Teacher: _____

North Sydney Girls High School

Student Number

2025 **HSC TRIAL EXAMINATION**

Mathematics Advanced

General Instructions

- Reading Time – 10 minutes
- Working Time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (pages 2 – 5)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – **90 marks** (pages 7 – 43)

- Attempt Questions 11 – 31
- Allow about 2 hours and 45 minutes for this section

Question	1–10	11–19	20-26	27-31	Total
Mark	/10	/32	/28	/30	/100

Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section.

Use the multiple choice answer sheet for Questions 1-10.

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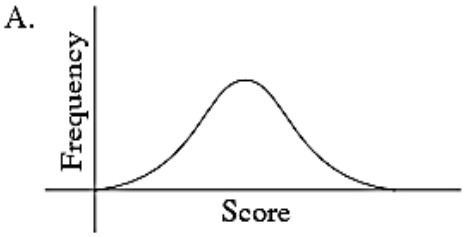
1 Which graph represents a negatively skewed distribution?

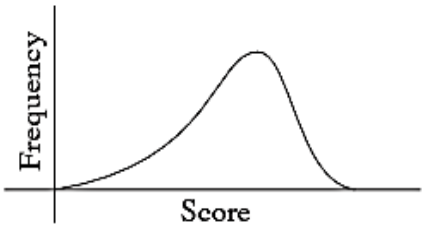
A.



B.

C.

D.



2 For what values of k does $x^2 - kx + 4 = 0$ have two distinct real roots?

A. $k > 4$

B. $k \geq 4$

C. $k < -4, k > 4$

D. $k \leq -4, k \geq 4$

3 A particle is moving along the x-axis. The displacement of the particle at time t seconds is x metres. At a certain time, the velocity is $-3\ \text{ms}^{-1}$ and the acceleration is $2\ \text{ms}^{-2}$.

Which statement describes the motion of the particle at that time?

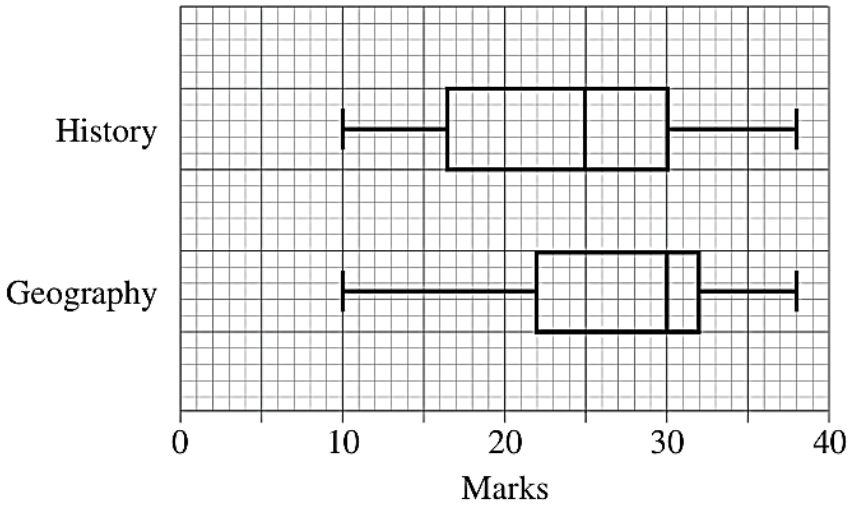
A. The particle is moving to the right with increasing speed.

B. The particle is moving to the left with decreasing speed.

C. The particle is moving to the right with decreasing speed.

D. The particle is moving to the left with increasing speed.

4 The boxplots show the results of a History test and a Geography test.



In History, 112 students completed the test. The number of students who scored below 30 marks was the same for the History test and the Geography test.

How many students completed the Geography test?

A. 56

B. 50

C. 168

D. 112

5 The equation of a circle is:

$$x^2 + y^2 - 8x + 6y + 9 = 0$$

The circle is then reflected in the x-axis.
What is the equation of the resulting circle?

A. $(x-4)^2 + (y+3)^2 = 16$

B. $(x+4)^2 + (y-3)^2 = 16$

C. $(x+4)^2 + (y+3)^2 = 16$

D. $(x-4)^2 + (y-3)^2 = 16$

6 Which of the following is an expression for $\int \left(\frac{1}{5}\right)^x dx$?

A. $\frac{1}{\ln 5 \cdot 5^x} + C$

B. $\frac{-1}{\ln 5 \cdot 5^x} + C$

C. $\frac{-1}{\ln \frac{1}{5} \cdot \left(\frac{1}{5}\right)^x} + C$

D. $\frac{1}{5} \ln x + C$

7 Given that $f(x)$ is an even function and $g(x)$ is an odd function, which of the following statements is true?

A. $f(x) \times g(x)$ is an even function

B. $f(x) + g(x)$ is an odd function

C. $f(g(x))$ is an even function

D. $g(f(x))$ is an odd function.

8 If $b > 1$ and $a = \log_{12} b,$ which expression is equal to $\frac{1}{a}$?

A. $-\log_{12} b$

B. $\log_b 12$

C. $-\log_{12} \frac{1}{b}$

D. $\frac{1}{\log_b 12}$

9 The curve $y = \ln x$ is translated to the left by π units and then dilated horizontally by a scale factor of 3. What is the equation that describes this new curve?

A. $y = \ln\left(\frac{x}{3} + \pi\right)$

B. $y = \ln\left(\frac{x}{3} - \pi\right)$

C. $y = \ln(3x + \pi)$

D. $y = \ln\left(\frac{x + \pi}{3}\right)$

10 Let A and B be two independent events from a sample space.
If $P(A) = p,\ P(B) = p^2,$ then which of the following is an expression for $P\left(\overline{A} \cup B\right)$?

A. $1 - p + p^2$

B. $p^2 - p^3$

C. $1 - p + p^3$

D. $p - p^3$

End of Section I

Student Number

Mathematics Advanced
Section II Answer Booklet 1

1

Section II

90 marks
Attempt Questions 11–31
Allow about 2 hours and 45 minutes for this section

Booklet 1 – Attempt Questions 11–19 (32 marks)
Booklet 2 – Attempt Questions 20–26 (28 marks)
Booklet 3 – Attempt Questions 27–31 (30 marks)

Instructions

- Write your student number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on pages at the end of each booklet. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 11 (2 marks)

Solve the equation $|2x-1|=3$. **2**

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Question 12 (4 marks)

For a geometric series the third term (T_3) is 20 and the sixth term (T_6) is -160.

(a) Find the common ratio and the first term. **2**

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(b) Find the sum of the terms from T_{10} to T_{20} inclusive. **2**

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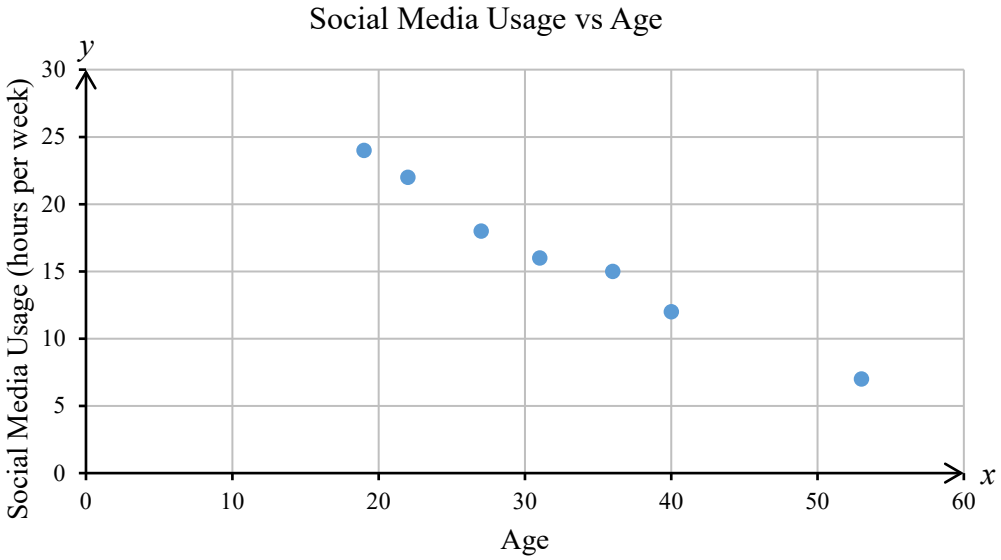
Question 13 (5 marks)

A group of adults across a range of ages were surveyed about their social media usage.
The scatterplot below shows the hours spent each week on social media vs age.



(a) The data is presented in a table below. Fill in the missing value in the table. **1**

Age (x)	19	22	27	31	36	40	53
Hours per week (y)	24	22	18	16		12	7

(b) Calculate Pearson's correlation coefficient, correct to 4 significant figures. **1**

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(c) Describe the correlation between social media use and age suggested by this data. **1**

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(d) Determine the equation of the least squares regression line, rounding coefficients to 2 decimal places. **1**

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(e) Should the regression line be used to predict the social media usage of 11 year-olds? By referring to the context, give a reason for your answer. **1**

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Question 14 (2 marks)

Find $\int \frac{x}{\sqrt{x^2+1}}\,dx$. **2**

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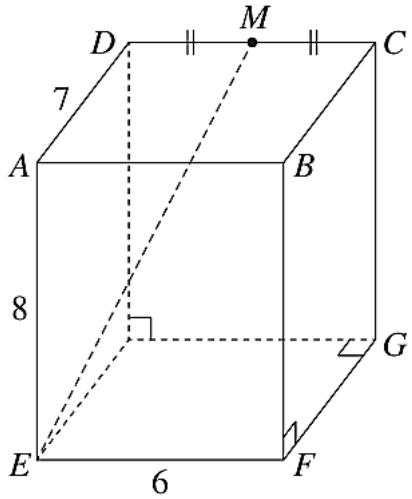
Question 15 (3 marks)

In the rectangular prism shown, $AD = 7$ cm, $AE = 8$ cm, $EF = 6$ cm. . Point M is the midpoint of CD. **3**



NOT TO SCALE

Find $\angle AEM$, to the nearest degree.

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Question 16 (4 marks)

A courier company tracks the delivery times (in days) for packages sent across the country. Packages arrive in 1 to 5 days. During a morning system check, the data analyst found that some of the delivery time probabilities were missing due to a database error.

The partial probability distribution is shown below:

Delivery Time X (days)	1	2	3	4	5
$P(X = x)$	0.1	a	b	0.2	0.2

The analyst recalls that the expected value for the delivery time is 3.2 days.

(a) By first forming two equations, calculate the missing probabilities a and b. **3**

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Later that day, the full dataset was recovered from a backup, and the corrected distribution was confirmed to be:

Delivery Time X (days)	1	2	3	4	5
$P(X = x)$	0.1	0.2	0.3	0.2	0.2

(b) Using the corrected distribution, calculate the variance of the delivery time. **1**

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Question 17 (3 marks)

Find the equation of the normal to the curve $y = \left(x + \frac{2}{x} \right)^2$ when $x = 2$. **3**

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Question 18 (5 marks)

The velocity v ms^{-1} of a moving particle is given after t seconds by $v = 8t - 16$.
Initially the particle is located at the origin.

(a) What is the acceleration of the particle? **1**

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(b) Determine when the particle is at rest. **1**

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(c) How far does the particle travel in the first 5 seconds? **3**

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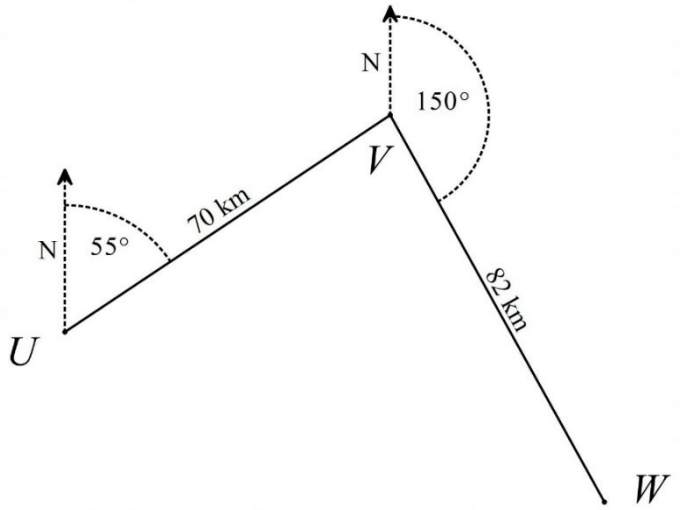
Question 19 (4 marks)

A helicopter leaves Underwood and flies 70 km on a bearing of 055° to Vacy.
It then flies 82 km on a bearing of 150° to Woodville.

The diagram below illustrates the journey.



(a) The helicopter then plans to fly directly to Underwood. **2**
Calculate, to the nearest km, the distance that it will fly.

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Question 19 continues on page 15

Question 19 (continued

(b) Determine the bearing (from Woodville) on which it should fly to return to Underwood. **2**
Give your answer to the nearest degree.

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End of Question 19

Proceed to Booklet 2 for Questions 20-27

Student Number

Mathematics Advanced
Section II Answer Booklet 2

2

Booklet 2 – Attempt Questions 20–26 (28 marks)

Instructions

- Write your student number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on pages at the end of each booklet. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 20 (2 marks)

It is given that a quantity y varies inversely with the square of x. **2**

When $x = 4,\ y = 10$.

Find the value of y when $x = 5$.

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Question 21 (4 marks)

(a) Show that $\dfrac{\sec^2 3x}{\tan 3x} = \sec 3x \operatorname{cosec} 3x$ **2**

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(b) Hence find $\int \sec 3x \operatorname{cosec} 3x \, dx$ **2**

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Question 22 (5 marks)

The population of two towns Melbtown and Brisbay are given by $P_M = 1200e^{0.04t}$ and $P_B = 1500e^{0.025t}$ respectively, where t is the number of years elapsed since January 2010.

(a) In which year will the population of Melbtown exceed 1500? **2**

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(b) In which year will the rate of growth of the population in Melbtown be double the rate of growth of Brisbay? **3**

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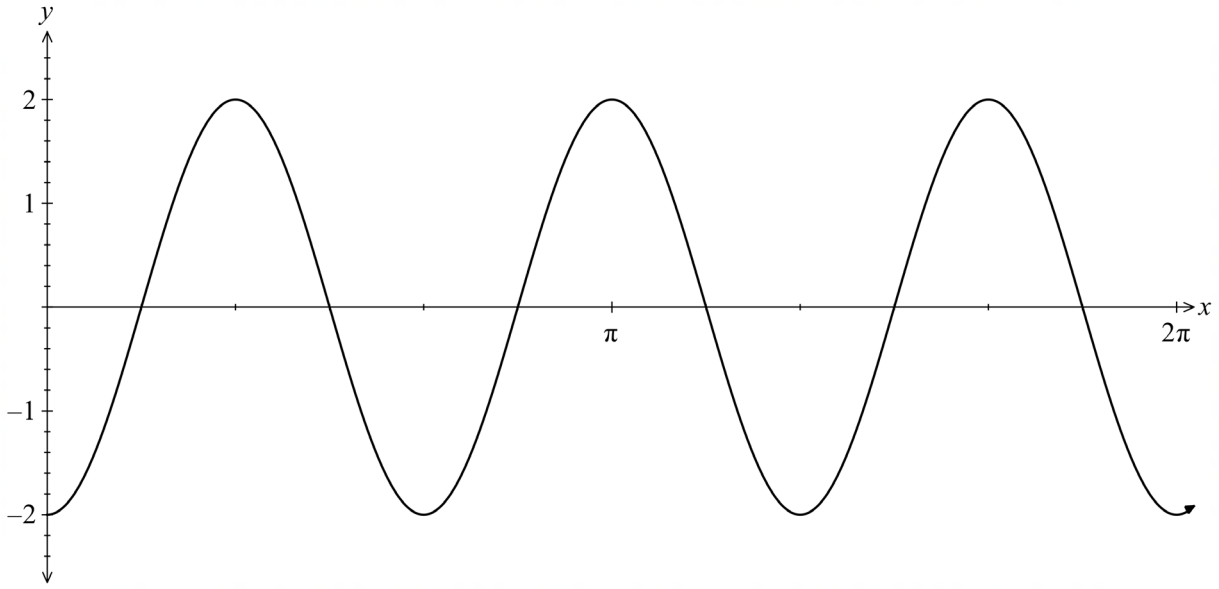
Question 23 (3 marks)

Ben sketched the curve shown below to model a tidal experiment in his Science class. **3**
Ben knows the curve is in the form $y = k\cos(ax+b)$ where $k > 0,\ a > 0$ and $0 \le b \le \pi$.

By finding *k*, *a* and *b*, write the equation for Ben's tidal experiment.



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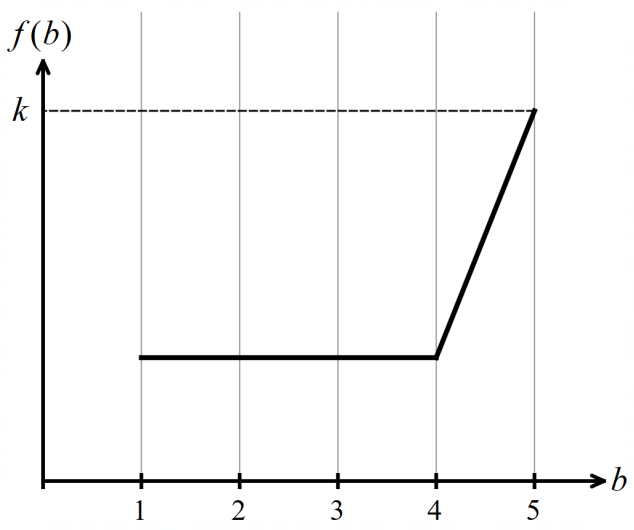
Question 24 (6 marks)

Liz is having a rare book restored at *Binding Repairs*. She drops the book off in the morning and is told that the book will be ready for pickup sometime between 1 pm and 5 pm that day.

Let the random variable B represent the time after noon (12 pm) that Liz's book is ready for collection. The probability density function for B is shown in the graph below.



The probability of a book being ready for collection between 2 pm and 3 pm is 0.2.

(a) Determine the value of k. **1**

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(b) An incomplete expression for the probability density function of B is given below. **1**
Fill in the box to complete the missing part of the expression.

$$f(b) = \begin{cases} 0.2, & 1 \le b < 4 \\ \boxed{}, & 4 \le b \le 5 \\ 0, & \text{otherwise} \end{cases}$$

Question 24 continues on page 25

(c) Determine the median time that Liz's book will be ready for collection. **1**

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Andrew is also having a book restored but he uses a different company called *Folio Fixers.* He drops his book off in the morning and is told that his book will be ready for collection sometime between 1 pm and 5 pm that day.

Let the random variable A represent the time after noon (12 pm) that Andrew's book is ready for collection. The cumulative distribution function for A is given by

$$P(A \le a) = \begin{cases} 0, & a < 1 \\ \dfrac{10a - a^2 - 9}{16}, & 1 \le a \le 5 \\ 1, & a > 5 \end{cases}$$

(d) Determine the probability that Andrew's book will be ready to collect by 4 pm. **1**

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(e) Determine the probability that Andrew's book will be ready to collect by 3 pm given that it is ready to collect by 4 pm. **2**

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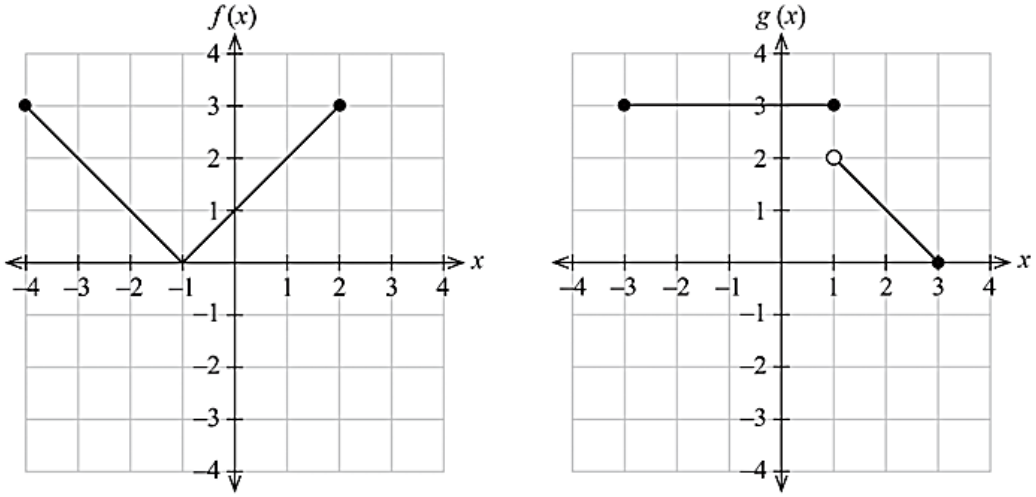
End of Question 24

Question 25 (4 marks)

The graphs of functions $f(x)$ and $g(x)$ are shown below.



(a) State the domain of $f(g(x))$. **1**

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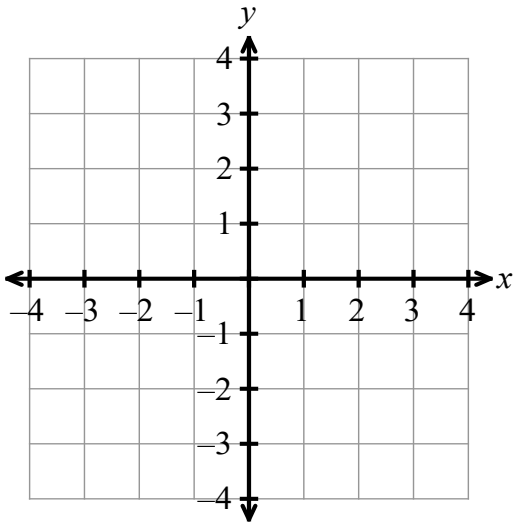
(b) Sketch $h(x) = g(-x) - 1$ on the grid below. **2**



(c) Hence solve $f(h(x)) = 0$. **1**

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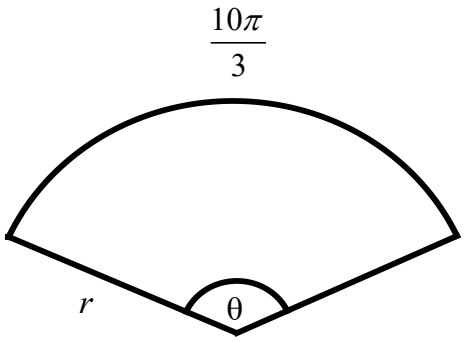
Question 26 (4 marks)

The diagram shows a sector with radius r and angle θ where $\frac{\pi}{2} < \theta \leq \pi$.

The arc length is $\frac{10\pi}{3}$.



(a) Find the range of possible values for r, expressing your answer in interval notation. **3**

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(b) Calculate the area of the sector when $r = 4$. **1**

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Proceed to Booklet 3 for Questions 27-31

Student Number

Mathematics Advanced
Section II Answer Booklet 3

3

Booklet 3 – Attempt Questions 27–31 (30 marks)

Instructions

- Write your student number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on pages at the end of each booklet. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 27 (4 marks)

The numbers $4e^{-k}$, 5, $4e^{k}$ are consecutive terms in an arithmetic sequence.

(a) Show that $2e^{k} - 5 + 2e^{-k} = 0$. **1**

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(b) Hence find in exact form any possible values of k. **3**

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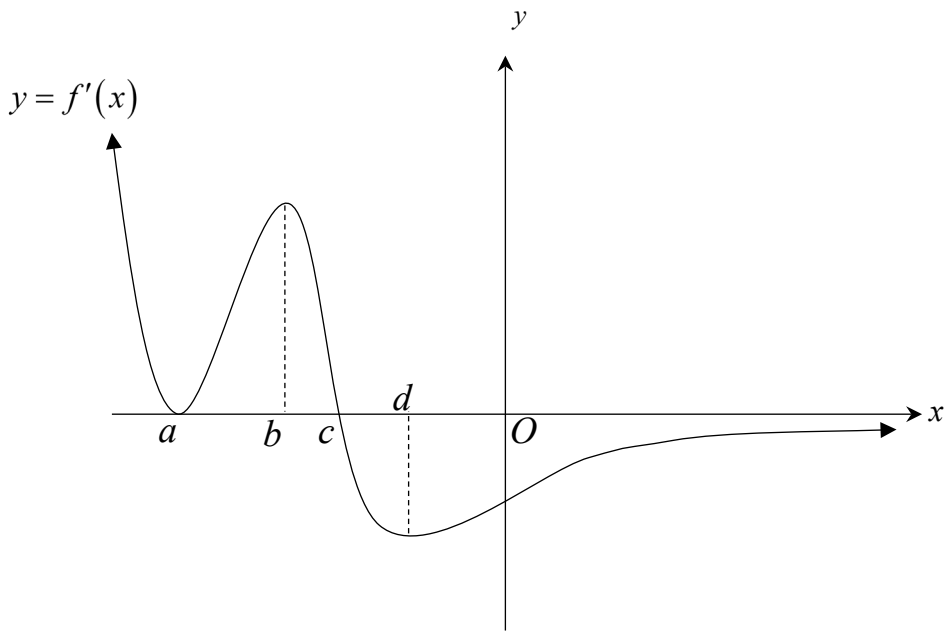
Question 28 (5 marks)

The graph of the gradient function $y = f'(x)$ is given below.
Specific x-values a, b, c and d are as indicated in the diagram.



(a) Justify why $f(x)$ has a maximum stationary point at $x = c$. **1**

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(b) For what value(s) of x is the graph of $y = f(x)$ concave up? **2**

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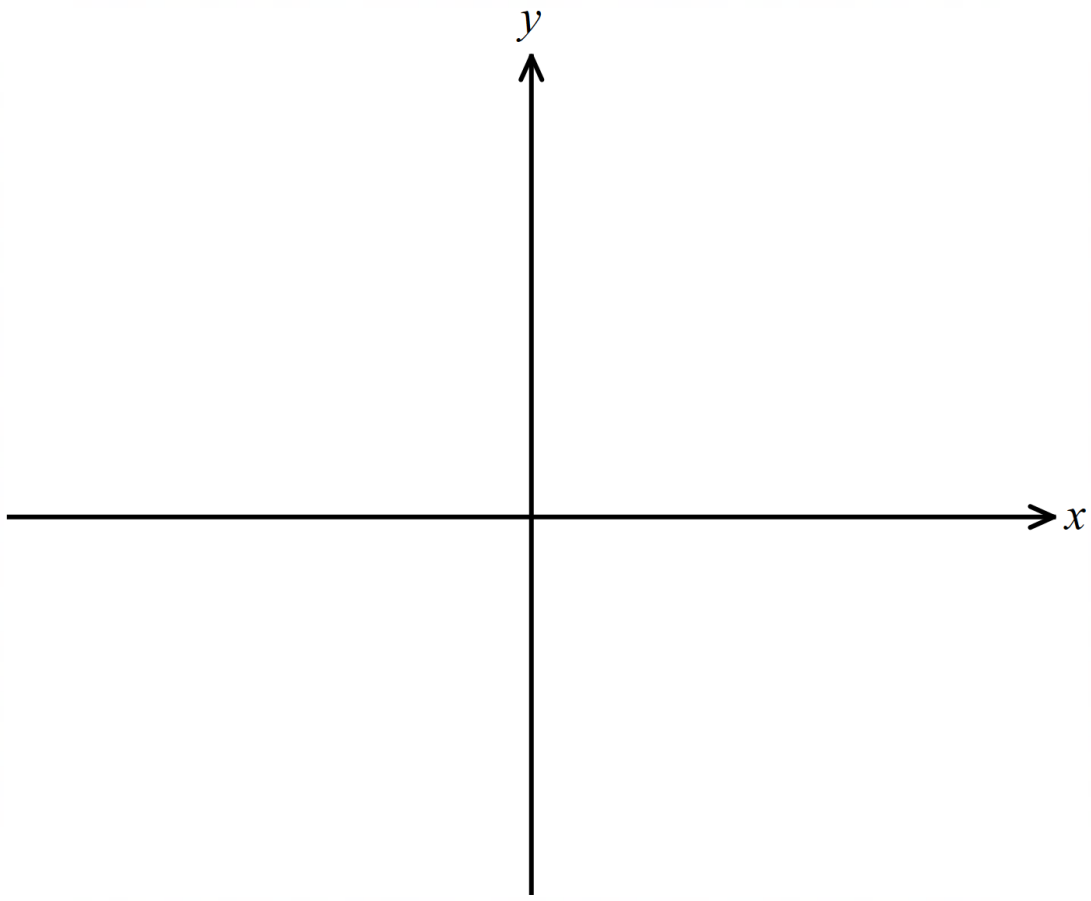
Question 28 continues on page 35

Question 28 (continued)

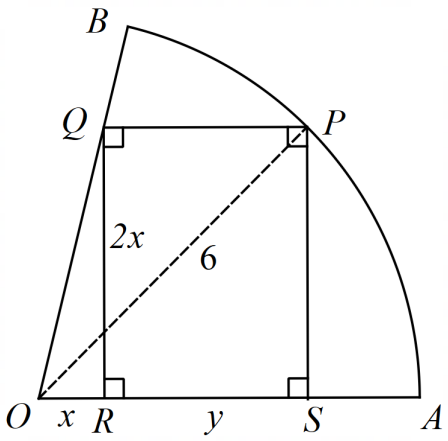
(c) Draw a possible graph of $y = f(x)$ showing all important features. **2**



End of Question 28

Question 29 (6 marks)

The diagram below shows a rectangle $PQRS$ drawn in a sector OAB of a circle with radius 6. $OR = x$, $QR = 2x$ and $RS = y$. All units are in metres.



NOT TO SCALE

(a) Show that $(x + y)^2 = 36 - 4x^2$. **1**

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(b) Show that the perimeter, L, of the rectangle is given by **2**

$$L = 2x + 2\sqrt{36 - 4x^2}$$

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Question 29 continues on page 37

Question 29 (continued)

(c) Find the value of x which gives the maximum perimeter of the rectangle. **3**

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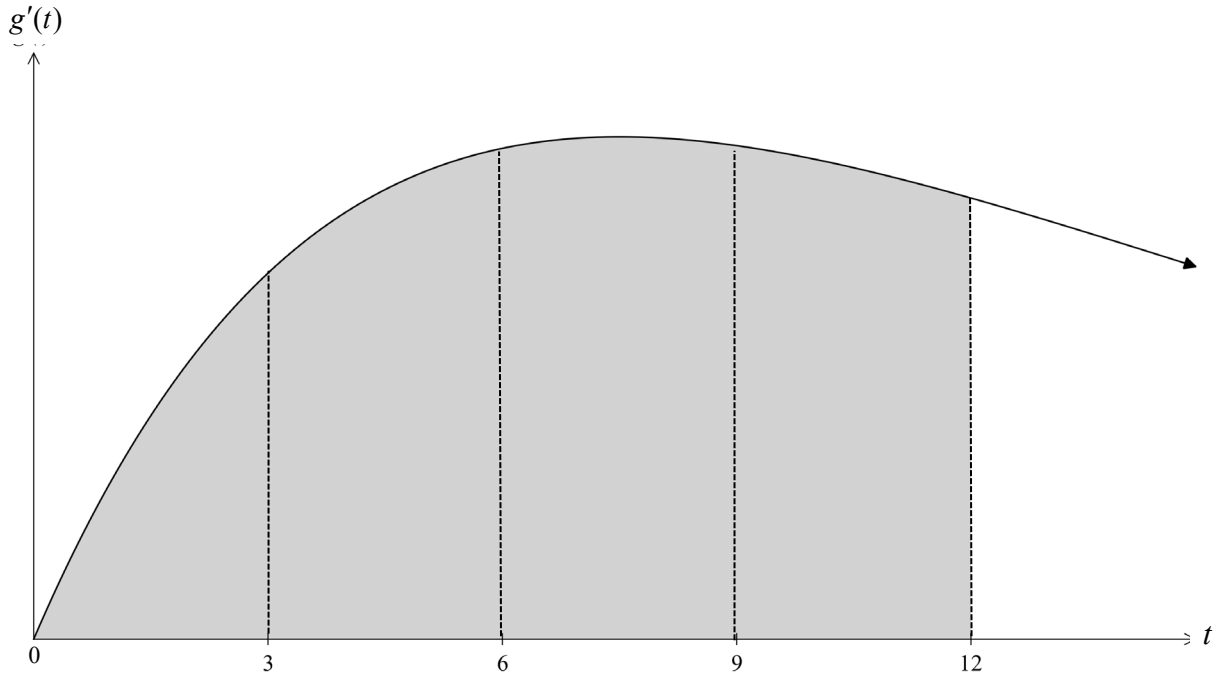
End of Question 29

Question 30 (5 marks)

Let $g(t)$ be a function defined for all $t \geq 0$. The gradient function is given by

$$g'(t) = \frac{33}{10} te^{-kt}.$$

where k is a positive constant. The diagram below shows a sketch of $g'(t)$ against t.
It is given that $g'(t)$ attains the greatest value at $t = 7.5$ and $g(0) = 0$.



(a) Show that $k = \frac{2}{15}$. **2**

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Question 30 continues on page 39

(b) Use the trapezoidal rule with four sub-intervals to estimate the shaded area in the diagram. **2**

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(c) Explain why your answer to part (b) is an estimate for $g(12)$. **1**

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End of Question 30

Please turn over for Question 31

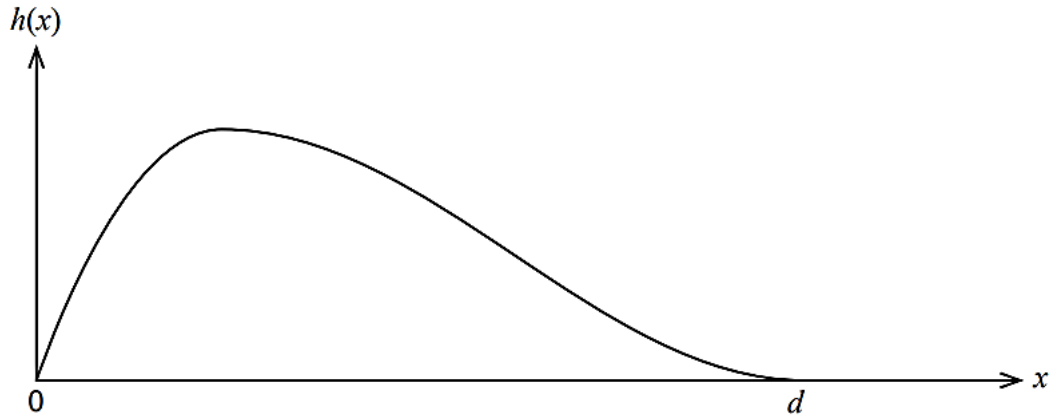
Question 31 (10 marks)

A new luxury hotel in Byron Bay is being designed to feature a stunning wave-shaped window that stretches across the main lobby wall, inspired by the rolling surf of the nearby beaches. The top edge of this window is modelled mathematically to reflect both a smooth curve and a gentle wave.

The height, $h(x)$, of the top edge of the window above ground level, in metres, is described by the piecewise function:

$$h(x) = \begin{cases} a\left(1-\left(x-1\right)^2\right), & 0 \le x \le 1 \\ 2\cos\left(x-1\right)+2, & 1 < x \le d \end{cases}$$

where a and d are constants and x is the horizontal distance from the leftmost point of the window (which is at ground level).



(a) Determine the value of a in the function $h(x)$. **1**

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Question 31 continues on page 41

(b) By first showing that the length of the bottom edge of the window d is $(\pi+1)$ metres, determine the area of the window. **5**

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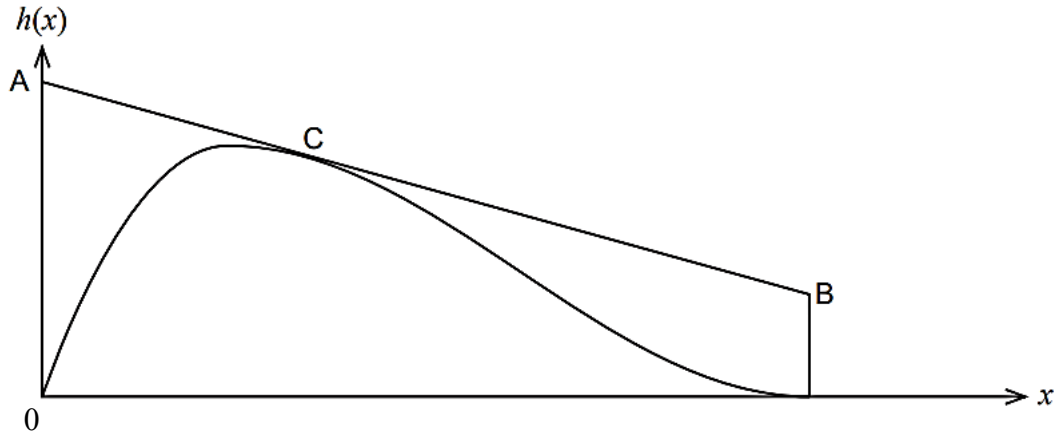
Question 31 continues on page 42

(c) The top edge of the wall, shown as the line AB below, is to just touch the window at point C as shown below. **4**

The wall is 1.39 m higher at point A than point B. How high is point C above the ground?



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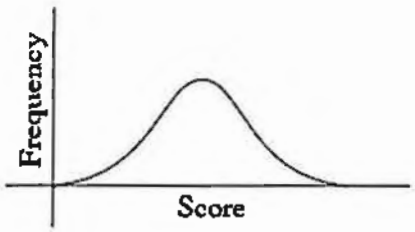
North Sydney Girls High School

2025 HSC TRIAL EXAMINATION

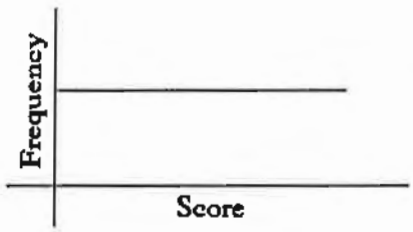
Mathematics Advanced- Solutions

1 Which graph represents a negatively skewed distribution?

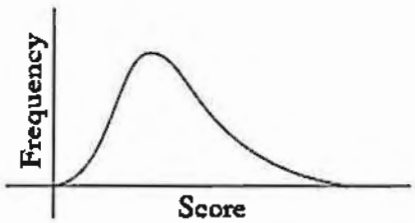
A.



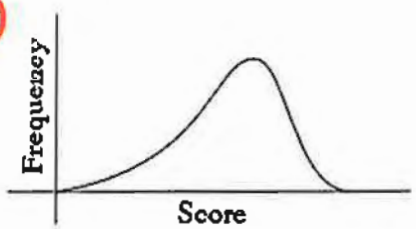
B.



C.



D.



2 For what values of k does $x^2 - kx + 4 = 0$ have two distinct real roots?

A. $k > 4$

B. $k \geq 4$

C. $k < -4, k > 4$

D. $k \leq -4, k \geq 4$

$\Delta = (-k)^2 - 4(1)(4)$

$k^2 - 16 > 0$

$(k-4)(k+4) > 0$

$k < -4, \quad k > 4$

3 A particle is moving along the x-axis. The displacement of the particle at time t seconds is x metres. At a certain time, the velocity is -3 ms^{-1} and the acceleration is 2 ms^{-2}.

Which statement describes the motion of the particle at that time?

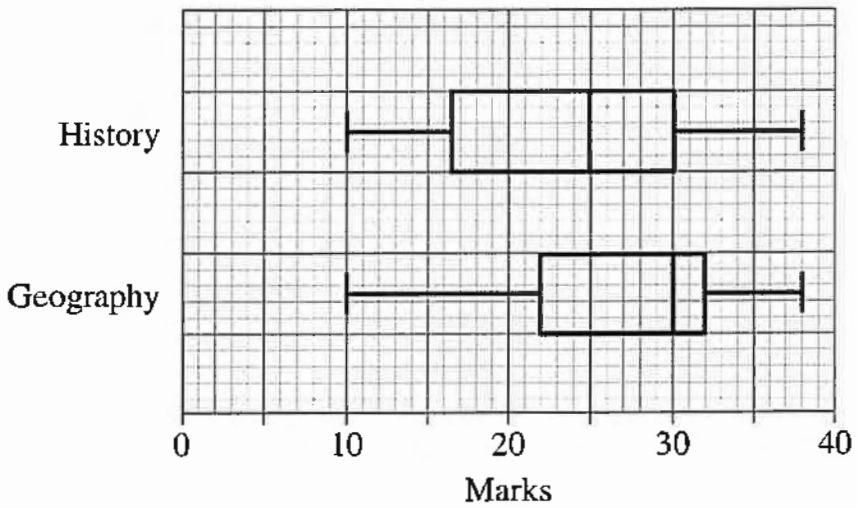
A. The particle is moving to the right with increasing speed.

B. The particle is moving to the left with decreasing speed.

C. The particle is moving to the right with decreasing speed.

D. The particle is moving to the left with increasing speed.

4 The boxplots show the results of a History test and a Geography test.



In History, 112 students completed the test. The number of students who scored below 30 marks was the same for the History test and the Geography test.

How many students completed the Geography test?

A. 56

B. 50

C. 168

D. 112

5 The equation of a circle is:

$$x^2+y^2-8x+6y+9=0$$

The circle is then reflected in the x-axis.
What is the equation of the resulting circle?

$x^2-8x+(-4)^2+y^2+6y+3^2=16$

$(x-4)^2+(y+3)^2=16$

then reflect in x-axis:

$(x-4)^2+(y-3)^2=16$

A. $(x-4)^2+(y+3)^2=16$

B. $(x+4)^2+(y-3)^2=16$

C. $(x+4)^2+(y+3)^2=16$

D. $(x-4)^2+(y-3)^2=16$

6 Which of the following is an expression for $\int\left(\frac{1}{5}\right)^x dx$?

$\int\left(\frac{1}{5}\right)^x dx = \frac{\left(\frac{1}{5}\right)^x}{\ln\left(\frac{1}{5}\right)}$

$= \frac{\left(\frac{1}{5}\right)^x}{\ln(1)-\ln 5}$

$= -\frac{1}{5^x \ln 5}$

A. $\frac{1}{\ln 5 \cdot 5^x}+C$

B. $\frac{-1}{\ln 5 \cdot 5^x}+C$

C. $\frac{-1}{\ln\frac{1}{5}\cdot\left(\frac{1}{5}\right)^x}+C$

D. $\frac{1}{5}\ln x+C$

7 Given that $f(x)$ is an even function and $g(x)$ is an odd function, which of the following statements is true?

$f(g(-x)) = f(-g(x))$

$= f(g(x))$

A. $f(x)\times g(x)$ is an even function

B. $f(x)+g(x)$ is an odd function

C. $f(g(x))$ is an even function

D. $g(f(x))$ is an odd function.

8 If $b > 1$ and $a = \log_{12} b$, which expression is equal to $\frac{1}{a}$?

A. $-\log_{12} b$

B. $\log_b 12$

C. $-\log_{12} \frac{1}{b}$

D. $\frac{1}{\log_b 12}$

$$\frac{1}{\log_{12} b} = \frac{1}{\frac{\log_b b}{\log_b 12}} = \log_b 12$$

9 The curve $y = \ln x$ is translated to the left by π units and then dilated horizontally by a scale factor of 3. What is the equation that describes this new curve?

A. $y = \ln\left(\frac{x}{3} + \pi\right)$

B. $y = \ln\left(\frac{x}{3} - \pi\right)$

C. $y = \ln(3x + \pi)$

D. $y = \ln\left(\frac{x + \pi}{3}\right)$

$y = \ln x$

translate left by π units:

$y = \ln(x + \pi)$

dilate horizontally by 3

$y = \ln\left(\frac{x}{3} + \pi\right)$

10 Let A and B be two independent events from a sample space.
If $P(A) = p$, $P(B) = p^2$, then which of the following is an expression for $P(\overline{A} \cup B)$?

A. $1 - p + p^2$

B. $p^2 - p^3$

C. $1 - p + p^3$

D. $p - p^3$

$$P(\overline{A} \cup B) = P(\overline{A}) + P(B) - P(\overline{A} \cap B)$$
$$= P(\overline{A}) + P(B) - P(\overline{A})P(B)$$

(since A & B are independent)

$$= (1 - p) + p^2 - (1 - p)p^2$$
$$= 1 - p + p^2 - p^2 + p^3$$
$$= 1 - p + p^3$$

End of Section I

Question 11 (2 marks)

Solve the equation $|2x-1|=3$. **2**

$2x-1=3$ $\quad 2x-1=-3$

$2x=4$ $\quad 2x=-2$

$x=2$ $\quad x=-1$

$x=2,-1$

Question 12 (4 marks)

For a geometric series the third term (T_3) is 20 and the sixth term (T_6) is −160.

(a) Find the common ratio and the first term. **2**

$T_3: ar^2=20$ (1)

$T_6: ar^5=-160$ (2)

(2) ÷ (1) $\quad r^3=-8$

$r=-2$

sub $r=-2$ into (1) $\quad a(-2)^2=20$

$a=5$

$a=5,\ r=-2$

(b) Find the sum of the terms from T_{10} to T_{20} inclusive. **2**

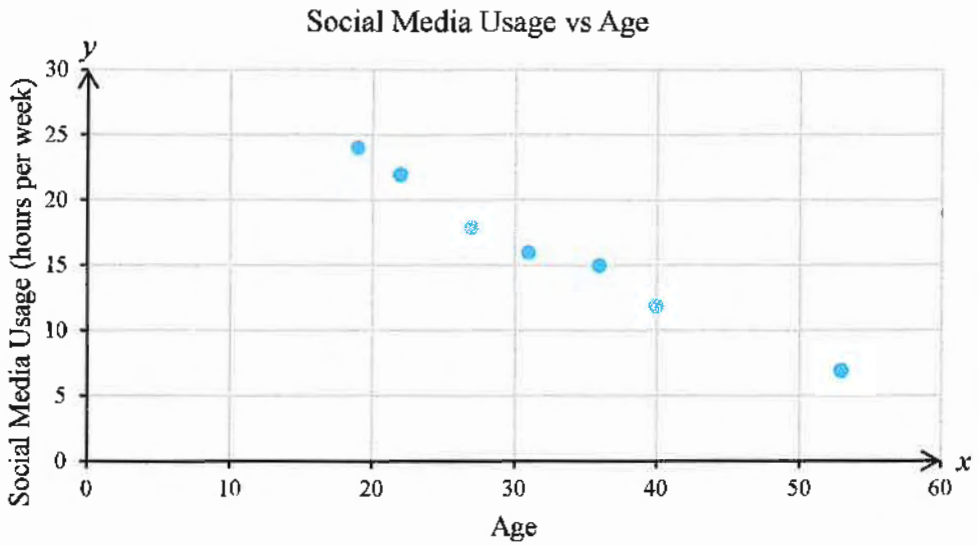
$T_{10}=5(-2)^9 \quad n=11$

$$S=\frac{5(-2)^9\left[(-2)^{11}-1\right]}{-2-1}$$

$=-1,748,480$

Question 13 (5 marks)

A group of adults across a range of ages were surveyed about their social media usage.
The scatterplot below shows the hours spent each week on social media vs age.



(a) The data is presented in a table below. Fill in the missing value in the table. **1**

Age (x)	19	22	27	31	36	40	53
Hours per week (y)	24	22	18	16	15	12	7

(b) Calculate Pearson's correlation coefficient, correct to 4 significant figures. **1**

$r = -0.9884$ (4 s.f.)

(c) Describe the correlation between social media use and age suggested by this data. **1**

There is a strong, linear, negative correlation between social media use and age

(d) Determine the equation of the least squares regression line, rounding coefficients to 2 decimal places. **1**

$y = 32.30 - 0.49x$

(e) Should the regression line be used to predict the social media usage of 11 year-olds? By referring to the context, give a reason for your answer. **1**

No, because the relationship will not continue to be linear as age decreases. Younger children like 11 year-olds are less likely to use social media due to age restrictions, parental controls or limited access.

Question 14 (2 marks)

Find $\int \frac{x}{\sqrt{x^2+1}}dx$. **2**

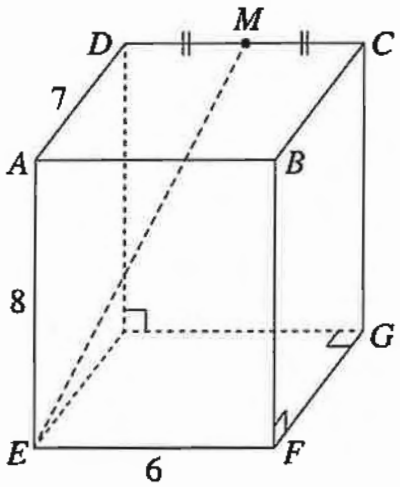
$\frac{1}{2}\int 2x(x^2+1)^{-1/2}\,dx$

$= \frac{1}{2}\times 2\frac{(x^2+1)^{1/2}}{1} + C$

$= \sqrt{x^2+1} + C$

Question 15 (3 marks)

In the rectangular prism shown, $AD=7$ cm, $AE=8$ cm, $EF=6$ cm. Point M is the midpoint of CD. **3**



NOT TO SCALE

Find $\angle AEM$, to the nearest degree.

$DM = 3$

In $\triangle ADM$, $7^2 + 3^2 = AM^2$ (Pythagoras Theorem)

$AM = \sqrt{58}$

In $\triangle AME$, $\tan\angle AEM = \frac{\sqrt{58}}{8}$

$\angle AEM = 43.59...$

$= 44^\circ$ (nearest degree)

Question 16 (4 marks)

A courier company tracks the delivery times (in days) for packages sent across the country. Packages arrive in 1 to 5 days. During a morning system check, the data analyst found that some of the delivery time probabilities were missing due to a database error.

The partial probability distribution is shown below:

Delivery Time X (days)	1	2	3	4	5
$P(X = x)$	0.1	a	b	0.2	0.2

The analyst recalls that the expected value for the delivery time is 3.2 days

(a) By first forming two equations, calculate the missing probabilities a and b. **3**

$\sum p(x) = 1 \Rightarrow 0.1 + a + b + 0.2 + 0.2 = 1$

$a + b = 0.5$ (1)

$\sum xp(x) = 3.2 \Rightarrow 1 \times 0.1 + 2a + 3b + 4 \times 0.2 + 5 \times 0.2 = 3.2$

$2a + 3b = 1.3$ (2)

Rearrange (1) $a = 0.5 - b$

sub. in (2) $2(0.5 - b) + 3b = 1.3$

$1 - 2b + 3b = 1.3 \Rightarrow b = 0.3, a = 0.2$

Later that day, the full dataset was recovered from a backup, and the corrected distribution was confirmed to be:

Delivery Time X (days)	1	2	3	4	5
$P(X = x)$	0.1	0.2	0.3	0.2	0.2

(b) Using the corrected distribution, calculate the variance of the delivery time. **1**

$Var(X) = E(X^2) - \mu^2$

$E(X^2) = 0.1 + 4 \times 0.2 + 9 \times 0.3 + 16 \times 0.2 + 25 \times 0.2$

$= 11.8$

$Var(X) = 11.8 - 3.2^2$

$= 1.56$ * using calculator okay too

Question 17 (3 marks)

Find the equation of the normal to the curve $y=\left(x+\frac{2}{x}\right)^2$ when $x=2$. **3**

$y' = 2\left(x+\frac{2}{x}\right) \times \left(1-\frac{2}{x^2}\right)$ (chain rule)

when $x=2$, $y=\left(2+\frac{2}{2}\right)$

$= 9$

when $x=2$, $y' = 2\left(2+\frac{2}{2}\right)\left(1-\frac{2}{2^2}\right)$

$= 2 \times 3 \times \left(\frac{1}{2}\right)$

$= 3$

$\therefore m_N = -\frac{1}{3}$

Equation of normal: $y-9 = -\frac{1}{3}(x-2)$

$-3y+27 = x-2$

$x+3y-29=0$

Question 18 (4 marks)

The velocity v ms^{-1} of a moving particle is given after t seconds by $v = 8t - 16$.

(a) What is the acceleration of the particle? 1

$\frac{dv}{dt} = 8$

$a = 8 \text{ ms}^{-2}$

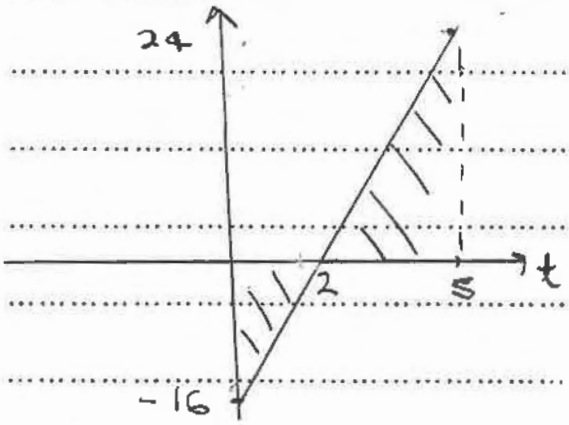
(b) Determine when the particle is at rest. $v = 0$ 1

$8t - 16 = 0$

$t = 2$

(c) How far does the particle travel in the first 5 seconds? 2

distance travelled $= \left| \int_0^2 (8t-16)\,dt \right| + \int_2^5 (8t-16)\,dt$



$= \frac{1}{2} \times 2 \times 16 + \frac{1}{2} \times 3 \times 24$

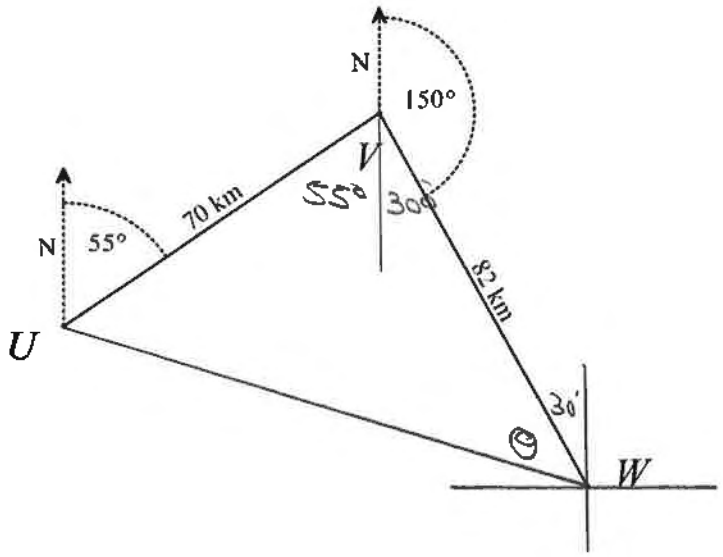
$= 16 + 36$

$= 52 \text{ m}$

Question 19 (4 marks)

A helicopter leaves Underwood and flies 70 km on a bearing of 055° to Vacy.
It then flies 82 km on a bearing of 150° to Woodville.

The diagram below illustrates the journey.



(a) It then plans to fly directly to Underwood. **2**
Calculate, to the nearest km, the distance that it will fly.

$UW^2 = 70^2 + 82^2 - 2 \times 70 \times 82 \cos 85^\circ$

$= 10623.45$

$UW = 103.07$

It will fly 103 km (nearest km)

Question 19 (continued

(b) Determine the bearing (from Woodville) on which it should fly to return to Underwood. Give your answer to the nearest degree. **2**

Let $\angle UWV = \theta$

$\cos\theta = \dfrac{103^2 + 82^2 - 70^2}{2 \times 103 \times 82}$

$= 0.736$

$\theta = 42.605$

$= 43^\circ$ (nearest degree)

Bearing from W to U is $360 - 30 - 43$

$= 287^\circ$ T

End of Question 19

Question 20 (2 marks)

It is given that a quantity y varies inversely with the square of x. **2**

When $x = 4$, $y = 10$.

Find the value of y when $x = 5$.

$y \propto \frac{1}{x^2}$

$y = \frac{k}{x^2}$

sub $(4, 10)$ $10 = \frac{k}{4^2}$

$k = 160$

$y = \frac{160}{x^2}$

sub $x = 5$,

$y = \frac{160}{5^2}$

$= 6.4$

Question 21 (4 marks)

(a) Show that $\dfrac{\sec^2 3x}{\tan 3x} = \sec 3x \operatorname{cosec} 3x$ **2**

LHS = $\dfrac{\sec^2 3x}{\tan 3x}$

$= \dfrac{1}{\cos^2 3x} \times \dfrac{\cos 3x}{\sin 3x}$

$= \dfrac{1}{\cos 3x} \times \dfrac{1}{\sin 3x}$

$= \sec 3x \operatorname{cosec} 3x$

$= RHS$

(b) Hence find $\int \sec 3x \operatorname{cosec} 3x \, dx$ **2**

$= \int \dfrac{\sec^2 3x}{\tan 3x} \, dx$ (from (a))

$= \dfrac{1}{3} \int \dfrac{3\sec^2 3x}{\tan 3x} \, dx$

$= \dfrac{1}{3} \ln |\tan 3x| + C$

Question 22 (5 marks)

The population of two towns Melbtown and Brisbay are given by $P_M = 1200e^{0.04t}$ and $P_B = 1500e^{0.025t}$ respectively, where t is the number of years elapsed since January 2010.

(a) In which year will the population of Melbtown exceed 1500? **2**

$1200e^{0.04t} > 1500$

$e^{0.04t} > \frac{5}{4}$

$0.04t > \ln\left(\frac{5}{4}\right)$

$t > 5.57\ldots$

In 2015, the population will exceed 1500.

(b) In which year will the rate of growth of the population in Melbtown be double the rate of growth of Brisbay? **3**

$\frac{dP_M}{dt} = 0.04 \times 1200e^{0.04t}$ $\qquad \frac{dP_B}{dt} = 0.025 \times 1500e^{0.025t}$

$= 48e^{0.04t}$ $\qquad = 37.5e^{0.025t}$

Find t when $\frac{dP_M}{dt} = 2\frac{dP_B}{dt}$

$48e^{0.04t} = 75e^{0.025t}$

$e^{(0.04 - 0.025)t} = \frac{75}{48}$

$e^{0.015t} = 1.5625$

$0.015t = \ln(1.5625)$

$t = \frac{1}{0.015}\ln(1.5625)$

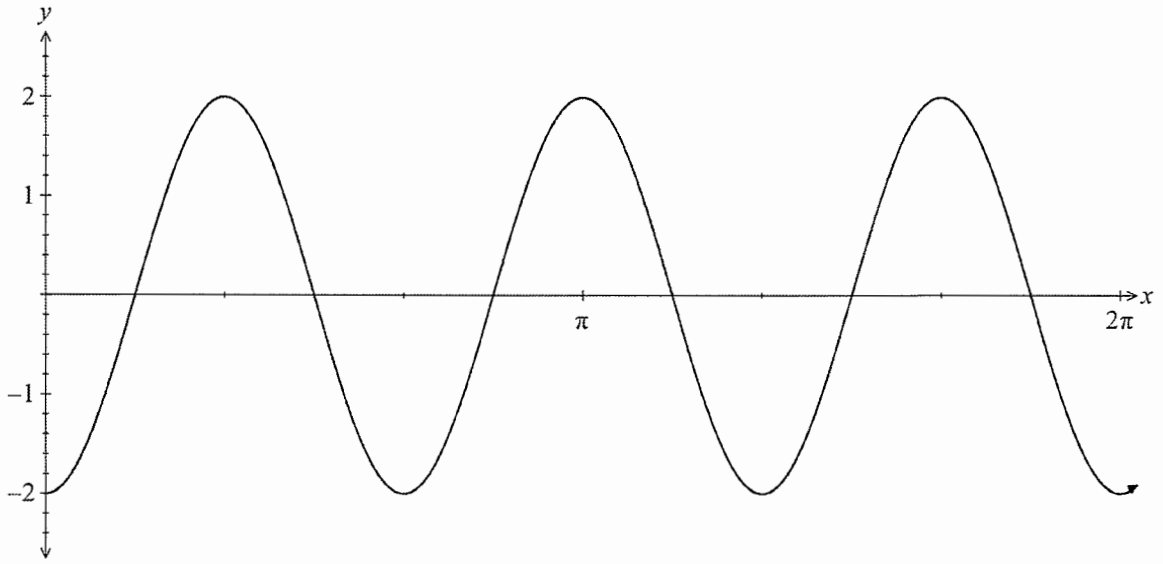
$= 29.75247\ldots$

During the year 2039, the rate of growth of Melbtown will be double that of Brisbay.

Question 23 (3 marks)

Ben sketched the curve shown below to model a tidal experiment in his Science class. **3**
Ben knows the curve is in the form $y = k\cos(ax+b)$ where $k > 0$, $a > 0$ and $0 \le b \le \pi$.

By finding k, a and b, write the equation for Ben's tidal experiment.



3 cycles = 2π

period = $\frac{2\pi}{3}$ $\Rightarrow$ $\frac{2\pi}{a} = \frac{2\pi}{3}$

$a = 3$

Amplitude = 2 $\therefore$ $k = 2$ since $k > 0$

$y = 2\cos(3x + b)$

sub $(0, -2)$ $-2 = 2\cos b$

$\cos b = -1$

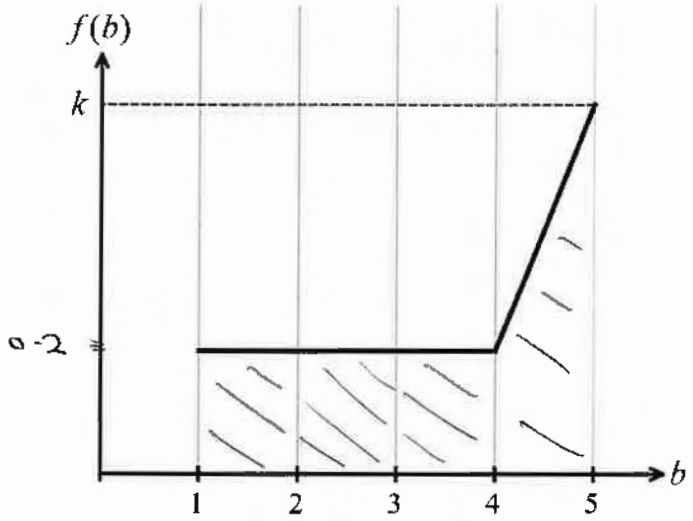
$b = \pi$ (since $0 \le x \le \pi$)

$y = 2\cos(3x + \pi)$

Question 24 (6 marks)

Liz is having a rare book restored at *Binding Repairs*. She drops the book off in the morning and is told that the book will be ready for pickup sometime between 1 pm and 5 pm that day.

Let the random variable B represent the time after noon (12 pm) that Liz's book is ready for collection. The probability density function for B is shown in the graph below.



The probability of a book being ready for collection between 2 pm and 3 pm is 0.2.

(a) Determine the value of k. **1**

$3 \times 0.2 + \frac{1}{2} \times 1 (0.2 + k) = 1$

$\frac{0.2+k}{2} = 0.4$

$0.2 + k = 0.8$

$k = 0.6$

(b) An incomplete expression for the probability density function of B is given below. **1**
Fill in the box to complete the missing part of the expression.

$m = 0.4$

$$f(b) = \begin{cases} 0.2, & 1 \le b < 4 \\ \boxed{0.4b - 1.4}, & 4 \le b \le 5 \\ 0, & \text{otherwise} \end{cases}$$

Question 24 continues on page 25

(c) Determine the median time that Liz's book will be ready for collection. **1**

$b = 3.5$ (by inspection)

$\therefore$ 3:30 pm

Andrew is also having a book restored but he uses a different company called *Folio Fixers*. He drops his book off in the morning and is told that his book will be ready for collection sometime between 1 pm and 5 pm that day.

Let the random variable A represent the time after noon (12 pm) that Andrew's book is ready for collection. The cumulative distribution function for A is given by

$$P(A \leq a) = \begin{cases} 0, & a < 1 \\ \dfrac{10a - a^2 - 9}{16}, & 1 \leq a \leq 5 \\ 1, & a > 5 \end{cases}$$

(d) Determine the probability that Andrew's book will be ready to collect by 4 pm. **1**

$P(A \leq 4) = \dfrac{10(4) - 4^2 - 9}{16}$

$= \dfrac{15}{16}$

(e) Determine the probability that Andrew's book will be ready to collect by 3 pm given that it is ready to collect by 4 pm. **2**

$P(A \leq 3 \mid A \leq 4) = \dfrac{P(A \leq 3)}{P(A \leq 4)}$

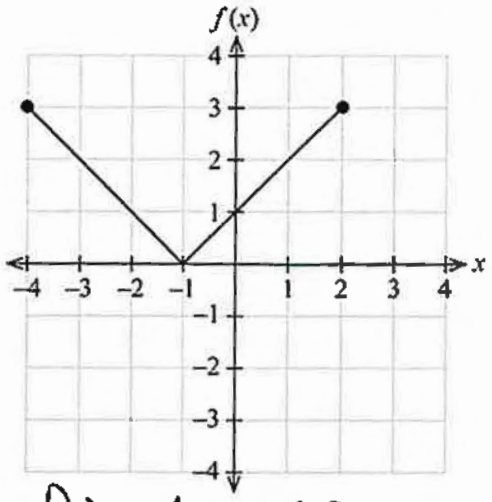
$= \dfrac{10(3) - 3^2 - 9}{16} \times \dfrac{16}{15}$

$= 0.8$

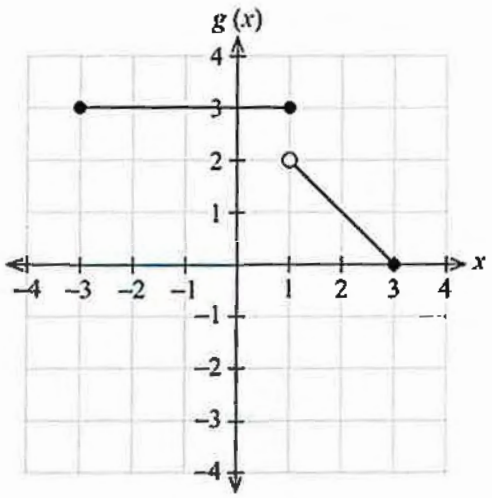
End of Question 24

Question 25 (4 marks)

The graphs of functions $f(x)$ and $g(x)$ are shown below.



D: $-4 \leq x \leq 2$

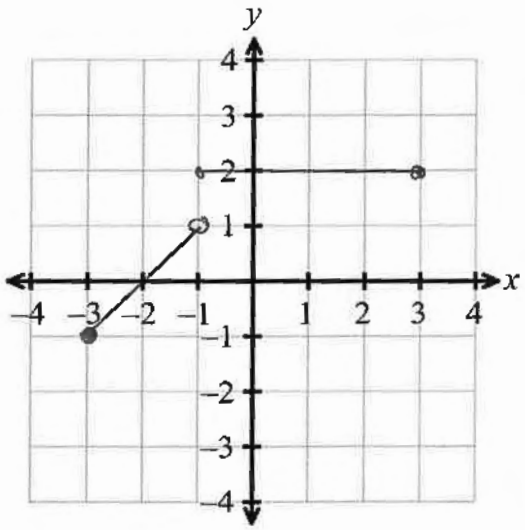


Range: $0 \leq y < 2$, $y = 3$

(a) State the domain of $f(g(x))$. **1**

Domain of $f(g(x))$ is $1 < x \leq 3$

(b) Sketch $h(x) = g(-x) - 1$ on the grid below. **2**



(c) Hence solve $f(h(x)) = 0$. **1**

$f(a) = 0$ when $x = -1$

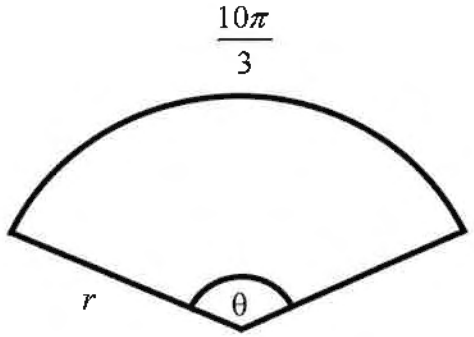
$h(x) = -1$

$x = -3$

Question 26 (4 marks)

The diagram shows a sector with radius r and angle θ where $\frac{\pi}{2} < \theta \leq \pi$.

The arc length is $\frac{10\pi}{3}$.



(a) Find the range of possible values for r, expressing your answer in interval notation. **3**

$r\theta = \frac{10\pi}{3}$

when $\theta = \frac{\pi}{2}$, $\frac{\pi}{2}r = \frac{10\pi}{3}$

$\therefore r = \frac{20}{3}$

when $\theta = \pi$, $\pi r = \frac{10\pi}{3}$

$r = \frac{10}{3}$

$r \in \left[\frac{10}{3}, \frac{20}{3}\right)$

(b) Calculate the area of the sector when $r = 4$. **1**

$A = \frac{1}{2}r^2\theta$

$= \frac{1}{2}r \times r\theta$

$= \frac{1}{2} \times 4 \times \frac{10\pi}{3}$

$= \frac{20\pi}{3}$ u^2

Proceed to Booklet 3 for Questions 27-31

Question 27 (4 marks)

The numbers $4e^{-k}$, 5, $4e^{k}$ are consecutive terms in an arithmetic sequence.

(a) Show that $2e^{k}-5+2e^{-k}=0$. **1**

$T_3 - T_2 = T_2 - T_1$

$4e^{k} - 5 = 5 - 4e^{-k}$

$4e^{k} - 10 + 4e^{-k} = 0$

$2e^{k} - 5 + 2e^{-k} = 0$

(b) Hence find in exact form any possible values of k. **3**

$2e^{k} - 5 + \frac{2}{e^{k}} = 0$

$2e^{2k} - 5e^{k} + 2 = 0$

Let $u = e^{k}$

$2u^{2} - 5u + 2 = 0$

$(2u - 1)(u - 2) = 0$

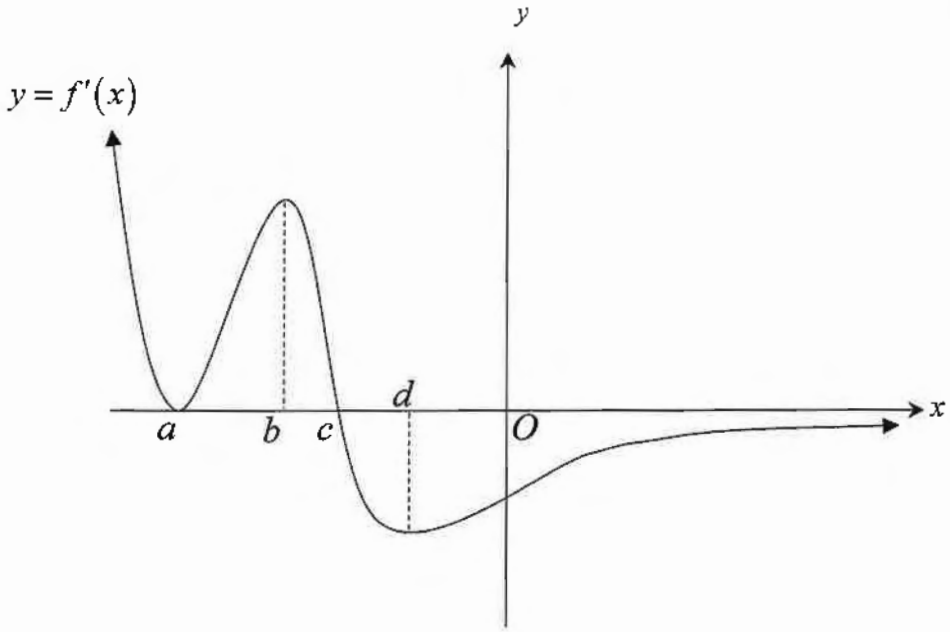
$u = \frac{1}{2}, 2$

$e^{k} = \frac{1}{2}, 2$

$k = \ln\left(\frac{1}{2}\right), \ln 2$

Question 28 (5 marks)

The graph of the gradient function $y = f'(x)$ is given below.
Specific x-values a, b, c and d are as indicated in the diagram.



(a) Justify why $f(x)$ has a maximum stationary point at $x = c$. **1**

At $x = c$, $f'(x) = 0$

and

x	b	c	d
$f'(x)$	+	0	−
	/	—	\

$\therefore$ maximum stationary point at $x = c$

(b) For what value(s) of x is the graph of $y = f(x)$ concave up? **2**

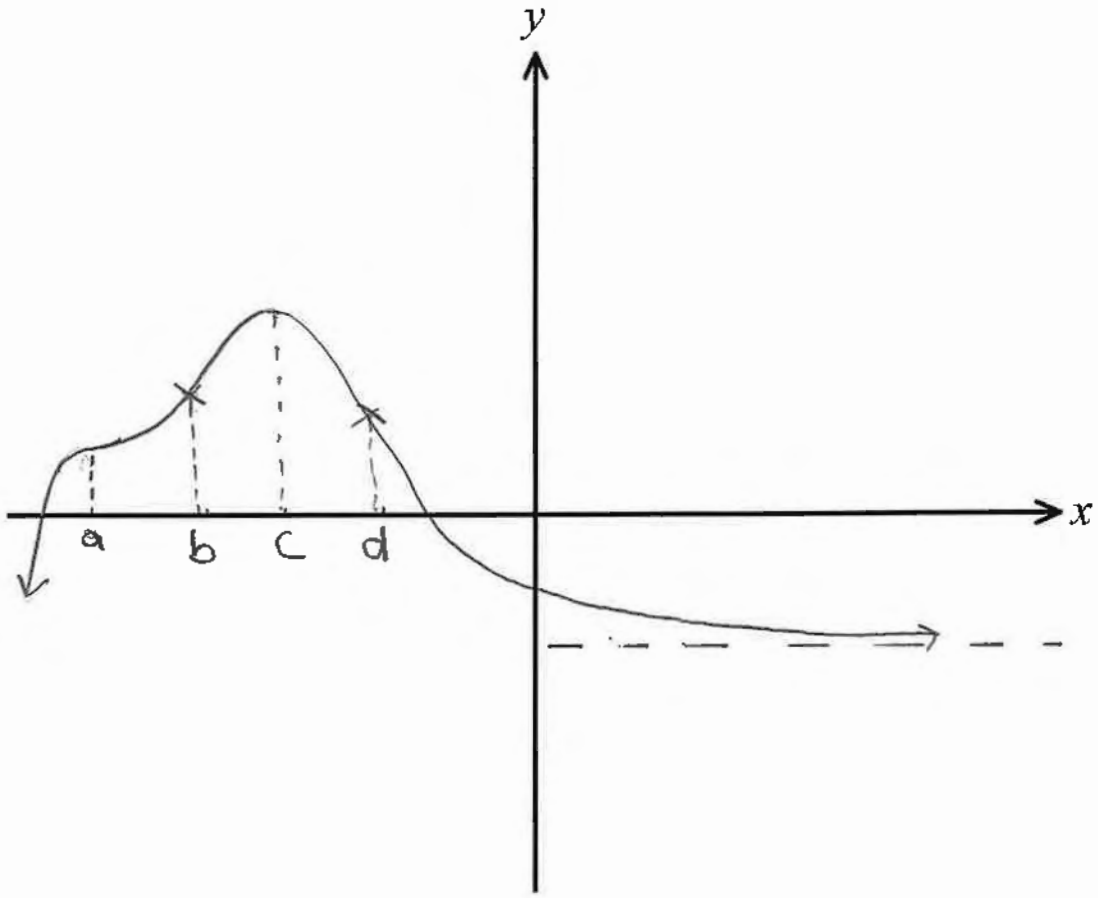
concave up when $f'(x)$ increasing

$a < x < b$, $x > d$

Question 28 continues on page 35

Question 28 (continued)

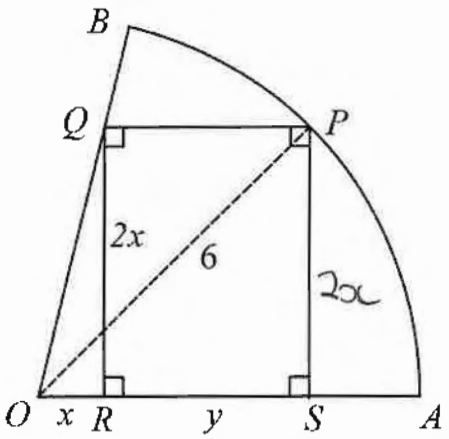
(c) Draw a possible graph of $y = f(x)$ showing all important features. 2



End of Question 28

Question 29 (6 marks)

The diagram below shows a rectangle $PQRS$ drawn in a sector OAB of a circle with radius 6. $OR = x$, $QR = 2x$ and $RS = y$. All units are in metres.



NOT TO SCALE

(a) Show that $(x+y)^2 = 36-4x^2$. **1**

In $\triangle OPS$, $(x+y)^2 + (2x)^2 = 6^2$ (Pythagoras' Theorem)

$(x+y)^2 = 36 - 4x^2$

(b) Show that the perimeter, L, of the rectangle is given by **2**

$$L = 2x + 2\sqrt{36-4x^2}$$

$L = 2x + 2x + y + y$

$= 4x + 2y$

$= 2(x+y) + 2x$

$= 2x + 2(x+y)$ $\quad (x+y)^2 = 36-4x^2$

$= 2x + 2\sqrt{36-4x^2}$ $\quad x+y = \sqrt{36-4x^2}$ $\quad (x+y) > 0$

Question 29 continues on page 37

(c) Find the value of x which gives the maximum perimeter of the rectangle. **3**

$$\frac{dL}{dx} = 2 + 2 \times \frac{1}{2}\left(36 - 4x^2\right)^{-\frac{1}{2}} \times -8x$$

$$= 2 - \frac{8x}{\sqrt{36-4x^2}}$$

For max/min set $\frac{dL}{dx} = 0$

$$2 - \frac{8x}{\sqrt{36-4x^2}} = 0$$

$$\frac{8x}{\sqrt{36-4x^2}} = 2$$

$$8x = 2\sqrt{36-4x^2}$$

$$4x = \sqrt{36-4x^2}$$

$$16x^2 = 36 - 4x^2$$

$$20x^2 = 36$$

$$x^2 = \frac{36}{20}$$

$$x^2 = \frac{9}{5}$$

$$x = \frac{3}{\sqrt{5}} \quad (x > 0)$$

x	1	$\frac{3}{\sqrt{5}}$	2
$\frac{dL}{dx}$	0.58...	0	-1.57...
	/	—	\

$\therefore$ max L when $x = \frac{3}{\sqrt{5}}$ m

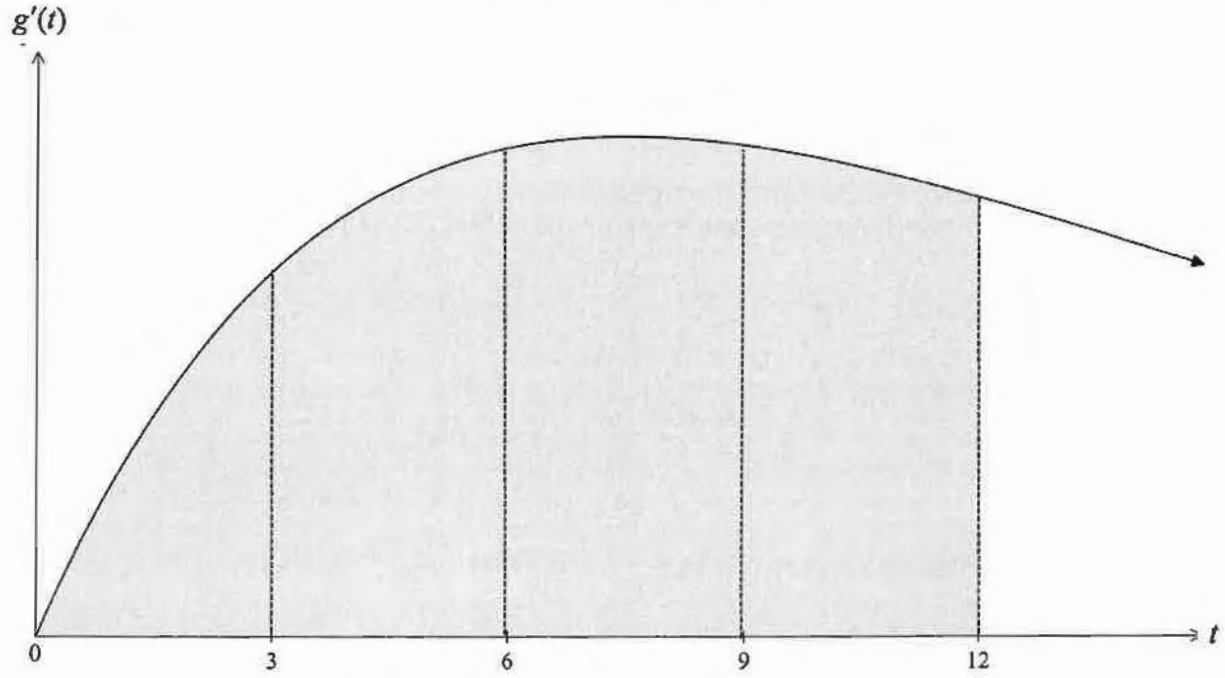
End of Question 29

Question 30 (5 marks)

Let $g(t)$ be a function defined for all $t \geq 0$. The gradient function is given by

$$g'(t) = \frac{33}{10} te^{-kt}.$$

where k is a positive constant. The diagram below shows a sketch of $g'(t)$ against t.
It is given that $g'(t)$ attains the greatest value at $t = 7.5$ and $g(0) = 0$.



(a) Show that $k = \frac{2}{15}$. **2**

$g''(t) = \frac{33}{10}\left(e^{-kt} + t(-k)e^{-kt}\right)$ (product rule)

$= \frac{33}{10}e^{-kt}(1 - kt)$

when $t = 7.5$ $g''(t) = 0$

$\frac{33}{10}e^{-7.5k}(1 - 7.5k) = 0$

$\therefore 1 - 7.5k = 0 \quad (e^{-7.5k} \neq 0)$

$7.5k = 1$

$k = \frac{1}{7.5}$

$= \frac{2}{15}$

Question 30 continues on page 39

(b) Use the trapezoidal rule with four sub-intervals to estimate the shaded area in the diagram above. **2**

x	0	3	6	9	12
$g'(t)$	0	$\frac{33}{10}\times 3e^{-\frac{6}{15}}$	$\frac{33}{10}\times 6e^{-\frac{12}{15}}$	$\frac{33}{10}\times 9e^{-\frac{18}{15}}$	$\frac{33}{10}\times 12e^{-\frac{24}{15}}$

$$A = \frac{12-0}{2\times 4}\left[0 + \frac{33}{10}\times 12e^{-24/5} + 2\times\frac{33}{10}\left(3e^{-\frac{6}{15}} + 6e^{-\frac{12}{15}} + 9e^{-\frac{18}{15}}\right)\right]$$

$$= \frac{3}{2} \times 56.9516\ldots$$

$$= 85.43 \text{ u}^2 \quad (2 \text{ d.p.})$$

(c) Explain why your answer to (b) is an estimate for $g(12)$. **1**

Using trapezoidal rule: Area $\doteq \int_0^{12} g'(t)\,dt$

$$= \left[g(t)\right]_0^{12}$$

$$= g(12) - g(0)$$

$$= g(12)$$

$\therefore$ 85.43 is an estimate for $g(12)$

End of Question 30

Please turn over for Question 31

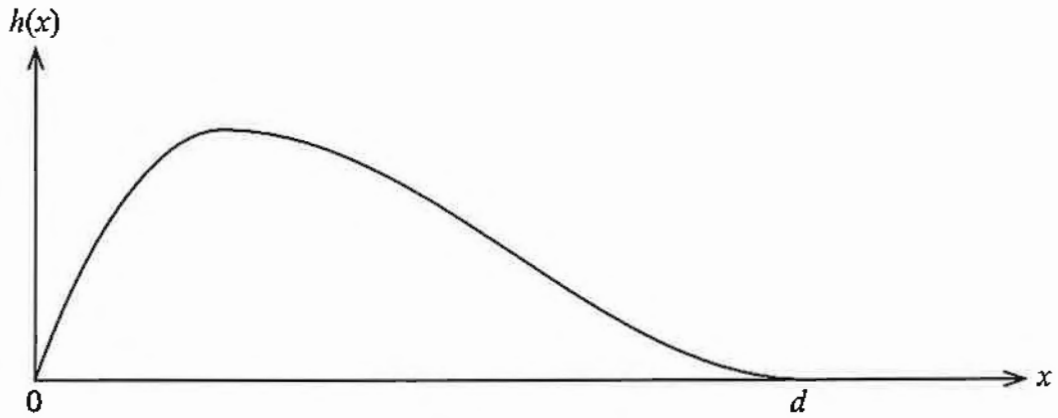
Question 31 (10 marks)

A new luxury hotel in Byron Bay is being designed to feature a stunning wave-shaped window that stretches across the main lobby wall, inspired by the rolling surf of the nearby beaches. The top edge of this window is modelled mathematically to reflect both a smooth curve and a gentle wave.

The height, $h(x)$, of the top edge of the window above ground level, in metres, is described by the piecewise function:

$$h(x)=\begin{cases} a\left(1-(x-1)^2\right), & 0\le x\le 1 \\ 2\cos(x-1)+2, & 1<x\le d \end{cases}$$

where a and d are constants and x is the horizontal distance from the leftmost point of the window (which is at ground level).



(a) Determine the value of a in the function $h(x)$. **1**

Since $h(x)$ is continuous at $x = 1$

$a(1-(1-1)^2) = 2\cos(1-1) + 2$

$a \times 1 = 2\cos 0 + 2$

$a = 2 \times 1 + 2$

$a = 4$

Question 31 continues on page 41

(b) By first showing that the length of the bottom edge of the window d is $(\pi+1)$ metres, determine the area of the window. **5**

At $x = d$, $2\cos(d-1) + 2 = 0$

$2\cos(d-1) = -2$

$\cos(d-1) = -1$

$d - 1 = \pi$

$d = \pi + 1$

$$A = \int_0^1 4\left(1-(x-1)^2\right)dx + \int_1^{\pi+1}\left(2\cos(x-1)+2\right)dx$$

$$= 4\int_0^1 \left(2x - x^2\right)dx + 2\int_1^{\pi+1}\left(\cos(x-1)+1\right)dx$$

$$= 4\left[\frac{2x^2}{2} - \frac{x^3}{3}\right]_0^1 + 2\left[\sin(x-1) + x\right]_1^{\pi+1}$$

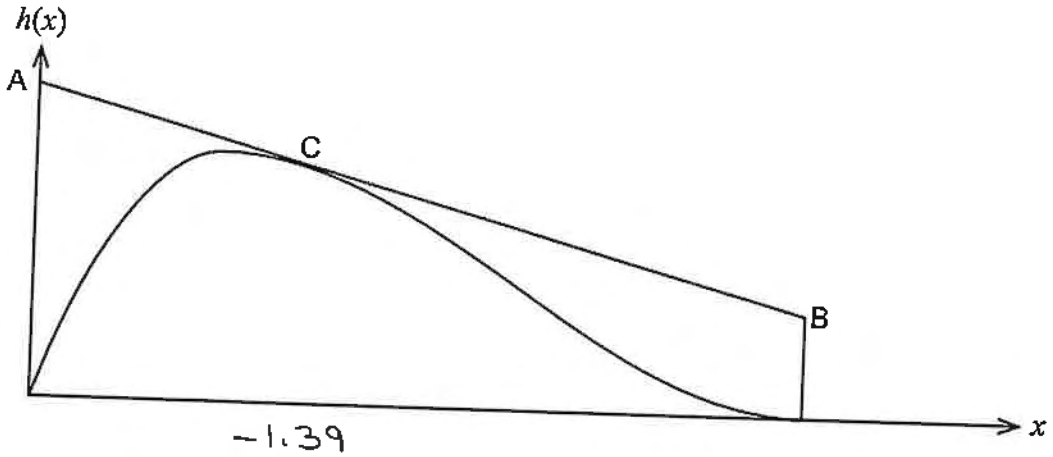
$$= 4\left[\left(1-\frac{1}{3}\right) - (0-0)\right] + 2\left[\sin\pi + \pi + 1 - (\sin 0 + 1)\right]$$

$$= \left(\frac{8}{3} + 2\pi\right) \text{ m}^2$$

Question 31 continues on page 42

(c) The top edge of the wall, shown as the line AB below, is to just touch the window at point C as shown below. **4**

The wall is 1.39 m higher at point A than point B. How high is point C above the ground?



$m_{AB} = \frac{-1.39}{\pi + 1}$

At C, $h'(x) = -2\sin(x-1)$

$-2\sin(x-1) = \frac{-1.39}{\pi + 1}$

$\sin(x-1) = \frac{1.39}{2(\pi+1)}$

$x - 1 = 0.1686$ (1st solution)

$x = 1.1686$

when $x = 1.1686$

$h(1.1686) = 2\cos(1 - 1.1686) + 2$

$= 3.97$

$\therefore$ C is 3.97 m above ground