KIRRAWEE HIGH

MEASURE BY ACHIEVEMENT

Student Name ....

Teacher Name: ....

2018 | **HIGHER SCHOOL CERTIFICATE TRIAL EXAMINATION**

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided at the back of this paper
- All necessary working should be shown in Questions 11-16
- Write your student name at the top of the relevant pages

Total marks: 100

Section I – 10 marks (pages 2 – 3)

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 4 – 9)

- Attempt Questions 11-16
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for questions 1-10

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1 What is the value of $9.8\times10^{7}-2.3\times10^{4}$?

A. 7.5×10^{3}
B. 7.5×10^{4}
C. 9.7977×10^{6}
D. 9.7977×10^{7}

2 What is the value of $\sum_{k=1}^{4}(-1)^{k}k^{2}$?

A. -30
B. -10
C. 10
D. 30

3 Which of the following quadratic expressions is positive definite?

A. x^2+5x+2
B. x^2+5x+4
C. x^2+5x+6
D. x^2+5x+8

4 Which of the following trigonometric expressions is equivalent to $\tan\left(\frac{\pi}{2}-x\right)$?

A. $\tan x$
B. $\cot x$
C. $-\tan x$
D. $-\cot x$

5 What is the range of the function $f(x)=\sqrt{1-x^2}$?

A. $0<y<1$
B. $0\le y\le 1$
C. $-1<y<1$
D. $-1\le y\le 1$

6 What is the value of x if $\log x^2-\log 3x=\log 9$?

A. 3
B. 18
C. 27
D. 81

7 What is the value of $\int_{-2}^{2}|x|\,dx$?

A. 0
B. 2
C. 4
D. 8

8 What are the amplitude and period of the function $f(x)=2-\sin 2x$?

A. Amplitude 1, period π
B. Amplitude 1, period 2π
C. Amplitude 2, period π
D. Amplitude 2, period 2π

9 Which of the following is an expression for $\dfrac{d}{dx}\left(e^{2x}\tan x\right)$?

A. $(1+\tan x)^2 e^{2x}$
B. $2\left(1+\tan^2 x\right)e^{2x}$
C. $2e^{2x}\tan x$
D. $e^{2x}\sec^2 x$

10 What is the number of real roots of the equation $x(x-2)\log_e x=0$?

A. 0
B. 1
C. 2
D. 3

Section II

90 marks
Attempt Questions 11-16
Allow about 2 hours and 45 minutes for this section

Answer the questions in writing booklets provided. Use a separate writing booklet for each question. In Questions 11-16 your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a separate writing booklet. **Marks**

(a) Find in simplest exact form the value of $\left(3-2\sqrt{2}\right)^2$. **2**

(b) Solve the quadratic equation $2x^2-5x-3=0$. **2**

(c) Differentiate with respect to x

(i) $\sin(1-x)$. **1**

(ii) $x\log_e x$. **1**

(d) Find the equation of the tangent to the curve $y=\dfrac{x}{x-1}$ at the point $(2,2)$ on the curve. **3**

(e) Evaluate in simplest exact form $\int_1^2 \dfrac{x+1}{x}\,dx$. **3**

(f) The region bounded by the curve $y=\dfrac{1}{2x+1}$ and the x axis between $x=0$ and $x=1$ is rotated about the x axis to form a solid. Find the volume of the solid of revolution in simplest exact form. **3**

Question 12 (15 marks) **Use a separate writing booklet** **Marks**

(a) Find $\int \sec x\left(\cos x + \sec x\right)dx$. **2**

(b) Find the coordinates of the point of inflexion on the curve $y = x^3 - 3x^2 + 6x$. **3**

(c) A parabola has equation $8y = x^2 - 6x + 1$.

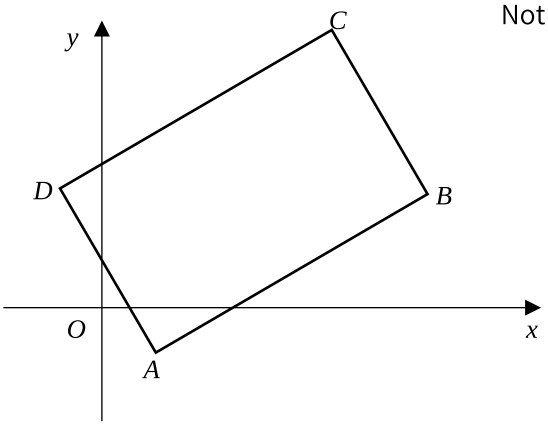
(i) Write the equation in the form $\left(x-h\right)^2 = 4a\left(y-k\right)$. **1**

(ii) Find the vertex of the parabola. **1**

(iii) Sketch the parabola showing vertex, directrix and focal point (label correctly and use a ruler). **3**

(d) In the diagram below, *ABCD* is a parallelogram. Side *AB* has equation $x - 2y - 3 = 0$, side *BC* has equation $2x + y - 16 = 0$ and vertex *D* has coordinates $\left(-1, 3\right)$.

Not to scale



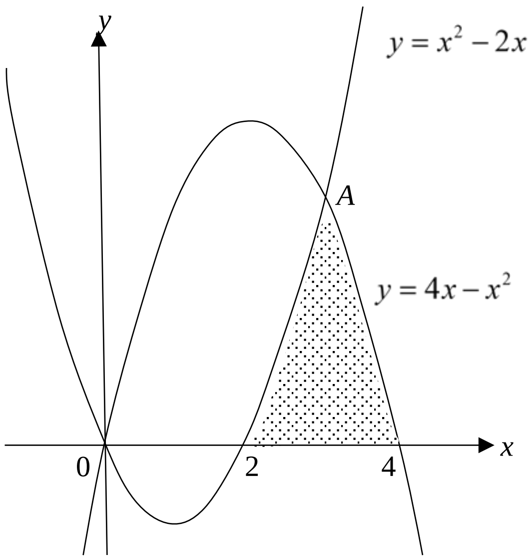
(i) Show that *ABCD* is a rectangle. **2**

(ii) Find the coordinates of the vertex *C*. **3**

Question 13 (15 marks) **Use a separate writing booklet** **Marks**

(a) The gradient function of a curve $y = f(x)$ is given by $f'(x) = 4x^3 + x^2 + 7$. **3**
The curve passes through the point $(4,5)$. Find the equation of the curve.

(b) The diagram below shows the parabolas $y = 4x - x^2$ and $y = x^2 - 2x$. The graphs intersect at the origin *O* and the point *A*.



(i) Find the *x* coordinate of the point *A*. **1**

(ii) Find the area of the shaded region bounded by the two parabolas and the *x* axis. **3**

(c) The population, *P*, of Booby Birds on Christmas island is decreasing at a rate proportional to *P*.

That is $\frac{dP}{dt} = -kP$, where *k* is a positive integer and *t* is measured in years and $P = Ae^{kt}$.

In January 2000 there were 3000 Booby Birds and by January 2010 the population had decreased to 2750.

(i) Show that the value of *k* is 0.0087, correct to 4 decimal places. **2**

(ii) If the population continues to decrease at this rate what will be the expected population in 2020? **2**

(d) Use Simpson's Rule with 5 function values to approximate $\int_1^9 (\log_e x)^2 \, dx$, **4**
giving your answer correct to 2 significant figures.

Question 14 (15 marks) **Use a separate writing booklet** **Marks**

(a) Find the values of x for which the function $y = x - x^2$ is decreasing. **2**

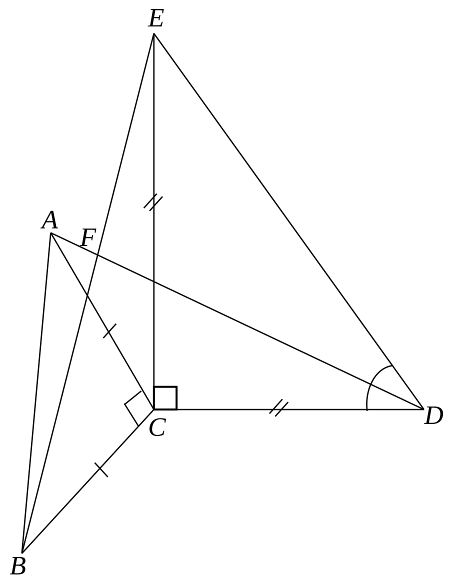
(b) The 8th term of an Arithmetic Progression is 23 and the 11th term is four times the 3rd term. Find the first term and the common difference. **3**

(c) After time t years the value $\$V$ of a car is given by $V = 24000e^{-0.1t}$.

(i) Find the decrease in the value of the car during the fourth year. **2**

(ii) Find the percentage decrease in the value of the car during the fourth year (to 2 significant figures). **2**

(d) In ΔABC, $AC = BC$ and $\angle BCA = 90°$. In ΔCDE, $DC = EC$ and $\angle ECD = 90°$. DA and BE intersect at F.



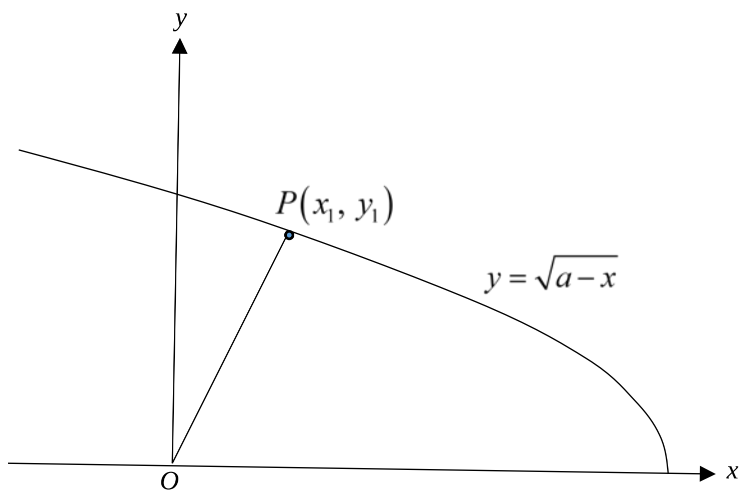
(i) Prove that $\Delta BCE \equiv \Delta ACD$. **3**

(ii) Show that $DA \perp BE$. **3**

Question 15 (15 marks) **Use a separate writing booklet** **Marks**

(a) Find in simplest exact form the value of $\int_0^{\log_e 3} e^{2x}\,dx$. **2**

(b) The diagram shows the graph of the curve $y = \sqrt{a-x}$ where $a > 0$. The normal to the curve at the point $P\left(x_1,\ y_1\right)$ passes through the origin $O(0,0)$. By considering the gradient of *OP* in two different ways, find the value of x_1. **3**



(c) (i) Solve the equation $\sin x = \cos x$ for $0 \le x \le 2\pi$. **2**

(ii) One the same diagram, sketch the graphs of the curves $y = \cos x$ and $y = \sin x$ for $0 \le x \le 2\pi$, showing clearly the intercepts on the coordinate axes. **2**

(iii) Find in simplest exact form the area of the region enclosed by the curves $y = \sin x$ and $y = \cos x$ for $0 \le x \le \dfrac{\pi}{4}$. **3**

(d) Solve the equation $2\log_2 x - \log_2\left(x+4\right) = 1$. **3**

Question 16 (15 marks) **Use a separate writing booklet** **Marks**

(a) Ozzie takes out a loan of \$400 000. The loan is to be repaid in equal monthly repayments of \$3600. Reducible interest is charged at 6% p.a. calculated monthly. Let $\$A_n$ be the amount owing after the n^{th} repayment.

(i) Show that $A_3 = 400\ 000(1.005)^3 - 3600(1 + 1.005 + 1.005^2)$. **2**

(ii) Find correct to the nearest month the time taken for Ozzie to repay the loan. **3**

(b) A cylindrical container closed at both ends is to be made from thin sheet metal. The container is to have a radius of *r* cm and a height of *h* cm such that its volume is $2000\pi\ cm^3$.

(Such a cylinder closed at both ends has surface area *S* and volume *V* given by the formulae $S = 2\pi r^2 + 2\pi rh$ and $V = \pi r^3 h$.)

(i) Show that the area of sheet metal required to make the container is $\left(2\pi r^2 + \dfrac{4000\pi}{r}\right)cm^2$. **2**

(ii) Hence find the minimum area of sheet metal required to make the container (leave answer in terms of π). **4**

(c) Given that the limiting sums S_1 and S_2 of the series both exist, where

$S_1 = 1 + \sin^2 x + \sin^4 x + \sin^6 x + ...$

$S_2 = 1 + \cos^2 x + \cos^4 x + \cos^6 x + ...$

(i) Show that $S_1 = \sec^2 x$ and $S_2 = \operatorname{cosec}^2 x$. **2**

(ii) Show that $S_1 + S_2 = S_1 S_2$. **2**

End of paper

2018 HSC Trial <u>Mathematics</u>.

MC.

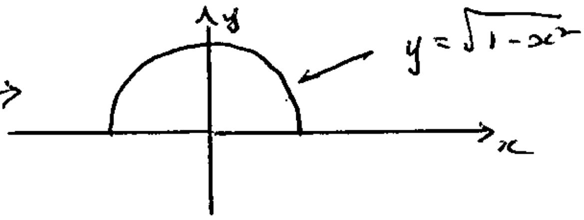
① D

② C → $-1 + (-1)^2(2)^2 + (-1)^3(3)^2 + (-1)^4(4)^2 =$
$-1 + 4 - 9 + 16$ $= 10$

③ D → for +ve definite $b^2 - 4ac < 0$
∴ D

④ B

⑤ B →



⑥ C

$\log x^2 - \log 3x = \log 9$

$\log \frac{x^2}{3x} = \log 9$

$x^2 = 27x$

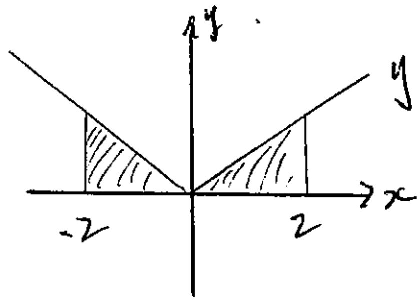
$x^2 - 27x = 0$

$x(x-27) = 0$

$\therefore x = 0$ or 27

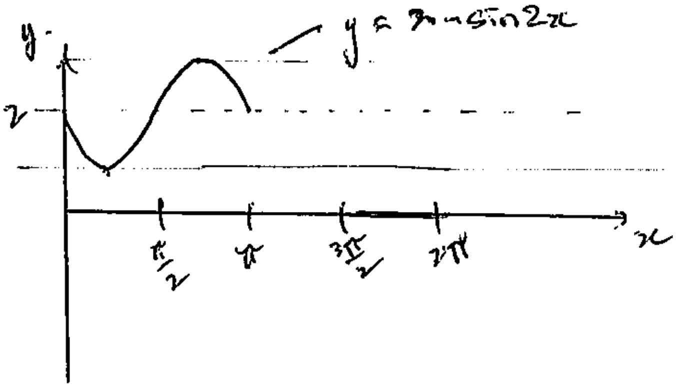
$x \neq 0 \therefore \underline{x = 27}$

⑦ C



$\therefore \int_{-2}^{2} |x|\,dx = 2\int_{2}^{0} x\,dx$
$= 4$

(8) A



(9) A.

$$\frac{d}{dx}\left(\overset{u}{e^{2x}}\ \overset{v}{\tan x}\right)$$

$u = e^{2x}$

$u' = 2e^{2x}$

$v = \tan x$

$v' = \sec^2 x$

$e^{2x}\sec^2 x + \tan x\, 2e^{2x}$

$e^{2x}(\sec^2 x + 2\tan x)$

from $\sin^2 x + \cos^2 x = 1$

then $\tan^2 x + 1 = \sec^2 x$

subs for $\underline{\sec^2 x}$

$e^{2x}(\tan^2 x + 2\tan x + 1)$

(perfect square)

$\underline{e^{2x}(\tan x + 1)^2 = A.}$

(10) C

$x(x-2)\log_e x = 0 \quad \therefore \quad \left.\begin{array}{l} x = 0 \\ x - 2 = 0 \\ \log_e x = 0 \end{array}\right\} \quad \begin{array}{l} x = 0 \\ x = 2 \\ \therefore x = 1 \\ x \neq 0 \end{array}$

$\therefore$ 2 roots

Question 11

(a) $(3-2\sqrt{2})^2 = 3^2 - 2(3)(2\sqrt{2}) + (2\sqrt{2})^2$

$= 9 - 12\sqrt{2} + 8$

$= \boxed{17 - 12\sqrt{2}}$ ✓ 2

(b) $2x^2 - 5x - 3 = 0$

$\left.\begin{matrix} \times & -6 \\ + & -5 \end{matrix}\right\}$ PSF

$\therefore \dfrac{(2x-6)(2x+1)}{2} = 0$ (or similar techniques)

$(x-3)(2x+1) = 0$

$\boxed{\therefore x = 3 \text{ or } -\tfrac{1}{2}}$ ✓ 2

(c) (i) $\dfrac{d(\sin(1-x))}{dx} = \cos(1-x)(-1)$

$\boxed{= -\cos(1-x)}$ ✓ 1

(ii) $\dfrac{d\, x \log_e x}{dx}$ let $u = x$

$u' = 1$

$v = \log_e x$

$v' = \dfrac{1}{x}$

$\dfrac{x}{x} + \log_e x\,(1)$

$\boxed{= 1 + \log_e x}$ ✓ 1

Q11 (cont'd)

(d) $y = \dfrac{x}{x-1}$ $\quad \begin{matrix} u \\ v \end{matrix}$ $\quad \begin{aligned} u &= x \\ u' &= 1 \\ v &= x-1 \\ v' &= 1 \end{aligned}$

then $y' = \dfrac{(x-1)(1) - (x)(1)}{(x-1)^2}$

$= \dfrac{\cancel{x} - 1 - \cancel{x}}{(x-1)^2}$

$y' = \dfrac{-1}{(x-1)^2}$

$f'(2) = \dfrac{-1}{1^2} = \underline{-1}$ $\therefore \underline{m = -1}$

$y = mx + b$

$b = y - mx$

$= 2 - (-1)(2)$

$= 4$

$\therefore \boxed{y = -x + 4}$

3

(e) $\int_1^2 \frac{x+1}{x}\, dx = \int_1^2 \frac{x}{x} + \frac{1}{x}\, dx$

$= \int_1^2 1 + \frac{1}{x}\, dx$

$= \int_1^2 1\, dx + \int \frac{1}{x}\, dx$

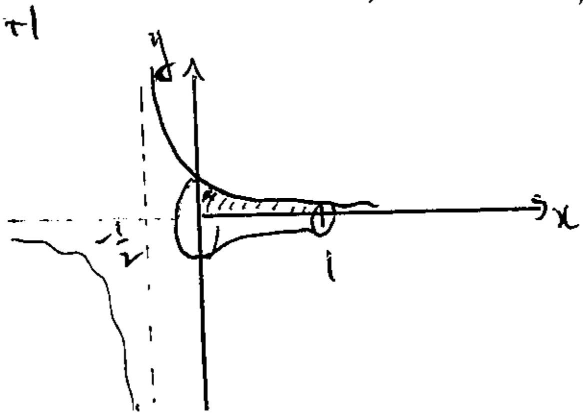
$= [x]_1^2 + [\ln x]_1^2$

$= (2-1) + \ln 2 - \ln 1$

$\boxed{= 1 + \ln 2}$

3

(d) $y = \dfrac{1}{2x+1}$ x axis, $x=0$, $x=1$,



$$V = \pi \int_b^a y^2 \, dx$$

$$= \pi \int_0^1 \frac{1}{(2x+1)^2} \, dx$$

$$= \pi \int_0^1 (2x+1)^{-2} \, dx$$

$$= \pi \left[\left(-\tfrac{1}{2}\right)(2x+1)^{-1} \right]_0^1$$

$$= \frac{-\pi}{2} \left[\frac{1}{2x+1} \right]_0^1$$

$$= \frac{-\pi}{2} \left[\frac{1}{3} - \frac{3}{3} \right]$$

$$= \frac{-\pi}{2} \left(\frac{-2}{3} \right)$$

$$\boxed{= \frac{\pi}{3} \text{ units}^3}$$

3

Q.12

(a) find $\int \sec x (\cos x + \sec x)\, dx$.

$$= \int \frac{1}{\cos x}\left(\cos x + \frac{1}{\cos x}\right) dx$$

$$= \int \left(1 + \frac{1}{\cos^2 x}\right) dx$$

$$= \int (1 + \sec^2 x)\, dx$$

$$= \boxed{x + \tan x + c}$$

(b) $y = x^3 - 3x^2 + 6x$

$y' = 3x^2 - 6x + 6$

$y'' = 6x - 6$

for point of inflexion $y'' = 0$

$6x - 6 = 0$

$x - 1 = 0$

$\therefore x = 1$

$f(1) = (1)^3 - 3(1)^2 + 6(1)$

$= 1 - 3 + 6$

$= 4$

$\therefore$ pt of inflexion is $(1, 4)$.

12

(c) $8y = x^2 - 6x + 1$

(i) $8y = (x^2 - 6x + 9) - 9 + 1$

$8y + 8 = (x-3)^2$

$8(y+1) = (x-3)^2$

$\boxed{4(2)(y+1) = (x-3)^2}$

$\therefore h = 3$

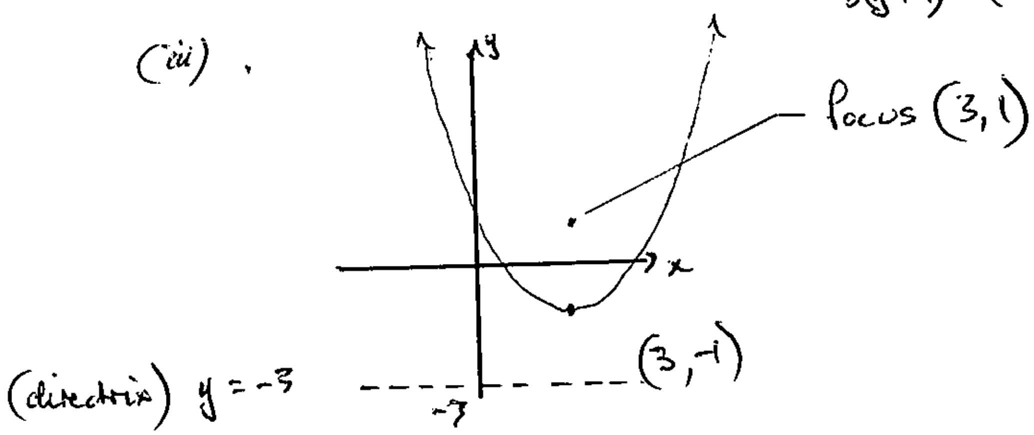
$k = -1$

(ii) $\therefore$ vertex $\boxed{(3, -1)}$

focal length (a) $= \boxed{2 \text{ units}}$

$8(y+1) = (x-3)^2$

(iii)



3

12 (d)

(i). ABCD is parallelogram (given)
then (opposite angles of parallelogram are equal)

eq BC $\quad 2x + y - 16 = 0 \quad$ ~ (1)
$\quad y = -2x + 16 \quad$ ~ (1a)
AB $\quad x - 2y - 3 = 0 \quad$ ~ (2)
$\quad y = \frac{x}{2} - \frac{3}{2} \quad$ (2a)

for $\perp$ lines

$$m_1 m_2 = -1$$

$$m_{AB}\, m_{BC} = \left(\frac{1}{2}\right)(-2)$$
$$= -1$$

(2)

$\therefore$ perpendicular

& opposite angles are equal so

ABCD is a rectangle

(ii) for eqn of DC (passes through (-1, 3)
grad $\frac{1}{2}$, // to AB).

$$\therefore \quad y = mx + b$$
$$b = y - mx$$
$$= 3 - \left(\tfrac{1}{2}\right)(-1)$$
$$b = 3\tfrac{1}{2}.$$
$$y = \tfrac{1}{2}x + \tfrac{7}{2}$$

Q12 d (cont'd)

$\therefore$ for DC intersects BC at C.

$$\therefore \quad y = \frac{1}{2}x + \frac{7}{2} \quad \sim DC$$

$$2y = x + 7$$

$$x - 2y + 7 = 0 \quad ① \; (DC)$$

$\times 2$ $$2x - 4y + 14 = 0 \quad (1a)$$

$$2x + y - 16 = 0 \quad ②$$

1a − 2 $$-5y + 30 = 0$$

$$\underline{y = +6}$$

then subs into ①

$$x - 2(6) + (7) = 0$$

$$x = 5$$

$$\therefore \; C = (5, 6).$$

Q13

$y = f(x)$ ~~&~~ passing through $(4,5)$

$y' = 4x^3 + x^2 + 7$

$\therefore y = \int 4x^3 + x^2 + 7 \; dx$

$y = x^4 + \frac{x^3}{3} + 7x + c$ passing through $(4,5)$

$5 = 4^4 + \frac{4^3}{3} + 7(4) + c$

$5 = 256 + \frac{64}{3} + 28 + c$

$\therefore c = 5 - 256 - \frac{64}{3} - 28$

$c = -300.\dot{3}$

$\therefore \boxed{y = x^4 + \frac{x^3}{3} + 7x - 300.\dot{3}.}$

3

Q13 (b)

$y = 4x - x^2$ ①

$y = x^2 - 2x$ ②

①-② $0 = -2x^2 + 6x$

$\therefore 0 = -2x(x-3)$

$\therefore x = 0$ or 3.

$\boxed{\therefore x = 3}$

(ii) $\text{Area} = \int_2^3 (x^2 - 2x)\,dx + \int_3^4 (4x - x^2)\,dx$

$= \left[\frac{x^3}{3} - x^2\right]_2^3 + \left[2x^2 - \frac{x^3}{3}\right]_3^4$

$= \left[(9-9) - \left(\frac{8}{3} - \frac{12}{3}\right)\right] + \left[2(4)^2 - \left(\frac{4^3}{3}\right)\right] - \left[2(3^2) - 9\right]$

$= \left[0 + \frac{4}{3}\right] + \left[32 - \frac{64}{3}\right] - [9]$

$\boxed{= 3 \text{ units}^2}$

13

(c) (i) $\dfrac{dP}{dt} = -kP$

$P = Ae^{-kt}$

then $\dfrac{P}{A} = e^{-kt}$

$\ln \dfrac{P}{A} = -kt$

$\dfrac{\ln \frac{P}{A}}{t} = -k$

$\dfrac{\ln\left(\frac{3000}{2750}\right)}{10} = \boxed{0.0087}$

2

(ii) $P = Ae^{-kt}$

$P = 3000e^{-0.0087(20)}$

$P = 2520.89$

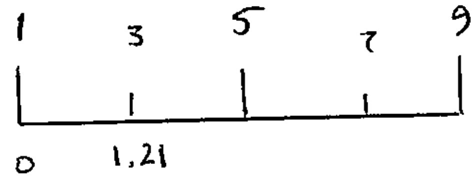
$\boxed{P = 2520}$

2

13 (d)

Simpson's Rule, $A \simeq \frac{h}{3}\left(f_L + 4f_m + f_R\right)$.

5 function values.



$$A \approx \frac{h}{3}\left[1 \times f_{1} + 4\left[f_3 + f_7\right] + 2\left[f_5\right] + f_9\right]$$

$$\approx \frac{2}{3}\left[(1 \times 0) + \right.$$

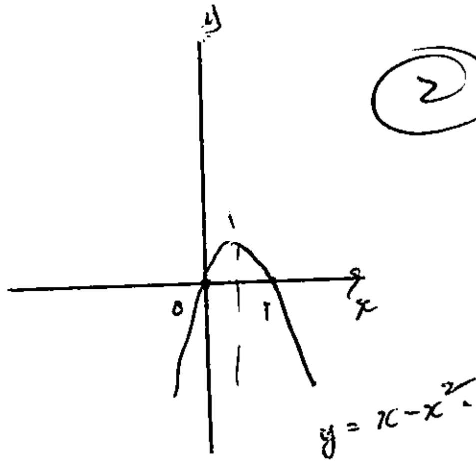
$$A \approx \frac{2}{3}(29.98)$$

$$\boxed{A \approx 19.99 \text{ units}^2}$$ (4 s.f.)

Q 14

(a). for decreasing then $y' < 0$

$$y = x - x^2$$
$$y' = 1 - 2x$$
$$0 > 1 - 2x$$



2

$$2x > 1$$
$$\boxed{x > \tfrac{1}{2}.}$$

(b) $T_n = a + (n-1)d.$

$T_8 = 23$

$T_{11} = 4T_3$. $T_{11} = T_8 + 3d$

$T_8 = T_3 + 5d$

$T_3 = T_8 - 5d$

$\therefore 4T_3 = T_8 + 3d$

$4(T_8 - 5d) = T_8 + 3d$

$4(23) - 20d = T_8 + 3d.$

$92 - 20d = 23 + 3d.$

$23d = 69$ $\therefore \underline{d = 3}$

14(b) (cont'd).

$T_8 = a + n - 1\ d$

$T_8 - (n-1)d = a$

$23 - 7(3) = a = 2$

$\boxed{\therefore\ a = 2 \quad d = 3}$

14(c)(i) $V = 24000e^{-0.1t}$

$\therefore\ V_4 = 24000\,e^{-0.1(4)}$

$V_4 = 16{,}087.68$.

2

$V_3 = 24000\,e^{-0.1(3)}$

$V_3 = 17{,}779.64$

$\therefore$ depreciation in 4^{th} year is. $17{,}779.64 - 16087.68$

$\boxed{\$1691.96}$

% dec (in 4^{th} year) $= \dfrac{1691.96}{17{,}779.64}$

$= 0.0952$

$\boxed{= 9.5\%}$

$\boxed{14\,d}$

$\triangle BCE \equiv \triangle ACD$.

let $x = \angle ECA$

then SAS

eg $/\ ,\ (90+x),\ //$

$\therefore$ congruent.

let F be intersection of $\overline{AD}$ & $\overline{EC}$

then $\angle MDC = a$
& $\angle MEF = a$ } corresponding sides of congruent triangles

$\angle CMD = \angle EMF$ } vertically opp. angles.
$= \underline{90 - a}$

$\boxed{\text{then } \angle EFM = 90^\circ}$ interior angle sum of a triangle

Q15

(a) $\int_0^{\log_e 3} e^{2x}\,dx$

$= \left[\frac{e^{2x}}{2} \right]_0^{\ln 3}$

$= \frac{e^{2\ln 3}}{2} - \frac{e^0}{2}$

$= \frac{e^{\ln 3^2}}{2} - \frac{1}{2}$

$= \frac{9}{2} - \frac{1}{2}$ (2)

$\boxed{= 4}$

(b) $y = \sqrt{a-x} = (a-x)^{\frac{1}{2}}$

$m_{OP} = \frac{\text{rise}}{\text{run}} = \frac{y_1}{x_1} = \frac{\sqrt{a-x}}{x}$ ✓ (1)

$m_{tang} = \quad f'(x)$

$f'(x) = -\frac{1}{2}(a-x)^{-\frac{1}{2}}$ ✓ (2)

$(m_{OP})(m_{tang}) = -1$ (for perpendicular lines)

$\therefore \left(\frac{\cancel{\sqrt{a-x}}}{x}\right) \times \frac{-1}{2\cancel{\sqrt{a-x}}} = -1$

Q15

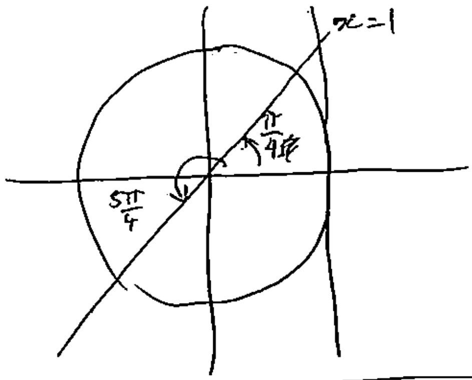
$$-\frac{1}{2x} = -1$$

$$\boxed{x = \frac{1}{2}}$$

(c) (i) $\sin x = \cos x \quad \{x : 0 \le x \le 2\pi\}$

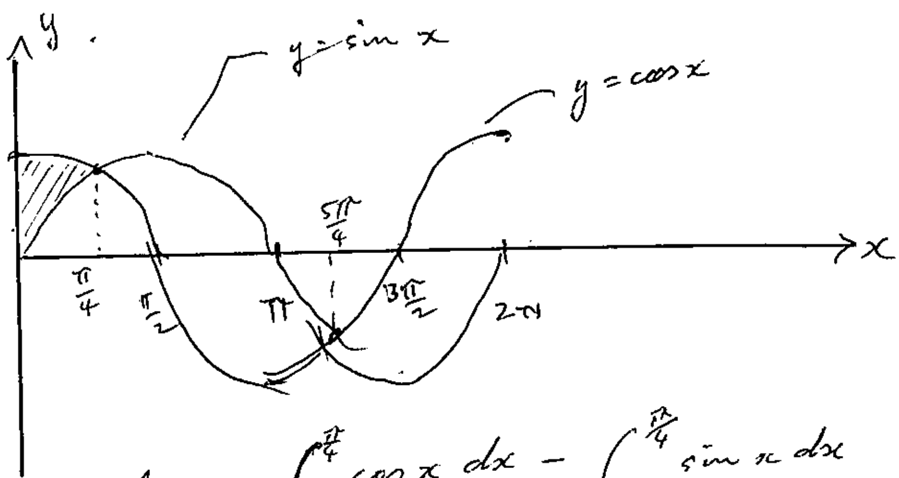
$\div \cos x \qquad \tan x = 1$

Using unit circle (or otherwise)



$$\boxed{\text{hence } x = \frac{\pi}{4} \text{ or } \frac{5\pi}{4}.}$$

(ii)



$$\text{Area} = \int_0^{\frac{\pi}{4}} \cos x \, dx - \int_0^{\frac{\pi}{4}} \sin x \, dx$$

$$= \left[+\sin x\right]_0^{\frac{\pi}{4}} - \left[-\cos x\right]_0^{\frac{\pi}{4}}$$

Q15 (c) cont'd

$$\frac{1}{\sqrt{2}} - 0 - \left[-\frac{1}{\sqrt{2}} - -1\right]$$

$$\frac{2}{\sqrt{2}} - 1$$

$$\boxed{= \sqrt{2} - 1 \quad u^2}$$

3

(d) $2\log_2 x - \log_2(x+4) = 1$

$\log_2 x^2 - \log_2(x+4) = 1$

$= \log_2\left(\frac{x^2}{x+4}\right) = 1$

$\frac{x^2}{x+4} = 2$

$x^2 = 2x + 8$

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$\therefore x = 4$ or -2

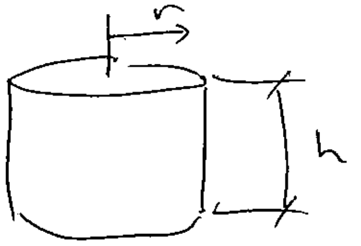
$x \neq -2$

$$\boxed{\therefore x = 4}$$

3

Q16

(b)



$$\underline{V = 2000\pi \text{ cm}^3} \quad \sim (1)$$

$$S = 2\pi r^2 + 2\pi r h \quad (2)$$

$$V = \pi r^2 h \quad (3)$$

(1) into (3) $2000\cancel{\pi} = \cancel{\pi} r^2 h.$

$$h = \frac{2000}{r^2} \quad \sim (4)$$

subs (4) into (2)

$$S = 2\pi r^2 + 2\pi r \frac{2000}{r^2} \quad \sim (2)$$

$$\boxed{= 2\pi r^2 + \frac{4000\pi}{r}.}$$

(ii) for $A = \min$ $A' = 0$

$$\frac{dA}{dr} = 4\pi r - \frac{4000\pi}{r^2}$$

$$\frac{dA}{dr} = 0 \quad \therefore \quad 4\cancel{\pi} r = \frac{4000\cancel{\pi}}{r^2}$$

$$r^3 = 1000$$

$$r = 10$$

for $r = 10$ $S = 2\pi(100) + \frac{4000\pi}{10} = \boxed{600\pi \text{ cm}^2}$

(c) (i)

$$S_1 = 1 + \sin^2 x + \sin^4 x + \cdots$$

$$S_2 - 1 + \cos^2 x + \cos^4 x + \cdots$$

$$S_n = \frac{a}{1-r}$$

$$= \frac{1}{1-\sin^2 x}$$

$$= \frac{1}{\cos^2 x}$$

$$\boxed{= \sec^2 x}$$

$$S_n = \frac{a}{1-r}$$

$$= \frac{1}{1-\cos^2 x}$$

$$= \frac{1}{\sin^2 x}$$

$$\boxed{= \operatorname{cosec}^2 x}$$

2

(ii) then $S_1 + S_2 = S_1 S_2$

$$\sec^2 x + \operatorname{cosec}^2 x = \sec^2 x \operatorname{cosec}^2 x$$

$\sin^2 x + \cos^2 x = 1$

$\div \sin^2 x$ $1 + \cot^2 x = \operatorname{cosec}^2 x$

$\div \cos^2 x$ $\tan^2 x + 1 = \sec^2 x$.

$$\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} = \frac{1}{\cos^2 x} \cdot \frac{1}{\sin^2 x}$$

$$= \frac{\sin^2 x + \cos^2 x}{\cos^2 x \sin^2 x} = \frac{1}{\cos^2 x \sin^2 x}$$

$$= \frac{1}{\cos^2 x \sin^2 x} = \frac{1}{\cos^2 x \sin^2 x}.$$