} - m \left(\frac{1.08^{20} - 1}{0.08} \right)$$

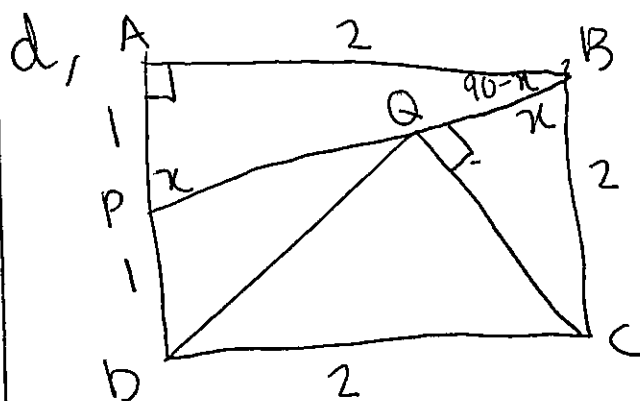
(iii)

$$m \left(\frac{1.08^{20} - 1}{0.08} \right) = 50000(1.08)^{20}$$

$$45.76196m = 50000(1.08)^{20}$$

$$m = \frac{50000(1.08)^{20}}{45.76196}$$

$$= \$5092.61$$

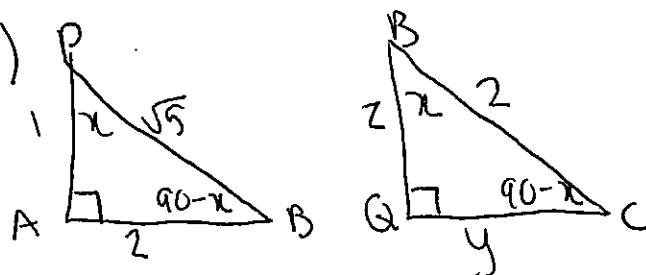


$$(1) \angle PAB = 90^\circ \text{ (angles of a square)}$$

$$\angle ABQ = 90 - x \text{ (angle sum of triangle)}$$

$$\angle QBC = x \text{ (angles of a square)}$$

$$= \angle APB$$



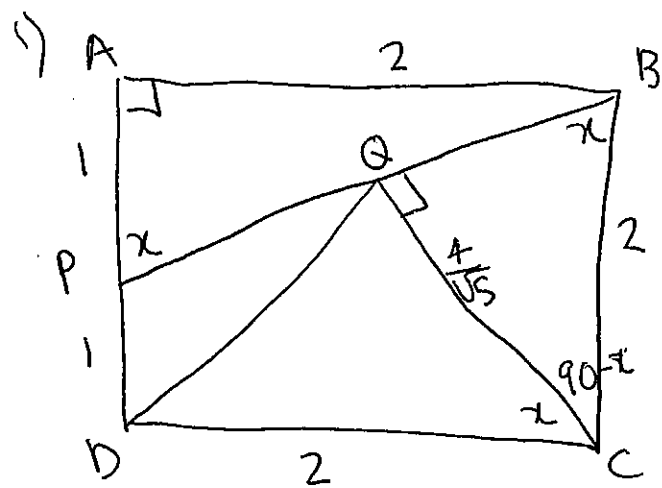
$$\angle PAB = \angle BQC \text{ (angles of a square)}$$

$$\angle APB = \angle QBC \text{ (from (1))}$$

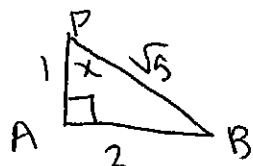
$$\therefore \triangle APB \parallel \triangle BQC \text{ (equiangular)}$$

$$\therefore \frac{QC}{2} = \frac{2}{\sqrt{5}}$$

$$\therefore QC = \frac{4}{\sqrt{5}} \text{ units} \quad (7)$$



from $\triangle APB$



$$\cos x = \frac{1}{\sqrt{5}}$$

$$QC = \frac{4}{\sqrt{5}}, \angle QCD = x$$

$$\begin{aligned} QD^2 &= DC^2 + QC^2 - 2 \times DC \times QC \times \cos x \\ &= 2^2 + \left(\frac{4}{\sqrt{5}}\right)^2 - 2 \times 2 \times \frac{4}{\sqrt{5}} \times \frac{1}{\sqrt{5}} \end{aligned}$$

$$= 4 + \frac{16}{5} - \frac{16}{\sqrt{5}} \times \frac{1}{\sqrt{5}}$$

$$= 4 + \frac{16}{5} - \frac{16}{5}$$

$$= 4$$

$$QD = \sqrt{4}$$

$$= 2 \text{ units}$$

$$= CD$$