NORTH SYDNEY BOYS HIGH SCHOOL

MATHEMATICS

2010 HSC Assessment Task 3 (Trial HSC)

General instructions

- Working time – 3 hours.
- Commence each new question on a new sheet.
- Write using blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- Board approved calculators may be used.
- All necessary working should be shown in every question.
- Attempt **all** questions.
- At the conclusion of the examination, bundle the sheets used in the correct order within this paper and hand to examination supervisors.

Class (please ✔)

- ◯ 12M21 – Mr Berry
- ◯ 12M22 – Mr Fletcher
- ◯ 12M31 – Mr Ireland
- ◯ 12M32 – Mr Lowe
- ◯ 12M33 – Mr Rezcallah

NAME: .... **# PAGES USED**: ....

Marker's use only.

QUESTION	1	2	3	4	5	6	7	8	9	10	Total	%
MARKS	$\overline{12}$	$\overline{12}$	$\overline{12}$	$\overline{12}$	$\overline{12}$	$\overline{12}$	$\overline{12}$	$\overline{12}$	$\overline{12}$	$\overline{12}$	$\overline{120}$	

Question 1 (12 Marks) Commence a NEW page. **Marks**

(a) Simplify $2\sqrt{11} \times 5\sqrt{11}$. **2**

(b) Solve correct to 2 decimal places: $x^2 - 3x - 7 = 0$ **3**

(c) If **2**

$$f(x) = \begin{cases} x - 2 & \text{for } x < 0 \\ x^2 & \text{for } x \geq 0 \end{cases}$$

Evaluate $f(-1) + f(0) + f(2)$.

(d) Give the exact value of

i. $\sin \frac{7\pi}{6}$. **1**

ii. $\log_2 1$. **1**

(e) A rectangle has sides of $\dfrac{1}{\sqrt{2}+1}$ cm and $\dfrac{1}{\sqrt{2}-1}$ cm. Find

i. its area **1**

ii. its perimeter in simplest surd form. **2**

Question 2 (12 Marks) Commence a NEW page. **Marks**

(a) Differentiate with respect to x:

i. $\dfrac{4x^2 - 3x + 2}{x}$. **2**

ii. e^{3x^2-5}. **2**

iii. $x^3 e^{-x}$. **2**

iv. $\ln(\sin x)$. **2**

(b) Explain, using calculus, why the curve $y = (3x-2)^4$ has no points of inflexion. **2**

(c) Solve for x, $|3 - 2x| \leq 5$. **2**

Question 3 (12 Marks) Commence a NEW page. **Marks**

(a) What is the period and amplitude of the curve $y = 3\cos 2x$? Sketch the curve for $0 \leq x \leq 2\pi$. **4**

(b) Write down a quadratic equation which is negative within the interval $-1 < x < 2$ and non negative for al other values of x. **2**

(c) Find the primitive of:

i. $\dfrac{2x}{x^2+1}$. **2**

ii. $\dfrac{2}{x} + 5e^x$. **2**

iii. $\dfrac{e^{2x}}{e^{2x}+4}$. **2**

Question 4 (12 Marks) Commence a NEW page. **Marks**

(a) The volume V cm^3 of water remainingin a leaking bucket after t seconds is

$$V = 2\,000 - 40t + 0.2t^2$$

i. How much water was there initially in the bucket? **1**

ii. How long does it take to empty the bucket? **1**

iii. How fast is the volume of water in the bucket decreasing when $t = 30$? **1**

(b) Find the value of $8 - 4 + 2 - 1 + 0.5 - 0.25 + \cdots$. **2**

(c) Use Simpson's Rule with five function values to estimate the area bounded by the curve $y = \sqrt{x+2}$, the x axis and the lines $x = 1$ and $x = -1$, correct to 2 decimal places. **3**

(d) Simplify $\dfrac{\sin^2 30°}{1 - \sin^2 30°}$. **2**

(e) Find the coordinates of the point on the curve $y = 2x^2 + 4x - 11$ where the tangent is parallel to the line $y = 8x + 9$. **2**

Question 5 (12 Marks) Commence a NEW page. **Marks**

(a) Find the equation of the normal to the curve $y = x \sin x$ where $x = \frac{\pi}{2}$. **3**

(b) For the quadratic equation $x^2 - (2 + k)x + 3k = 0$, find the value of k if

i. the sum of roots is 5. **1**

ii. the product of roots is 12. **1**

(c) Given that $\sin \theta = \frac{3}{4}$ and θ is acute, find the value of

i. $\tan \theta$. **1**

ii. $\sec \theta$. **1**

iii. $\operatorname{cosec}^2 \theta - \cot^2 \theta$. **1**

(d) For the parabola $y = 5 + 2x - x^2$,

i. Find the equation of the axis of symmetry. **1**

ii. Find the coordinates of the vertex. **1**

iii. Find the focal length. **1**

iv. Sketch the parabola.

Question 6 (12 Marks) Commence a NEW page. **Marks**

(a) What is the equation of the line through $(-1, 3)$ and perpendicular to the line $5x - 2y - 3 = 0$? **2**

(b) For $y = 2x^3 - 3x^2 - 12x + 2$

i. Find all stationary points and determine their nature. **2**

ii. Find the point(s) of inflexion. **2**

iii. Sketch the curve. **2**

(c) Find the volume generated when the area between the curve $y = x^3$ and the line $y = x$ where $x \geq 0$ is rotated about the x axis. **4**

Question 7 (12 Marks) Commence a NEW page. **Marks**

(a) Inflation is still increasing, but with various policies to reduce inflation over the last few years had an effect. Draw a graph to represent this information with I (inflation) against t (time). **2**

(b) Show that $\dfrac{\cos\alpha}{1+\sin\alpha} + \dfrac{1+\sin\alpha}{\cos\alpha} = \dfrac{2}{\cos\alpha}$. **2**

(c) Draw the graph of $y = e^{-x}$. By drawing another graph on the same set of axes, show that $f(x) = e^{-x} - x + 1$ has exactly one root. **2**

(d) If A is $(1,3)$, B is $(5,6)$ and P is (x,y)

i. Find the lengths PA and PB in terms of x and y. **2**

ii. Find the locus of the points whose distance from A is double its distance from B. **2**

(e) The p-th term of a certain geometric progression is c times the first term. **2**

Show that the common ratio r is given by

$$\log r = \frac{\log_c}{p-1}$$

Question 8 (12 Marks) Commence a NEW page. **Marks**

(a) The series $7 + 13 + 31 + \cdots$ has $3^n + 4$ as its n-th term. Write down

i. the seventh term. **1**

ii. the sum of the first n terms. **2**

(b) Prove that the line $4x - y - 8 = 0$ is a tangent to the curve $x^2 = 2y$ and find the point of contact. **3**

(c) Solve the following where $0 \le x \le 2\pi$:

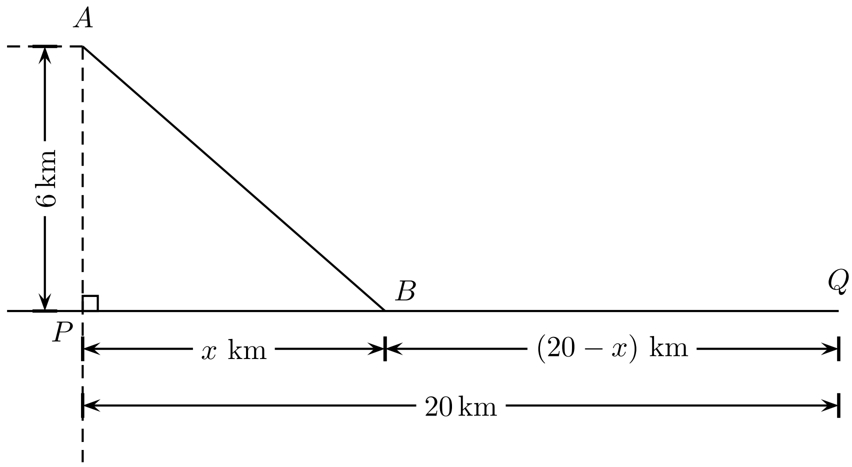
i. $(2\cos x + 1)(\cos x - 1)$. **2**

ii. $3\tan^2 x = 1$. **2**

iii. $2\sin\left(x - \frac{\pi}{3}\right) = -1$. **2**

Question 9 (12 Marks) Commence a NEW page. **Marks**

(a) A courier on a bike is in open country at the point A and is 6 km from the nearest point P on a straight sealed road. He wishes to get as quickly as possible to a point Q on the sealed road 20 km from P. If the maximum speed across the open country is 40 km/h and along the sealed road is 50 km/h,



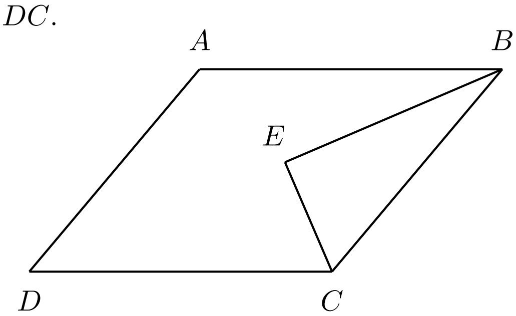
i. Write an expression for the times taken to traverse AB and BQ, if B is a point on the sealed road between P and Q. **2**

ii. Write an expression for the total time T for the journey. **2**

iii. Find the distance from P that he should find the road. **2**

(b) Find the volume of revolution if the curve $y = \sec x$ from $x = 0$ to $x = \frac{\pi}{4}$ is rotated about the x axis. **3**

(c) In the figure, $ABCD$ is a parallelogram (not to scale). EB bisects $\angle ABC$, $\angle ADC = 64°$ and $EC \perp DC$.



i. Draw a diagram showing all the information given. **1**

ii. Calculate the size of $\angle BEC$, giving reasons. **2**

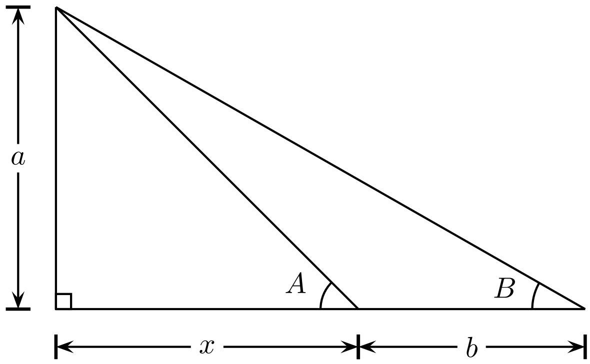
Question 10 (12 Marks) Commence a NEW page. **Marks**

(a) Evaluate $\tan\dfrac{\pi}{3} + \operatorname{cosec}\dfrac{\pi}{4}$. **1**

(b) i. Show that $\dfrac{1}{2x-3} - \dfrac{1}{2x+3} = \dfrac{6}{4x^2-9}$. **2**

ii. Hence find $\displaystyle\int \frac{dx}{4x^2-9}$. **2**

(c) From the diagram, $\tan A - \tan B = 1$.



i. Prove that $x^2 + bx - ab = 0$. **2**

ii. If $a = 3$ and $b = 6$, find x in simplest surd form. **2**

iii. Prove that $\tan A = \dfrac{\sqrt{3}+1}{2}$ and find $\tan B$ in similar form. **2**

iv. Find A and B to the nearest degree. **2**

End of paper.

Q1 $\qquad$ 2u - Task 3 - 2010.

(i) $2\sqrt{11} \times 5\sqrt{11} = 10 \times 11$

$= 110$ ✓✓

(ii) $x^2 - 3x - 7 = 0 \quad \therefore x = \dfrac{3 \pm \sqrt{9 - 4(1)(-7)}}{2}$

$= \dfrac{3 \pm \sqrt{37}}{2}$ ✓

$\doteqdot 4.54138, \ -1.54138$

$= 4.54, \ -1.54 \quad (2.dp)$ ✓✓

(iii) $f(-1) + f(0) + f(2)$

$= (-1-2) + 0^2 + 2^2$ ✓

$= -3 + 4$

$= 1$ ✓

(iv) (a) $\sin \frac{7\pi}{6} = -\sin \frac{\pi}{6}$

$= -\frac{1}{2}$ ✓

(b) $\log_2 1 = 0$ ✓

(v) (a) $A = \dfrac{1}{\sqrt{2}+1} \cdot \dfrac{1}{\sqrt{2}-1}$

$= \dfrac{1}{2 - 1}$

$= 1 \text{ cm}^2$ ✓

(b) $P = 2 \cdot \dfrac{1}{\sqrt{2}+1} + 2 \cdot \dfrac{1}{\sqrt{2}-1}$

$= \dfrac{2}{\sqrt{2}+1} \cdot \dfrac{\sqrt{2}-1}{\sqrt{2}-1} + \dfrac{2}{\sqrt{2}-1} \cdot \dfrac{\sqrt{2}+1}{\sqrt{2}+1}$

$= 2\sqrt{2} - 2 + 2\sqrt{2} + 2 \quad = 4\sqrt{2}.$ ✓✓

/12

(2) i a) $\frac{d}{dx}\left(4x - 5 + 2x^{-1}\right)$

$= 4 - 2x^{-2}$

$= 4 - \frac{2}{x^2}$ 2

b) $\frac{d}{dx}\left(e^{3x^2-5}\right) = 6x e^{3x^2-5}$ 2

c) $\frac{d}{dx}\left(x^3 e^{-x}\right) = 3x^2 e^{-x} - e^{-x}x^3$ 2

$= x^2 e^{-x}(3-x)$

d) $\frac{1}{\sin x} \times \cos x = \cot x$ 2

[cot x is worth 1 mark]

ii/ $y = (3x-2)^4$

$y' = 12(3x-2)^3$

$y'' = 108(3x-2)^2$ 1

As y'' is a square it will never change sign through the derivative $\therefore$ no inflexion 2

iii/ $-5 \leq 3 - 2x \leq 5$

$-8 \leq -2x \leq 2$

$4 \geq x \geq -1$ 2

iv $-1 \leq x \leq 4$

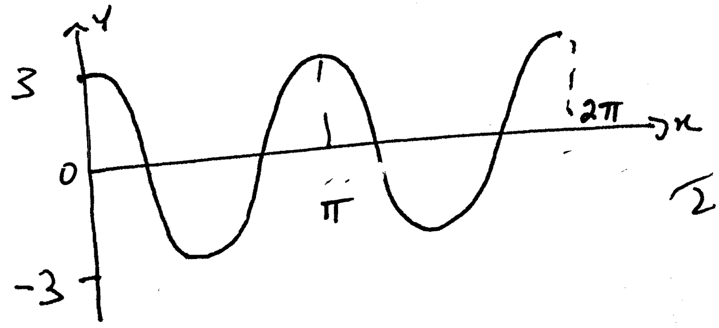
Accepted: $x \geq -1$ and $x \leq 4$.

2(ii) The second mark is for a valid explanation. i.e. $y'' \geq 0$ for all real x $\therefore$ no change in concavity.
OR even testing one point before + one point after.

v/

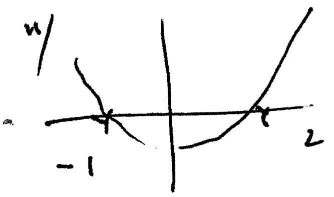
$p = \frac{2\pi}{2} = \pi$. 1

$a = 3$ 1



2

ii/



$y = k(x+1)(x-2)$ 2

iii/ a) $\ln(x^2+1) + c$ 2

b) $2\ln x + 5e^x + c$ 2

c) $\frac{1}{2}\ln(e^{2x}+4) + c$ 2

4) i/ $V = 2000 - 40t + 0.2t^2$

a) $t = 0$ $\underline{V = 2000\,\text{m}^3}$ 1

b) either solve quadratic to get $t = 100$ or $\frac{dV}{dt} = 0$ to get $t = 100$ 1

c) $\frac{dV}{dt} = .4t - 40$ 1

when $t = 30$

$\frac{dV}{dt} = -28$

Decreasing at rate of 28 cm³/s

ii/ $S_\infty = \frac{a}{1-r}$ G.P $r = -\frac{1}{2}$, $a = 8$ 1

$\doteq \frac{8}{3/2} = \underline{\frac{16}{3}}$ 1

iii/

x	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1
$f(x)$	1	$\sqrt{\frac{3}{2}}$	$\sqrt{2}$	$\sqrt{\frac{5}{2}}$	$\sqrt{3}$

1

$A \doteq \frac{1}{6}\left[\sqrt{2}+1+4\sqrt{\frac{3}{2}}\right] + \frac{1}{6}\left[\sqrt{2}+\sqrt{3}+4\sqrt{\frac{5}{2}}\right]$

$\doteq 2.80$ (2dp) 1

(only 2.80 for last mark).

iv/ $\frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$ 2

v/ $y' = 4x + 4$

when $4x + 4 = 8$ → worth 1

$4x = 4$

$x = 1$

∴ at $(1, -8)$ 1

(5) i) $y' = x\cos x + \sin x$

when $x = \frac{\pi}{2}$ $\quad y' = 1 \quad y = \frac{\pi}{2}$

$\therefore$ Normal is

$$y - \frac{\pi}{2} = -1\left(x - \frac{\pi}{2}\right)$$

$$y - \frac{\pi}{2} = -x + \frac{\pi}{2}$$

$$\underline{x + y = \pi}$$

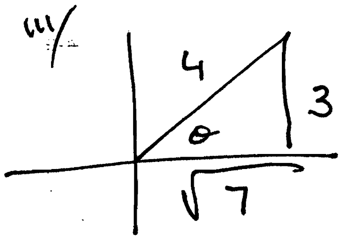
ii) a) $2 + k = 5$

$\underline{k = 3}$

b) $3k = 12$

$\underline{k = 4}$

iii)



a) $\tan\theta = \frac{3}{\sqrt{7}}$ $\quad$ 1.13

b) $\sec\theta = \frac{4}{\sqrt{7}}$ $\quad$ 1.51

c) $\csc^2\theta - \cot^2\theta = 1$

iv) $y = -x^2 + 2x + 5$

a) $x = -\frac{b}{2a} = \frac{-2}{-2}$

$\underline{x = 1}$

b) vertex is $(1, 6)$

c) $y - 5 = -x^2 + 2x$

$-y + 5 + 1 = x^2 - 2x + 1$

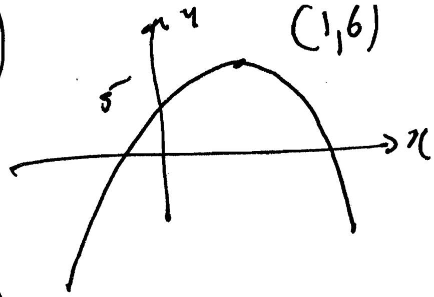
$-y + 6 = (x-1)^2$

$(x-1)^2 = -1(y-6)$

$4a = 1$

$a = \frac{1}{4}$

d)



(6) i) $2x + 5y + k = 0$ $\left(\text{as } m = -\frac{2}{5}\right)$

through $(-1, 3)$

$-2 + 15 + k = 0$

$k = -13$

$\therefore 2x + 5y - 13 = 0$

ii) $y = 2x^3 - 3x^2 - 12x + 2$

$y' = 6x^2 - 6x - 12$

$y'' = 12x - 6$

a) stat when $y' = 0$

i.e. $x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = -1$ or $x = 2$

$y'' < 0$ $\qquad$ $y'' > 0$

$\therefore$ max at $\qquad$ min at

$(-1, 9)$ $\qquad$ $(2, -18)$

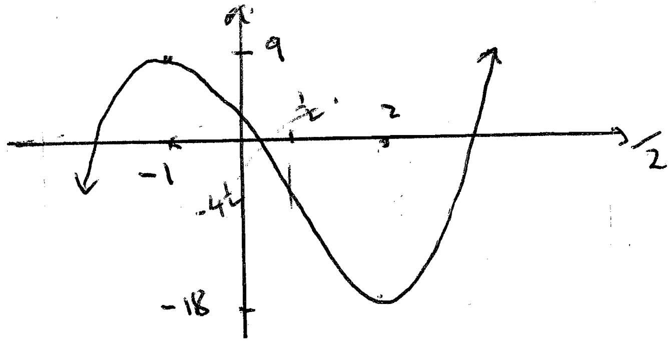
b) inflexion when $y'' = 0$ & sign changes

i.e. $x = \frac{1}{2}$

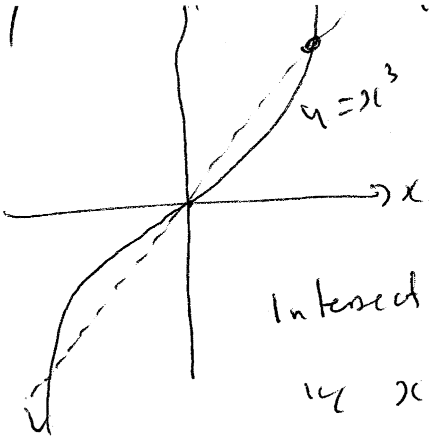
test $f''\left(\frac{1}{2} - \epsilon\right)$ −ve
$f''\left(\frac{1}{2} + \epsilon\right)$ +ve

sign change $\therefore$ inflexion at $\left(\frac{1}{2}, -4\frac{1}{2}\right)$

c)



i)



Intersect when $x^3 = x$ T

i.e. $x = 0, x = 1$

$$V = \pi \int_0^1 y^2 \, dx$$

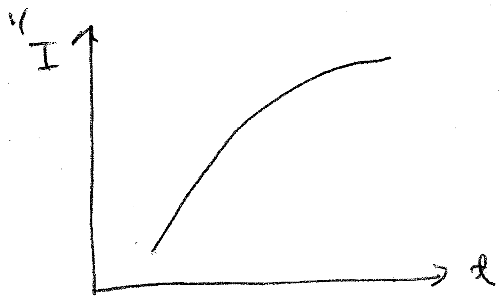
$$= \pi \int_0^1 x \, dx - \pi \int_0^1 x^3 \, dx$$ T

$$= \pi \left[\frac{x^2}{2} \right]_0^1 - \pi \left[\frac{x^4}{4} \right]_0^1$$ T

$$= \frac{\pi}{2} - \frac{\pi}{4}$$

$$= \frac{\pi}{4} \text{ u}^3$$ T

7) i)



1 - increasing
1 - concavity

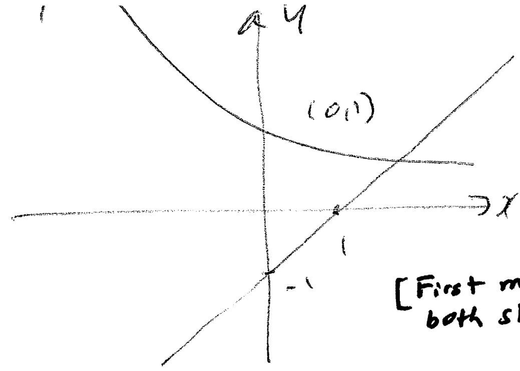
ii) $$LHS = \frac{\cos^2\alpha + 1 + 2\sin\alpha + \sin^2\alpha}{\cos\alpha(1+\sin\alpha)}$$ T

$$= \frac{2 + 2\sin\alpha}{\cos\alpha(1+\sin\alpha)}$$

$$= \frac{2(1+\sin\alpha)}{\cos\alpha(1+\sin\alpha)}$$ T

$$= \frac{2}{\cos\alpha}$$

$= RHS$



T

[First mark for both shapes] ✓

$e^{-x} - x + 1 = 0$. Because 1 point of intersection, 1 root. ✓

other graph is $y = x - 1$

iv) a) $PA = \sqrt{(x-1)^2 + (y-3)^2}$ T

$PB = \sqrt{(x-5)^2 + (y-6)^2}$ T

b) $PA = 2PB$

$PA^2 = 4PB^2$ T

$$(x-1)^2 + (y-3)^2 = 4\left[(x-5)^2 + (y-6)^2\right]$$

$$x^2 - 2x + 1 + y^2 - 6y + 9 = 4(x^2 - 10x + 25 + y^2 - 12y + 36)$$

$$3x^2 - 38x + 3y^2 - 42y + 234 = 0$$ T

v) $T_p = ar^{p-1}$

$\therefore ca = ar^{p-1}$ ✓

$c = r^{p-1}$

$\log c = \log r^{p-1}$

$\log r \, (p-1) = \log c$

$$\log r = \frac{\log c}{p-1}$$ T

(8) i) $T_n = 3^n + 4$

a) $T_7 = 3^7 + 4$

b) $S_n =$ Sum of $3^1 + 3^2 + \dots + 3^n + 4n$

$= \dfrac{3(3^n - 1)}{3 - 1} + 4n$

$= \dfrac{3(3^n - 1)}{2} + 4n$

ii) $y = 4x - 8 \cap x^2 = 2y$

when $x^2 = 8x - 16$

i.e. $x^2 - 8x + 16 = 0$

$(x - 4)^2 = 0$

∴ $x = 4$

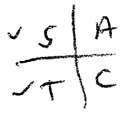
$y = 8$

$(4, 8)$ ∴ 1 point of intersection

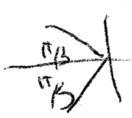
∴ line is a tangent

iii) a) $\cos x = -\frac{1}{2}$ or $\cos x = 1$

$x = 0, 2\pi$



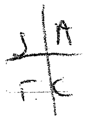
Base $x = \frac{\pi}{3}$



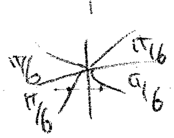
∴ $x = \frac{2\pi}{3}, \frac{4\pi}{3}$

b) $\tan^2 x = \frac{1}{3}$

$\tan x = \pm \frac{1}{\sqrt{3}}$

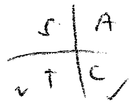


$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

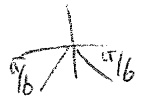


c) $\sin(x - \pi/3) = -\frac{1}{2}$

Base ∠ $\pi/6$



$x - \frac{\pi}{3} = \frac{7\pi}{6}$ or $\frac{11\pi}{6}$ 1



i.e. $x = \frac{7\pi}{6} + \frac{\pi}{3}$ or $\frac{11\pi}{6} + \frac{\pi}{3}$

$= \frac{9\pi}{6}$ $\quad \frac{13\pi}{6}$ 1

$= \frac{3\pi}{2}$ $\quad$ i.e. $\frac{\pi}{6}$

to fit Domain

9(a)(i) $T = \frac{D}{S}$

$T_{AB} = \frac{\sqrt{x^2+36}}{40}$ ✓

Accepted in minutes.

$T_{BQ} = \frac{20-x}{50}$ ✓ (2)

(b) $T = \frac{\sqrt{x^2+36}}{40} + \frac{20-x}{50}$ ✓ (1)

(c) $\frac{dT}{dx} = \frac{\frac{1}{2}(x^2+36)^{-\frac{1}{2}} 2x}{40} - \frac{1}{50}$

$= \frac{x}{40\sqrt{x^2+36}} - \frac{1}{50}$ ✓

Let $\frac{x}{40\sqrt{x^2+36}} - \frac{1}{50} = 0$

$\frac{x}{40\sqrt{x^2+36}} = \frac{1}{50}$

$5x = 4\sqrt{x^2+36}$

$25x^2 = 16(x^2+36)$

$9x^2 = 16 \cdot 36$

$x^2 = 16 \cdot 4$

$x = 8 \quad (x>0)$ ✓

$f'(8-\varepsilon) < 0$

$f'(8) = 0$ (3)

$f'(8+\varepsilon) > 0$

$\therefore$ ~~Max~~ at $x = 8$ ✓

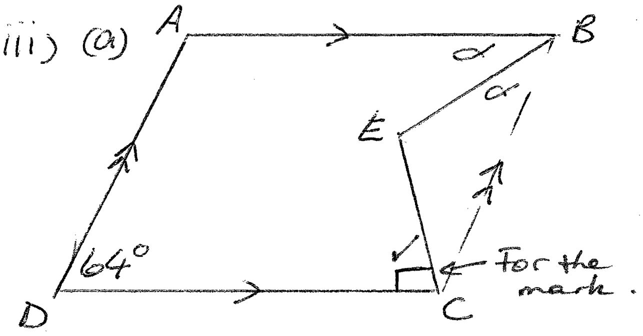
Min

(ii) (a) $V = \pi \int_0^{\frac{\pi}{4}} \sec^2 x \, dx$ ✓

$= \pi \left[\tan x\right]_0^{\frac{\pi}{4}}$ ✓ (3)

$= \pi u^3$ ✓

(iii) (a)



(b) $\angle ECB = 180 - (90+64)$ (coint. $\angle$'s, parallel lines)

$= 26°$

$\angle ABC = 64°$ (opp. $\angle$'s of parm)

$\angle EBC = 32°$ (data)

$\therefore \angle BEC = 180 - (32+26)$ ($\angle$ sum of $\triangle$)

$= 122°$ ✓ ✓

If right angle not between EC and DC, question much easier (3)

$\frac{2}{3}$ km/min

$\sqrt{36+x^2}$

p 5 C10 Williams

Q10

(i) $\tan\frac{\pi}{3} + \operatorname{cosec}\frac{\pi}{4} = \sqrt{3} + \sqrt{2}$ ✓

(ii) (a) $\dfrac{1}{2x-3} - \dfrac{1}{2x+3} = \dfrac{2x+3 - 2x+3}{(2x-3)(2x+3)}$

$= \dfrac{6}{4x^2-9}$ ✓

(b) $\displaystyle\int \frac{dx}{4x^2-9} = \frac{1}{6}\int \frac{6}{4x^2-9}\,dx$

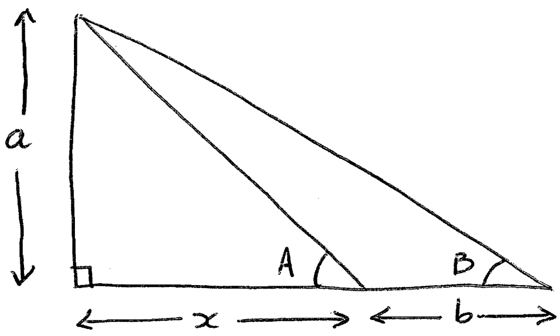
$\displaystyle= \frac{1}{6}\int\left(\frac{1}{2x-3} - \frac{1}{2x+3}\right)dx$ ✓

$= \frac{1}{12}\ln(2x-3) - \frac{1}{12}\ln(2x+3) + c$

$= \frac{1}{12}\ln\left(\frac{2x-3}{2x+3}\right) + c$ ✓

Q10 (cont'd)

(iii)



Given:
$\tan A - \tan B = 1$

(a) $\tan A - \tan B = 1$

$\therefore \frac{a}{x} - \frac{a}{b+x} = 1$ ✓

$\therefore \frac{ab + \cancel{ax} - \cancel{ax}}{x(x+b)} = 1$

$\therefore ab = x(x+b)$ ✓

$\therefore x^2 + bx - ab = 0$.

(b) $x^2 + 6x - 18 = 0$

$x = \frac{-6 \pm \sqrt{36 - 4(1)(-18)}}{2}$ ✓

$= \frac{-6 \pm \sqrt{108}}{2} = \frac{-6 \pm 6\sqrt{3}}{2}$

$= -3 \pm 3\sqrt{3}$ ✓

[But $x > 0$ $\therefore$ $x = -3 + 3\sqrt{3}$]

(c) $\tan A = \frac{a}{x} = \frac{3}{-3+3\sqrt{3}} = \frac{1}{\sqrt{3}-1}$

$= \frac{\sqrt{3}+1}{2}$ ✓

$\tan B = \frac{3}{6 + (-3+3\sqrt{3})} = \frac{3}{3+3\sqrt{3}} = \frac{1}{1+\sqrt{3}}$

$= \frac{\sqrt{3}-1}{2}$ ✓

(d) $A \doteqdot 53.79^\circ$, $B \doteqdot 20.104^\circ$

ie $A = 54^\circ$ (nearest degree) ✓
$B = 20^\circ$ ✓

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