Write your student exam number in the boxes								

Northern Beaches Secondary College - Manly Campus

2025 Higher School Certificate Trial Examination

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/ or calculations

Total Marks:

100

Section I – 10 marks (pages 3-6)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 7-42)

- Attempt Questions 11–34
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks

Attempt Questions 1 – 10

Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1–10.

1. An amount of \$6000 is invested at an interest rate of 3% per annum. What is the future value of the investment after five years if the interest is compounded monthly?

   A. \$6075.38

   B. \$6900.00

   C. \$6955.64

   D. \$6969.70

2. Which of the following functions is neither even nor odd?

   A. $f(x) = \sin(x) - x$

   B. $f(x) = \cos(x) + 2x$

   C. $f(x) = \tan(3x) + x^3$

   D. $f(x) = \sin(x^2 + 4)$

3. Which of the following is equivalent to $\log_a\left(\frac{2xy^3}{\sqrt{z}}\right)$?

   A. $\log_a 2x + (\log_a y)^3 - \sqrt{\log_a z}$

   B. $\frac{\log_a 2x \log_a y^3}{\log_a z^{\frac{1}{2}}}$

   C. $2\log_a x + \log_a y^3 - \log_a \sqrt{z}$

   D. $\log_a 2x + 3\log_a y - \frac{1}{2}\log_a z$

4. A deck of cards contains 6 black cards and 5 red cards. Oliver randomly draws two cards at the same time.
Given that one of the cards Oliver has drawn is red, what is the probability that the other card he has drawn is also red?

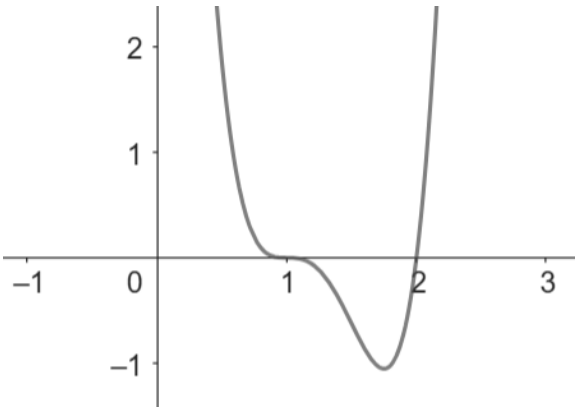
A. $\frac{1}{4}$

B. $\frac{1}{3}$

C. $\frac{2}{5}$

D. $\frac{2}{11}$

5. In the diagram, the graph of $y = f'(x)$ is given.



Which of the following is correct for the graph of $y = f(x)$?

A. It has a local maximum at $x = 1$

B. It has a local minimum at $x = 1$

C. It has a local maximum at $x = 2$

D. It has a horizontal point of inflection at $x = 1$

6. What is $\int (\pi + \sin(\pi x))dx$?

A. $\frac{1}{2}\pi^2 - \pi\cos(\pi x) + C$

B. $\pi x + \frac{1}{\pi}\cos(\pi x) + C$

C. $\pi x - \frac{1}{\pi}\cos(\pi x) + C$

D. $\pi x - \pi\cos(\pi x) + C$

7. What is the range of the function $f(x) = \sqrt{-3x^2 + 6x + 5}$?

A. $(0, \infty)$

B. $[-0.6, 2.6]$

C. $\left[0, 2\sqrt{2}\right]$

D. $(0, 2.6]$

8. The graph of $y = \frac{1}{x-2} + 1$ is reflected about the x-axis. This is then translated two units vertically downwards.
What is the equation of the resulting graph?

A. $y = \frac{-3x + 5}{x - 2}$

B. $y = \frac{-3x - 7}{x + 2}$

C. $y = \frac{x + 1}{x + 2}$

D. $y = \frac{x - 3}{x - 2}$

9. Let $0 < q < p$. If p and q are the 1st and 2nd terms of a geometric progression. What is the limiting sum?

A. $\frac{p}{1-q}$

B. $\frac{pq}{q-p}$

C. $\frac{p^2}{p+q}$

D. $\frac{p^2}{p-q}$

10. In a geometric sequence $T_3 = -45$ and $S_5 = -305$ and $S_6 = 910$.
What is the 9th term in the sequence.

A. -32805

B. -10935

C. 10935

D. 32805

End of Section I

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Mathematics Advanced

Section II Answer Booklet 1

Section II

90 marks
Attempt questions 11 – 34.
Allow about 2 hours and 45 minutes for this section.

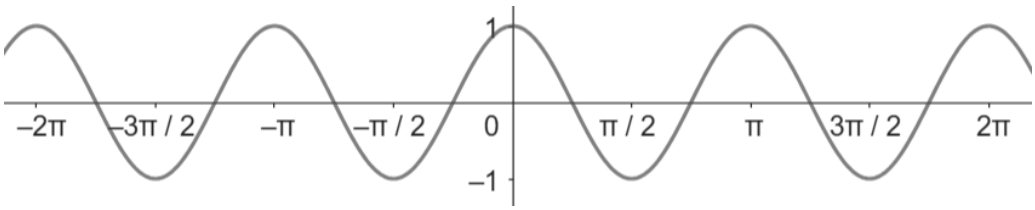
Booklet 1 – Attempt Questions 11 – 17 (22 marks)
Booklet 2 – Attempt Questions 18 – 23 (22 marks)
Booklet 3 – Attempt Questions 24 – 29 (23 marks)
Booklet 4 – Attempt Questions 30 – 34 (23 marks)

Instructions

- Write your Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the end of this booklet. If you use this space, clearly indicate which question you are answering.

Question 11 (2 marks)

State the amplitude and period of the trigonometric graph below.



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Question 12 (3 marks)

The function $f(x) = 3\ln(x+1) + 2$ undergoes the following transformations in the order given to form a new function $g(x)$.

1) Horizontal translation 2 units to the right,
2) Vertical translation 3 units up,
3) Vertical dilation by a factor of 2.

By showing these transformations, find the algebraic expression for $g(x)$. **3**

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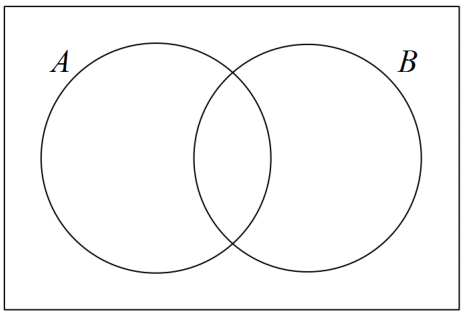
Question 13 (2 marks)

(a) The Venn diagram shows two events, A and B, which are not mutually exclusive. Shade the portion of the diagram corresponding to $\overline{A} \cup B$.



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(b) If $P(A) = 0.3$, $P(B) = 0.5$, and $P(A \cap B) = 0.2$, find $P\left(\overline{A} \cup B\right)$.

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Question 14 (2 marks)

The probability distribution for discrete random variable X is shown in the table.

x	0	1	2	3	4
$P(X = x)$	0.1	0.15	0.2	0.25	0.3

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Find the expected value and variance of X.

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Question 15 (2 marks)

The table shows the future values of an annuity of $1 for different interest rates for 7, 8 and 9 years. The contributions are made at the end of each year.

Future value of an annuity of $1

Years	*Interest rate per annum*			
	2.0%	*2.5%*	*3.0%*	*3.5%*
7	7.434	7.547	7.662	7.779
8	8.583	8.736	8.892	9.052
9	9.755	9.955	10.159	10.368

(a) What is the future value of an annuity based on a contribution of $600 per year for eight years at 3% per annum?

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(b) What is the yearly amount that needs to be contributed to provide a future value of $15 000 after seven years at 2.5% per annum?

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Question 16 (4 mark)

Solve $\log_2(1-x)+\log_2(3-x)=3$. **4**

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Question 17 (7 marks)

For the curve $y = x^4 - 2x^2$.

(a) Find any stationary points and determine their nature. **3**

Question 17 continues on the next page

Question 17 (continued)

(b) Find any points of inflection. **2**

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(c) Sketch the curve showing all stationary points and inflection points. **2**

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Mathematics Advanced

Section II Answer Booklet 2

Section II

90 marks
Attempt questions 11 – 34.
Allow about 2 hours and 45 minutes for this section.

Booklet 1 – Attempt Questions 11 – 17 (22 marks)
Booklet 2 – Attempt Questions 18 – 23 (22 marks)
Booklet 3 – Attempt Questions 24 – 29 (23 marks)
Booklet 4 – Attempt Questions 30 – 34 (23 marks)

Instructions

- Write your Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
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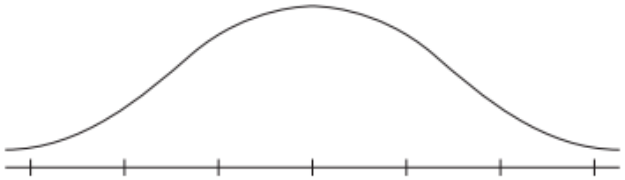
Question 18 (4 marks)

The mean and standard deviation of the test marks of a class are μ and σ respectively.

The table below shows the test marks and the standard z-scores of 2 students, Chloe and Tiffany.

	Test mark	Z-score
Chloe	42	-0.5
Tiffany	60	1

(a) By labelling the normal distribution curve below, or otherwise, find the values of μ and σ. **2**



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(b) The result of a student who got 48 was not included in the calculations. How will the z-scores of Chloe and Tiffany change when the missing mark is now included? **2**

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Question 19 (3 marks)

Find $\int_1^4 \frac{4x-2}{x^2-x+3}\,dx$ in its simplest exact form. **3**

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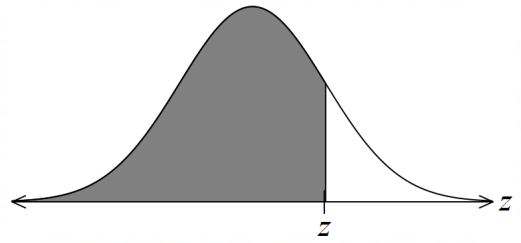
Question 20 (4 marks)

Data on a random sample of 1000 melons grown on a farm using a particular brand of chemical fertilizer is collected by an agricultural scientist. The mean melon weight was 3400 grams with a standard deviation of 530 grams. This data is normally distributed.

The following table gives the probability that a normally distributed random variable with a mean of 0 and a standard deviation of 1 lies below z for some positive values of z.

z	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
%	0.841	0.864	0.885	0.903	0.919	0.933	0.945	0.955	0.964	0.971

The shaded part of the diagram below illustrates the percentage given by the table.



(a) Melons weighing below 2500 grams are considered underweight which indicates that they cannot be sold. **2**

Use z-scores and calculations to show that 4.5% of the melons are expected to be underweight.

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Question 20 continues on the next page

Question 20 (continued)

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(b) In a later study, weights of melons grown using a different fertilizer were found to have a mean of 3220 grams with a standard deviation of 600 grams.
By referring to the data, justify a recommendation to a farmer whether they should consider using this type of fertilizer, rather than the one used in the first study.

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Question 21 (3 marks)

Solve $3\tan x = \tan x \sec^2 x$ for $0 \leq x \leq 2\pi$. Give your answers correct to 2 decimal places. **3**

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Question 22 (3 marks)

If $y = \ln\left(\frac{\cos x}{1-\sin x}\right)$ show that $\frac{dy}{dx} = \sec x$. **3**

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Question 23 (5 marks)

A probability density function is defined by:

$$f(x)=\begin{cases} b & 2\leq x\leq 5 \\ 2b & 5<x\leq 7 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of b. **1**

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(b) Sketch the probability density function. **2**

(c) Find an expression for the cumulative density function. **2**

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Mathematics Advanced

Section II Answer Booklet 3

Section II

90 marks
Attempt questions 11 – 34.
Allow about 2 hours and 45 minutes for this section.

Booklet 1 – Attempt Questions 11 – 17 (22 marks)
Booklet 2 – Attempt Questions 18 – 23 (22 marks)
Booklet 3 – Attempt Questions 24 – 29 (23 marks)
Booklet 4 – Attempt Questions 30 – 34 (23 marks)

Instructions

- Write your Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
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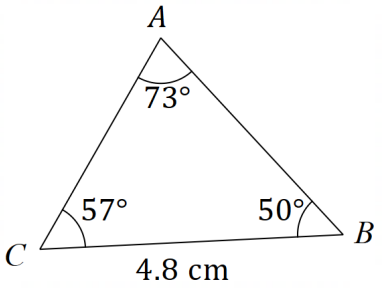
Question 24 (3 marks)

Find the area of the triangle *ABC,* correct to two decimal places.

NOT TO SCALE



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Question 25 (3 marks)

Find, in exact form, the equation of the tangent to the curve $y = 4^x$ at $x = \frac{1}{2}$. **3**

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Question 26 (5 marks)

(a) Differentiate $x^2 \ln x$.

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(b) Hence, find $\int x \ln x \, dx$.

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Question 27 (3 marks)

A virus infects 120 people on the first day. On each following day, it infects 10 more new people than it infected the previous day. That is, it infects 130 new people on the second day, 140 new people on the third day and so on.

How many days will it take for the virus to infect a total of more than 10 000 people? **3**

Question 28 (6 marks)

A worker invests \$$M$ into an account at the beginning of each month. The account earns 3% p.a. interest, compounded monthly.

(a) Show that the total amount in the account at the end of the second month, A_2, is given by

$$A_2 = M \times (1.0025 + 1.0025^2) .$$

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(b) Show that the amount in the account after n months, A_n is given by

$$A_n = 401M(1.0025^n - 1) .$$

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Question Continued overpage.

(c) Find the amount ($\$M$) that needs to be invested at the beginning of each month so that the worker has \$500000 after investing for 25 years. **2**

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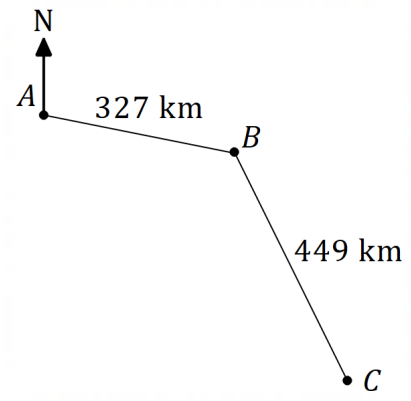
Question 29 (3 marks)

A plane is to make a trip between three islands. Balsam Island is on a bearing of 097° from Aster Island. Calendula Island is on a bearing of 132° from Aster Island.

The plane flies 327 km straight from Aster Island to Balsam Island. It then flies 449 km straight to Calendula Island. The plane's path can be represented with the diagram below.

NOT TO SCALE



What is the bearing of Calendula Island from Balsam Island, to the nearest degree? **3**

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Mathematics Advanced

Section II Answer Booklet 4

Section II

90 marks
Attempt questions 11 – 34.
Allow about 2 hours and 45 minutes for this section.

Booklet 1 – Attempt Questions 11 – 17 (22 marks)
Booklet 2 – Attempt Questions 18 – 23 (22 marks)
Booklet 3 – Attempt Questions 24 – 29 (23 marks)
Booklet 4 – Attempt Questions 30 – 34 (23 marks)

Instructions

- Write your Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the end of this booklet. If you use this space, clearly indicate which question you are answering.

Question 30 (2 marks)

A sequence is defined recursively as follows.

$T_1 = 256 \qquad T_n = \frac{T_{n-1}}{4}$

(a) List the first four terms of the sequence. **1**

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(b) Find the limiting sum of the sequence, if it exists. If it does not exist, explain why it does not. **1**

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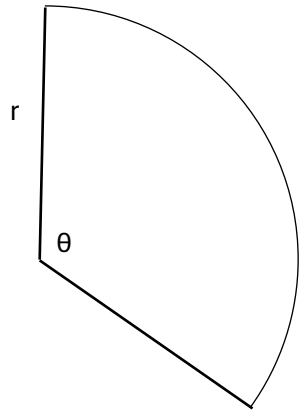
Question 31 (6 marks)

A sector of a circle of radius, r and included angle, θ has a perimeter of 48 centimetres.



(a) Show that the area of the sector is $A = r(24 - r)$. **2**

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(b) Find the radius, r and included angle, θ of the sector if the area is a maximum. **4**

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Question 32 (6 marks)

The number of daylight hours varies throughout the course of a year and can be modelled with a function of the form

$$H(t) = k\cos(a(t+b)) + c$$

where H is the number of hours of daylight and t is the number of days from the 1st of January.

In Canberra, the day with the shortest daylight hours has 10.7 hours of daylight.
The day with the longest daylight hours has 15.5 hours of daylight.

(a) Given that

$$H(t) = k\cos\left(\frac{2\pi}{365}(t+10)\right) + c$$

find the values of k and c needed to complete the equation of the function $H(t)$ that gives the number of daylight hours t days from the 1st of January.
(Assume one year is 365 days.)

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Question Continued overpage.

(b) A nature photographer needs at least 13 hours of daylight to take his photographs. Over what days of the year will the photographer be able to take photographs of nature? Answer in number of days from the 1st of January. **4**

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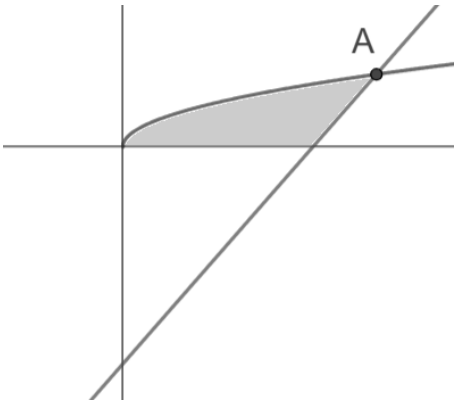
Question 33 (5 marks)

The graphs of $y = 2x - 3$ and $y = \sqrt{\frac{x}{2}}$ intersect at point A as shown in the diagram.



(a) Find the coordinates of point A. **2**

Question Continued overpage.

(b) Hence, find the area of the shaded region. **3**

Question 34 (4 marks)

A block of ice is removed from the refrigerator. The amount M grams of ice remaining after t minutes is given by the equation $M = M_0 e^{-kt}$ where M_0 is the initial amount of ice.

(a) After 35 minutes, only half of the ice remains. Find the value of k to 3 significant figures. **2**

Question Continued overpage.

(b) After how many hours and minutes will 5% of the original amount of ice remain? **2**

End of Paper

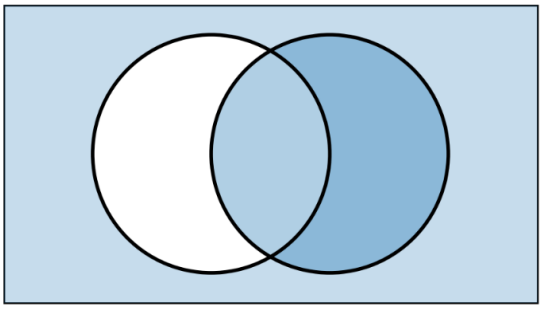
Multiple Choice

1	$6000\left(1+\frac{0.03}{12}\right)^{5\times12} = 6969.70$	D
2	$y = Cosx$ is an even and $y = 2x$ is an odd function, hence their sum is neither.	B
3	$\log_a\left(\frac{2xy^3}{\sqrt{z}}\right) = \log_a 2x + \log_a y^3 - \log_a z^{\frac{1}{2}}$ $= \log_a 2x + 3\log_a y - \frac{1}{2}\log_a z$	D
4	$P(A\|B) = P\frac{(A\cap B)}{P(B)}$ $P(R\ R) = \frac{5}{11}\times\frac{4}{10} = \frac{2}{11}$ P(at least one *red*) = 1 – *no* red $= 1 - \frac{6}{11}\times\frac{5}{10} = \frac{8}{11}$ $\therefore \quad = \frac{\frac{2}{11}}{\frac{8}{11}} = \frac{2}{8} = \frac{1}{4}$ $\therefore$ A	A
5	at $x = 1$ Grad is + 0 – so max at $x = 2$ Grad is – 0 + so min *D* is a distraction, inflection of Grad function does not mean a horizontal pt of inflection $\therefore$ A	A
6	Using reference sheet and remembering to divide by pi answers is C	C
7	$\sqrt{-3x^2+6x+5} = \sqrt{-3(x^2-2x+1)+8}$ $= \sqrt{-3(x-1)^2+8}$ is concave down with vertex (1,8). the $\sqrt{}$ only exists for $y \geq 0$, in this case $0 \leq y \leq 8$ so the range is $\sqrt{0} \leq y \leq \sqrt{8}$ $0 \leq y \leq 2\sqrt{2}$	C

8	$y = \frac{1}{x-2} + 1$ refect in $x$axis $\rightarrow y = -\left(\frac{1}{x-2} + 1\right)$ shift down 2 $\rightarrow y = -\left(\frac{1}{x-2} + 1\right) - 2$ $= -\frac{1}{x-2} - 1 - 2$ $= -\frac{1}{x-2} - 3$ $-\frac{1}{x-2} - \frac{3(x-2)}{x-2}$ $= \frac{-1-3x+6}{x-2}$ $= \frac{-3x+5}{x-2}$	A
9	$a = p$ $r = \frac{q}{p}$ $S = \frac{a}{1-r}$ $= \frac{p}{\left(1-\frac{q}{p}\right)}$ $= \frac{p^2}{p-q}$	D

10	$T_6 = S_6 - S_5$ $= 910 - (-305)$ $= 1\,215$ $T_n = ar^{n-1}$ $T_3 = ar^2$ $-45 = ar^2$ ① $T_6 = ar^5$ $1\,215 = ar^5$ ② ② ÷ ① $\frac{1\,215}{-45} = \frac{ar^5}{ar^2}$ $r^3 = -27$ $T_9 = ar^8$ $= ar^5 \times r^3$ $= 1\,215 \times (-27)$ $= -32\,805$	A

Booklets

11	Amplitude = 1, period = π	Mostly done well. Some attempts at applying $\frac{2\pi}{n}$ did not succeed.
12	$g(x) = \left(3\ln((x+1)-2)+2+3\right) \times 2$ $g(x) = 6\ln(x-1)+10$	Mostly done well if order of the transformations were followed. Some errors due to dilation factor of 2 not multiplied by all terms in the function. Showing each of the transformations in the order given gained a mark.
13	a)  b) $P(\overline{A} \cup B) = 1 - P(only A)$ $= 1 - (0.3 - 0.2)$ $= 0.9$	a) mostly shaded correct. b) mostly done well, especially if the given information was correctly included in the diagram. CEF mark given if the shading was incorrect in part a)
14	$E(X) = 2.5$, V(X)=1.75	Mostly done well, especially using the calculator. Some errors due to incorrect value for mean in finding variance.
15	a) $600 \times 8.892 = \$5\,335.20$ b) $15\,000 \div 7.547 = \$1\,987.54$	Mostly done well. These working outs are required. Any indication of the cell used in

		the table was given the mark. Bold answers do not get marks.
16	$x<1$ and $x<3$, hence x<1 $\log_2(1-x)(3-x)=3$ 1 mark for log law $(1-x)(3-x)=2^3$ 1 mark for correct eq $x^2-4x+3=8$ $x^2-4x-5=0$ $(x+1)(x-5)=0$ $x=-1$, x=5 1 mark for correct values but $x<1$, $\therefore x=-1$ only solution *OR* test the x-values to discard $x=5$ 1 mark for discarding with reason	Most solutions applied the correct index law. Some solutions had 3^2 in stead of 2^3 hence lost a mark. Most solutions solved the quadratic with both solutions, however, not discarding of *x=5* cost them a mark. Discarding *x=5* without a reason did not get the last mark of the question. If the reason for discarding 5 was that *x<3* the mark was not awarded, as in that case *x=1* or *2* will be acceptable which is incorrect.
17	a) $y=x^4-2x^2$ $\frac{dy}{dx}=4x^3-4x$ $=4x(x^2-1)$ $\frac{dy}{dx}=0$, x=-1,0,1 $f''(x)=12x^2-4$ $f''(-1)=8>0$ *concave* up $\therefore$ (-1,-1) min turning point $f''(0)=-4<0$ *concave* down $\therefore$ (0,0) min turning point $f''(1)=8>0$ *concave* up $\therefore$ (1,-1) min turning point Or use the first derivative table, with values for *x* and *y'*.	Most solutions applied a correct method to find the nature of the turning points. One mark for each point, and showing nature correctly. This time not stating concavity for nature of point, was not deducted. This may not be the case in the HSC.

b)

$f''(x) = 0$

$12x^2 - 4 = 0$

$x = \pm\frac{1}{\sqrt{3}}$

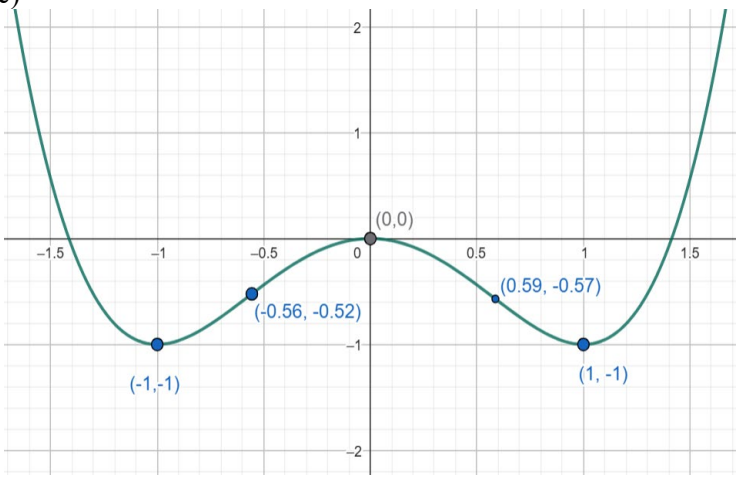
$f\left(\pm\frac{1}{\sqrt{3}}\right) = -\frac{5}{9}$

x	0	$\frac{1}{\sqrt{3}}$	1
$f''(x)$	-4	0	8
	Concave down		Concave up

x	-1	$-\frac{1}{\sqrt{3}}$	0
$f''(x)$	8	0	-4
	Concave up		Concave down

Since concavity changes, both points are points of inflection.

c)



Most solutions found the x-coordinates of the points of inflection, however, did not check change of concavity.

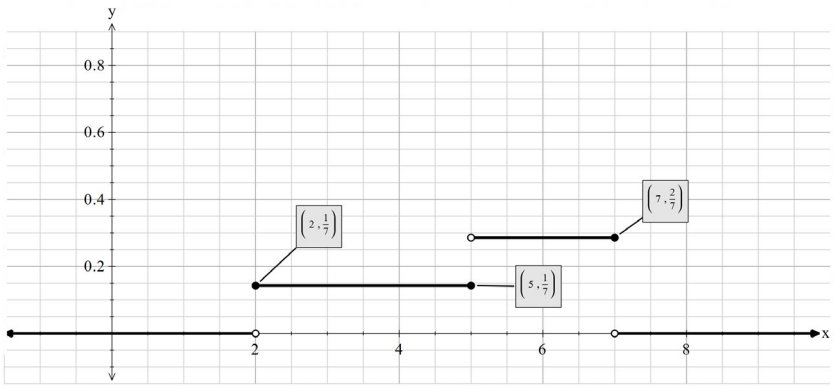
Some solutions indicated the points are between a max and min (or min and max) hence concavity changes resulting in a point of inflection. The mark was given however, note that the answer must include that the function is continuous for this reasoning to be used successfully.

1 mark for points and 1 mark for correct shape. Note that the scale on each of the axes must be consistent, however, not the same. Use the space well.

18 a	using z scores from 1 to -0.5 = $1.5z$ $z = \frac{x-\mu}{\sigma}$ $1.5z = 60 - 42$ $1.5z = 18$ $\sigma = 12$ since $x = 60$ is 1σ above mean $\therefore \quad \mu = 60 - 12 = 48$	2marks both correct 1 mark if only 1 correct
18 b	Since mean = 48 the mean stays the same And the standard deviation reduces because the extra score is at the mean So **Tiffiany's z score increases** because $(60-48) \div$ by a smaller value gets larger But **Chloe's z score decreases** because $(42-48) \div$smaller value becomes more negative so it actually redu e.g if $\sigma < 12$ say 10 then $-\frac{6}{10}$ = -0.6 which is smaller then -0.5	2 marks for both correct. 1 mark for 1 person correct
19	$\int_1^4 \frac{4x-2}{x^2-x+3}\,dx$ $= 2\int_1^4 \frac{2x-1}{x^2-x+3}\,dx$ $= 2\left[\log_e(x^2-x+3)\right]^4,1$ $= 2(\log_e(15) - \log_e(3))$ $= 2\log_e(5)$ or $\log_e(25)$	3marks correct solution in simplified from 2 marks for 2nd last line or equivalent 1 mark for correct integration
20 a	$z = \frac{x-\mu}{\sigma}$ $z = \frac{2\,500 - 3\,400}{530}$ $z = -1.698$ $z \approx -1.7$ $\therefore 95.\% < 1.7z$ or $4.5\% > 1.7z$ by symmetry of normal distribution $4.5\% < -1.7z$	2 marks correct solution with z=-1.7 and showing why z=-1,7 = z=1.7 1 mark for getting z=-1.7 only with no proper reasons. Note: Just saying z=1.7 gives 1-0.955 is **NOT enough.**

20 b	$\because z = \frac{x-\mu}{\sigma}$ $z = \frac{2\,500 - 3\,200}{600}$ $z = -1.2$ $\therefore 88.5\% < 1.2z$ and $11.5\% > 1.2z$ $\therefore$ using symmetry of normal distribution $11.5\% < -1.2z$ hence No, would not recommend this fertilizer $\because$ more than twice as many would be rejected based on the 2 500g as underweight	2marks correct full solution with 2 full reasons **Just saying mean higher and std deviation higher not enough.** 1 mark for one correctly explained reason
21	$3\tan x = \tan x \sec^2 x$ for $0 \le x \le 2\pi$ $\tan x \sec^2 x - 3\tan x = 0$ using $\sec^2 x = \tan^2 x + 1$ $\tan x(\tan^2 x + 1) - 3\tan x = 0$ $\tan x(\tan^2 x + 1 - 3) = 0$ $\tan x(\tan^2 x - 2) = 0$ $\therefore \quad \tan x = 0$ or $\tan x = \pm\sqrt{2}$ **Note it is radians!** $\tan^{-1}(0) = 0$ $x = 0, \pi, 2\pi$ $\tan^{-1}\left(\sqrt{2}\right) = 0.96$ $x = 0.96, \pi - 0.953.., \pi + 0.953.., 2\pi - 0.95$ $x = 0.96, 2.19, 4.10, 5.33$ (2 Decimal Pl) $x = 0, \pi, 2\pi, 0.96, 2.19, 4.10, 5.33$	**3 marks** correct full solution in radians **2 marks** for 2 parts correct, either all 4 values for tan(x)=sr2, but missing x=0, pi, & 2pi Or only 2 values from tanx=sr2 but has x=0, pi ,2pi or values in degrees 1 mark if at least 2 x values correct
21	OR $\sec^2 x \tan x - 3\tan x = 0$ $\tan x(\sec^2 x - 3) = 0$ $\therefore \quad \tan x = 0$ or $\sec^2 x = 3$ $\tan x = 0$ or $\sec x = \pm\sqrt{3}$ $\tan x = 0$ or $\cos x = \pm\frac{1}{\sqrt{3}}$ $\tan^{-1}(0), \cos^{-1}\left(= -\frac{1}{\sqrt{3}}\right)$ $x = 0, \pi, 2\pi, 0.96, 2.19, 4.10, 5.33$	See above for marking criteria

22	$y = \ln\left(\frac{\cos x}{1-\sin x}\right)$ $= \ln \cos x - \ln(1-\sin x)$ $\frac{dy}{dx} = \frac{-\sin x}{\cos x} - \frac{1}{1-\sin x} \times -\cos x$ $= \frac{-\sin x}{\cos x} + \frac{\cos x}{1-\sin x}$ $= \frac{-\sin x + \sin^2 x + \cos^2 x}{\cos x(1-\sin x)}$ Since $\sin^2 x + \cos^2 x = 1$ $= \frac{1-\sin x}{\cos x(1-\sin x)}$ $= \frac{1}{\cos x}$ $= \sec x$ Or $y = \ln\left(\frac{\cos x}{1-\sin x}\right)$ $\frac{dy}{dx} = \frac{1}{\frac{\cos x}{1-\sin x}} \times \frac{d}{dx}\left(\frac{\cos x}{1-\sin x}\right)$ $= \frac{1-\sin x}{\cos x} \times \frac{(1-\sin x)(-\sin x) - (\cos x)(-\cos x)}{(1-\sin x)^2}$ $= \frac{1}{\cos x} \times \frac{\sin^2 x - \sin x + \cos^2 x}{1-\sin x}$ $= \frac{1}{\cos x} \times \frac{\sin^2 x - \sin x + 1 - \sin^2 x}{1-\sin x}$ $= \frac{1}{\cos x} \times \frac{1-\sin x}{1-\sin x}$ $= \frac{1}{\cos x} \times 1$ $= \sec x$	**3 marks** correct solution **2 marks** if 1 error **1 mark** if only 2 errors, but with working showing how arrived at answer not just writing given info from the question
23 a	Using fact that $PDF = 1$ $\int_2^5 bx\,dx + \int_5^7 2bx\,dx = 1$ $(5b - 2b) + (14b - 10b) = 1$ $3b + 4b = 1$ $7b = 1$ $b = \frac{1}{7}$	1 mark correct solution
23 b		**2 mark**s full correct solution with **open and**

		**closed circles correct.** **1 mark** correct graph lines for 2<=x<=5 and 5<x<=7, but must have correct open and closed circles Carry error possible if correctly used their b value from part a
23 c	See below for expression $$F(x)=\int_2^x \frac{1}{7}\,dx \qquad 2\le x\le 5$$ $=\frac{x}{7}-\frac{2}{7}$ $=\frac{x-2}{7} \qquad 2\le x\le 5$ + $$F(x)=\int_5^x \frac{2}{7}\,dx+\frac{3}{7} \qquad 5<x\le 7$$ (must have the $\frac{3}{7}$ which is the part under curve between $2\le x\le 5$ $=\frac{2x}{7}-\frac{10}{7}+\frac{3}{7}$ $=\frac{2x}{7}-1$ $=\frac{2x-7}{7} \qquad 5<x\le 7$ hence $$F(x)=\begin{cases}\frac{x-2}{7} & 2\le x\le 5\\ \frac{2x-7}{7} & 5<x\le 7\end{cases}$$	**2 marks** full correct solution for both parts of the piecewise function Partial marks for the correct **integration and sub into the expression** **1 mark** for either the correct expression for 2<= x<=5 OR 5<x<=7. **No marks for the 0 or 1**

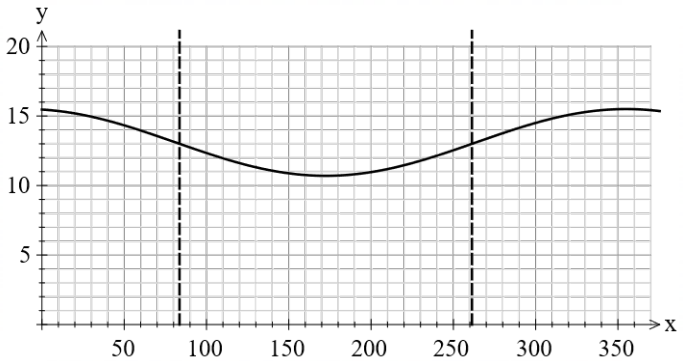
24	$\frac{x}{\sin 50} = \frac{4.8}{\sin 73}$ $x = \frac{4.8 \sin 50}{\sin 73}$ $= 3.845 \text{ cm}$ $A = \frac{1}{2} a b \sin C$ $= \frac{1}{2} \times 4.8 \times 3.845 \sin 57° = 7.74 \text{ cm}^2$ Markers Note – students who rounded to 2dp (or less) in the middle of the calculation end up with the final answer being incorrect, therefore do not have the "correct solution".	3 marks – correct solution 2 marks – attempts to use Area formula to find Area 1 mark – correctly finds a side
25	$y = 4^x$ $\frac{dy}{dx} = \ln 4 \times 4^x$ when $x = \frac{1}{2}$ $\frac{dy}{dx} = \ln 4 \times 4^{\frac{1}{2}}$ $= 2 \ln 4$ and $y = 4^{\frac{1}{2}} = 2$ (not -2) so $m = 2\ln 4$ and point $\left(\frac{1}{2}, 2\right)$ $y = mx + c$ $2 = 2\ln 4 \times \frac{1}{2} + c$ $2 = \ln 4 + c$ $c = 2 - \ln 4$ $y = (2\ln 4)x + 2 - \ln 4$	3 marks – correct solution 2 marks – finds the gradient an y-value at x=1/2 1 mark – differentiates correctly
26 a	$y' = uv' + vu'$ $= x^2 \frac{1}{x} + 2x \ln x$ $= x + 2x\ln x$	2 marks – correct solution 1 mark – progress towards solution

26 b	from a) $\int (x + 2x\ \ln x)dx = x^2 \ln x$ $\int x\,dx + 2\int x\ln x\,dx = x^2 \ln x$ $2\int x\ln x\,dx = x^2 \ln x - \int x\,dx$ $\int x\ln x\,dx = \frac{1}{2}\left(x^2 \ln x - \int x\,dx\right)$ $\int x\ln x\,dx = \frac{1}{2}\left(x^2 \ln x - \frac{x^2}{2}\right) + c$	3 marks – correct solution 2 marks – one element of integral correct.
27	$S = 120 + 130 + 140....$ A.P. with $a = 120$, $d = 10$ $S_n = \frac{n}{2}(2a + (n-1)d)$ $10\,000 = \frac{n}{2}(240 + 10(n-1))$ $20\,000 = 240n + 10n^2 - 10n$ $10n^2 + 230n - 20\,000 = 0$ $n^2 + 23n - 2\,000 = 0$ $n = -\frac{23 \pm \sqrt{23^2 + 4 \times 2\,000}}{2}$ = -57.7 , 34.67 days can't be negative. 35 days to infect more than 10,000 people.	
28 a	A\n=amount at end of month n monthly interest rate = $\frac{3\%}{12} = \frac{0.03}{12} = 0.0025$ $A_1 = M \times 1.0025$ $A_2 = A_1 \times 1.0025 + M \times 1.0025$ $= M \times 1.0025^2 + M \times 1.0025$ $= M(1.0025 + 1.0025^2)$	

28 b	$A_n = M(1.0025 + 1.0025^2 + ... + 1.0025^n)$ $= M\,\frac{a(r^n - 1)}{r - 1}$ where $a = r = 1.0025$ $= M \times 1.0025\,\frac{1.0025^n - 1}{1.0025 - 1}$ $= M \times \frac{1.0025}{0.0025} \times (1.0025^n - 1)$ $= 401M((1.0025^n - 1)$	2 marks – correct solution clearly shown 1 mark – develops GP and uses sum of GP formula
28 c	25 years is 300 months $500\,000 = 401M(1.0025^{300} - 1)$ $M = \frac{500\,000}{401(1.0025^{300} - 1)}$ $= \$1\,118.26$ per month (If you incorrectly did yearly the answer was \$19 358.10 per year)	
29	$\angle BAC$ in $\Delta ABC = 132° - 97° = 35°$ $\frac{\sin \angle BCA}{327} = \frac{\sin 35°}{449}$ $\angle BCA = 24.69°$ $\angle ABC$ in $\Delta ABC = 180 - 35 - 24.69° = 120.3°$ bearing of C from B is $360° - 120.3° - 83° = 156.69° \approx 157°$	3 marks – correct solution 2 marks – finds angle ABC 1 mark – finds angle BCA
30 a	$256, 64, 16, 4$	1 mark correct solution.
30 b	$a = 256, r = \frac{1}{4}$ $S = \frac{a}{1 - r}$ $= \frac{256}{\left(1 - \frac{1}{4}\right)}$ $= \frac{256}{\left(\frac{3}{4}\right)}$ $= \frac{1\,024}{3}$	1 mark correct limiting sum with correct working shown. No mark for a correct answer without working as this answer may have been arrived at using an incorrect process.

31 a	arc length, $l = r\theta$ perimeter, $P = 2r + l$ $48 = 2r + r\theta$ $48 - 2r = r\theta$ $\theta = \frac{48 - 2r}{r}$ ① area of a sector, $A = \frac{1}{2} r^2 \theta$ ② sub ① into ② $A = \frac{1}{2} r^2 \left(\frac{48 - 2r}{r} \right)$ $= r(24 - r)$	2 marks for correct solution showing the required expression with correct working shown. 1 mark for a correct expression for theta or equivalent progress towards the solution with correct working shown.
31 b	$A = 24r - r^2$ $\frac{dA}{dr} = 24 - 2r$ maximum area when $\frac{dA}{dr} = 0$ $0 = 24 - 2r$ $r = 12$ $\frac{d^2A}{dx^2} = -2$ $< 0 \therefore$ concave down $\therefore \quad r = 12$ is a maximum when $r = 12$, $\theta = \frac{48 - 24}{12}$ $= 2$ radians $\therefore$ maximum area when $r = 12$ and $\theta = 2$ radians.	4 marks correct solution with first derivative of A and giving r and theta and proving that the value of r is a maximum with correct working shown. 3 marks for correct solution with first derivative of A and giving r and theta and proving that the value of r is a maximum but with one error or one of these elements of the solution missing with correct working shown. 2 marks for correct solution with first derivative of A and giving r and theta and proving that the value of r is a maximum but with two error or two of these elements of the

		solution missing with correct working shown. 1 mark for correct expression for dA/dr.
32 a	$c = \frac{15.5 - 10.7}{2}$ $= 13.1$ $k = \frac{15.5 - 10.7}{2}$ $= 2.4$	2 marks for c=13.1 and k=2.4 with correct working shown. 1 mark for one of these values correct with correct working or two correct values with no working.

32 b	Solve $H(t) = 13$ $13 = 2.4\cos\left(\frac{2\pi}{365}(t+10)\right) + 13.1$ $-0.1 = 2.4\cos\left(\frac{2\pi}{365}(t+10)\right)$ $\cos\left(\frac{2\pi}{365}(t+10)\right) = -\frac{1}{24}$ Let $\theta = \frac{2\pi}{365}(t+10)$ $\cos\theta = -\frac{1}{24}$ related angle, $\theta = \cos^{-1}\left(\frac{1}{24}\right)$ $\approx 1.529117\ldots$ radians $\theta \approx \pi - 1.529117\ldots$ $\approx 1.61247\ldots$ $\frac{2\pi}{365}(t+10) = 1.61247$ $t = 83.67188\ldots$ days *OR* $\theta \approx \pi + 1.529117\ldots$ $\approx 4.670710\ldots$ $\frac{2\pi}{365}(t+10) = 4.670710$ $t = 261.3288\ldots$ days  From the graph, for $H(t) \geq 13$, $0 \leq t \leq 83$ or $262 \leq t \leq 365$ $\therefore$ The photographer can take nature photographs for t: $[0, 83] \cup [262, 365]$.	4 marks correct solution with whole number of days with correct working. 3 marks correct working to find the two boundary days as 83.671... and 261.328... 2 marks correct working to find the related angle theta=1.529117... or 1 correct value for t. 1 mark correct working to find that cos(theta)=-1/24

33 a	$y = 2x - 3$ ① $y = \sqrt{\frac{x}{2}}$ ② Note $y \geq 0$ sub ① into ② $\sqrt{\frac{x}{2}} = 2x - 3$ $\frac{x}{2} = (2x-3)^2$ $x = 2(4x^2 - 12x + 9)$ $= 8x^2 - 24x + 18$ $0 = 8x^2 - 25x + 18$ $= (x-2)(8x-9)$ $x = 2$ or $x = \frac{9}{8}$ for $x = 2$, $y = 4 - 3$ for $x = \frac{9}{8}$, $y = \frac{9}{4} - 3 = -\frac{3}{4}$ $= 1$ $x = \frac{9}{8}$ is not a solution since $y \geq 0$ $\therefore A(2,1)$	2 marks correct solution with correct working shown. 1 mark finds x=2 or x=9/8 with correct working.
33 b	Area, $A = \int_0^2 \sqrt{\frac{x}{2}}\,dx$ – area of triangle for $y = 2x - 3$, when $y = 0$, $0 = 2x - 3$ $3 = 2x$ $x = \frac{3}{2}$ $\therefore$ triangle has a base of $\frac{1}{2}$ and a height of 1. $A = \frac{1}{\sqrt{2}}\int_0^2 x^{\frac{1}{2}}\,dx - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(1)$ $= \frac{1}{\sqrt{2}}\left[\frac{2}{3}x^{\frac{3}{2}}\right] - \frac{1}{4}$ $= \frac{2}{3\sqrt{2}}\left(2\sqrt{2}\right) - \frac{1}{4}$ $= \frac{4}{3} - \frac{1}{4}$ $= \frac{13}{12}\,units^2$	3 marks correct solution with correct working shown. 2 marks correct integral statement for the shaded area and correct integral found from this statement. 1 mark correct integral statement for the shaded area.

34 a	$M = M_0 e^{-kt}$ at $t = 35$, $M = \frac{1}{2} M_0$ $\frac{1}{2} M_0 = M_0 e^{-35k}$ $\frac{1}{2} = e^{-35k}$ $\ln\left(\frac{1}{2}\right) = -35k$ $k = \frac{-1}{35} \ln\left(\frac{1}{2}\right)$ $= 0.0198042 \ldots$ $= 0.0198 \ (3s.f.)$ Full marks are now awarded here, if correct exact value of k was written, even if decimal value was not given. This was due to the precision mark allocated elsewhere in this paper. Good rule of thumb; writing the calculator value to many decimal places, before rounding your answer.	2 marks correct solution with correct working shown. 1 mark for the expression $1/2M_0 = M_0e^{-35k}$ No loss of mark for incorrect 3s.f. rounding if correct unrounded value shown.
34 b	$5\% = 0.05$ $M = M_0 e^{-kt}$ $0.05\ M_0 = M_0 e^{-kt}$ $\ln(0.05) = e^{-kt}$ $t = \frac{\ln 0.05}{-k}$ $= 151.26748 \ldots$ minutes $= 2$ hours 31 minutes $\therefore$ 5% of original amount of ice will remain after 2 hours and 31 minutes.	2 marks for correct solution in hours and minutes with correct working shown. 1 mark for correct value of t in minutes with correct working shown.