KNOX
GRAMMAR
SCHOOL

2024 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes.
- Working time – 3 hours.
- Write using black pen.
- NESA approved calculators may be used.
- A reference sheet is provided with this paper.
- For questions in Section II, show relevant mathematical reasoning and/or calculations.

Total marks: 100

Section I – 10 marks (pages 2 – 5)

- Attempt Questions 1 – 10.
- Allow about 15 minutes for this section.

Section II – 90 marks (pages 6 – 33)

- Attempt Questions 11 – 31.
- Allow about 2 hours and 45 minutes for this section.

Question	**MC**	**11-15**	**16-17**	**18-20**	**21-22**	**23-25**	**26-27**	**28-29**	**30-31**	**Total**
Mark	/10	/9	/11	/8	/11	/13	/11	/12	/15	/100

Teachers:

- Charlier G.
- Cheah S.
- Dempsey L.
- Ekanayake P.
- Meli S.
- Siu A.
- Willcocks A.
- Zaidi S.

Examiner: SIU

Section I

10 Marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

1. A marketing team is investigating the relationship between fuel consumption (km/L) and a car's weight (kg). Data has been collected and a least squares regression line is used to predict this relationship.

   The regression line predicts that a car weighting 800 kg will travel 11.7 km per litre of fuel, while a car weighing 1600 kg will travel 9.7 km per litre of fuel.
   What is the slope of the least squares regression line?

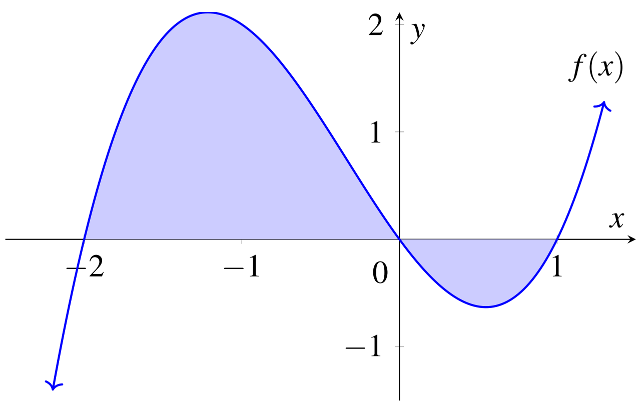
   A. -400

   B. -0.0025

   C. 0.0025

   D. 400

2. The graph of $y = f(x)$ is shown.



NOT TO SCALE

   Which of the following gives the area of the shaded region between $y = f(x)$ and the x-axis between $x = -2$ and $x = 1$?

   A. $-\int_{-2}^{0} f(x)dx + \int_{0}^{1} f(x)dx$

   B. $\int_{-2}^{1} f(x)dx$

   C. $\int_{-2}^{0} f(x)dx + \int_{1}^{0} f(x)dx$

   D. $\int_{-2}^{0} f(x)dx + \int_{0}^{1} f(x)dx$

3. There are ten pieces of fruit in a basket. Four are apples and six are oranges. Mia randomly picks a piece from the basket and eats it. Liam then randomly picks and eats one of the remaining fruits.

   What is the probability that Liam ate an orange?

   A. $\frac{3}{5}$

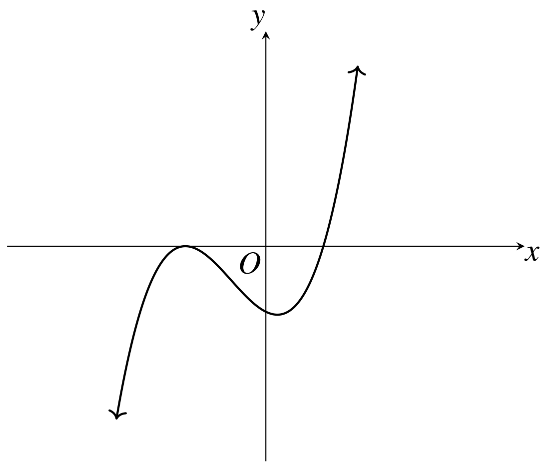
   B. $\frac{4}{5}$

   C. $\frac{27}{50}$

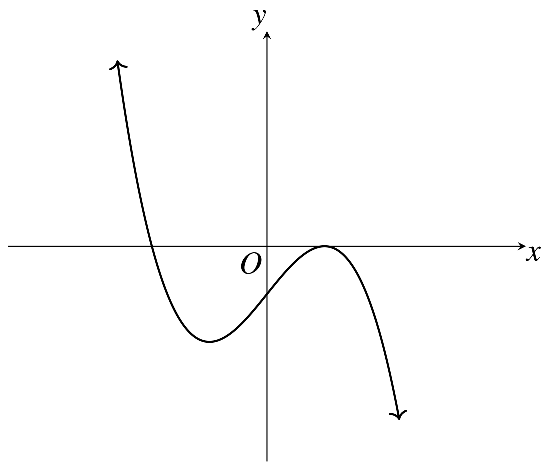
   D. $\frac{22}{45}$

4. Which of the following show the graph of $f(x) = (b - x)^2(a + x)$, if $a > 0$ and $b > 0$?

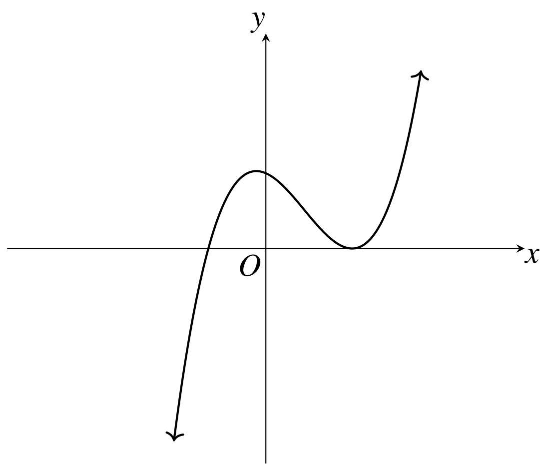
A.



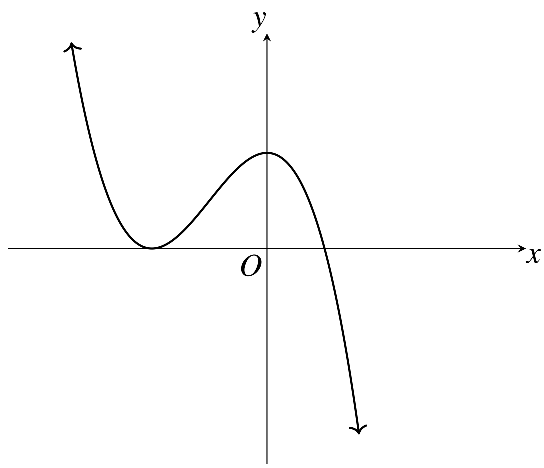
C.



B.



D.



5. A cylindrical tank with a radius of 3 metres is being filled with water. The height of the water in the tank, $h(t)$ in metres, is given by the function $h(t) = 5\sin(t) + 6$, where t is the time in hours.

   What is the rate of change of the height of water in the tank at $t = \dfrac{\pi}{2}$ hours?

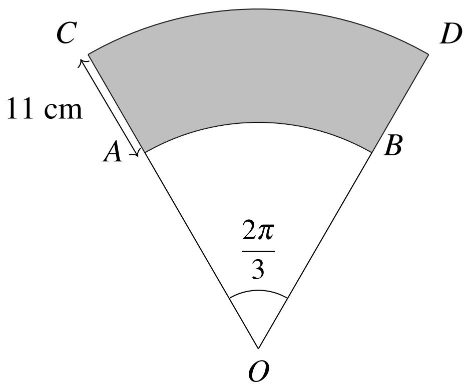
   A. $0\ \text{m}^3/\text{h}$

   B. $15\pi\ \text{m}^3/\text{h}$

   C. $15\pi\sqrt{2}\ \text{m}^3/\text{h}$

   D. $15\pi\sqrt{3}\ \text{m}^3/\text{h}$

6. In the figure, OAB and OCD are sectors with centre O . It is given that the area of the shaded region is $143\pi\ \text{cm}^2$. If $AC = 11\text{cm}$ and $\angle AOB = \dfrac{2\pi}{3}$, what is the value of OA correct to the nearest cm?



NOT TO SCALE

   A. 12 cm

   B. 14 cm

   C. 20 cm

   D. 25 cm

7. What is the domain of $y = \ln(x+1) + \sqrt{4-x}$?

   A. $(-1, 4)$

   B. $[4, \infty)$

   C. $(-\infty, 4)$

   D. $(-1, 4]$

8. A set of scores in a data set has a median of 65, an interquartile range of 16 and a variance of 5.

   Each score has 2 added to it and each resulting score is then doubled to form a new set of scores.

   What is the median, interquartile range, and variance of the new set of scores?

	Median	Interquartile Range	Variance
A.	130	36	10
B.	134	36	20
C.	134	32	10
D.	134	32	20

9. The table below shows the probability distribution of a discrete random variable X.

X	-2	-1	0	3
$P(X=x)$	k^2	$5k$	k	$-k^2-6k+1$

   What is the maximum possible value for μ?

   A. 0

   B. 1

   C. 3

   D. $\dfrac{589}{20}$

10. Let $f(x) = k\sin(mx+l)$, $m > 0$ where $f(x)$ is continuous and increasing for $a \le x \le b$, $f(a) = -k$, and $f(b) = k$.
    Let $g(x) = k\cos(mx+l)$. Which of the following statements is true for $g(x)$ in the domain $a \le x \le b$?

    A. $g(x)$ is always increasing

    B. $g(x)$ is always decreasing

    C. There exists a point c such that $g(c) = k$

    D. There exists a point c such that $g(c) = -k$

End of Section I

Section II

90 marks

Attempt Questions 11 – 31
Allow about 2 hours and 45 minutes for this section

Answer the questions in the spaces provided.
Show all necessary working.

Question 11 (2 marks)

A certain chemical is used to control the population growth of an invasive plant species in the city. The amount of the chemical in the soil decays exponentially according to the function $C = C_0 e^{kt}$, where C_0 is the initial amount of the chemical in kilograms, and t is the time in years. Initially, 50 kg of chemical is applied. After one year, 10 kg of chemical remains in the soil.

(a) Show that $k = \ln\left(\frac{1}{5}\right)$. **1**

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(b) Find the amount of chemical in soil after five years. **1**

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Question 12 (2 marks) **2**

Find the sum of the arithmetic series $-17-9-1+7+15+...+495$.

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Question 13 (1 marks) **1**

Find the derivative function of $7^x+\cos(x)$

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Question 14 (1 marks) **1**

Find $\int \frac{2x+4}{x^2+4x}dx$.

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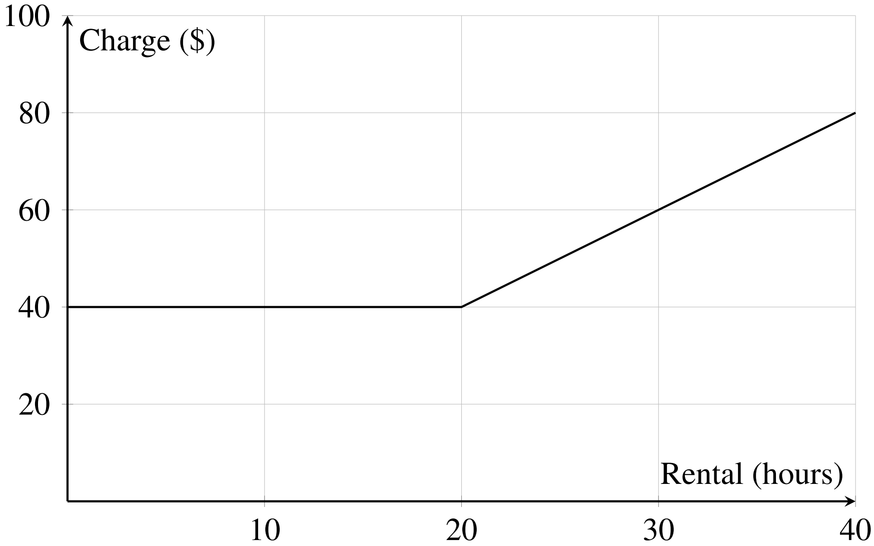
Question 15 (3 marks)

The graph below shows the charge imposed by Company A for an electric bike. Company A charges a fixed cost of \$40 for the first 20 hours and an additional rate of \$2 per hour thereafter.



(a) How much does the company charge for renting an electric bike for 35 hours? **1**

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(b) Company B charges a rate of \$4 per hour, with no fixed cost. On the same grid above, draw the graph which represents Company B's pricing model. **1**

(c) Hence, determine after how many hours, will Company B cost more than Company A. **1**

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Question 16 (6 marks)

Greg is a student who conducted a study to investigate whether there was a correlation between the number of texts a person sends each day and the amount of reading they completed each day.

A group of seven people participated in the study. On a particular day, each person reported how many texts they had sent, and how many pages of a book they had read.

Greg released the results shown at the conclusion of the study.

- The number of texts sent by each participant was: $15, 20, 24, 25, 28, 28, 28$.
- The total number of pages read by the participants was 98.
- If a participant sends 0 texts in a day, it is predicted that they would read 17 pages on that day.

Let x be the number of texts sent in a day and y be the number of pages of a book read in a day.

(a) Greg predicts that the least-squares regression line would pass through the $y-$intercept and the point $(\bar{x}, \bar{y})$, where $\bar{x}$ is the sample mean of the number of texts sent in a day and $\bar{y}$ is the sample mean of the number of pages of a book read in a day. Find the equation of Greg's regression line. **2**

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Question 16 continues on page 10

Question 16 (continued)

(b) Use Greg's regression line to predict the number of texts sent by a person who has read six pages in a day. Comment on the accuracy of this prediction and the use of Greg's regression line. **2**

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(c) Viki, a fellow student, disagrees with Greg's results. She asks the same group of seven people to record the same data on a different day. The results of Viki's study are shown in the table. **2**

x	11	19	25	26	28	28	31
y	14	10	11	18	14	16	15

Using the data from Viki's study, determine the equation of the least-squares regression line. Comment on the accuracy of each student's equation.

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End of Question 16

Question 17 (5 marks)

(a) Show that $\dfrac{\cot x}{1+\operatorname{cosec} x}+\dfrac{1+\operatorname{cosec} x}{\cot x}=\dfrac{2}{\cos x}$ **2**

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(b) Hence, solve the equation for $\dfrac{\cot x}{1+\operatorname{cosec} x}+\dfrac{1+\operatorname{cosec} x}{\cot x}=1+3\cos x$ for $0\leq x\leq 2\pi$ **3**

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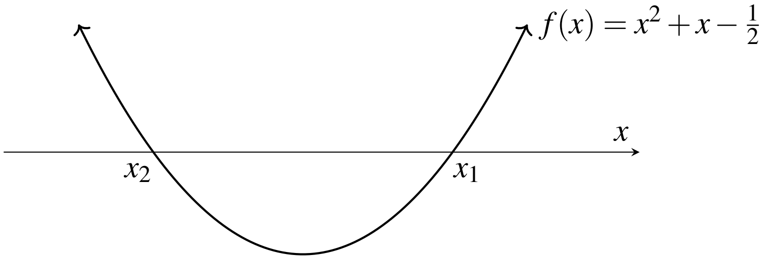
Question 18 (3 marks) **3**

The quadratic curve $f(x) = x^2 + x - \frac{1}{2}$ cuts the $x-$axis at x_1 and x_2, as shown in the diagram below. Show that $\frac{x_1}{x_2} = \sqrt{3} - 2$.



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Question 19 (2 marks) **2**

Solve $\log_3 5 = 2\log_3 10 - \log_3 x$.

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Question 20 (3 marks)

The table below shows the future value of a $1 annuity at different interest rates over different numbers of time periods

Time Period	Interest rate				
	1%	2%	3%	4%	5%
1	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500
3	3.0301	3.0604	3.0909	3.1216	3.1525
4	4.0604	4.1216	4.1836	4.2465	4.3101
5	5.1010	5.2040	5.3091	5.4163	5.5256
6	6.1520	6.3081	6.4684	6.6330	6.8019
7	7.2135	7.4343	7.6625	7.8983	8.1420
8	8.2857	8.5830	8.8923	9.2142	9.5491

(a) What is the value of an annuity that would provide $315 932 after 7 years at 4% per annum compound interest. **1**

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(b) An annuity of $5000 is invested at 4% per annum, bi-annually, compounded every six months for 3 years. What is the amount of interest earned? **2**

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Question 21 (5 marks)

(a) On the day that Alex was born, his grandmother deposited \$4000 into an account earning 4% per annum compounded annually. **2**

On each of Alex's birthdays after this, his grandmother deposited \$1200 into the same account, making her final deposit on Alex's 15^{th} birthday. That is, a total of 16 deposits were made.

Let B_n be the amount in the account on Alexs nth birthday, after the deposit is made.

Show that $B_4 = \$9775.19$

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(b) On his 15^{th} birthday, just after the final deposit is made, Alex has $\$31,232.08$ in his account. Alex then decides to leave all the money in the same account, continuing to earn interest at 4% per annum compounded annually. **3**

On his 16^{th} birthday, and on each birthday after this, Alex withdraws \$2500 from the account.

How many withdrawals of \$2500 will Alex be able to make?

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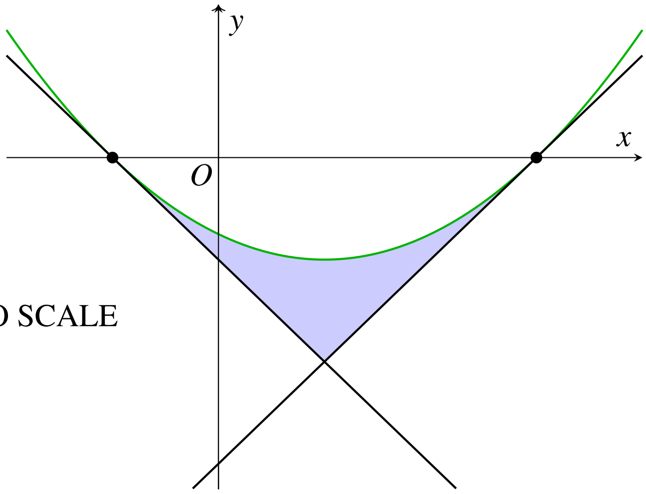
Question 22 (6 marks)

The graph of $y = x^2 - 2x - 3$ and its tangents through $x-$intercepts are shown.



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(a) Show the equation of the tangents are $y = 4x - 12$ and $y = -4x - 4$. **3**

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Question 22 continues on page 17

Question 22 (continued)

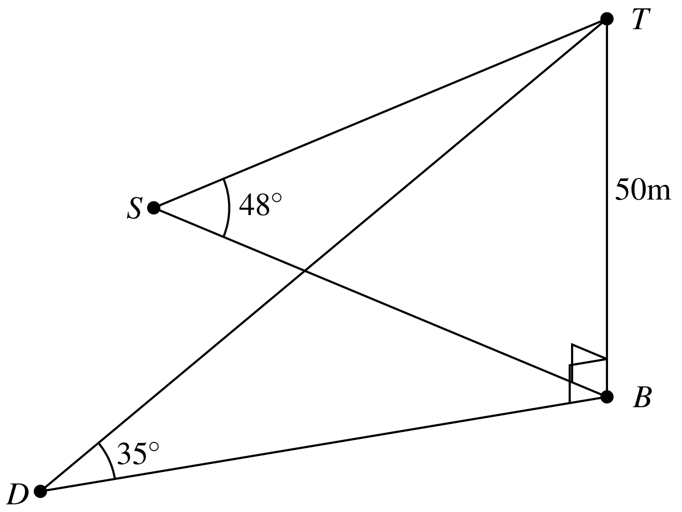
(b) Find the shaded area between the parabola and its tangents. **3**

End of Question 22

Question 23 (5 marks)

Darline and Saher visit Sydney Harbour on New Year's Eve to watch the fireworks.
Darline is standing outside the Sydney Opera House and Saher is on a yacht in the harbour. They can both see the fireworks at the top of the Sydney Harbour Bridge.

NOT TO SCALE



From Darline's position (D), the Sydney Harbour Bridge (B) is at a bearing of $78°$, and the angle of elevation to the top of the bridge (T) is $35°$.
From Saher's position (S), the Sydney Harbour Bridge is at a bearing of $140°$, and the angle of elevation to the top of the bridge is $48°$.
The top of the bridge is 50 m above the base of the bridge.

(a) Using appropriate diagrams and reasoning, show that $\angle SBD = 62°$ **2**

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Question 23 continues on page 19

Question 24 (5 marks)

A company produces boxes that contain various miniature chocolates. The chocolate boxes are labelled with a net weight of 200 g. However, the company aims to fill the chocolate boxes to a net weight of 205 g to avoid customer complaints. The weight of the chocolate boxes is normally distributed with a mean of 205 g and a standard deviation of 2 g.

(a) Approximately 30% of the boxes weigh less than 204 g. **3**
What is the approximate percentage of chocolate boxes that weigh between 206 g and 207 g?

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(b) The table gives the probability that a random variable that is normally distributed lies below z for some positive values of z. **2**

z	0	0.01	0.02	0.03	0.04	0.05	0.06
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500
2	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030

Find the probability that a chocolate box weighs more than 208.6 g.

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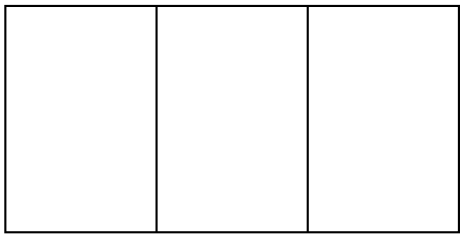
Question 25 (3 marks) **3**

A farmer wants to build a rectangular enclosure with a fence to keep livestock. The farmer has 120 metres of fencing material and wants to maximise the area of the enclosure.

The enclosure will be divided into three equal-sized smaller rectangular sections by using two additional fences parallel to one of the sides of the rectangle.



Find the dimensions that will maximise the area of the enclosure and give the maximum area.

Question 26 (7 marks)

A continuous random variable X has the probability density function $f(x)$ given by

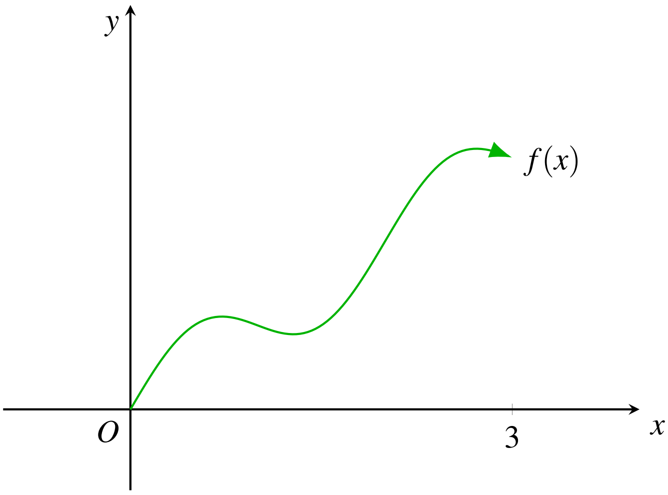
$$f(x) = \begin{cases} k\sin(\pi x) + 2kx, & \text{for } 0 \leq x \leq 3 \\ 0, & \text{for all other values of } x \end{cases}$$

(a) Show that $k = \dfrac{\pi}{2+9\pi}$ **3**

Question 26 continues on page 21

Question 26 (continued)

(b) The graph of the density function is given. **4**



Find the mode of X. Give your answer correct to two decimal places.

End of Question 26

Question 27 (4 marks)

A polynomial function $f(x)$ has the following properties:

	$x<0$	$x=0$	$0<x<1$	$x=1$	$1<x<3$	$x=3$	$x>3$
$f(x)$		-27		-16		0	
$f'(x)$	<0	0	>0	>0	>0	0	>0
$f''(x)$	<0	>0	>0	0	<0	0	>0

(a) Find the range of values where $f(x)$ is increasing and concave up. **1**

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(b) State the nature of the stationary points of $f(x)$. **1**

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(c) Sketch a possible graph of $y=f(x)$, using the properties in the table above. **2**

Question 28 (5 marks)

A particle starts from a fixed point A and travels in a straight line. As it passes a point B, the brakes are applied and the particle slows down at a constant rate of $\frac{1}{3}$ m/s^2, coming to a rest at point C. For the motion from A to B, the velocity, v m/s, of the particle, t s after leaving A, is given by $v = 1 + 3e^{\frac{1}{2}(t-4)}$ for $0 \leq t \leq 4$.

(a) Find the time taken for the object to reach a velocity of 3 m/s. **2**

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(b) Find the initial acceleration of the particle. **2**

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(c) Find the velocity of the particle at point B. **1**

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Question 29 (7 marks)

The water depth, y in metres, in a lake can be modelled by the function $y = 5 + 3\cos\left(\frac{\pi t}{6}\right)$, where t is the time in hours after 6:00 am.

(a) Determine the time between successive high tides as well as the maximum and minimum depth. **2**

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(b) Sketch the graph of $y(t)$ for $0 \leq t \leq 24$ **2**

Question 29 continues on page 27

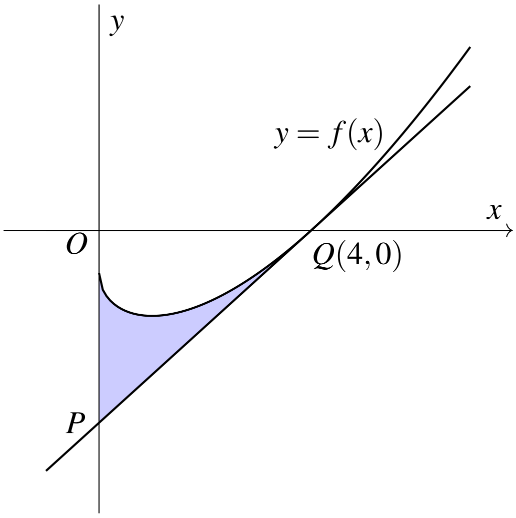
Question 29 (continued)

(c) A boat is to sail from a dock to a canal and then out to the main lake. An overhead bridge 7 metres above the bottom of the lake obstructs the boat's exit from the dock. The boat itself has a height of three metres from the waterline to its highest point. What is the earliest time that the boat can leave the dock? Give your answer correct to the nearest minute. **3**

End of Question 29

Question 30 (7 marks)

The diagram below shows part of a curve $y = f(x)$. The curve is such that $f'(x) = x^{\frac{1}{2}} - x^{-\frac{1}{2}}$ and it passes through the point $Q(4, 0)$. The tangent at Q meets the $y-$axis at the point P.



(a) Find $f(x)$. **2**

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Question 30 continues on page 29

Question 30 (continued)

(b) Show that the $y-$coordinate of P is -6. **2**

(c) Find the area of the shaded region. **3**

End of Question 30

Question 31 (8 marks)

In a 100 m race, the speeds, S_I m/s and S_H m/s, of two animals, an iguana and a hippopotamus respectively can be modelled by the functions.

$$S_I = \frac{6400}{953125}\left(\frac{t^3}{3} - \frac{77}{4}t^2 + 325t\right), \text{and}$$

$$S_H = \frac{2453}{1000}te^{-0.08t},$$

Where t is the time measured from the start in seconds.

It is known that the iguana finishes the race in 12.5 seconds. [Do Not Prove This]

(a) Use the trapezoidal rule with 5 sub-intervals to estimate the distance covered by the hippopotamus in 12.5 seconds. **2**

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Question 31 continues on page 31

Question 31 (continued)

(b) Find $\dfrac{d^2 S_H}{dt^2}$ and determine whether the hippopotamus finishes ahead of the iguana. **2**

Question 31 continues on page 32

Question 31 (continued)

(c) Differentiate $\left(\ln\left(\dfrac{t+2}{2}\right)\right)^2$ **2**

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Question 31 continues on page 33

Question 31 (continued)

In the same race, the speed S_C m/s, of a cow is modelled by

$$S_C = \frac{50[\ln(t+2) - \ln 2]}{t+2}$$

(d) Using the function above, show that the cow came last. **2**

End of Question 31

End of Paper

KNOX GRAMMAR SCHOOL

2024 **TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION**

Mathematics Advanced

General Instructions

- Reading time – 10 minutes.
- Working time – 3 hours.
- Write using black pen.
- NESA approved calculators may be used.
- A reference sheet is provided with this paper.
- For questions in Section II, show relevant mathematical reasoning and/or calculations.

Total marks: 100

Section I – 10 marks (pages 1 – 4)
- Attempt Questions 1 – 10.
- Allow about 15 minutes for this section.

Section II – 90 marks (pages 5 – 31)
- Attempt Questions 11 – 31.
- Allow about 2 hours and 45 minutes for this section.

Question	MC	11-15	16-17	18-20	21-22	23-25	26-27	28-29	30-31	Total
Mark	/10	/9	/11	/8	/11	/13	/11	/12	/15	/100

Teachers:
- Charlier G.
- Cheah S.
- Dempsey L.
- Ekanayake P.
- Meli S.
- Siu A.
- Willcocks A.
- Zaidi S.

Examiner: SIU

Number of Students in Course: 175

Section I

10 Marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

1. A marketing team is investigating the relationship between fuel consumption (km/L) and a car's weight (kg). Data has been collected and a least squares regression line is used to predict this relationship.

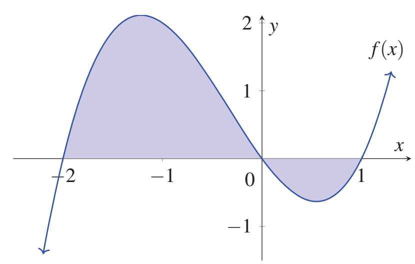
   The regression line predicts that a car weighting 800 kg will travel 11.7 km per litre of fuel, while a car weighing 1600 kg will travel 9.7 km per litre of fuel.
   What is the slope of the least squares regression line?

   A. -400
   B. -0.0025
   C. 0.0025
   D. 400

   $m = \dfrac{9.7 - 11.7}{1600 - 800}$

   $= -0.0025$

2. The graph of $y = f(x)$ is shown.



NOT TO SCALE

   Which of the following gives the area of the shaded region between $y = f(x)$ and the x-axis between $x = -2$ and $x = 1$?

   A. $-\int_{-2}^{0} f(x)dx + \int_{0}^{1} f(x)dx$

   B. $\int_{-2}^{1} f(x)dx$

   C. $\int_{-2}^{0} f(x)dx + \int_{1}^{0} f(x)dx \rightarrow \int_{-2}^{0} f(x)\,dx - \int_{0}^{1} f(x)\,dx$

   D. $\int_{-2}^{0} f(x)dx + \int_{0}^{1} f(x)dx$

3. There are ten pieces of fruit in a basket. Four are apples and six are oranges. Mia randomly picks a piece from the basket and eats it. Liam then randomly picks and eats one of the remaining fruits.

   What is the probability that Liam ate an orange?

   A. $\frac{3}{5}$

   B. $\frac{4}{5}$

   C. $\frac{27}{50}$

   D. $\frac{22}{45}$

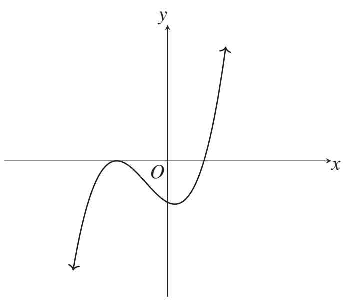
$\frac{4}{10}$ A, $\frac{3}{9}$ A, $\frac{6}{9}$ O

$\frac{6}{10}$ O, $\frac{4}{9}$ A, $\frac{5}{9}$ O

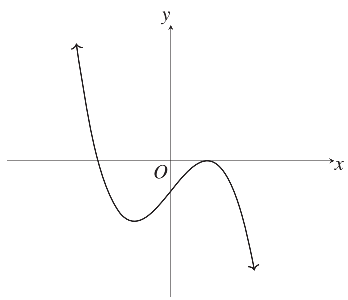
$P(AO) + P(OO)$

$= \frac{3}{5}$

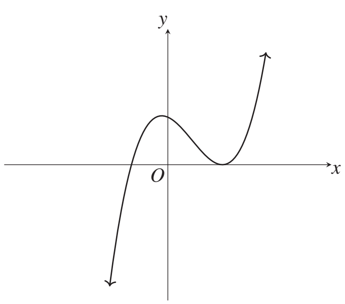
4. Which of the following show the graph of $f(x) = (b-x)^2(a+x)$, if $a > 0$ and $b > 0$?



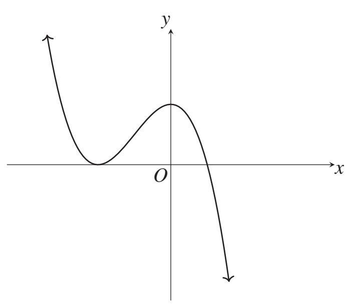
A.



C.



B.



D.

5. A cylindrical tank with a radius of 3 metres is being filled with water. The height of the water in the tank, $h(t)$ in metres, is given by the function $h(t) = 5\sin(t) + 6$, where t is the time in hours.

   What is the rate of change of the height of water in the tank at $t = \frac{\pi}{2}$ hours?

   A. $0\ \mathrm{m^3/h}$

   B. $15\pi\ \mathrm{m^3/h}$

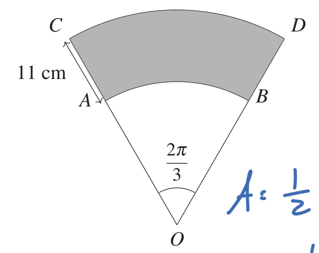
   C. $15\pi\sqrt{2}\ \mathrm{m^3/h}$

   D. $15\pi\sqrt{3}\ \mathrm{m^3/h}$

$h'(t) = 5\cos t$

$h'(\frac{\pi}{2}) = 5\cos\frac{\pi}{2} = 0$

6. In the figure, OAB and OCD are sectors with centre O . It is given that the area of the shaded region is $143\pi\ \mathrm{cm}^2$. If $AC = 11\mathrm{cm}$ and $\angle AOB = \frac{2\pi}{3}$, what is the value of OA correct to the nearest cm?



$A: \frac{1}{2} \times \frac{2\pi}{3}[(11+r)^2 - r^2] = 143\pi$

$\frac{1}{3}(121 + 22r) = 143$

$r = 14\mathrm{cm}$

   A. 12 cm

   B. 14 cm

   C. 20 cm

   D. 25 cm

7. What is the domain of $y = \ln(x+1) + \sqrt{4-x}$?

   A. $(-1, 4)$

   B. $[4, \infty)$

   C. $(-\infty, 4)$

   D. $(-1, 4]$

$x + 1 > 0$

$x > -1$

8. A set of scores in a data set has a median of 65, an interquartile range of 16 and a variance of 5.

Each score has 2 added to it and each resulting score is then doubled to form a new set of scores.

What is the median, interquartile range, and variance of the new set of scores?

	Median	Interquartile Range	Variance
A.	130	36	10
B.	134	36	20
C.	134	32	10
D.	134	32	20

Median $= 65+2 = 67 \times 2 = 134$

IQR $= 16 \times 2 = 32$

Variance $= 5 \times 2^2 = 20$

9. The table below shows the probability distribution of a discrete random variable X.

X	-2	-1	0	3
$P(X=x)$	k^2	$5k$	k	$-k^2-6k+1$

What is the maximum possible value for μ?

$\mu = -5k^2 - 23k + 3$

$k = -\frac{b}{2a} = \frac{23}{-10} = -2.3$ (but $k \geq 0$)

For max value of μ, $k=0$

$\mu = -5(0)^2 - 23(0) + 3$

$= 3$

A. 0

B. 1

C. 3

D. $\frac{589}{20}$

10. Let $f(x) = k\sin(mx+l)$, $m>0$ where $f(x)$ is continuous and increasing for $a \leq x \leq b$, $f(a) = -k$, and $f(b) = k$.

Let $g(x) = k\cos(mx+l)$. Which of the following statements is true for $g(x)$ in the domain $a \leq x \leq b$?

A. $g(x)$ is always increasing

B. $g(x)$ is always decreasing

C. There exists a point c such that $g(c) = k$

D. There exists a point c such that $g(c) = -k$

$f(x) = k\sin(mx+l)$

$f'(x) = km\cos(mx+l)$

$\therefore km\cos(mx+l) > 0$

As $m > 0$

$\therefore k\cos(mx+l) > 0$

$\therefore g(x) > 0$

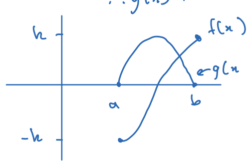
If $f(a) = -k$

$\therefore -k = k\sin(ma+l)$

$\therefore \sin(ma+l) = -1$

if $f(b) = k$

$\therefore \sin(mb+l) = 1$



End of Section I

Section II

90 marks

Attempt Questions 11 – 31

Allow about 2 hours and 45 minutes for this section

Answer the questions in the spaces provided.
Show all necessary working.

Question 11 (2 marks)

A certain chemical is used to control the population growth of an invasive plant species in the city. The amount of the chemical in the soil decays exponentially according to the function $C = C_0e^{kt}$, where C_0 is the initial amount of the chemical in kilograms, and t is the time in years. Initially, 50 kg of chemical is applied. After one year, 10 kg of chemical remains in the soil.

(a) Show that $k = \ln\left(\frac{1}{5}\right)$. **1**

$t=0$ $C_0 = 50$ $\rightarrow$ $C = 50e^{kt}$

when $t=1$, $C=10$

$\frac{1}{5} = e^k$ $\rightarrow$ $e^k = \frac{1}{5}$

$k = \ln\left(\frac{1}{5}\right)$

(b) Find the amount of chemical in soil after five years. **1**

$C = 50e^{t \times \ln\left(\frac{1}{5}\right)}$ at $t=5$,

$= 50e^{5\times\ln\left(\frac{1}{5}\right)}$

$= 16g$ or $0.016kg$

Question 12 (2 marks) **2**

Find the sum of the arithmetic series $-17-9-1+7+15+\ldots+495$.

$a=-17 \quad d=8$ ① For correct value of n.

$495=-17+8(n-1)$

$n=65$ ① Mark for correct answer

$\therefore S_{65}=\frac{65}{2}(-17+495)$

$=15535$

Question 13 (1 marks) **1**

Find the derivative function of $7^x+\cos(x)$

$\frac{d}{dx}(7^x+\cos(x))$

$=\ln 7 . 7^x-\sin x$

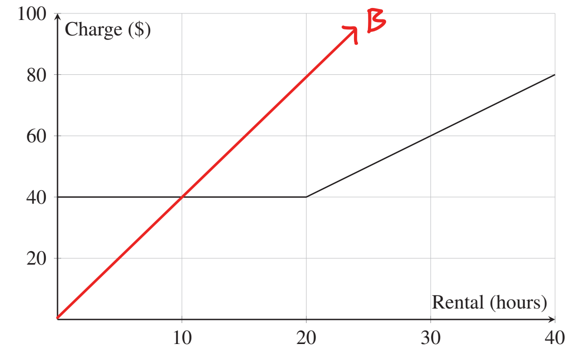
Question 14 (1 marks) **1**

Find $\int \frac{2x+4}{x^2+4x}dx$.

$=\ln(x^2+4x)+C$

Question 15 (3 marks)

The graph below shows the charge imposed by Company A for an electric bike. Company A charges a fixed cost of $40 for the first 20 hours and an additional rate of $2 per hour thereafter.



(a) How much does the company charge for renting an electric bike for 35 hours? **1**

$70

(b) Company B charges a rate of $4 per hour, with no fixed cost. On the same grid above, draw the graph which represents Company B's pricing model. **1**

(c) Hence, determine after how many hours, will Company B cost more than Company A. **1**

After 10 hours

Question 16 (6 marks)

Greg is a student who conducted a study to investigate whether there was a correlation between the number of texts a person sends each day and the amount of reading they completed each day.

A group of seven people participated in the study. On a particular day, each person reported how many texts they had sent, and how many pages of a book they had read.

Greg released the results shown at the conclusion of the study.

- The number of texts sent by each participant was: $15, 20, 24, 25, 28, 28, 28$.
- The total number of pages read by the participants was 98.
- If a participant sends 0 texts in a day, it is predicted that they would read 17 pages on that day.

Let x be the number of texts sent in a day and y be the number of pages of a book read in a day.

(a) Greg predicts that the least-squares regression line would pass through the y-intercept and the point $(\bar{x}, \bar{y})$, where $\bar{x}$ is the sample mean of the number of texts sent in a day and $\bar{y}$ is the sample mean of the number of pages of a book read in a day. Find the equation of Greg's regression line. **2**

$\bar{x} = 24$

$\bar{y} = 14$

① Mark for finding $\bar{x}$ and $\bar{y}$

① Mark for correct answer

$\therefore$ The line passes through $(0, 17)$ and $(24, 14)$

$m = \dfrac{14-17}{24-0} = -\dfrac{3}{24} = -\dfrac{1}{8}$

$\therefore y = -\dfrac{1}{8}x + 17$

Question 16 continues on page 9

Question 16 (continued)

(b) Use Greg's regression line to predict the number of texts sent by a person who has read six pages in a day. Comment on the accuracy of this prediction and the use of Greg's regression line. **2**

Sub in $y = 6$

① Mark for finding x.

$-\dfrac{1}{8}x + 17 = 6$

① Mark for providing correct reasoning.

$-\dfrac{1}{8}x = -11$

$x = 88$

$\therefore$ Inaccurate as 88 lies outside the range.

(c) Viki, a fellow student, disagrees with Greg's results. She asks the same group of seven people to record the same data on a different day. The results of Viki's study are shown in the table. **2**

x	11	19	25	26	28	28	31
y	14	10	11	18	14	16	15

Using the data from Viki's study, determine the equation of the least-squares regression line. Comment on the accuracy of each student's equation.

① Mark for equation.

$y = 0.143x + 10.571$

① Mark for correct reason.

$\therefore$ More accurate Viki uses exact values for x and y.

End of Question 16

Question 17 (5 marks)

(a) Show that $\dfrac{\cot x}{1+\operatorname{cosec} x}+\dfrac{1+\operatorname{cosec} x}{\cot x}=\dfrac{2}{\cos x}$ **2**

$$L.H.S: \left(\frac{\frac{\cos x}{\sin x}}{1+\frac{1}{\sin x}}+\frac{1+\frac{1}{\sin x}}{\frac{\cos x}{\sin x}}\right)\frac{\sin x}{\sin x}$$

$$=\frac{\cos x}{\sin x+1}+\frac{\sin x+1}{\cos x}$$

① Mark for simplifying into this form

$$=\frac{\cos^2 x+\sin^2 x+2\sin x+1}{\cos x(\sin x+1)}$$

① Mark for using identity.

$$=\frac{2+2\sin x}{\cos x(\sin x+1)}=\frac{2}{\cos x} \quad \therefore L.H.S=R.H.S$$

(b) Hence, solve the equation for $\dfrac{\cot x}{1+\operatorname{cosec} x}+\dfrac{1+\operatorname{cosec} x}{\cot x}=1+3\cos x$ for $0\le x\le 2\pi$ **3**

Using part (i):

$$\frac{2}{\cos x}=1+3\cos x$$

① Mark for multiplying cos x over.

① Mark equating cos x to -1 and $\frac{2}{3}$

S A
T C

$$2=\cos x+3\cos^2 x$$

$$0=3\cos^2 x+\cos x-2$$

$$0=(\cos x+1)(3\cos x-2)$$

① Mark for correct solutions.

$\cos x=-1$ $\quad\quad$ $\cos x=\frac{2}{3}$

$x=\pi$ $\quad\quad$ $x=0.841,\ 2\pi-0.841$

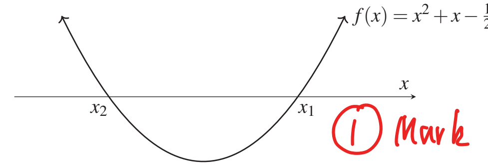
$x=0.841,\ 5.442$

$\therefore x=0.841,\ \cancel{\pi},\ 5.442$

Question 18 (3 marks) **3**

The quadratic curve $f(x)=x^2+x-\frac{1}{2}$ cuts the x−axis at x_1 and x_2, as shown in the diagram below.

Show that $\frac{x_1}{x_2}=\sqrt{3}-2$.



For the x-int, let $f(x)=0$:

$$x^2+x-\frac{1}{2}=0$$

① Mark for applying the quadratic formula.

① Mark for correct x_1 and x_2

$$x=\frac{-1\pm\sqrt{1-4(-\frac{1}{2})(1)}}{2}$$

① Mark for rationalising correctly.

$$x=\frac{-1\pm\sqrt{3}}{2} \qquad \therefore x_2=\frac{-1-\sqrt{3}}{2} \text{ and } x_1=\frac{-1+\sqrt{3}}{2}$$

$$\therefore \frac{x_1}{x_2}=\frac{-1+\sqrt{3}}{-1-\sqrt{3}}\times\frac{(-1+\sqrt{3})}{(-1+\sqrt{3})}$$

$$=\frac{1-2\sqrt{3}+3}{-2}=\sqrt{3}-2 \text{ as required.}$$

Question 19 (2 marks) **2**

Solve $\log_3 5=2\log_3 10-\log_3 x$.

① Mark for using log law correctly

$$\log_3 5+\log_3 x=\log_3 100$$

$$\log_3 5x=\log_3 100$$

① Mark for correct answer

$$5x=100$$

$$x=20$$

Question 20 (3 marks)

The table below shows the future value of a $1 annuity at different interest rates over different numbers of time periods

Time Period	Interest rate				
	1%	2%	3%	4%	5%
1	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500
3	3.0301	3.0604	3.0909	3.1216	3.1525
4	4.0604	4.1216	4.1836	4.2465	4.3101
5	5.1010	5.2040	5.3091	5.4163	5.5256
6	6.1520	6.3081	6.4684	6.6330	6.8019
7	7.2135	7.4343	7.6625	7.8983	8.1420
8	8.2857	8.5830	8.8923	9.2142	9.5491

(a) What is the value of an annuity that would provide $315 932 after 7 years at 4% per annum compound interest. **1**

FV factor = 7.8983

$\therefore 315932 \div 7.8983$

$= \$40\,000$

(b) An annuity of $5000 is invested at 4% per annum, bi-annually, compounded every six months for 3 years. What is the amount of interest earned? **2**

$r = \frac{4\%}{2}$, $n = 3 \times 2$ FV factor = 6.3081

$\$5000 \times 6.3081 = \$31540.50$ 1 Mark for correct FV factor

interest = $31540.50 - 5000 \times 6$

$= \$1540.50$ 1 Mark for correct answer.

Question 21 (5 marks)

(a) On the day that Alex was born, his grandmother deposited $4000 into an account earning 4% per annum compounded annually. **2**

On each of Alex's birthdays after this, his grandmother deposited $1200 into the same account, making her final deposit on Alex's 15^{th} birthday. That is, a total of 16 deposits were made.

Let B_n be the amount in the account on Alexs nth birthday, after the deposit is made.

Show that $B_4 = \$9775.19$

1 Finds $4000(1.04)+1200$

1 Correct answer

$B_0 = 4000$

$B_1 = 4000(1.04) + 1200$

$B_2 = 4000(1.04)^2 + 1200(1.04) + 1200$

$B_3 = 4000(1.04)^3 + 1200(1.04)^2 + 1200(1.04) + 1200$

$= 4000(1.04)^3 + 1200(1 + 1.04 + 1.04^2)$

$\therefore B_4 = 4000(1.04)^4 + 1200(1 + 1.04 + 1.04^2 + 1.04^3)$

$= \$9775.19$

(b) On his 15^{th} birthday, just after the final deposit is made, Alex has $\$31,232.08$ in his account. Alex then decides to leave all the money in the same account, continuing to earn interest at 4% per annum compounded annually. **3**

On his 16^{th} birthday, and on each birthday after this, Alex withdraws $2500 from the account.

How many withdrawals of $2500 will Alex be able to make?

1 Mark for A_1 established.

1 Mark for geometric series

1 Mark for correct withdrawals

$A_1 = 31232.08(1.04) - 2500$

$A_2 = (A_1)(1.04) - 2500$

$= 31232.08(1.04)^2 - 2500(1.04) - 2500$

$= 31232.08(1.04)^2 - 2500(1 + 1.04)$

$A_n = 31232.08(1.04)^n - 2500(1 + 1.04 + \dots + 1.04^{n-1})$

let $A_n = 0$ $\therefore 2500\left(\frac{1.04^n - 1}{0.04}\right) = 31232.08(1.04)^n$

$62500(1.04^n - 1) = 31232.08(1.04)^n$

$31267.92 \times 1.04^n = 62500$

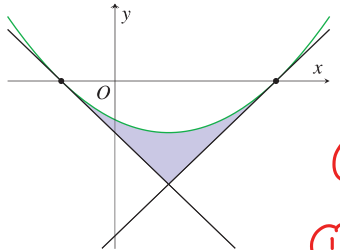
$1.04^n = 1.99885$

$n = 17.65\ldots$

$n = 17$ withdrawals.

Question 22 (6 marks)

The graph of $y = x^2 - 2x - 3$ and its tangents through x–intercepts are shown.



① Mark for identifying x-intercepts
① Mark for finding either gradient.
① Mark for correctly finding equations.

(a) Show the equation of the tangents are $y = 4x - 12$ and $y = -4x - 4$. **3**

$y' = 2x - 2$

x:int: $0 = x^2 - 2x - 3$
$0 = (x-3)(x+1)$
$x = 3 \quad x = -1$

$\therefore$ Tangents at $(-1, 0)$ and $(3, 0)$

For $y' = 2x - 2 \rightarrow m_1 = 2(-1) - 2 = -4$
$m_2 = 2(3) - 2 = 4$

$\therefore y - 0 = -4(x+1) \rightarrow y = -4x - 4$
$y - 0 = 4(x-3) \rightarrow y = 4x - 12$

(b) Find the shaded area between the parabola and its tangents. **3**

P.O.I: $4x - 12 = -4x - 4$
$x = 1$

Area $= 2\int_1^3 (x^2 - 2x - 3) - (4x - 12)\, dx$

$= 2\int_1^3 x^2 - 6x + 9 \, dx$

$= 2\int_1^3 (x-3)^2 \, dx$

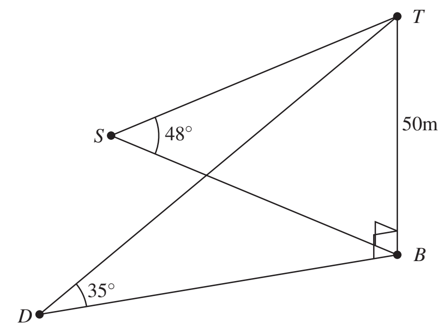
$= \frac{2}{3}\left[(x-3)^3\right]_1^3$

$= \frac{2}{3}\left[(3-3)^3 - (1-3)^3\right] = \frac{16}{3}\ u^2$

① Mark for finding P.O.I
① Mark for correct integration
① Mark for correct answer.

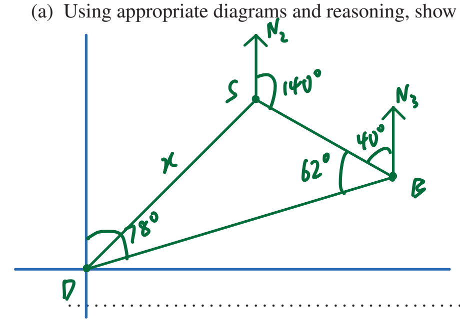
Question 23 (5 marks)

Darline and Saher visit Sydney Harbour on New Year's Eve to watch the fireworks.
Darline is standing outside the Sydney Opera House and Saher is on a yacht in the harbour. They can both see the fireworks at the top of the Sydney Harbour Bridge.



From Darline's position (D), the Sydney Harbour Bridge (B) is at a bearing of 78°, and the angle of elevation to the top of the bridge (T) is 35°.
From Saher's position (S), the Sydney Harbour Bridge is at a bearing of 140°, and the angle of elevation to the top of the bridge is 48°.
The top of the bridge is 50 m above the base of the bridge.

(a) Using appropriate diagrams and reasoning, show that $\angle SBD = 62°$ **2**



① Mark for correct diagram with accurate annotations
① Mark for correct answer

$\angle SBN_3 = 180 - 140 = 40°$ $\quad \angle DBN_3 = 180 - 78 = 102°$ $\quad \angle SBD = 102 - 40 = 62°$

Question 23 continues on page 17

Question 23 (continued)

(b) Find the distance between Saher and Darline. Give your answer correct to two decimal places. **3**

① Mark for correct length of SB OR DB

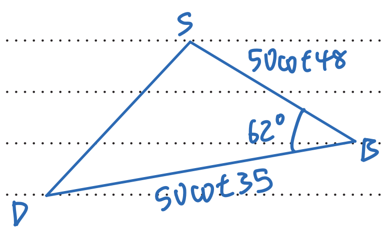
$\tan 48 = \frac{50}{SB}$ $\tan 35 = \frac{50}{DB}$

① Mark for length of SB AND DB.

$SB = \frac{50}{\tan 48}$ $DB = \frac{50}{\tan 35}$

① Mark for correct answer.

$= 50\cot 48$ $= 50\cot 35$



$\therefore SD^2 = SB^2 + DB^2 - 2 \times SB \times DB \cos 62$

$= [50\cot(48)]^2 + [50\cot(35)]^2 - 2 \times 50\cot(48) \times 50\cot 35 \times \cos 62$

$= 4107.3439\ldots$

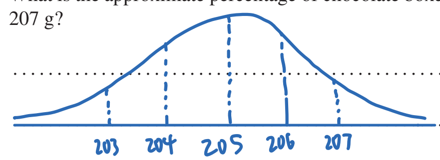
$\therefore SD = \sqrt{4107.3439\ldots} = 64.09\text{m}$ $(SD > 0)$

End of Question 23

Question 24 (5 marks)

A company produces boxes that contain various miniature chocolates. The chocolate boxes are labelled with a net weight of 200 g. However, the company aims to fill the chocolate boxes to a net weight of 205 g to avoid customer complaints. The weight of the chocolate boxes is normally distributed with a mean of 205 g and a standard deviation of 2 g.

(a) Approximately 30% of the boxes weigh less than 204 g. **3**
What is the approximate percentage of chocolate boxes that weigh between 206 g and 207 g?



① Mark for identifying any relevant %

① Mark for identifying $P(205 < w < 207) = 34\%$

$P(205 < w < 207) = 34\%$

① Mark for correct answer

$P(206 < w < 207) = 34\% - 20\%$

$= 14\%$

(b) The table gives the probability that a random variable that is normally distributed lies below z for some positive values of z. **2**

z	0	0.01	0.02	0.03	0.04	0.05	0.06
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500
2	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030

Find the probability that a chocolate box weighs more than 208.6 g.

$z = \frac{x - \mu}{6}$

$= \frac{208.6 - 205}{2}$

$= 1.8$

$P(z < 1.8) = 0.96407$

$\therefore P(z > 1.8) = 1 - 0.96407$

$= 0.03593$

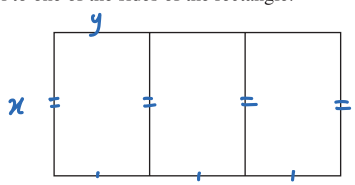
① Mark for correct z-score

① Mark for correct answer

Question 25 (3 marks) **3**

A farmer wants to build a rectangular enclosure with a fence to keep livestock. The farmer has 120 metres of fencing material and wants to maximise the area of the enclosure.

The enclosure will be divided into three equal-sized smaller rectangular sections by using two additional fences parallel to one of the sides of the rectangle.



Find the dimensions that will maximise the area of the enclosure and give the maximum area.

$4x + 2y = 120$

$y = 60 - 2x$

$A = (60-2x)x$

$= 60x - 2x^2$

$A' = 60 - 4x \rightarrow$ let $A' = 0 \rightarrow 0 = 60 - 4x$

$x = 15$

$A'' = -4$ ($\therefore$ st. pt. is max)

$\therefore 15 \times 30 = 450\text{m}^2$

1 Mark for correct area equation

1 Mark for correct x value

1 Mark for correct max area.

Question 26 (7 marks)

A continuous random variable X has the probability density function $f(x)$ given by

$$f(x) = \begin{cases} k\sin(\pi x) + 2kx, & \text{for } 0 \le x \le 3 \\ 0, & \text{for all other values of } x \end{cases}$$

(a) Show that $k = \dfrac{\pi}{2+9\pi}$ **3**

$k\int_0^3 \sin(\pi x) + 2x \, dx = 1$

$\left[-\frac{1}{\pi}\cos(\pi x) + x^2\right]_0^3 = \frac{1}{k}$

$\left(-\frac{1}{\pi}\cos(3\pi) + 9\right) - \left(-\frac{1}{\pi}\cos(0) + 0^2\right) = \frac{1}{k}$

$\frac{1}{\pi} + 9 + \frac{1}{\pi} = \frac{1}{k}$

$\frac{2}{\pi} + 9 = \frac{1}{k}$

$\frac{2+9\pi}{\pi} = \frac{1}{k}$

$\therefore k = \frac{\pi}{2+9\pi}$

1 Mark for correct equation of integral

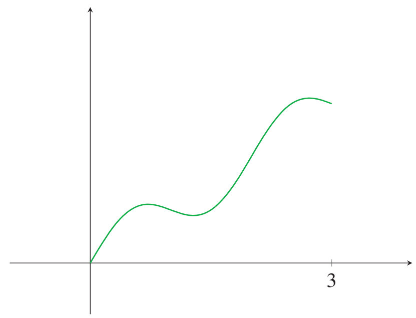
1 Mark for correct integration

1 Mark for correct substitution

Question 26 continues on page 21

Question 26 (continued)

(b) The graph of the density function is given. **4**



Find the mode of X. Give your answer correct to two decimal places.

For $0 \le x \le 3$:

$f'(x) = k[\pi \cos(\pi x) + 2]$

$f'(x) = 0 \rightarrow \cos(\pi x) = -\frac{2}{\pi}$

$\pi x = \cos^{-1}(-\frac{2}{\pi})$

$x = 0.72, 1.28, 2.72$

From graph, the largest x-value gives the maximum value.

$\therefore x = 2.72$ is the mode

① Mark for attempt at derivative

① Mark for equation involving $\cos(\pi x)$

① Mark for finding at least one st. pt.

① Mark for correct answer.

End of Question 26

Question 27 (4 marks)

A polynomial function $f(x)$ has the following properties:

	$x<0$	$x=0$	$0<x<1$	$x=1$	$1<x<3$	$x=3$	$x>3$
$f(x)$		-27		-16		0	
$f'(x)$	<0	0	>0	>0	>0	0	>0
$f''(x)$	<0	>0	>0	0	<0	0	>0

(a) Find the range of values where $f(x)$ is increasing and concave up. **1**

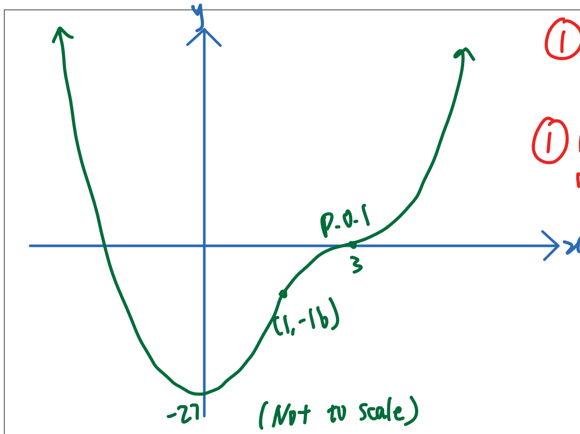
$(0, 1)$ and $(3, \infty)$

(b) State the nature of the stationary points of $f(x)$. **1**

$(0, -27)$ is a min

$(3, 0)$ is a horizontal P.O.I

(c) Sketch a possible graph of $y = f(x)$, using the properties in the table above. **2**



① Mark for correct shape

① Mark for labelling min, and $(1,-16)$ and $(3,0)$

Question 28 (5 marks)

A particle starts from a fixed point A and travels in a straight line. As it passes a point B, the brakes are applied and the particle slows down at a constant rate of $\frac{1}{3}$ m/s^2, coming to a rest at point C. For the motion from A to B, the velocity, v m/s, of the particle, t s after leaving A, is given by $v = 1 + 3e^{\frac{1}{2}(t-4)}$ for $0 \le t \le 4$.

(a) Find the time taken for the object to reach a velocity of 3 m/s. **2**

$1+3e^{\frac{1}{2}(t-4)} = 3$

$e^{\frac{1}{2}(t-4)} = \frac{2}{3}$

$\frac{1}{2}(t-4) = \log_e \frac{2}{3}$

$t = 2(\log_e \frac{2}{3} + 2)$

$\approx 3.19s$

① Mark for making the power the subject.

① Mark for correct ancwer

(b) Find the initial acceleration of the particle. **2**

$V = 1+3e^{\frac{1}{2}t-2}$

$a = \frac{3}{2}e^{\frac{1}{2}t-2}$

when $t=0$,

$a = \frac{3}{2}e^{-2}$

or

$\approx 0.203 m/s^2$

① Mark for finding acceleration

① Mark for correct answer.

Exact value or decimal accepted.

(c) Find the velocity of the particle at point B. **1**

at $t=4$ $V = 1+3e^0$

$V = 4m/s$

1

Question 29 (7 marks)

The water depth, y in metres, in a lake can be modelled by the function $y = 5 + 3\cos\left(\frac{\pi t}{6}\right)$, where t is the time in hours after 6:00 am.

(a) Determine the time between successive high tides as well as the maximum and minimum depth. **2**

period $= \frac{2\pi}{\frac{\pi}{6}} = 12$ hours

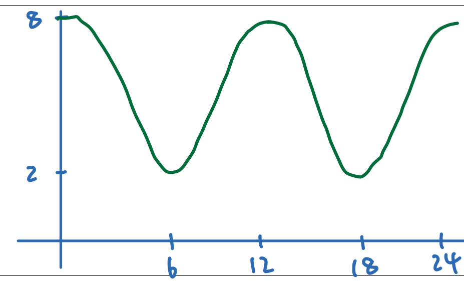
$\therefore$ max $= 5+3 = 8$

min $= 5-3 = 2$

① Mark for period

① Mark for max and min.

(b) Sketch the graph of $y(t)$ for $0 \le t \le 24$ **2**



① Mark for correct shape.

① Mark for correct turning points.

(c) A boat is to sail from a dock to a canal and then out to the main lake. An overhead bridge 7 metres above the bottom of the lake obstructs the boat's exit from the dock. The boat itself has a height of three metres from the waterline to its highest point. What is the earliest time that the boat can leave the dock? Give your answer correct to the nearest minute. **3**

Water level at most : $7-3=4m$

$5+3\cos(\frac{\pi t}{6}) = 4$

$\cos(\frac{\pi t}{6}) = -\frac{1}{3}$

$\frac{\pi t}{6} = 1.91..$

$t = 3.649..$ hrs $= 3$ hours $\frac{65}{100} \times 60$ mins

$= 6:00 + 3:39$

$= 9:39$ am

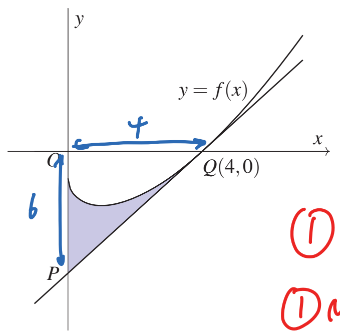
① Mark for correct equation

① Mark for correct t-value

① Mark for correct answer.

Question 30 (7 marks)

The diagram below shows part of a curve $y = f(x)$. The curve is such that $f'(x) = x^{\frac{1}{2}} - x^{-\frac{1}{2}}$ and it passes through the point $Q(4,0)$. The tangent at Q meets the $y-$axis at the point P.



① Mark for correct integration

① Mark for correct final equation.

(a) Find $f(x)$. **2**

$\int x^{\frac{1}{2}} - x^{-\frac{1}{2}}\,dx$

$f(x) = \frac{2}{3}x^{3/2} - 2x^{1/2} + C$

when $y = 0,\ x = 4$

$0 = \frac{2}{3}(4)^{3/2} - 2(4)^{1/2} + C$

$C = -4/3$

$\therefore f(x) = \frac{2}{3}x^{3/2} - 2x^{1/2} - \frac{4}{3}$

(b) Show that the $y-$coordinate of P is -6. **2**

$f'(x) = x^{1/2} - x^{-1/2}$

$m = 4^{1/2} - 4^{-1/2}$

$m = 2 - \frac{1}{2} = \frac{3}{2}$

$\therefore y = \frac{3}{2}(x-4)$

let $x = 0$ for y-int:

$y = \frac{3}{2}(0-4)$

$y = -6$

① Mark for gradient.

① Mark for finding equation and subsequently y-int.

(c) Find the area of the shaded region. **3**

① Mark for area of triangle

① Mark for correct integration

① Mark for correct final answer

$\text{Area}_{POQ} = \frac{1}{2} \times 6 \times 4 = 12$

$\text{Area}_{\text{curve}} = \left| \int_0^4 \frac{2}{3}x^{3/2} - 2x^{1/2} - \frac{4}{3}\,dx \right|$

$= \left| \left[\frac{4}{15}x^{5/2} - \frac{4}{3}x^{3/2} - \frac{4}{3}x \right]_0^4 \right|$

$= \left| \left(\frac{4}{15}(4)^{5/2} - \frac{4}{3}(4)^{3/2} - \frac{4}{3}(4) \right) - 0 \right| = \left| -\frac{112}{15} \right| = \frac{112}{15}$

$\therefore 12 - \frac{112}{15} = \frac{68}{15}$ #

Question 31 (8 marks)

In a 100 m race, the speeds, S_I m/s and S_H m/s, of two animals, an iguana and a hippopotamus respectively can be modelled by the functions.

$$S_I = \frac{6400}{953125}\left(\frac{t^3}{3} - \frac{77}{4}t^2 + 325t\right), \text{and}$$

$$S_H = \frac{2453}{1000}te^{-0.08t},$$

Where t is the time measured from the start in seconds.

It is known that the iguana finishes the race in 12.5 seconds. [Do Not Prove This]

(a) Use the trapezoidal rule with 5 sub-intervals to estimate the distance covered by the hippopotamus in 12.5 seconds. **2**

t	0	2.5	5	7.5	10	12.5
S_H	0	5.0209	8.2215	10.0968	11.0220	11.2801

$\int_0^{12.5} S_H \approx \frac{2.5}{2}\left[0 + 11.2801 + 2(5.0209 + 8.2215 + 10.0968 + 11.0220)\right]$

≈ 100.003125

① Mark for attempt at using trapezoidal rule

① Mark for correct answer

Question 30 continues on page 28

Question 30 (continued)

(b) Find $\frac{d^2S_H}{dt^2}$ and determine whether the hippopotamus finishes ahead of the iguana. **2**

$S_H = \frac{2453}{1000} t e^{-0.08t}$

① Mark for finding $\frac{d^2S_H}{dt^2}$

① Mark for correct solution.

$\frac{dS_H}{dt} = \frac{2453}{1000} e^{-0.08t} - \frac{2453}{12500} t e^{-0.08t}$

$\frac{d^2S_H}{dt^2} = \frac{(4906t - 61325)e^{-0.08t}}{312500} - \frac{2453}{12500} t e^{-0.08t}$

$\frac{d^2S_H}{dt^2} = \frac{(2453t - 61325)e^{-0.08t}}{156250}$

$\frac{d^2S_H}{dt^2} = 0$ at $t = 25$

$\frac{d^2S_H}{dt^2} < 0$ when $t < 25$

$\therefore S_H$ is an underestimate. H covers more than 100m in 12.5 seconds.

H finishes ahead of I.

Question 30 continues on page 28

Question 30 (continued)

(c) Differentiate $\left(\ln\left(\frac{t+2}{2}\right)\right)^2$ **2**

$\frac{d}{dt}\left(\ln\left(\frac{t+2}{2}\right)\right)^2$

① Mark for attempt at the chain rule

$= 2\ln\left(\frac{t+2}{2}\right) \times \frac{\frac{1}{2}}{\frac{t+2}{2}}$

① Mark for correct solution.

$= 2\ln\left(\frac{t+2}{2}\right) \times \frac{1}{t+2}$

$= \frac{2\ln\left(\frac{t+2}{2}\right)}{t+2}$

Question 30 continues on page 29

Question 30 (continued)

In the same race, the speed S_C m/s, of a cow is modelled by

$$S_C = \frac{50[\ln(t+2) - \ln 2]}{t+2}$$

(d) Determine the finishing position of all 3 animals. **2**

$$\int_0^{12.5} \frac{50[\ln(t+2) - \ln 2]}{t+2}\, dt$$

$$= \int_0^{12.5} \frac{50\left[\ln\left(\frac{t+2}{2}\right)\right]}{t+2}\, dt$$

$$= 25\int_0^{12.5} \frac{2\left[\ln\left(\frac{t+2}{2}\right)\right]}{t+2}\, dt$$

$$= 25\left[\left[\ln\left(\frac{t+2}{2}\right)\right]^2\right]_0^{12.5}$$

$$= 25\left[(\ln 7.25)^2 - (\ln 1)^2\right]$$

$$= 98.1092$$

$\therefore$ Cow covers 98.1092m in 12.5s

$\therefore$ H finishes first, I second, and C last.

(1) Mark for correctly integrating $\int \frac{[\ln(t+2) - \ln 2]}{t+2}$

(1) Mark for proving and identifying that Cow finishes last.

End of Question 30

End of Paper