



2011 HSC Course Assessment Task 3
(Trial HSC Examination)

- Working time – 3 hours (plus 5 minutes reading time)
- **Commence each new question on a new page.**
- Write using blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- Board approved calculators may be used.
- All necessary working should be shown in every question.
- Attempt **all** questions.
- At the conclusion of the examination, bundle the booklets used in the correct order within this paper and hand to examination supervisors.

- ☐ 12M2A – Ms Ziaziaris
- ☐ 12M2B – Mr Berry
- ☐ 12M3A – Mr Lam
- ☐ 12M3B – Mr Weiss
- ☐ 12M3C – Mr Lin

STUDENT NUMBER **# BOOKLETS USED:**

[illegible]

Question 1 (12 Marks)	Commence a NEW page.	Marks
(a) Evaluate $\frac{\sqrt{25 - e^3}}{\pi^2}$ correct to 3 significant figures.		2
(b) Write down the exact value of $\tan 315^\circ$.		1
(c) Find x when $\frac{3}{5} + \frac{2x - 1}{3} = 1$.		2
(d) Write $1.2\overline{87}$ as a fraction.		2
(e) Rationalise the denominator of $\frac{8}{3 - \sqrt{7}}$.		2
(f) Solve $ 3x + 2 < 11$ and sketch your solution on a number line.		3

Question 2 (12 Marks)	Commence a NEW page.	Marks
(a) Differentiate: i. $\frac{2}{\sqrt{x^3}}$. ii. $\cos^2(3x)$. iii. $\frac{x^2}{e^x + 1}$.		1 1 2
(b) Find the equation of the normal to $y = x^3 - 7x^2 + 4x + 11$ when $x = 2$.		4
(c) A plane flew 250 km on a bearing of 070°T and then 100 km due east. Find: i. Its distance from the starting point. ii. Its bearing from the starting point to the nearest degree.		2 2

Question 3 (12 Marks)

Commence a NEW page.

Marks

- (a) $A(2, 1)$, $B(-5, -6)$, $C(-6, -1)$ and $D(1, 6)$ form a parallelogram.
- Plot the points A , B , C and D on a number plane. **1**
 - Find the gradient of AB . **1**
 - Show that the equation of AB is $x - y - 1 = 0$. **1**
 - Find the exact length of AB . **1**
 - Find the coordinates where the diagonals of $ABCD$ intersect. Label it as point E on your diagram. **2**
 - Find the exact area of $\triangle ABE$. **3**
- (b) Find the values of A , B and C for the identity **3**

$$A(x - 1)^2 + B(x - 1) + C \equiv 3x^2 - x + 3$$

Question 4 (12 Marks)

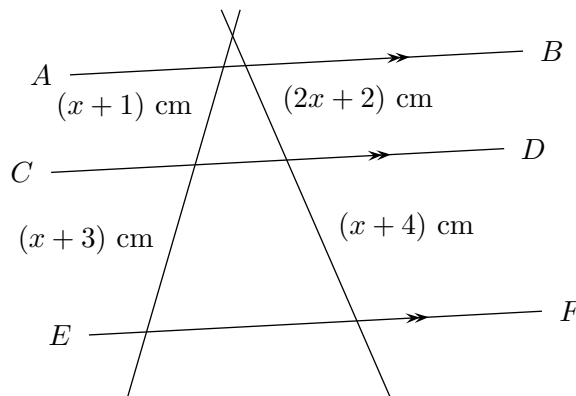
Commence a NEW page.

Marks

- (a) Evaluate:
- $\int \frac{x^3 + 1}{x^2} dx$. **2**
 - $\int \frac{x^2}{x^3 + 1} dx$. **2**
- (b) Evaluate $\int_2^5 e^{3x} dx$. Round your answer to 2 decimal places. **2**
- (c) For the function $f(x) = x^3$:
- Evaluate $\int_{-1}^1 f(x) dx$. **1**
 - Find the area between $y = f(x)$, $x = 1$, $x = -1$ and the x axis. **2**
 - Is the answer for part (i) the same as (ii)? Give a reason. **1**
- (d) Sketch the graph of $y = 2 + \cos 2x$ for $0 \leq x \leq 2\pi$. **2**

Question 5 (12 Marks) Commence a NEW page. **Marks**

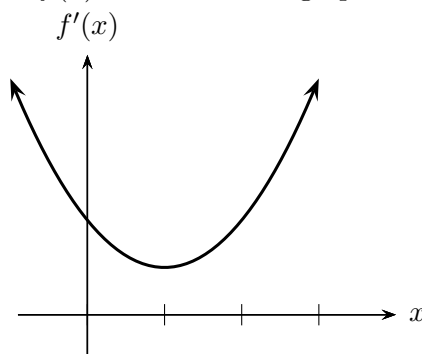
- (a) Find the value(s) of k in $x^2 + (k + 6)x - 2k = 0$ such that
- 3 is a root of the quadratic. **1**
 - The roots are equal in magnitude but opposite in sign. **1**
 - The roots are reciprocals of one another. **1**
 - The roots are real. **3**
- (b) AB , CD and EF are parallel lines. Find the exact length(s) of x , giving reasons. **3**



- (c) Find the possible values of θ when $3 \tan^2 \theta - 1 = 0$ and $0 \leq \theta \leq 2\pi$. **3**

Question 6 (12 Marks) Commence a NEW page. **Marks**

- (a) For the function $f(x) = x^3 + 2x^2 - 4x - 8$
- Find the y intercept. **1**
 - Factorise in pairs to find the x intercept(s). **2**
 - Find the stationary point(s) and determine their nature. **4**
 - Find the point(s) of inflexion. **2**
 - Draw a neat sketch of the curve showing all the above features. **1**
- (b) The gradient of a function $f(x)$ is shown in the graph below such that $y = f'(x)$. **2**



Sketch a possible graph for $y = f(x)$.

Question 7 (12 Marks)

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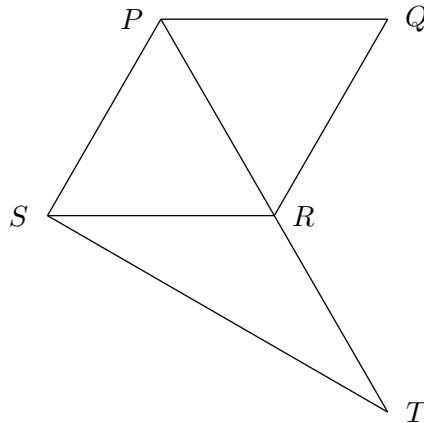
Marks

- (a) George plays a game where he rolls 2 dice. The first die has 3 red faces, 2 blue faces and 1 green face. The other die has 2 red faces, 2 blue faces, and 2 green faces.
- Find the probability that both dice show red. **1**
 - Find the probability that one shows red and one shows blue. **1**
 - Find the probability that both dice do not show red, nor do both show green. **1**

- (b) Evaluate $\int_0^1 \pi^x dx$ using Simpson's Rule with 5 function values. **3**

Give you answer to two decimal places.

- (c) $PQRS$ is a rhombus. PR is produced to T such that $SR = TR$.



- Show that $\angle SPQ = 4\angle STR$. **3**
- Show that R is the midpoint of PT , given that $\angle PST = 90^\circ$. **3**

Question 8 (12 Marks)	Commence a NEW page.	Marks
(a) Evaluate $\lim_{x \rightarrow 4} \frac{x^2 - 16}{x - 4}$.		2
(b) For the series $8 + 12x + 18x^2 + 27x^3 + \dots$		
i. For what value(s) of x will this series have a limiting sum?		2
ii. Find the limiting sum if $x = \frac{1}{4}$.		2
(c) A parabola has a focus of $(3, 2)$ and a directrix $x = 5$.		
i. Find the vertex.		1
ii. State the equation of the parabola.		1
iii. Show that the points of intersection of the parabola and the line $2x + y - 6 = 0$ are $(3, 0)$ and $(0, 6)$.		1
iv. Find the area between the parabola and the line in the first quadrant.		3

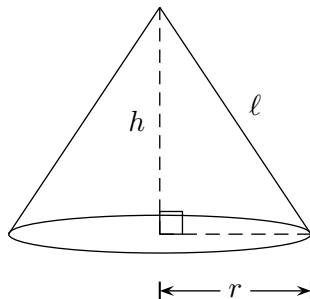
Question 9 (12 Marks)	Commence a NEW page.	Marks
(a) Evaluate $\sum_{n=0}^4 \cos^2\left(\frac{n\pi}{3}\right)$.		2
(b) For the function $y = 2 \tan x$		
i. Sketch $y = 2 \tan x$ for $-\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$.		2
ii. State the range of the curve.		1
iii. On your graph, shade the regions bounded by the function and the x axis.		1
iv. Show that $\frac{d}{dx} \ln(\cos x) = -\tan x$.		1
v. Hence find the exact shaded area from part (iii).		2
vi. Using the identity $1 + \tan^2 x = \sec^2 x$, find the volume of the solid generated when the area bound by the curve and the x -axis is rotated about the x axis.		3

Question 10 (12 Marks)

Commence a NEW page.

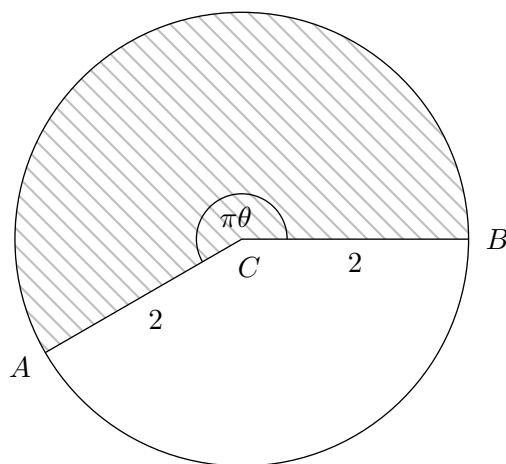
Marks

- (a) Find the sum of the first 50 terms in the series $\ln 3 + \ln 9 + \ln 27 + \ln 81 + \dots$ **3**
- (b) A cone has radius r , height h and slant height ℓ . The volume of the cone is given by $V = \frac{\pi}{3}\pi r^2 h$. **2**



Show that the volume of the cone can be expressed as $V = \frac{\pi}{3}\sqrt{\ell^2 r^4 - r^6}$.

- (c) The angle at the centre C of a circle of radius 2 cm is $\pi\theta$ radians, $0 < \theta < 2$, as shown on the diagram.



- i. Write down the length of the arc of the shaded sector. **1**
- ii. The sector is cut from the circle along the radii CA and CB and folded to make a cone. **1**

Find the radius of the cone.

- iii. Show that the volume of the cone is given by $V = \frac{\pi}{3}\sqrt{4\theta^4 - \theta^6}$. **1**
- iv. Find the value of θ to 2 decimal places, for which the volume of the cone is maximised. **4**

End of paper.

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + C, \quad n \neq -1; \quad x \neq 0 \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x + C, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + C, \quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a} \sin ax + C, \quad a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax + C, \quad a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax + C, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax + C, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a} + C, \quad a > 0, -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln \left(x + \sqrt{x^2 - a^2} \right) + C, \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln \left(x + \sqrt{x^2 + a^2} \right) + C$$

NOTE: $\ln x = \log_e x$, $x > 0$

Question 1

$$e) \frac{8}{3-\sqrt{7}} \times \frac{3+\sqrt{7}}{3+\sqrt{7}} \checkmark$$

$$= \frac{8(3+\sqrt{7})}{3^2 - (\sqrt{7})^2}$$

$$= \frac{24 + 8\sqrt{7}}{2}$$

$$= 12 + 4\sqrt{7} \checkmark$$

$$f) |3x + 2| < 11$$

$$\text{case 1: } 3x + 2 < 11$$

$$3x < 9$$

$$x < 3 \checkmark$$

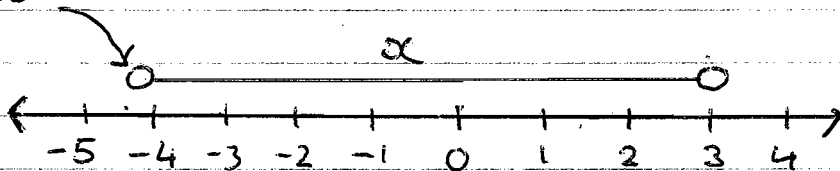
$$\text{case 2 } -(3x + 2) < 11$$

$$-3x - 2 < 11$$

$$-3x < 13$$

$$x > -\frac{13}{3} \text{ or } -4\frac{1}{3} \checkmark$$

$-4\frac{1}{3}$



Question 1

a) 0.225 (3 s.f.)

✓ answer
✓ rounding

b) -1

✓

c) $\frac{3}{5} + \frac{2x-1}{3} = 1$

$$\frac{9}{15} + \frac{5(2x-1)}{15} = 1$$

✓

$$\therefore 9 + 10x - 5 = 15$$

$$10x + 4 = 15$$

$$10x = 11$$

$$x = \frac{11}{10}$$

✓

d) let $x = 1.287878\ldots$

$$100x = 128.7878\ldots$$

$$99x = 128.7878\ldots - 1.2878\ldots$$

✓

$$99x = 127.5$$

$$\therefore x = \frac{127.5}{99}$$

$$\therefore x = \frac{85}{66}$$

✓

Question 2

a) i) $\frac{d}{dx} \frac{2}{\sqrt{x^3}}$

$$= \frac{d}{dx} 2x^{-3/2}$$

$$= -3x^{-5/2} \quad \checkmark$$

ii) $\frac{d}{dx} \cos^2(3x)$

$$= -6 \sin 3x \cos 3x \quad \checkmark$$

iii) let $y = \frac{x^2}{e^x + 1}$

$$u = x^2 \quad \text{and} \quad u' = 2x$$

$$v = e^x + 1 \quad \text{and} \quad v' = e^x \quad \checkmark$$

$$y' = \frac{u'v - v'u}{v^2}$$

$$= \frac{2x(e^x + 1) - (e^x)(x^2)}{(e^x + 1)^2}$$

$$= \frac{2xe^x + 2x - x^2e^x}{(e^x + 1)^2} \quad \checkmark$$

$$= \frac{x(2e^x + 2 - xe^x)}{(e^x + 1)^2}$$

Question 2

$$b) y = x^3 - 7x^2 + 4x + 11$$

$$\text{when } x = 2$$

$$y = 2^3 - 7(2)^2 + 4(2) + 11$$

$$y = -1$$

$$y' = 3x^2 - 14x + 4$$

$$\therefore m_{\text{gt}} = 3(2)^2 - 14(2) + 4$$

$$= -12$$

$$\therefore m_{\text{norm}} = \frac{1}{12}$$

$$y - y_1 = m(x - x_1)$$

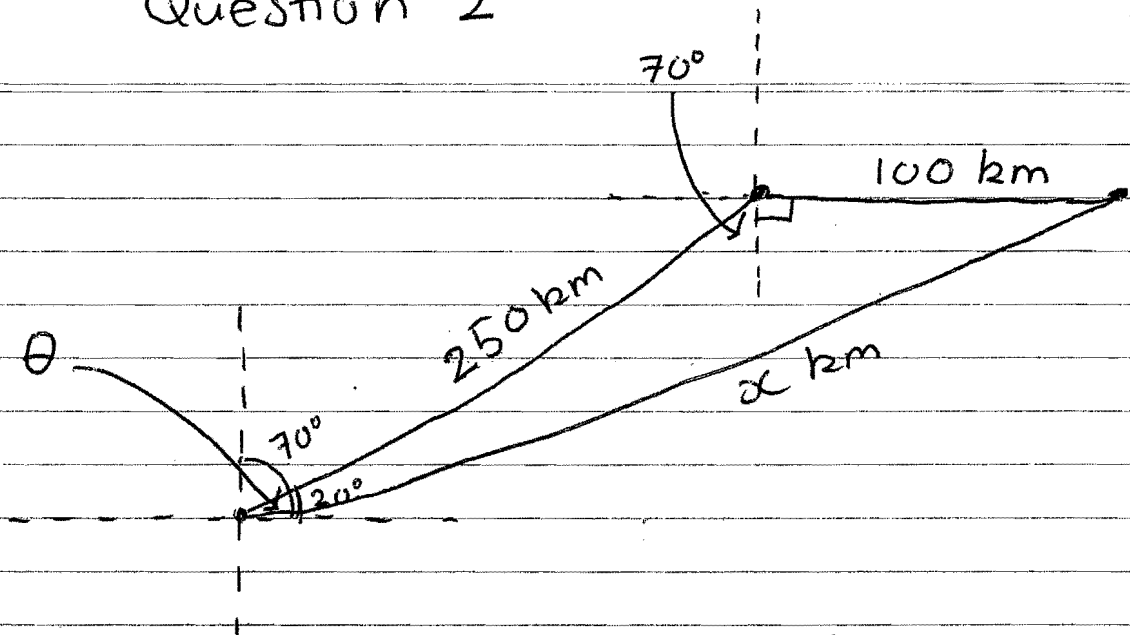
$$y + 1 = \frac{1}{12}(x - 2)$$

$$12y + 12 = x - 2$$

$$\therefore x - 12y - 14 = 0$$

Question 2

c)



i) let x be the distance

$$x^2 = 250^2 + 100^2 - 2 \times 250 \times 100 \times \cos 160^\circ$$

$$\therefore x = 346 \text{ km (nearest km)}$$

ii) let θ be as shown.

$$\frac{\sin \theta}{100} = \frac{\sin 70^\circ}{x}$$

$$\therefore \sin \theta = \frac{100 \sin 70^\circ}{x}$$

$$\therefore \theta = \sin^{-1} \left(\frac{100 \sin 70^\circ}{x} \right)$$

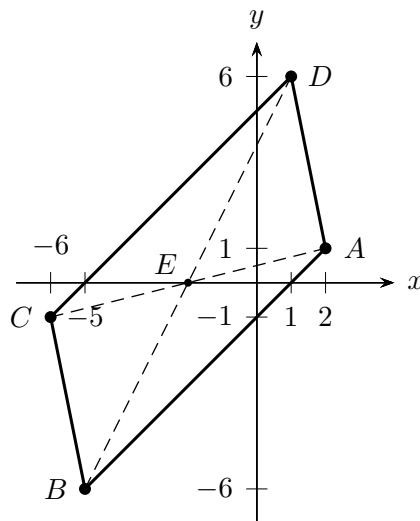
$$\theta = 6^\circ \text{ (nearest degree)}$$

$$\therefore \text{bearing} = 70^\circ + 6^\circ$$

$$= 076^\circ \text{T (nearest degree)}$$

Question 3 (Lam)

(a) i. (1 mark)



ii. (1 mark)

$$m = \frac{1 - (-6)}{2 - (-5)} = \frac{7}{7} = 1$$

iii. (1 mark)

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 1 &= 1(x - 2) \\ y - 1 &= x - 2 \\ x - y - 1 &= 0 \end{aligned}$$

iv. (1 mark)

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(2 - (-5))^2 + (1 - (-6))^2} \\ &= \sqrt{98} = 7\sqrt{2} \end{aligned}$$

v. (2 marks)

Diagonals intersect at midpoints (property of a parallelogram). Find midpoint E of AB :

$$E = \left(\frac{2 + (-6)}{2}, \frac{1 + (-1)}{2} \right) = (-2, 0)$$

vi. (3 marks)

- ✓ [1] for correct substitution into perpendicular dist formula.
- ✓ [1] for perpendicular height
- ✓ [1] for area of $\triangle ABE$

$$A_{\triangle ABE} = \frac{1}{2}bh$$

Perpendicular height from $E(2, 0)$ to $x - y - 1 = 0$

$$\begin{aligned} h &= \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}} \\ &= \frac{|1(2) + (-1)(0) + (-1)|}{\sqrt{1^2 + (-1)^2}} \\ &= \frac{3}{\sqrt{2}} \\ A &= \frac{1}{2} \times 7\sqrt{2} \times \frac{3}{\sqrt{2}} = \frac{21}{2} \end{aligned}$$

(b) (3 marks)

✓ [1] each for A , B and C

$$A(x - 1)^2 + B(x - 1) + C \equiv 3x^2 - x + 3$$

By inspection,

$$A = 3$$

Letting $x = 1$,

$$\begin{aligned} 0 + 0 + C &= 3 - 1 + 3 \\ \therefore C &= 5 \end{aligned}$$

Letting $x = 2$,

$$\begin{aligned} A + B + C &= 3(2)^2 - 2 + 3 \\ 3 + B + 5 &= 13 \\ \therefore B &= 5 \end{aligned}$$

Question 4

a) i) $\int \frac{x^3 + 1}{x^2} dx$

$$= \int (x + x^{-2}) dx \quad \checkmark$$

$$= \frac{1}{2} x^2 + (-1)x^{-1} + C$$

$$= \frac{x^2}{2} - \frac{1}{x} + C \quad \checkmark$$

ii) $\int \frac{x^2}{x^3 + 1} dx$

$$= \frac{1}{3} \int \frac{3x^2}{x^3 + 1} dx \quad \checkmark$$

$$= \frac{1}{3} \log_e(x^3 + 1) + C \quad \checkmark$$

b) $\int_2^5 e^{3x} dx$

$$= \left[\frac{1}{3} e^{3x} \right]_2^5 \quad \checkmark$$

$$= \frac{1}{3} [e^{15} - e^6]$$

$$= 1\,089\,538 \quad (\text{nearest whole number}) \quad \checkmark$$

Question 4

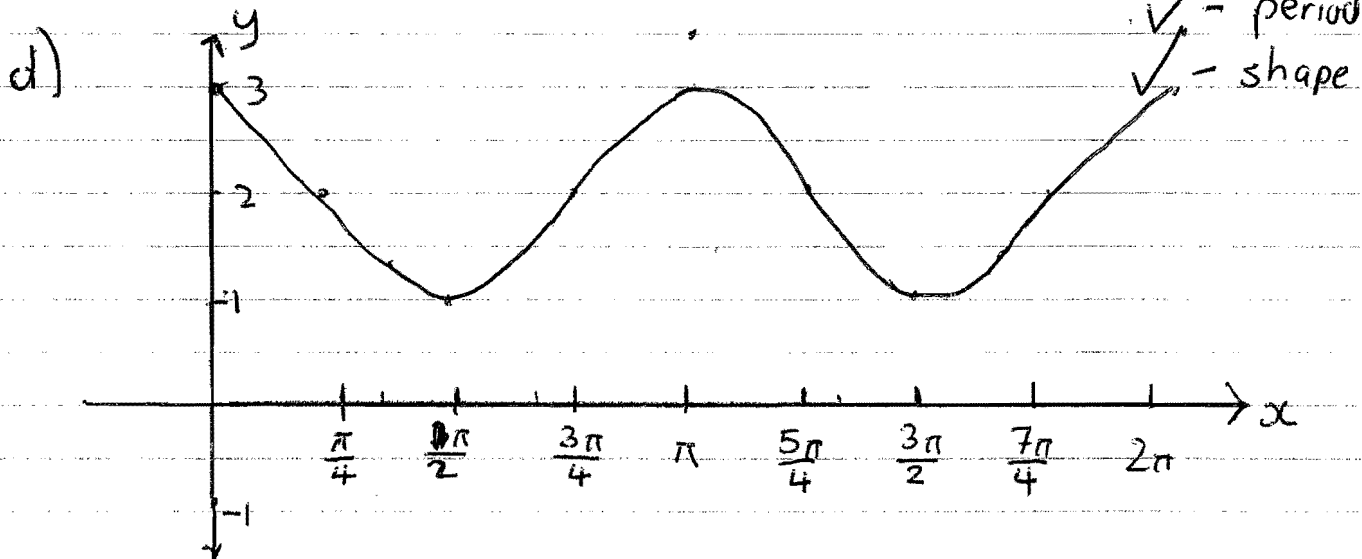
c) i) $\int_{-1}^1 x^3 dx$

$$= \left[\frac{1}{4} x^4 \right]_{-1}^1$$
$$= \frac{1}{4} (1)^4 - \frac{1}{4} (-1)^4$$
$$= \frac{1}{4} - \frac{1}{4}$$
$$= 0 \quad \checkmark$$

ii) $A = 2 \int_0^1 x^3 dx$ (since it's an odd function) $\checkmark$

$$= 2 \left[\frac{1}{4} x^4 \right]_0^1$$
$$= 2 \left(\frac{1}{4} (1)^4 - \frac{1}{4} (0)^4 \right)$$
$$= 2 \times \frac{1}{4}$$
$$= \frac{1}{2} u^2 \quad \checkmark$$

iii) Because half of the area is below the x -axis it will cancel out when evaluating the definite integral. $\checkmark$



Question 5

a) i) $3^2 + (k+6)3 - 2k = 0$

$$9 + 3k + 18 - 2k = 0$$

$$27 + k = 0$$

$$k = -27 \quad \checkmark$$

ii) let the roots be α and $-\alpha$

$$\therefore \alpha + -\alpha = \frac{-b}{a}$$

$$\therefore 0 = \frac{-(k+6)}{1}$$

$$\therefore k = -6 \quad \checkmark$$

iii) let the roots be α and $\frac{1}{\alpha}$

$$\therefore \alpha \times \frac{1}{\alpha} = \frac{c}{a}$$

$$\therefore 1 = \frac{-2k}{1}$$

$$\therefore -2k = 1$$

$$k = -\frac{1}{2} \quad \checkmark$$

Question 5

$$\text{iv) } b^2 - 4ac \geq 0 \quad \checkmark$$

$$(k+6)^2 - 4(1)(-2k) \geq 0 \quad \checkmark$$

$$k^2 + 12k + 36 + 8k \geq 0$$

$$k^2 + 20k + 36 \geq 0$$

$$(k+18)(k+2) \geq 0$$

$$\therefore k \leq -18 \quad \text{or} \quad k \geq -2 \quad \checkmark$$

$$\text{b) } \frac{x+1}{x+3} = \frac{x+4}{2x+2} \quad (\text{ratio of intercepts}) \quad \checkmark$$

$$(x+1)(2x+2) = (x+4)(x+3)$$

$$2x^2 + 2x + 2x + 2 = x^2 + 3x + 4x + 12$$

$$2x^2 + 4x + 2 = x^2 + 7x + 12$$

$$x^2 - 3x - 10 = 0 \quad \checkmark$$

$$(x-5)(x+2) = 0$$

$$\therefore x = 5 \text{ cm} \quad (\text{since } x \text{ cannot be negative}) \quad \checkmark$$

$$\text{c) } 3 \tan^2 \theta - 1 = 0 \quad \checkmark$$

$$3 \tan^2 \theta = 1 \quad \checkmark$$

$$\tan^2 \theta = \frac{1}{3} \quad \checkmark$$

$$\tan \theta = \pm \frac{1}{\sqrt{3}} \quad \checkmark$$

$$\therefore \theta = \pi/6, 5\pi/6, 7\pi/6, 11\pi/6$$

Question 6

$$\begin{aligned} \text{a) i) } y &= 0^3 + 2(0)^2 - 4(0) - 8 \\ &= -8 \quad \checkmark \end{aligned}$$

$$\begin{aligned} \text{ii) } y &= x^2(x+2) - 4(x+2) \\ &= (x+2)(x^2-4) \\ &= (x+2)(x+2)(x-2) \\ &= (x+2)^2(x-2) \end{aligned}$$

$$\therefore x = -2 \quad \checkmark \text{ and } x = 2 \quad \checkmark$$

$$\begin{aligned} \text{iii) } y' &= 3x^2 + 4x - 4 \quad \checkmark \\ &= (3x-2)(x+2) \end{aligned}$$

$$\therefore x = -2 \quad \text{or} \quad x = \frac{2}{3} \quad \checkmark$$

$$y'' = 6x + 4$$

when $x = -2$

$$\begin{aligned} y &= (-2)^3 + 2(-2)^2 - 4(-2) - 8 \\ &= 0 \end{aligned}$$

$$\begin{aligned} y'' &= 6(-2) + 4 \\ &= -8 \end{aligned}$$

$$< 0$$

$\therefore (-2, 0)$ is a maximg $\checkmark$

Question 6

when $x = \frac{2}{3}$

$$y = \left(\frac{2}{3}\right)^3 + 2\left(\frac{2}{3}\right)^2 - 4\left(\frac{2}{3}\right) - 8$$
$$= -9\frac{13}{27}$$

$$y'' = 6\left(\frac{2}{3}\right) + 4$$
$$= 8$$

$$> 0$$

$\therefore \left(\frac{2}{3}, -9\frac{13}{27}\right)$ is a minima. ✓

iv) $y'' = 0$

$$\therefore 6x + 4 = 0$$

$$6x = -4$$

$$x = -\frac{2}{3}. \quad \checkmark$$

$$y = \left(-\frac{2}{3}\right)^3 + 2\left(-\frac{2}{3}\right)^2 - 4\left(-\frac{2}{3}\right) - 8$$
$$= -4\frac{20}{27}$$

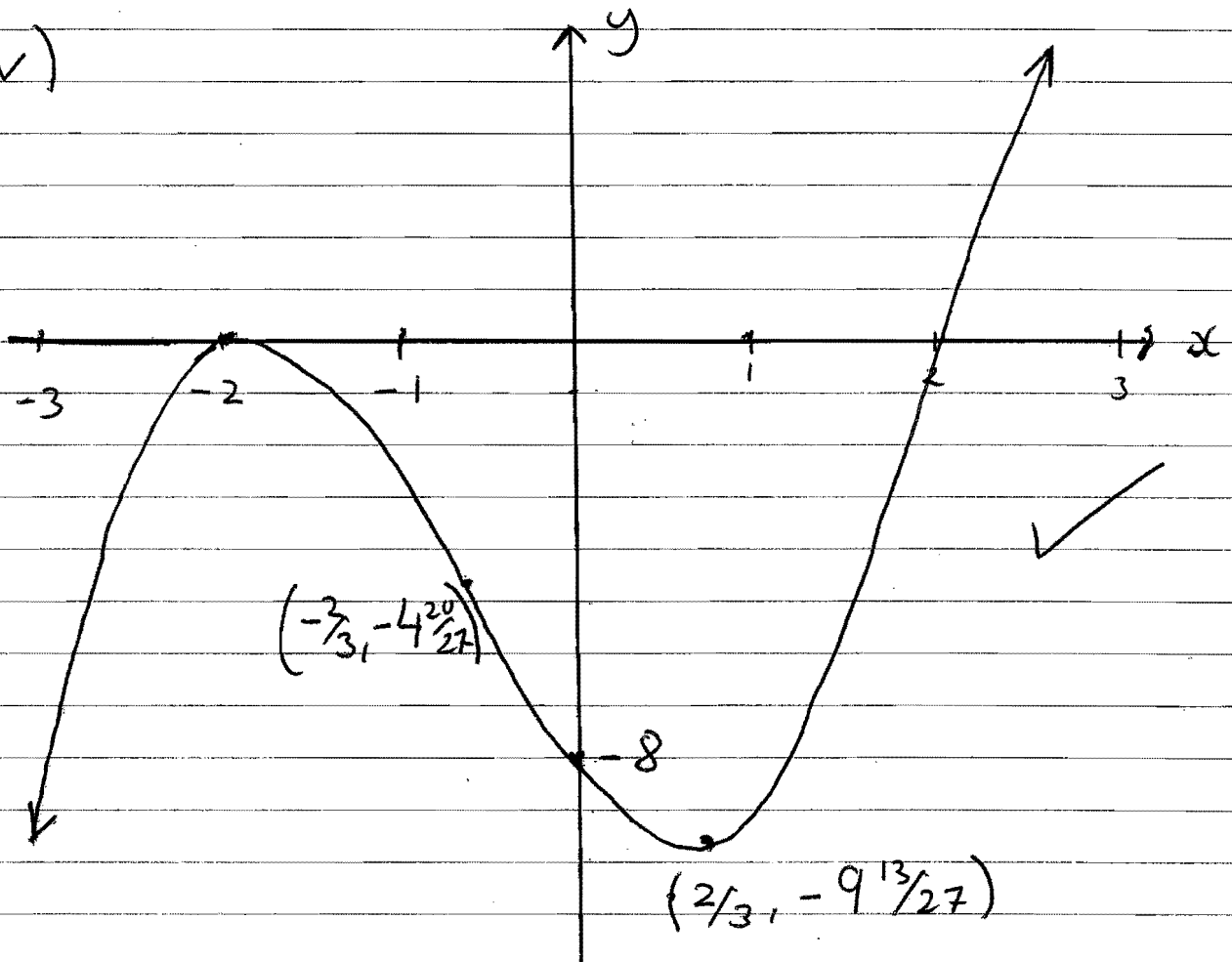
x	$-\frac{2}{3}$	$-\frac{2}{3}$	0
y''	$-$	0	$+$

$\therefore$ concavity change ✓

$\therefore \left(-\frac{2}{3}, -4\frac{20}{27}\right)$ is a point of inflexion

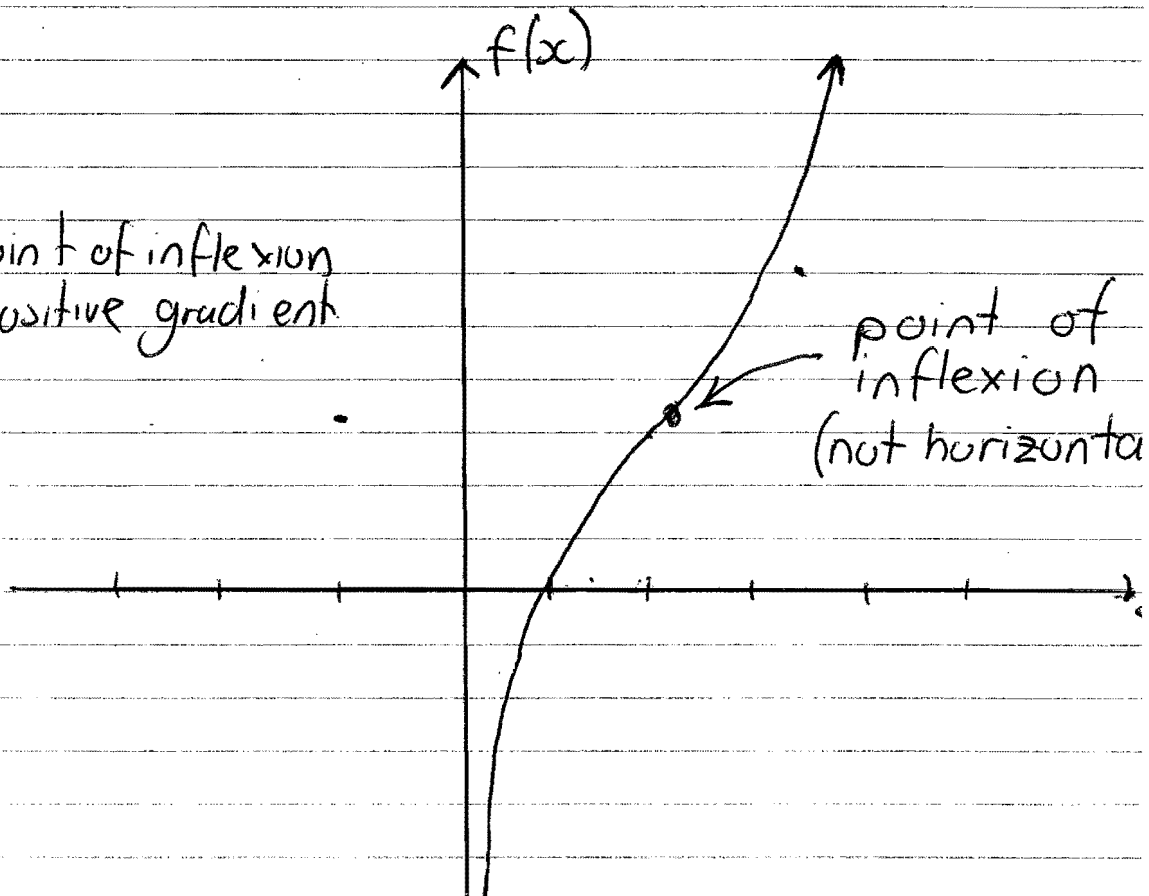
Question 6

v)

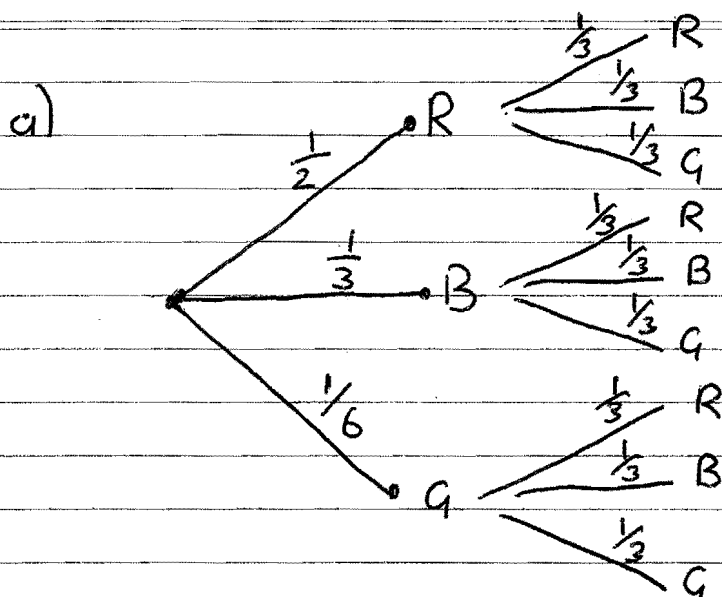


b)

✓ - point of inflexion
✓ - positive gradient



Question 7



$$\text{i) } P(RR) = \frac{1}{2} \times \frac{1}{3}$$

$$= \frac{1}{6} \quad \checkmark$$

$$\text{ii) } P(RB) + P(BR) = \left(\frac{1}{2} \times \frac{1}{3} \right) + \left(\frac{1}{3} \times \frac{1}{3} \right)$$

$$= \frac{5}{18} \quad \checkmark$$

$$\text{iii) } 1 - (P(RR) + P(GG)) = 1 - \left(\left(\frac{1}{2} \times \frac{1}{3} \right) + \left(\frac{1}{6} \times \frac{1}{3} \right) \right)$$

$$= \frac{7}{9} \quad \checkmark$$

b) sub-interval width = 0.25 $\checkmark$

$$A \doteq \frac{h}{3} (f + 4m + l)$$

$$\therefore A \doteq \frac{1}{12} (\pi^0 + 4\pi^{0.25} + \pi^{0.5}) \quad \checkmark$$

$$+ \frac{1}{12} (\pi^{0.5} + 4\pi^{0.75} + \pi^1)$$

$$\doteq 1.87 \quad (2 \text{ d.p.}) \quad \checkmark$$

Question 7

c) i) let $\angle STR = x$

$\therefore \angle TSR = x$ (since $SR = TR$ $\triangle RST$ is isosceles and base angles are equal)

✓ $\therefore \angle PRS = 2x^\circ$ (external angle of $\triangle RST$)

✓ $\therefore \angle SPR = 2x$ (since $PQRS$ is a rhombus $PS = SR$ and $\triangle PRS$ is isosceles and base angles are equal)

✓ $\therefore \angle SPQ = 4x$ (diagonals bisect the angles of a rhombus)

$\therefore \angle SPQ = 4 \angle STR$ as required

ii) $x + 2x + 90^\circ = 180^\circ$ (angle sum of $\triangle PST$) ✓
 $\therefore x = 30^\circ$

$\angle PSR = 90^\circ - 30^\circ$ (since $TSR = x$ from i)
 $= 60^\circ$

$\therefore \angle PSR = \angle SPR = \angle PRS$ (from i)

$\therefore \triangle PRS$ is equilateral ✓

$\therefore PR = SR$ (sides of equilateral triangle)

$\therefore PR = TR$ (both = SR)

$\therefore R$ is the midpoint of PT ✓

Question 8 (Lam)

(a) (2 marks)

✓ [1] correctly factorises numerator.

✓ [1] final answer.

$$\lim_{x \rightarrow 4} \frac{x^2 - 16}{x - 4} = \lim_{x \rightarrow 4} \frac{\cancel{(x-4)}(x+4)}{\cancel{x-4}} = 8$$

(b) i. (2 marks)

✓ [1] for observing $r = \frac{3x}{2}$.

✓ [1] for final answer.

$$8 + 12x + 18x^2 + 27x^3 + \dots$$

$$r = \frac{T_2}{T_1} = \frac{12x}{8} = \frac{3x}{2}$$

A limiting sum exists when $|r| < 1$,

$$\left| \frac{3x}{2} \right| < 1$$

$$\frac{-1}{\times 2} < \frac{3x}{2} < \frac{1}{\times 2}$$

$$\frac{-2}{\div 3} < \frac{3x}{\div 3} < \frac{2}{\div 3}$$

$$\therefore -\frac{2}{3} < x < \frac{2}{3}$$

ii. (2 marks)

✓ [1] recall limiting sum formula.

✓ [1] for final answer.

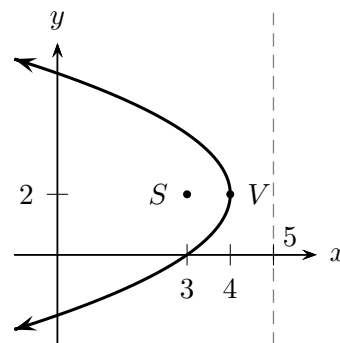
$$x = \frac{1}{4}$$

$$\therefore r = \frac{3 \times \frac{1}{4}}{2} = \frac{3}{8}$$

$$S = \frac{a}{1-r} = \frac{8}{1-\frac{3}{8}}$$

$$= \frac{8}{\frac{5}{8}} = \frac{64}{5}$$

(c) i. (1 mark)

 $S(3, 2)$ directrix $x = 5$ $\therefore V(4, 2)$ 

ii. (1 mark)

$$(y - 2)^2 = -4(x - 4)$$

iii. (1 mark)

$$\begin{cases} (y - 2)^2 = -4(x - 4) \\ 2x + y - 6 = 0 \end{cases}$$

Straight line: $y = -2x + 6$.

Substitute into equation of parabola to find pts of intersection:

$$(-2x + 6 - 2)^2 = -4(x - 4)$$

$$(-2x + 4)^2 = -4(x - 4)$$

$$\cancel{4}(x - 2)^2 = \cancel{4}(x - 4)$$

$$x^2 - 4x + 4 = \underset{+x}{-x} + 4$$

$$x^2 - 3x = 0$$

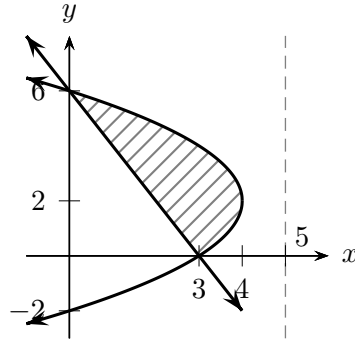
$$\therefore x = 0, 3$$

$$\therefore y = 6, 0$$

Alternatively substitute points into line and parabola and verify.

iv. (3 marks)

- ✓ [1] for equation of parabola in terms of y .
- ✓ [1] for equation of line in terms of y .
- ✓ [1] for final answer.
- ✓ [0] for any attempt to integrate w.r.t. x .



$$\begin{aligned}
 (y - 2)^2 &= -4(x - 4) \\
 (y^2 - 4y + 4) &= -4(x - 4) \\
 \therefore x &= -\frac{1}{4}(y^2 - 4y + 4) + 4 \\
 &= -\frac{1}{4}y^2 + y + 3
 \end{aligned}$$

Change subject of the line to x ,

$$\begin{aligned}
 2x + y - 6 &= 0 \\
 2x &= -y + 6 \\
 \therefore x &= -\frac{1}{2}y + 3
 \end{aligned}$$

Shaded area is between the parabola and the y axis, subtracting the area between the line and the y axis.

$$\begin{aligned}
 A &= \int_0^6 \left(-\frac{1}{4}y^2 + y + 3 \right) dy \\
 &\quad - \int_0^6 \left(-\frac{1}{2}y + 3 \right) dy \\
 &= \int_0^6 \left(-\frac{1}{4}y^2 + \frac{3}{2}y \right) dy \\
 &= \left[-\frac{1}{12}y^3 + \frac{3}{4}y^2 \right]_0^6 \\
 &= -\frac{1}{12}(6^3) + \frac{3}{4}(6^2) \\
 &= 9
 \end{aligned}$$

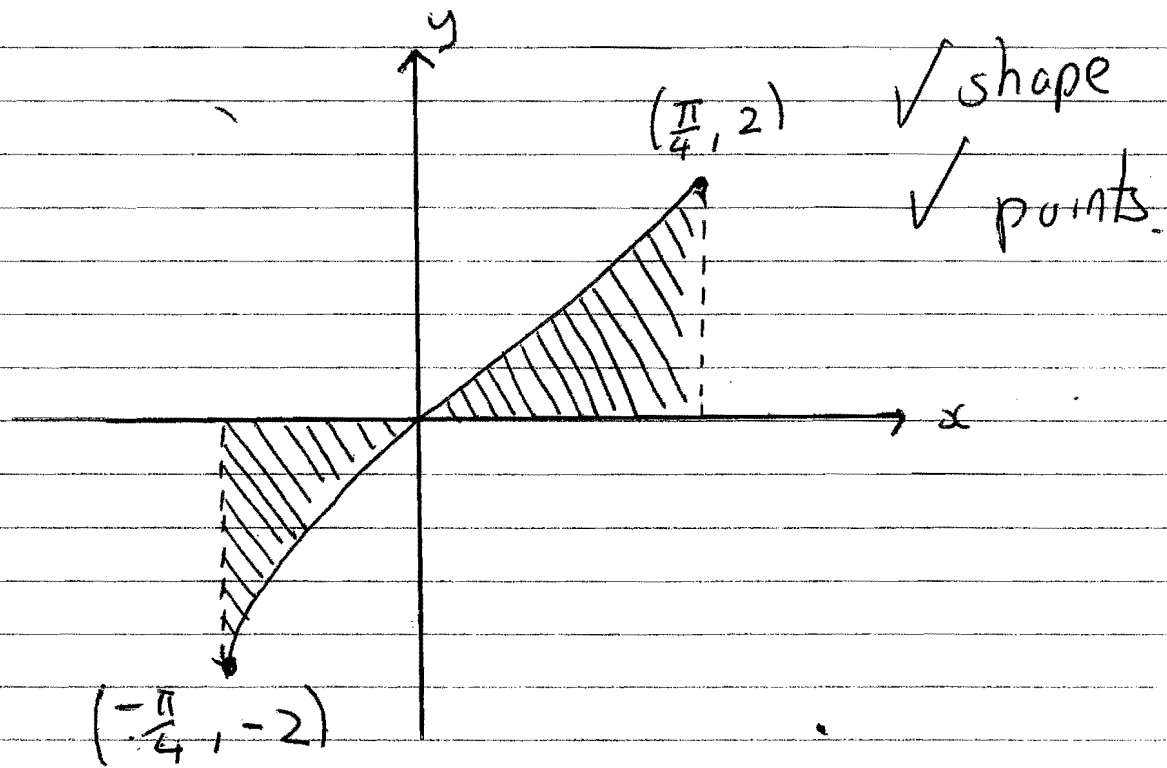
Question 9

$$a) \cos^2(0) + \cos^2\left(\frac{\pi}{3}\right) + \cos^2\left(\frac{2\pi}{3}\right) + \cos^2\left(\frac{3\pi}{3}\right) + \cos^2\left(\frac{4\pi}{3}\right)$$

$$= 1 + \left(\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^2 + (-1)^2 + \left(-\frac{1}{2}\right)^2 \quad \checkmark$$

$$= 2\frac{3}{4} \quad \checkmark$$

b) i)



ii) $-2 \leq y \leq 2$ ✓

iii) see graph ✓

Question 9

$$\text{iv) } \frac{d}{dx} \ln(\cos x) = \frac{-\sin x}{\cos x} \quad \checkmark$$

$$= -\tan x$$

$$\text{v) } A = 2 \int_0^{\pi/4} (2 \tan x) dx \quad (\text{since it's an odd function})$$

$$= 2 [-2 \ln(\cos x)]_0^{\pi/4} \quad \checkmark$$

$$= 2 [-2 (\ln 1/\sqrt{2} - \ln 1)]$$

$$= -4 \ln 2^{-1/2}$$

$$= 2 \ln 2 \text{ units}^2 \quad \checkmark$$

$$\text{vi) } V = 2\pi \int_0^{\pi/4} (2 \tan x)^2 dx \quad \checkmark (\text{odd function})$$

$$= 8\pi \int_0^{\pi/4} (\sec^2 x - 1) dx$$

$$= 8\pi [\tan x - x]_0^{\pi/4} \quad \checkmark$$

$$= 8\pi [(\tan(\pi/4) - \pi/4) - (\tan(0) - 0)]$$

$$= 8\pi (1 - \pi/4)$$

$$= (8\pi - 2\pi^2) \text{ units}^3 \quad \checkmark$$

Question 10

$$a) \ln 3 + \ln 9 + \ln 27 + \ln 81$$

$$= \ln 3 + 2 \ln 3 + 3 \ln 3 + 4 \ln 3 + \dots \checkmark$$

this is an AP where $a = \ln 3$ and $d = \ln 3$

$$S_n = \frac{n}{2} (a + l) \checkmark$$

$$= \frac{50}{2} (\ln 3 + 50 \ln 3)$$

$$= 25 \times 51 \ln 3$$

$$= 1275 \ln 3. \checkmark$$

$$b) l^2 = r^2 + h^2 \text{ (Pythagoras' Theorem)}$$

$$\therefore h^2 = l^2 - r^2 \checkmark$$

$$h = \sqrt{l^2 - r^2} \text{ (since } h > 0)$$

$$V = \frac{\pi}{3} r^2 h$$

$$= \frac{\pi}{3} r^2 \sqrt{l^2 - r^2}$$

$$= \frac{\pi}{3} \sqrt{r^4} \sqrt{l^2 - r^2}$$

$$= \frac{\pi}{3} \sqrt{l^2 r^4 - r^6}, \text{ as required.} \checkmark$$

Question 10

i) $l = 2\pi\theta$ ✓

ii) $l = 2\pi r$

$\therefore 2\pi\theta = 2\pi r$ (since AB is the circumference)

$\therefore r = \theta$ ✓

iii) $V = \frac{\pi}{3} \sqrt{l^2 r^4 - r^6}$ ✓
 $= \frac{\pi}{3} \sqrt{2^2 \theta^4 - \theta^6}$
 $= \frac{\pi}{3} \sqrt{4\theta^4 - \theta^6}$ as required.

iv) $V = \frac{\pi}{3} (4\theta^4 - \theta^6)^{1/2}$
 $V' = \frac{1}{2} \times \frac{\pi}{3} (16\theta^3 - 6\theta^5) (4\theta^4 - \theta^6)^{-1/2}$ ✓
 $= \frac{\pi (16\theta^3 - 6\theta^5)}{6 \sqrt{4\theta^4 - \theta^6}}$
 $= \frac{\pi \theta^3 (16 - 6\theta^2)}{6 \sqrt{4\theta^4 - \theta^6}}$

when $V' = 0$ ✓

$16 - 6\theta^2 = 0$ (since $\theta \neq 0$)

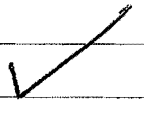
$6\theta^2 = 16$

$\theta = \sqrt{16/6}$

$\therefore \theta = 1.63$ (2 d.p.) ✓

Question 10

θ	1	1.63	2
V'	+	0	-



$\therefore 1.63$ maximises the volume.