NORTH SYDNEY BOYS HIGH SCHOOL

2025 YEAR 12 HSC ASSESSMENT TASK 3

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – **3 hours**
- Write using blue or black pen
- DoE approved calculators may be used
- All necessary working should be shown in every question
- Attempt all questions

Class Teacher:
(Please tick or highlight)

- ○ Ms Sarofim
- ○ Ms Cai
- ○ Mr Ireland
- ○ Mr Umakanthan

Student Number:

(To be used by the exam markers only.)

Question No	1-10	11- 33	Total
Mark	$\frac{\square}{10}$	$\frac{\square}{90}$	$\frac{\square}{100}$

Outcomes – Section I

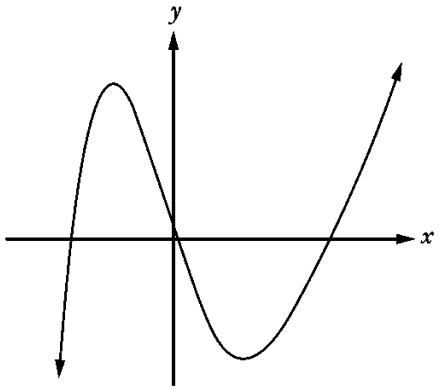
Q.	MA11-2	MA11-3	MA11-4	MA11-6	MA11-7	MA12-1	MA12-3	MA12-4	MA12-5	MA12-6	MA12-7	
1	/1											
2											/1	
3							/1					
4		/1										
5					/1							
6									/1			
7				/1								
8								/1				
9						/1						
10										/1		
Total	**/1**	**/1**		**/1**	**/1**	**/1**	**/1**	**/1**	**/1**	**/1**	**/1**	**/10**

Section I

10 marks

Allow about 15 minutes for this section. Use the multiple-choice answer sheet.

1 The graph of $y = f(x)$ is shown below



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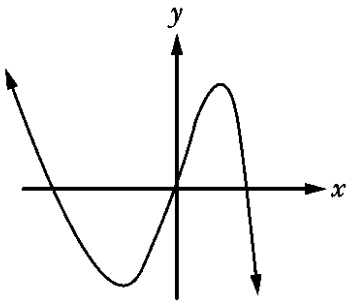

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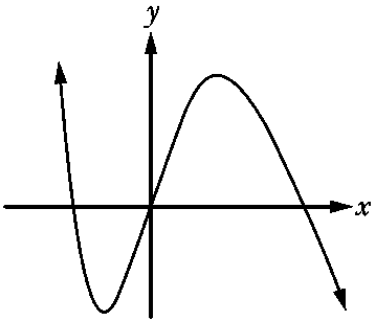

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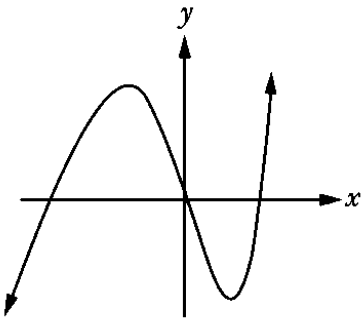

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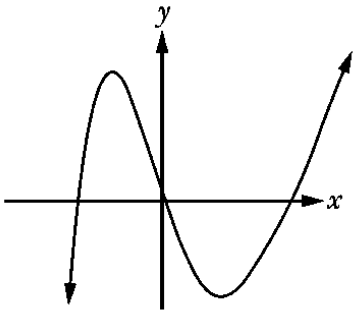

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(B) $2\ln|x^2+3| + C$

(C) $\frac{1}{2(x^2+3)^2} + C$

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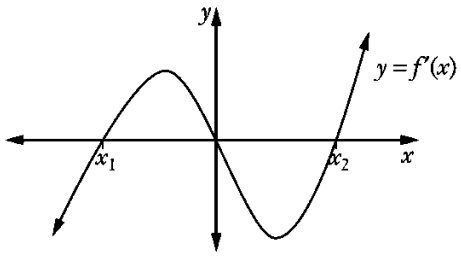

Which of the following graphs could be the graph of y = f(x)?

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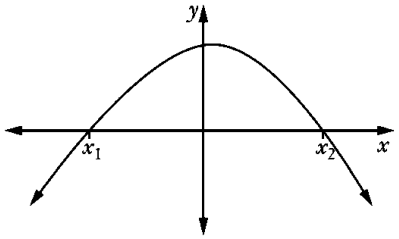

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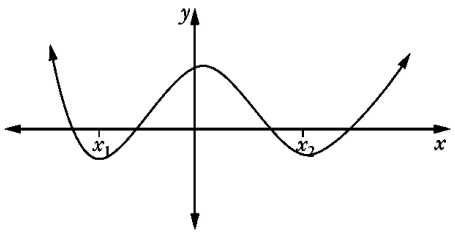

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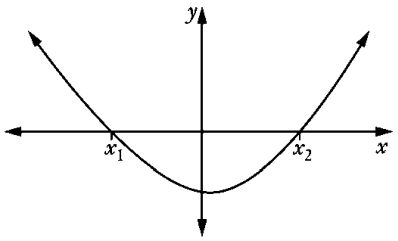

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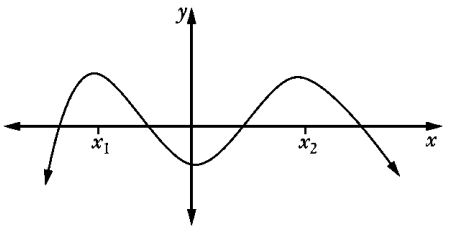

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(A) $\frac{\pi}{3}, \frac{2\pi}{3}$

(B) $\frac{2\pi}{3}, \frac{5\pi}{3}$

(C) $\frac{4\pi}{3}, \frac{5\pi}{3}$

(D) $\frac{7\pi}{3}, \frac{11\pi}{3}$

5 A card is randomly drawn from a set of 5 cards numbered from 1 to 5. After the card is returned to the set, another card is randomly drawn. What is the probability that the number on the first card is greater than the number on the second card?

(A) $\frac{1}{10}$

(B) $\frac{1}{5}$

(C) $\frac{3}{10}$

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6 What are the period and axis of symmetry for $y = |\tan x|$?

(A) $\pi, x = \frac{k\pi}{2} (k \in \mathbb{Z})$

(B) $\frac{\pi}{2}, x = k\pi (k \in \mathbb{Z})$

(C) $\pi, x = k\pi (k \in \mathbb{Z})$

(D) $\frac{\pi}{2}, x = \frac{k\pi}{2} (k \in \mathbb{Z})$

7 If $a > b > 1, 0 < c < 1$, then which of the following is always true?

(A) $a^c < b^c$ (B) $ab^c < ba^c$

(C) $log_a c < log_b c$ (D) $alog_b c < blog_a c$

8 For an Arithmetic Sequence, $T_4 + T_5 = 24, S_6 = 48$. What is the common difference d?

(A) 1 (B) 2

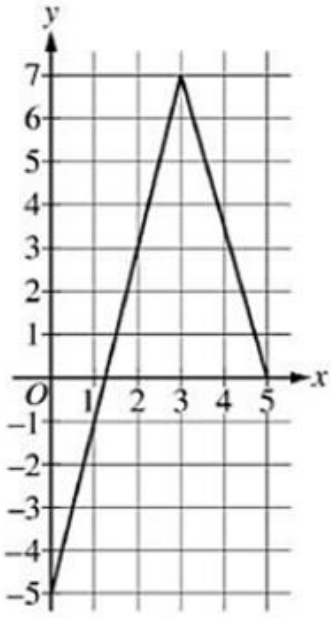
(C) 4 (D) 8

9 The function $f(x) = ax^2 - 2x + log_e x$ has two distinct extrema (max or min) at $x_1$and x_2. What is the range of a?

(A) $(0, \frac{1}{2})$ (B) $(\frac{1}{2}, +\infty)$

(C) $(0, e)$ (D) $(e, +\infty)$

10 The graph of a function $f(x)$ is shown below.



Graph of $f(x)$

If g is the function defined by $g(x) = \frac{x^2+1}{f(x)}$, what is the value of $g'(2)$?

(A) $-\frac{8}{9}$ (B) $\frac{1}{9}$

(C) 1 (D) $\frac{32}{9}$

2025 Y12 Adv - Assessment Task 3 – Outcomes – Section II														
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11	/2													
12				/2										
13			/3											
14						/2								
15				/2										
16												/5		
17													/2	
18(a)					/1									
18(b)													/2	
19					/3									
20(a)												/2		
20(b)													/1	
21(a)(b)(d)									/6					
21(c)											/2			
22									/8					
23										/5				
24		/2												
25									/3					
26								/4						
27							/5							
28													/4	
29												/6		
30		/3												
31					/5									
32											/7			
33										/3				
	/2	/5	/3	/4	/9	/2	/5	/4	/17	/8	/9	/13	/9	/90

Section II

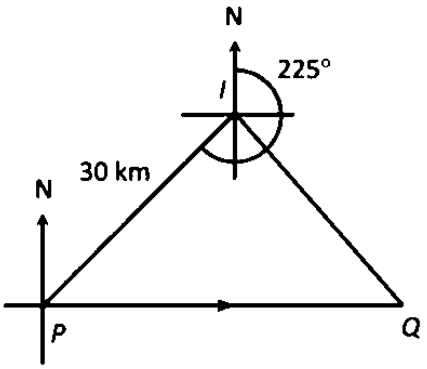
90 marks

Question 11(2 marks)
Solve the equation $|x| \geq |x - 6|$. **2**

Question 12(2 marks)
Show that $(1 - \tan x)^2 + (1 + \tan x)^2 = 2\sec^2 x$. **2**

Question 13 (3 marks)

At 12:00 noon, a ship sails due east from point P at an average speed of 20 km/h. The ship arrives at point Q at 2:00 PM. The bearing of point P from island I is 225°, and the distance between I and P is 30 km.



(a) Find the distance between I and Q, to the nearest km. 1

(b) Find the true bearing of Q from I. 2

Question 14(2 marks)

Solve $log_e x - \frac{3}{log_e x} = 2.$ 2

Question 15(2 marks)

Solve $2\cos\left(2x - \frac{\pi}{2}\right) = \sqrt{3}$ for $x \in [-\pi, \pi]$. 2

Question 16(5 marks)

Differentiate with respect to x:

(a) $y = x^4 lnx$ **1**

(b) $y = \frac{e^x}{x^2}$ **2**

(c) $y = \sin x^3$ **2**

Question 17(2 marks)

Find the integral $\int \frac{1}{5x+3} dx$. **2**

Question 18(3 marks)

(a) Differentiate $y = x log_e x$ with respect to x. **1**

(b) Hence, find the exact value of $\int_1^3 log_e x \, dx$. **2**

Question 19 (3 marks)

(a) Use the trapezoidal rule, with five function values to find the area bounded by the curve $y = \sqrt{x+1}$, the x-axis, and the lines $x = 0$ and $x = 4$. Give your answer to 2 decimal places. **2**

(b) Is your answer in part (a) an underestimate or overestimate of the actual integral? Give a reason for your answer. **1**

Question 20 (3 marks)

(a) Differentiate $y = \sin^4 x$. **2**

(b) Hence, evaluate $\int_0^{\frac{\pi}{2}} \sin^3 x \cos x \, dx$. **1**

Question 21(8 marks)

The acceleration of a particle moving along the *x*-axis is given by $\ddot{x} = 4sin2t$, where *x* is the displacement from the origin in meters and *t* is the time in seconds.
Initially the particle is at the origin moving to the left at 1 m/s.

(a) Show the velocity is given by $\dot{x} = 1 - 2cos2t$. **2**

(b) Find the time when the particle first comes to rest. **2**

(c) Sketch the graph of $\dot{x} = 1 - 2cos2t$ for $0 \leq t \leq 2\pi$. **2**

(d) Find the exact distance travelled by the particle in the first $\frac{\pi}{2}$ seconds. **2**

Question 22(8 marks)

Consider the curve given by the equation $y = x^3 - 6x^2 + 9x + 4$.

(a) Find the coordinates of the stationary points and determine their nature. **3**

(b) Find the coordinates of any point of inflection. **2**

(c) Sketch the curve, showing all the information in part (a) and (b). **2**

(d) Determine the values of x for which $\frac{dy}{dx} > 0$. **1**

Question 23(5 marks)

For the series $8 + 12x + 18x^2 + 27x^3 + \cdots$

(a) Show that the terms in this series form a GP. **1**

(b) For what value(s) of x will this series have a limiting sum? **2**

(c) Find the limiting sum if $x = \frac{1}{4}$. **2**

Question 24(2 marks)

Show that function$f(x)$ is an odd function for $x \in \mathbb{R}$, given that 2

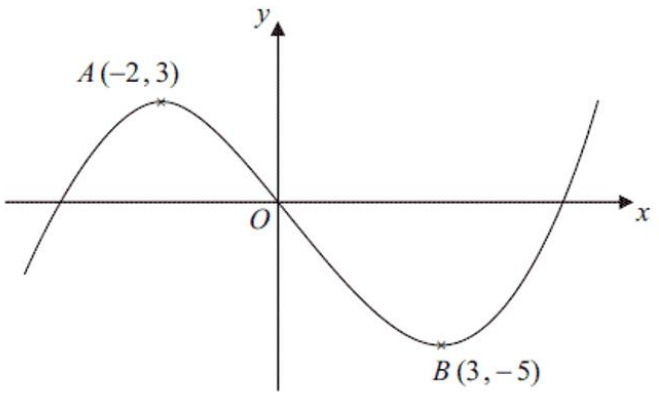
$$f(x) = \begin{cases} x^2 - 2x + 3, & (x > 0) \\ 0, (x = 0) \\ -x^2 - 2x - 3, (x < 0) \end{cases}$$

Question 25(3 marks)

For a curve with function $f(x) = \left(\frac{1}{x} - 1\right)\ln(1 + x)$, find the equation of the tangent to this curve at the point $(1, f(1))$. **3**

Question 26(4 marks)

(a) The diagram below shows the sketch of the curve with equation $y = f(x)$.



Sketch the curve with the equation: $y = 2f(x + 2)$ **2**

(b) State the period, amplitude, centre, and phase for the graph of $f(x) = 5 - 3\sin(2x - 9)$. **2**

Question 27(5 marks)

There are 5 red marbles and 4 blue marbles in a bag. Bill and Ben are playing a game in which they take turns drawing a marble from the bag and then replacing it.

To win the game, Ben must draw a red marble and for Bill to win he must draw a blue marble. They continue taking turns until there is a winner. Ben goes first.

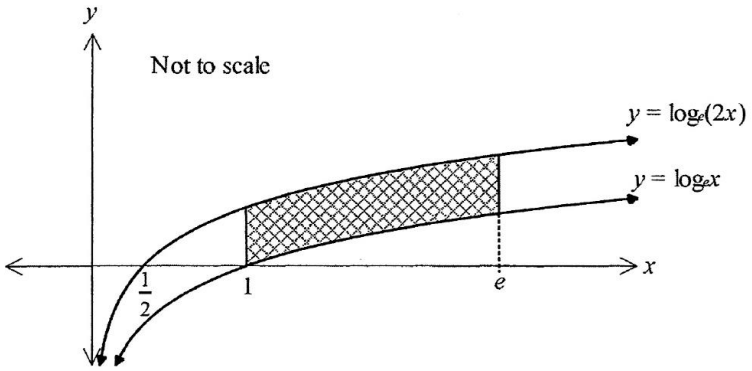
(a) Find the probability that Ben wins on his first draw. **1**

(b) Find the probability that Ben wins in three or less of his turns, in nearest percentage. **2**

(c) Find the probability that eventually Ben wins the game, in nearest percentage. **2**

Question 28(4 marks)

The curves $y = log_e x$ and $y = log_e(2x)$ are drawn below.



Find the shaded area between the two curves and the lines $x = 1$ and $x = e$. 4

Question 29(6 marks)

A rainwater tank attached to a house holds 5600 litres of water when full. The tank is initially empty.

During a recent thunderstorm the tank took 2 hours to fill.

The rate at which water flows off the roof into the tank after the storm commences is given by $\frac{dV}{dt} = \frac{7}{5}t - \frac{7}{900}t^2$.

(a) Find the time when the tank is filling at the fastest rate. **3**

(b) Find the amount of water which overflowed the tank if it rained for 3 hours. **3**

Question 30(3 marks)

We are given a quadratic function $y = A + Bx - x^2$. 3

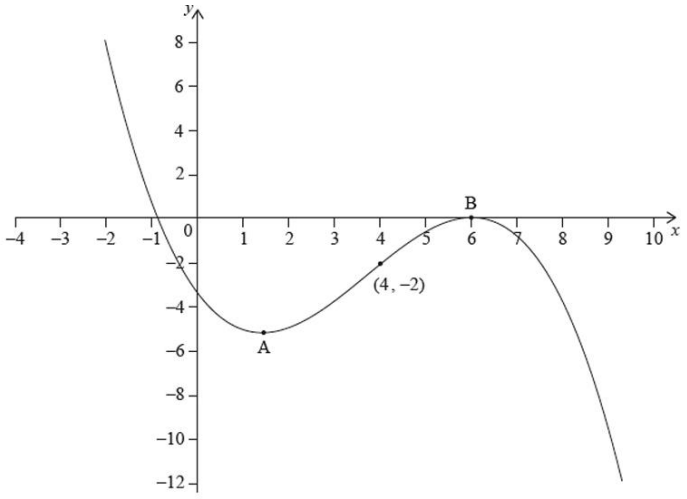
When the function is reflected across the *y*-axis, it is identical to shifting it 3 units to the left.

When the function is reflected across the *x*-axis, it meets the y-axis at (0, 4).

Find the equation of this quadratic function.

Question 31(5 marks)

The following diagram shows the graph of *f'(x)*, the derivative of *f(x)*.



The graph of *f'(x)* has a local minimum at A, a local maximum at B and passes through (4, -2).

The point $P(4, 3)$ lies on the graph of the function *f(x)*.

(a) Write down the gradient of the curve at *P*. **1**

(b) Find the equation of the normal to the curve of f*(x)* at P. **2**

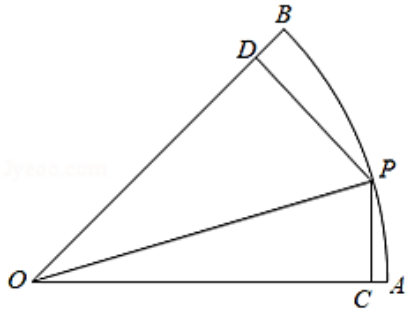
(c) Determine the concavity of the graph of *f(x)* when $4 < x < 5$ and justify your answer.

2

Question 32(7 marks)

A farm has a sector-shaped field, as shown in the diagram below.

∠BOA = $\frac{\pi}{4}$ and the radius is 80 meters. Point P lies on arc AB. PC ⊥ OA and PD ⊥ OB.



The farm plans to have two types of vegetables, α and β, planted respectively in regions △OPC and △OPD. The ratio of the annual production value per unit between α and β is $1:\sqrt{3}$.

Let ∠AOP = θ, where $0 < \theta < \frac{\pi}{4}$.

(a) Show that $A_{\triangle OPC} = 1600 \sin 2\theta$. **2**

(b) Let the annual production value of α and β be y, and the annual production value per unit be t. **3**

Show that $y = 3200t \sin(2\theta + \frac{\pi}{3})$.

(c) Hence, find the value of θ when the annual production value is at its maximum. **2**

Question 33(3 marks)

Given that three numbers form a geometric sequence, if the first two terms remain unchanged and the third term is decreased by 32, they form an arithmetic sequence. Then, if the second term of this arithmetic sequence is decreased by 4, the sequence becomes a geometric sequence again. Find the third term in the original geometric sequence. 3

END OF PAPER

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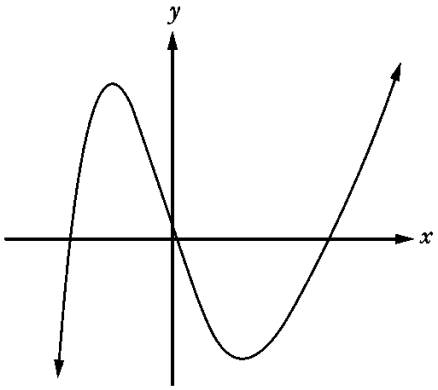

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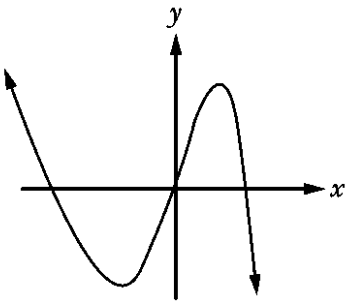

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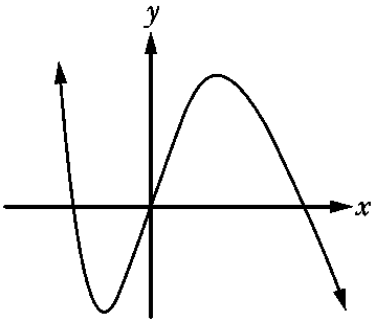

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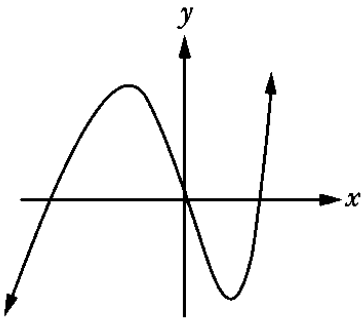

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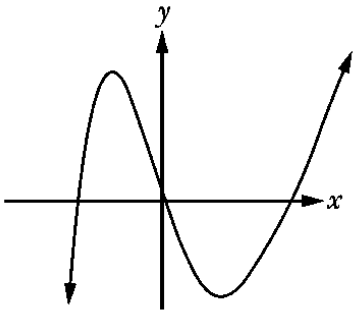

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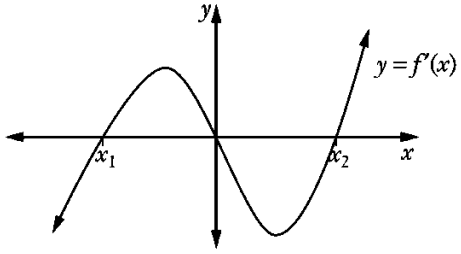

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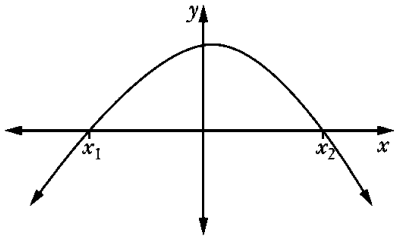

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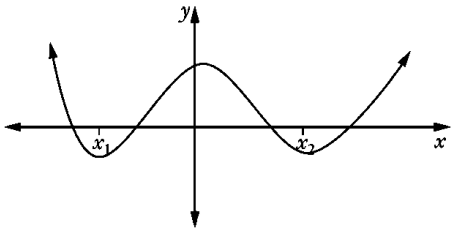

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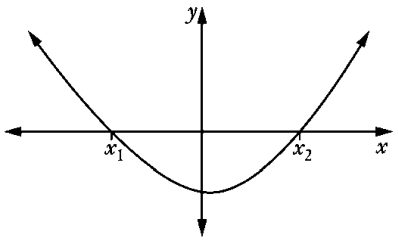

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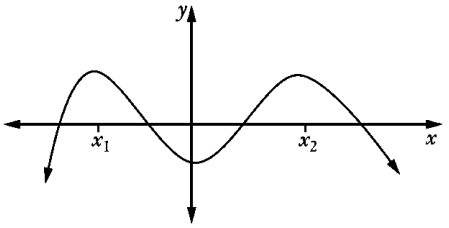

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(D) $\frac{\pi}{2}, x = \frac{k\pi}{2}\ (k \in \mathbb{Z})$

7 If $a > b > 1, 0 < c < 1$, then which of the following is always true?

(A) $a^c < b^c$ $y = x^c$ $(c>0)$ increasing $a > b$, ∴ False.

(B) $ab^c < ba^c \Leftrightarrow \left(\frac{b}{a}\right)^c < \frac{b}{a}$ $y = \left(\frac{b}{a}\right)^x$ decreasing. ∴ False.

(C) $log_a c < log_b c$ ✗

(D) $a log_b c < b log_a c$

8 For an Arithmetic Sequence $a_4 + a_5 = 24$, $S_6 = 48$. What is the common difference d?

$T_4 + T_5$

(A) 1

(B) 2

(C) 4

(D) 8

$a+3d+a+4d=24$

$2a+7d=24$ ① $\frac{6}{2}(2a+5d)=48$

① − ② $2d=8$

$d=4$

$2a+5d=16$ ②

$f'(x) = 2ax - 2 + \frac{1}{x} = \frac{2ax^2 - 2x + 1}{x}$ 2 roots.

$\Delta = 4 - 8a > 0$

$-8a > -4$

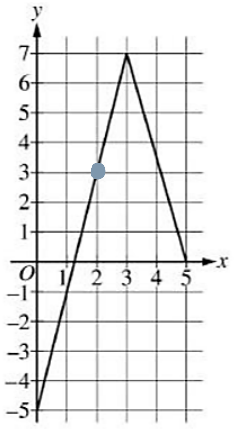
$a < \frac{1}{2}$.

9 The function $f(x) = ax^2 - 2x + log_e x$ has two distinct extrema (max or min) at x_1 and x_2. What is the range of a?

(A) $(0, \frac{1}{2})$ (B) $(\frac{1}{2}, +\infty)$

(C) $(0, e)$ (D) $(e, +\infty)$

10 The graph of a function f is shown below.



Graph of f

(3,7)
(2,3)

$m = \frac{7-3}{3-2} = \frac{4}{1}$

$f(2) = 3$.

$f'(2) = 4$

If g is the function defined by $g(x) = \frac{x^2+1}{f(x)}$, what is the value of $g'(2)$?

(A) $-\frac{8}{9}$ (B) $\frac{1}{9}$

(C) 1 (D) $\frac{32}{9}$

$u = x^2 + 1 \quad u' = 2x$

$v = f(x) \quad v' = f'(x)$

$g'(x) = \frac{2x f(x) - f'(x)(x^2+1)}{f^2(x)}$

$g'(2) = \frac{4f(2) - 5f'(2)}{f^2(2)}$

$= \frac{12 - 20}{9} = -\frac{8}{9}$.

Section II

90 marks

Question 11(2 marks)
Solve the equation $|x| \geq |x - 6|$. **2**

$x^2 \geq (x-6)^2$

$x^2 \geq x^2 - 12x + 36$

$12x \geq 36$

$x \geq 3$

1 mark for answer
1 mark for method
No method, No marks

Question 12(2 marks)
Show that $(1 - \tan x)^2 + (1 + \tan x)^2 = 2\sec^2 x$. **2**

LHS $= 1 - 2\tan x + \tan^2 x + 1 + 2\tan x + \tan^2 x$

$= 2 + 2\tan^2 x$

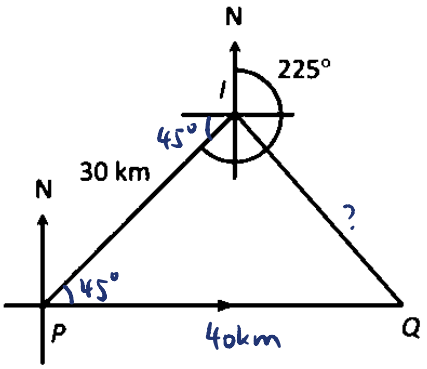
$= 2(1 + \tan^2 x)$

$= 2\sec^2 x$

$=$ RHS.

Question 13 (3 marks)

At 12:00 noon, a ship sails due east from point P at an average speed of 20 km/h. The ship arrives at point Q at 2:00 PM. The bearing of point P from island I is 225°, and the distance between I and P is 30 km.



(a) Find the distance between I and Q, to the nearest km. **1**

$IQ^2 = 30^2 + 40^2 - 2 \times 30 \times 40 \cos 45°$

$IQ = \sqrt{2500 - 1200\sqrt{2}}$

≈ 28 km (nearest km)

Working and answer 1 mark

(b) Find the true bearing of Q from I. **2**

$\frac{\sin Q}{30} = \frac{\sin 45°}{\sqrt{2500 - 1200\sqrt{2}}}$

$Q \approx 48°$ (nearest degree)

Find θ 1 mark

True bearing $= 225° - (180° - 45° - 48°)$

$= 138°$ T

Find bearing 1 mark

OR, use cosine rule: $\cos\theta = \frac{30^2 + 28^2 - 40^2}{2 \times 30 \times 28}$

$\theta \approx 87°$ (nearest degree)

True bearing $= 225° - 87°$

$= 138°$T

Question 14(2 marks)

Solve $log_e x - \frac{3}{log_e x} = 2$.

2

Let $u = \log_e x$

$u - \frac{3}{u} = 2$

$u^2 - 2u - 3 = 0$

$(u-3)(u+1) = 0$

$u = 3 \qquad u = -1$

$\log_e x = 3 \qquad \log_e x = -1$

$x = e^3$ or $x = e^{-1}$

There is a solution here when $x = e^{-1}$

1 mark for working

1 mark for 2 answers

Question 15(2 marks)

Solve $2\cos\left(2x - \frac{\pi}{2}\right) = \sqrt{3}$ for $x \in [-\pi, \pi]$.

2

$-\pi \leq x \leq \pi$

$-2\pi \leq 2x \leq 2\pi$

$-\frac{5\pi}{2} \leq 2x - \frac{\pi}{2} \leq \frac{3\pi}{2}$

$\cos\left(2x - \frac{\pi}{2}\right) = \frac{\sqrt{3}}{2}$

$2x - \frac{\pi}{2} = \frac{\pi}{6}, -\frac{\pi}{6}, -\frac{11\pi}{6}, -\frac{13\pi}{6}$

$2x = \frac{2\pi}{3}, \frac{\pi}{3}, -\frac{4\pi}{3}, -\frac{5\pi}{3}$

$x = \frac{\pi}{3}, \frac{\pi}{6}, -\frac{2\pi}{3}, -\frac{5\pi}{6}$.

2 marks for giving all 4 answers if 2 or 3 with correct working 1 mark

Question 16(5 marks)

Differentiate with respect to x:

(a) $y = x^4 lnx$ 1

$u = x^4 \quad u' = 4x^3$

$v = \ln x \quad v' = \frac{1}{x}$

$y' = x^4 \times \frac{1}{x} + 4x^3 \ln x$

$= x^3 + 4x^3 \ln x$

(b) $y = \frac{e^x}{x^2}$ 2

$u = e^x \quad u' = e^x$

$v = x^2 \quad v' = 2x$

$y' = \frac{x^2 e^x - 2xe^x}{x^4}$

$= \frac{xe^x - 2e^x}{x^3}$

(c) $y = \sin x^3$ 2

$y' = 3x^2 \cos x^3$

Question 17(2 marks)

Find the integral $\int \frac{1}{5x+3} dx$. **2**

$= \frac{1}{5} \ln|5x+3| + C$

Question 18(3 marks)

(a) Differentiate $y = x\log_e x$ with respect to x. **1**

$y' = \log_e x + 1$

(b) Hence, find the exact value of $\int_1^3 \log_e x \, dx$. **2**

$= \int_1^3 \left(\frac{d}{dx}(x\log_e x) - 1\right) dx$

$= [x\log_e x - x]_1^3$

$= 3\log_e 3 - 3 - \log_e 1 + 1$

$= 3\log_e 3 - 2$

Question 19 (3 marks)

(a) Use the trapezoidal rule, with five function values to find the area bounded by the curve $y = \sqrt{x+1}$, the x-axis, and the lines $x = 0$ and $x = 4$. Give your answer to 2 decimal places. **2**

$$A = \frac{4-0}{2\times 4}\{f(0) + f(4) + 2[f(1) + f(2) + f(3)]\}$$

$$= \frac{1}{2}[\sqrt{1} + \sqrt{5} + 2(\sqrt{2} + \sqrt{3} + \sqrt{4})]$$

$$= 6.7642\ldots$$

$$\approx 6.76 \text{ units}^2 \text{ (2 d.p.)}$$

1 mark for substituting correctly

1 mark for correct answer

(b) Is your answer in part (a) an underestimate or overestimate of the actual integral? Give a reason for your answer. **1**

$\therefore y = \sqrt{x+1}$ is concave down for $x \in [-1, +\infty)$

$\therefore$ It underestimate the area of the actual integral.

1 mark

Question 20 (3 marks)

(a) Differentiate $y = \sin^4 x$. **2**

$y' = 4\sin^3 x \cos x$

2 marks for working correctly and clearly

(b) Hence, evaluate $\int_0^{\frac{\pi}{2}} \sin^3 x \cos x \, dx$. **1**

$= \frac{1}{4}\int_0^{\frac{\pi}{2}} 4\sin^3 x \cos x \, dx$

$= \frac{1}{4}\left[\sin^4 x\right]_0^{\frac{\pi}{2}}$

$= \frac{1}{4}$.

1 mark for correct answer and working

Question 21(8 marks)

The acceleration of a particle moving along the *x*-axis is given by $\ddot{x} = 4sin2t$, where *x* is the displacement from the origin in meters and *t* is the time in seconds.
Initially the particle is at the origin moving to the left at 1 m/s.

(a) Show the velocity is given by $\dot{x} = 1 - 2cos2t$. **2**

$\ddot{x} = 4\sin 2t$

$\dot{x} = \int 4\sin 2t \, dt$

$= 2\int 2\sin 2t \, dt$

$= -2\cos 2t + C$

When $t=0$, $\dot{x} = -1$.

$-2\cos(0) + C = -1$

$-2 + C = -1$

$C = 1$

$\therefore \dot{x} = 1 - 2\cos 2t$

2 marks to show every step not skipping

(b) Find the time when the particle first comes to rest. **2**

At rest when $\dot{x} = 0$.

$1 - 2\cos 2t = 0$

$\cos 2t = \frac{1}{2}$

$2t = \frac{\pi}{3}, \frac{5\pi}{3}, \ldots$

$t = \frac{\pi}{6}, \frac{5\pi}{3}, \ldots$

$\therefore$ First at rest when $t = \frac{\pi}{6}$.

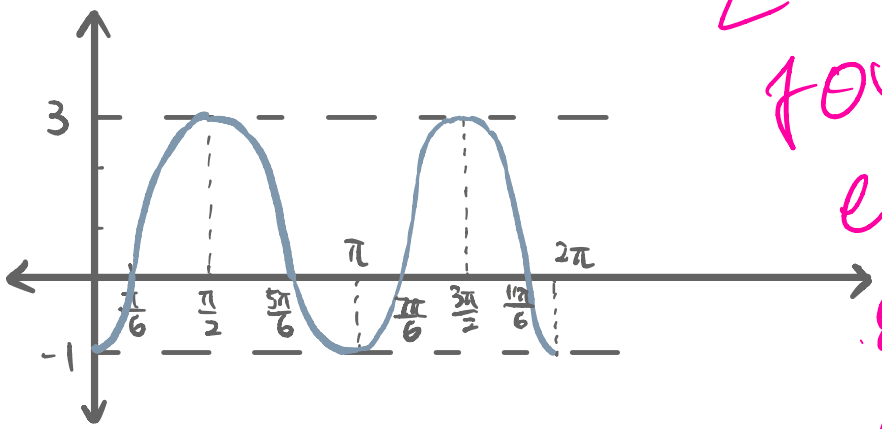
1 mark for solving
1 mark for stating the answer is $\frac{\pi}{6}$

(c) Sketch the graph of $\dot{x} = 1 - 2cos2t$ for $0 \leq t \leq 2\pi$. **2**

Period $= \frac{2\pi}{2} = \pi$

Amplitude : 2

reflect on x-axis & 1 unit up $\left\{\begin{array}{l} \text{max: 3} \\ \text{min: 1} \end{array}\right.$



2 marks for correct end points shape and x intercepts

(d) Find the exact distance travelled by the particle in the first $\frac{\pi}{2}$ seconds. **2**

$x = \int 1 - 2\cos 2t$

$= -\sin 2t + t + c.$

When $t = 0$, $x = 0$

$\therefore c = 0$

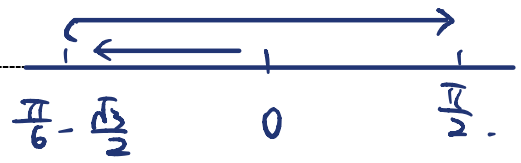
$\therefore x = -\sin 2t + t.$

1 mark for working

1 mark for correct answer

When $t=\frac{\pi}{6}$, $x=\frac{\pi}{6}-\frac{\sqrt{3}}{2}$

When $t=\frac{\pi}{2}$, $x=\frac{\pi}{2}$



$$d=\left|\frac{\pi}{6}-\frac{\sqrt{3}}{2}\right|\times 2+\frac{\pi}{2}$$

$$=2\left(\frac{\sqrt{3}}{2}-\frac{\pi}{6}\right)+\frac{\pi}{2}$$

$$=\sqrt{3}-\frac{\pi}{3}+\frac{\pi}{2}$$

$$=\left(\sqrt{3}+\frac{\pi}{6}\right)\text{ m}.$$

Question 22 (8 marks)

Consider the curve given by the equation $y=x^3-6x^2+9x+4$.

(a) Find the coordinates of the stationary points and determine their nature. **3**

$y'=3x^2-12x+9$.

Stationary point when $y'=0$,

$3x^2-12x+9=0$

$x^2-4x+3=0$

$(x-3)(x-1)=0$

$x=3$ or $x=1$.

$y''=6x-12$

When $x=1$, $y=8$, $f'(1)=0$,

$f''(1)=-6<0$

1 mark for $f'(x)$ and solve

2 marks for finding and checking max and min

∴ (1, 8) is a max turning point.

When $x = 3$, $y = 4$, $f'(3) = 0$,

$f''(3) = 6 > 0$

∴ (3, 4) is a min turning point.

(b) Find the coordinates of any point of inflection. **2**

Point of inflection when $f''(x) = 0$

$6x - 12 = 0$

$6x = 12$

$x = 2$

x	1	2	3
y''	−6	0	6
	<0		>0

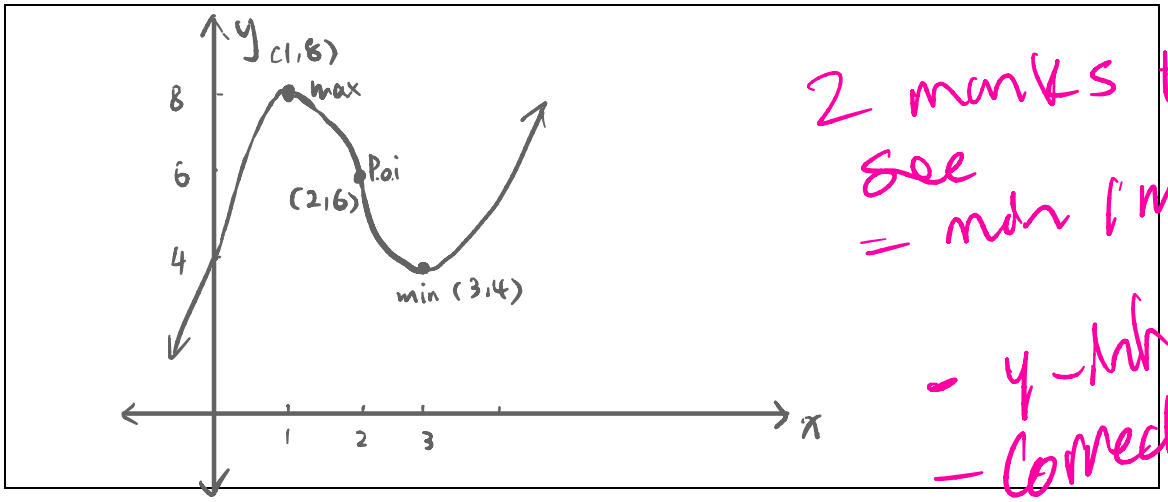
1 mark for getting the point

1 mark for using y'' table to check concavity change

∴ There is a change of concavity

∴ (2, 6) is a point of inflection.

(c) Sketch the curve, showing all the information in part (a) and (b). **2**



2 marks to see
= min / max
- y-int intercept
- Correct shape

(d) Determine the values of x for which $\frac{dy}{dx} > 0$. **1**

$x > 3$ or $x < 1$.

1 mark for giving both inequality

Question 23(5 marks)

For the series $8 + 12x + 18x^2 + 27x^3 + \cdots$

(a) Show that the terms in this series form a GP. **1**

$\frac{T_2}{T_1} = \frac{12x}{8} = \frac{3}{2}x$

$\frac{T_3}{T_2} = \frac{18x^2}{12x} = \frac{3}{2}x$

$\therefore \frac{T_2}{T_1} = \frac{T_3}{T_2}$

$\therefore$ It is a GP.

(b) For what value(s) of x will this series have a limiting sum? **2**

$r = \frac{3}{2}x$, limiting sum exist when $|r| < 1$

$\left|\frac{3}{2}x\right| < 1$

$-1 < \frac{3}{2}x < 1$

$\therefore -\frac{2}{3} < x < \frac{2}{3}$.

(c) Find the limiting sum if $x = \frac{1}{4}$. **2**

$r = \frac{3}{2} \times \frac{1}{4}$

$= \frac{3}{8}$

$S_\infty = \frac{a}{1-r}$

$= \frac{8}{1-\frac{3}{8}}$

$= \frac{64}{5}$

Question 24(2 marks)

Show that function $f(x)$ is an odd function for $x \in \mathbb{R}$, given that 2

$$f(x) = \begin{cases} x^2 - 2x + 3, & (x > 0) \\ 0, (x = 0) \\ -x^2 - 2x - 3, (x < 0) \end{cases}$$

When $x = 0$, $f(-x) = 0$

$-f(x) = 0$ which satisfies $f(-x) = -f(x)$

When $x > 0$, $f(x) = x^2 - 2x + 3$.

Then, $-x < 0$

$\therefore f(-x) = -(-x)^2 - 2(-x) - 3$

$= -x^2 + 2x - 3$

$= -f(x)$ for $x > 0$.

When $x < 0$, $f(x) = -x^2 - 2x - 3$

Then $-x > 0$

$\therefore f(-x) = (-x)^2 - 2(-x) + 3$

$= x^2 + 2x + 3$

$= -f(x)$, for $x < 0$.

Question 25(3 marks)

For a curve with function $f(x) = \left(\frac{1}{x} - 1\right)\ln(1+x)$, find the equation of the tangent to this curve at the point $(1, f(1))$. **3**

$u = \frac{1}{x} - 1 \qquad u' = -\frac{1}{x^2}$

$v = \ln(1+x) \qquad v' = \frac{1}{x+1}$

$f'(x) = \left(\frac{1}{x} - 1\right) \cdot \frac{1}{1+x} - \frac{1}{x^2}\ln(1+x)$

$f'(1) = 0 - \ln 2$

$= -\ln 2$

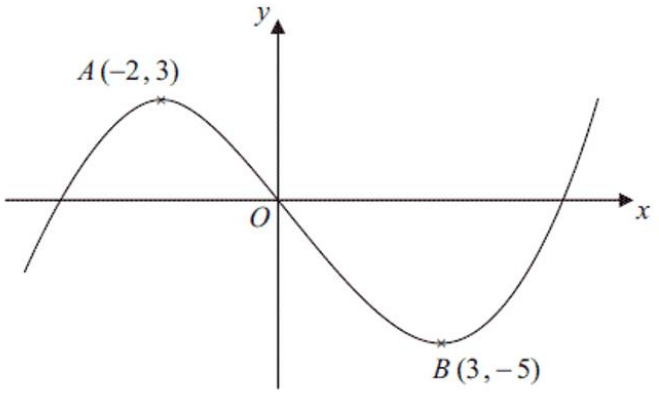
When $x = 1$, $f(1) = 0$

$y - 0 = (-\ln 2)(x - 1)$

$y = \ln 2 - x\ln 2.$

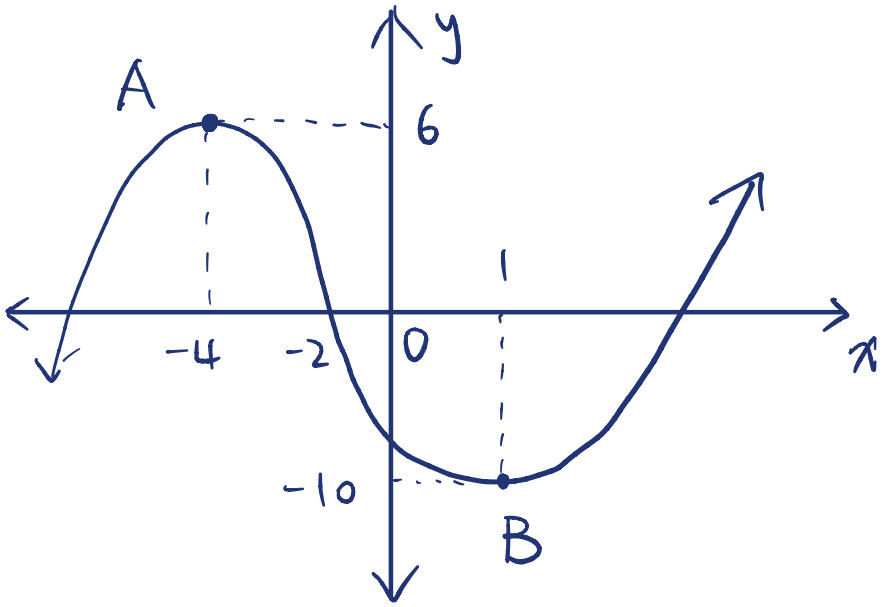
Question 26(4 marks)

(a) The diagram below shows the sketch of the curve with equation $y = f(x)$.



Sketch the curve with the equation: $y = 2f(x + 2)$ **2**

Shift 2 units to the left,
Stretch vertically by 2.



(b) State the period, amplitude, centre, and phase for the graph of $f(x) = 5 - 3\sin(2x - 9)$. **2**

Period $= \frac{2\pi}{2} = \pi$

Amplitude $= 3$

Centre : 5

Phase : $\sin(2x - 9) = \sin\left[2\left(x - \frac{9}{2}\right)\right]$

shift $\frac{9}{2}$ units to the right.

Question 27 (5 marks)

There are 5 red marbles and 4 blue marbles in a bag. Bill and Ben are playing a game in which they take turns drawing a marble from the bag and then replacing it.

To win the game, Ben must draw a red marble and for Bill to win he must draw a blue marble. They continue taking turns until there is a winner. Ben goes first.

(a) Find the probability that Ben wins on his first draw. **1**

$Pr(\text{Ben wins on 1st draw}) = \frac{5}{9}$

(b) Find the probability that Ben wins in three or less of his turns, in nearest percentage. **2**

$Pr(\text{Ben wins on 3rd or less turns})$

$= \frac{5}{9} + \frac{4}{9} \times \frac{5}{9} \times \frac{5}{9} + \frac{4}{9} \times \frac{5}{9} \times \frac{4}{9} \times \frac{5}{9} \times \frac{5}{9}$

$= 0.726...$

$\approx 73\%$

(c) Find the probability that eventually Ben wins the game, in nearest percentage. **2**

GP with $a = \frac{5}{9}$, $r = \frac{4}{9} \times \frac{5}{9} = \frac{20}{81}$

$\because |r| < 1$

$\therefore$ limiting sum exists.

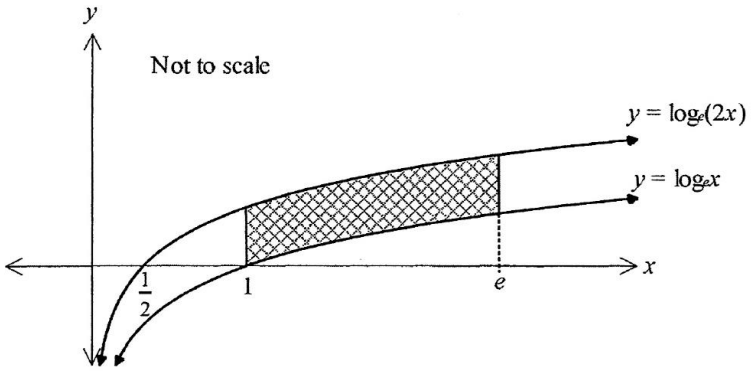
$$S_\infty = \frac{\frac{5}{9}}{1 - \frac{20}{81}}$$

$= 73.77...\%$

$\approx 74\%$

Question 28(4 marks)

The curves $y = log_e x$ and $y = log_e(2x)$ are drawn below.



Find the shaded area between the two curves and the lines $x = 1$ and $x = e$. 4

$$A = \int_1^e [\log_e(2x) - \log_e(x)]\, dx$$

$$= \int_1^e (\ln 2 + \ln x - \ln x)\, dx$$

$$= \int_1^e \ln 2$$

$$= [x \ln 2]_1^e$$

$$= e \ln 2 - \ln 2$$

$$= \ln 2 (e - 1)$$

Question 29 (6 marks)

A rainwater tank attached to a house holds 5600 litres of water when full. The tank is initially empty.

During a recent thunderstorm the tank took 2 hours to fill.

The rate at which water flows off the roof into the tank after the storm commences is given by $\frac{dV}{dt} = \frac{7}{5}t - \frac{7}{900}t^2$.

(a) Find the time when the tank is filling at the fastest rate. **3**

$\frac{d^2V}{dt^2} = \frac{7}{5} - \frac{7t}{450}$

When $\frac{d^2t}{dt^2} = 0$

$\frac{7t}{450} = \frac{7}{5}$

$t = 90$ mins

Check: $\frac{d}{dt}\left(\frac{d^2V}{dt^2}\right) = -\frac{7}{450} < 0$

∴ It's max.

(b) Find the amount of water which overflowed the tank if it rained for 3 hours. **3**

$V = \int \frac{7}{5}t - \frac{7}{900}t^2 \, dt$

$= \frac{7}{10}t^2 - \frac{7}{2700}t^3 + C$

When $t=0$, $V=0$.

∴ $C = 0$

∴ $V = \frac{7}{10}t^2 - \frac{7}{2700}t^3$.

When $t = 180$,

$V = \frac{7}{10} \times 180^2 - \frac{7}{2700} \times 180^3$

$= 7560\ \ell$.

Overflowed $= 7560 - 5600$

$= 1960\ \ell$.

Question 30(3 marks)

We are given a quadratic function $y = A + Bx - x^2$. 3

When the function is reflected across the *y*-axis, it is identical to shifting it 3 units to the left.

When the function is reflected across the *x*-axis, it meets the y-axis at (0, 4).

Find the equation of this quadratic function.

y-int : -4

$\therefore y = -x^2 + Bx - 4$ — 1 mark

$f(-x) = f(x+3)$ — 1 mark

$-x^2 - Bx - 4 = -(x+3)^2 + B(x+3) - 4$

$-x^2 - Bx - 4 = -x^2 - 6x - 9 + Bx + 3B - 4$

$-Bx - Bx + 6x = 3B - 9$

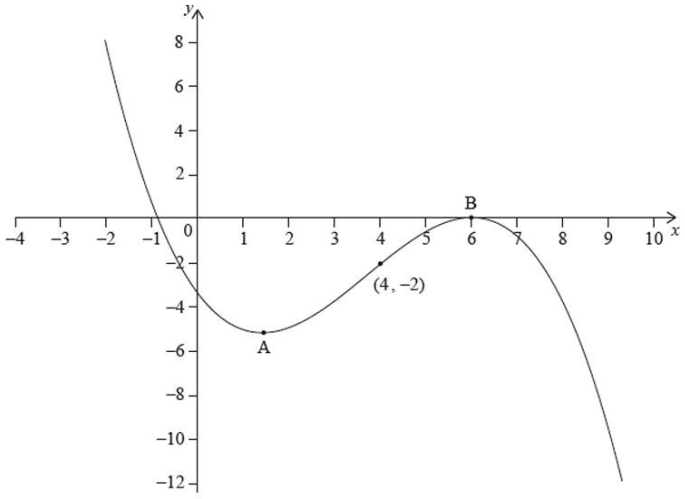
$(6 - 2B)x = 3B - 9$

$\therefore B = 3$

$\therefore$ The equation is $y = -x^2 + 3x - 4$ 1 mark.

Question 31 (5 marks)

The following diagram shows the graph of $f'(x)$, the derivative of $f(x)$.



The graph of $f'(x)$ has a local minimum at A, a local maximum at B and passes through (4, -2).

The point $P(4, 3)$ lies on the graph of the function $f(x)$.

(a) Write down the gradient of the curve at P. **1**

$m = -2$

(b) Find the equation of the normal to the curve of $f(x)$ at P. **2**

$m_{normal} = \frac{1}{2}$ — 1 mark for correct gradient

$y - 3 = \frac{1}{2}(x - 4)$

$2y - 6 = x - 4$

$2y = x + 2$

$y = \frac{1}{2}x + 1$ — 1 mark for the equation.

or $x - 2y + 2 = 0$

(c) Determine the concavity of the graph of *f(x)* when $4 < x < 5$ and justify your answer.

2

∵ The gradient of $f'(x)$ at $4 < x < 5$ is positive. 1 mark

∴ $f''(x)$ at $4 < x < 5$ is positive

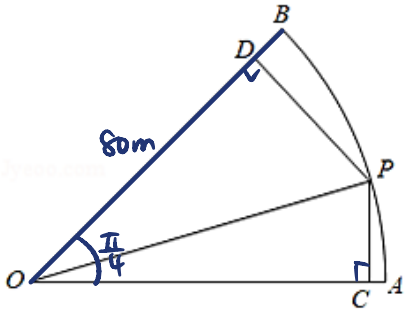
Therefore, concave up. 1 mark.

* articulate your response clearly with correct mathematical terms.

Question 32(7 marks)

A farm has a sector-shaped field, as shown in the diagram below.

∠BOA = $\frac{\pi}{4}$ and the radius is 80 meters. Point P lies on arc AB. PC ⊥ OA and PD ⊥ OB.



The farm plans to have two types of vegetables, α and β, planted respectively in regions △OPC and △OPD. The ratio of the annual production value per unit between α and β is $1:\sqrt{3}$.

Let ∠AOP = θ, where $0 < \theta < \frac{\pi}{4}$.

(a) Show that $A_{\triangle OPC} = 1600 \sin 2\theta$. **2**

In △OPC, OP = 80m, OC = OP cosθ

= 80 cosθ — 1 mark

$A_{\triangle OPC} = \frac{1}{2} \times 80 \times 80\cos\theta \times \sin\theta$

$= 3200 \sin\theta \cos\theta$

$= 1600 \sin 2\theta$. — 1 mark

(b) Let the annual production value of α and β be y, and the annual production value per unit be t. **3**

Show that $y = 3200t \sin(2\theta + \frac{\pi}{3})$.

$A_{\triangle OPC} = 1600 \sin 2\theta$

$A_{\triangle OPD} = 1600 \sin[2(\frac{\pi}{4} - \theta)]$

$= 1600 \sin(\frac{\pi}{2} - 2\theta)$

$= 1600 \cos 2\theta$ — 1 mark

use the ratio between α and β with t.

$y = 1600\sin 2\theta \times t + 1600\cos 2\theta \times \sqrt{3}t$ — 1 mark

$= 1600t(\sin 2\theta + \sqrt{3}\cos 2\theta)$

$= 3200t\sin\left(2\theta + \frac{\pi}{3}\right)$ successfully show with clear working out — 1 mark.

(c) Hence, find the value of θ when the annual production value is at its maximum. **2**

$\because 0 < \theta < \frac{\pi}{4}$

$\therefore \frac{\pi}{3} < 2\theta + \frac{\pi}{3} < \frac{5\pi}{6}$

Max when $2\theta + \frac{\pi}{3} = \frac{\pi}{2}$ — 1 mark

$2\theta = \frac{\pi}{6}$

$\theta = \frac{\pi}{12}$ — 1 mark

Question 33 (3 marks)

Given that three numbers form a geometric sequence, if the first two terms remain unchanged and the third term is decreased by 32, they form an arithmetic sequence. Then, if the second term of this arithmetic sequence is decreased by 4, the sequence becomes a geometric sequence again. Find the third term in the original geometric sequence.

3

Let the numbers form A.P be $a-d$, a, $a+d$.

original (form G.P) : $a-d$, a, $a+d+32$

G.P (again) : $a-d$, $a-4$, $a+d$

— 1 mark for setting variables

$$\frac{a}{a-d} = \frac{a+d+32}{a} \Rightarrow a^2 = (a-d)(a+d+32) \quad \cdots (1)$$

$$\frac{a-4}{a-d} = \frac{a+d}{a-4} \Rightarrow (a-4)^2 = (a+d)(a-d) \quad \cdots (2)$$

— 1 mark for setting simultaneous equations correctly with the information provided.

From (2), $a^2 - 8a + 16 = a^2 - d^2$

$$8a = d^2 + 16$$

$$a = \frac{d^2+16}{8} \quad \cdots (3)$$

From (1) $a^2 = a^2 + ad + 32a - ad - d^2 - 32d$

$$32a - d^2 - 32d = 0 \quad \cdots (4)$$

Sub (3) into (4) : $32 \times \frac{d^2+16}{8} - d^2 - 32d = 0$

$$4(d^2+16) - d^2 - 32d = 0$$

$$4d^2 + 64 - d^2 - 32d = 0$$

$$3d^2 - 32d + 64 = 0$$

$$d = \frac{32 \pm 16}{6}$$

$$d = 8 \text{ or } d = \frac{8}{3}$$

$$\begin{cases} a = 10 \\ d = 8 \end{cases} \text{ and } \begin{cases} a = \frac{26}{9} \\ d = \frac{8}{3} \end{cases}$$

∴ The 3rd term in the original GP are 50 or $\frac{338}{9}$

— 1 mark for correct answers.

END OF PAPER

Alternate Method.

Question 33 (3 marks)

Given that three numbers form a geometric sequence, if the first two terms remain unchanged and the third term is decreased by 32, they form an arithmetic sequence. Then, if the second term of this arithmetic sequence is decreased by 4, the sequence becomes a geometric sequence again. Find the third term in the original geometric sequence. 3

Let the original GP be a, ar and ar^2.

Then, the AP will be a, ar and ar^2-32

The second GP will be a, $ar-4$ and ar^2-32.

$ar-a = ar^2-32-ar$

$ar^2-2ar+a = 32$

$a(r^2-2r+1) = 32 \cdots (1)$

$\frac{ar-4}{a} = \frac{ar^2-32}{ar-4}$

$(ar-4)^2 = a(ar^2-32)$

$a^2r^2-8ar+16 = a^2r^2-32a$

$8ar-32a = 16$

$a(r-4) = 2$

$a = \frac{2}{r-4} \cdots (2)$

Sub (2) into (1),

$\frac{2}{r-4}(r^2-2r+1) = 32$

$2(r^2-2r+1) = 32(r-4)$

$r^2-2r+1 = 16r-64$

$r^2-18r+65 = 0$

$(r-5)(r-13) = 0$

$\therefore \begin{cases} a=2 \\ r=5 \end{cases}$ or $\begin{cases} a=\frac{2}{9} \\ r=13 \end{cases}$

$ar^2 = 50$, $ar^2 = \frac{338}{9}$

$\therefore$ The 3rd term in the original GP are 50 or $\frac{338}{9}$.

END OF PAPER