PENRITH SELECTIVE HIGH SCHOOL

2020 HSC TRIAL EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided with this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (pages 2–5)
- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 6–25)
- Attempt Questions 11–33
- Allow about 2 hours and 45 minutes for this section

Topic	Functions	Trig. Functions	Differential Calculus / App. of Differentiation	Integral Calculus	Financial Maths	Statistical Analysis	Total
MC Q.1-10	10 / 1	4 / 1	1, 3 / 2	5, 7 / 2	2, 9 / 2	6, 8 / 2	
Section II Q.11-18	11, 13, 14 / 10	18 / 6	15 / 3		17 / 3	12, 16 / 8	
Section II Q.19-25			19, 20, 21, 23 / 16	22, 24 / 9	25 / 3		
Section II Q.26-33		29 / 3	32, 33 / 9	27, 31 / 7	28 / 5	26, 30 / 8	
Total	/ 11	/ 10	/ 30	/ 18	/ 13	/ 18	/ 100

Student Number: ____________________ **Teacher Name:** ____________________________

This paper MUST NOT be removed from the examination room

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

---

1 The derivative of $\tan(e^{2x})$ is:

A. $2e^{2x}\tan(e^{2x})$

B. $2e^{2x}\sec^2(e^{2x})$

C. $\sec^2(2e^{2x})$

D. $2\sec^2(e^{2x})$

2 For what values of x does the series $1 + (x-1) + (x-1)^2 + (x-1)^3 + \cdots$ have a limiting sum?

A. $-1 < x < 1$

B. $-2 < x < 0$

C. $0 < x < 2$

D. $x < 1$

3 For what values of x is the curve $y = x^4 - 2x^3 + 5$ concave down?

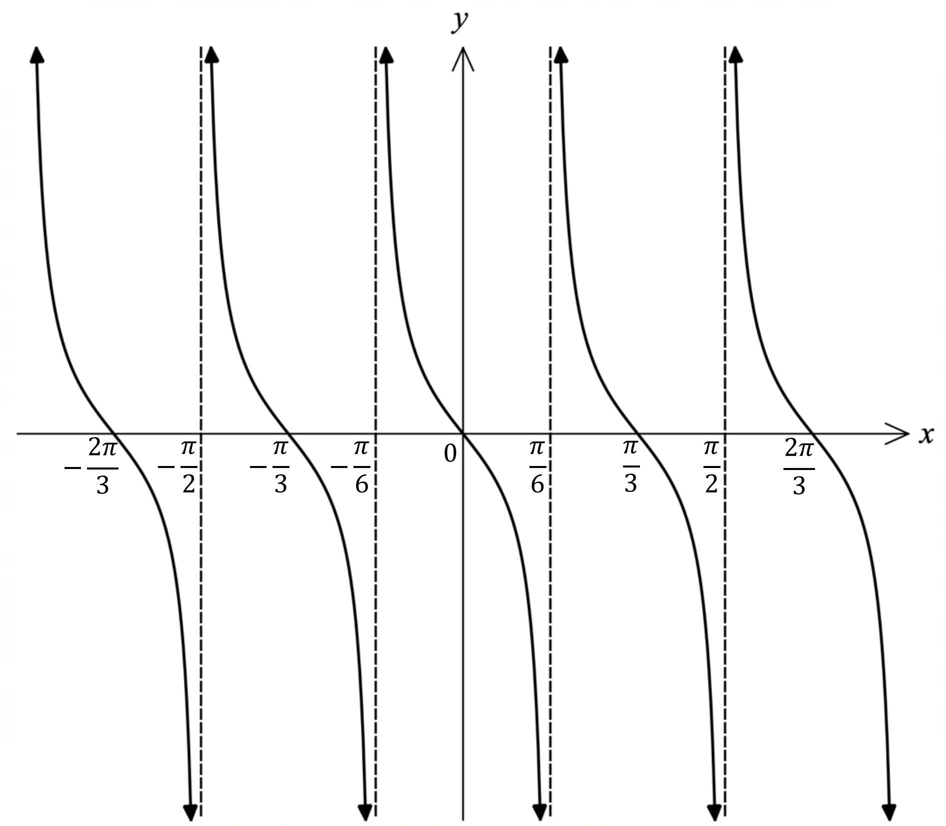
A. $0 < x < 1$

B. $x < 0$ or $x > 1$

C. $x = 0$ or $x = 1$

D. All real numbers

4 Which of the following equations best describes the graph shown below?



A. $y = \tan\left(\frac{x}{6}\right)$

B. $y = -\tan\left(\frac{x}{3}\right)$

C. $y = \tan\left(x - \frac{\pi}{6}\right)$

D. $y = -\tan(3x)$

5 When the tap of a water tank is turned on, water flows out at a rate of $\frac{dV}{dt} = 24 + 10t - t^2$ litres per minute, where t is the time in minutes.
When does the water stop flowing out of the tank?

A. 5 minutes

B. 6 minutes

C. 12 minutes

D. 24 minutes

6 Which one of the following descriptions of a Pareto chart is incorrect?

A. A Pareto chart has two vertical axes. Frequencies on the left and cumulative percentage on the right.

B. A Pareto chart consists of three graphs drawn on the same chart.

C. A Pareto chart is useful for displaying categorical data from the most to the least important.

D. The columns of the frequency histogram in a Pareto chart are arranged in descending order.

7 Find $\int \frac{x}{x^2 - 1} dx$.

A. $\ln(x^2 - 1) + C$

B. $2\ln(x^2 - 1) + C$

C. $\ln(2x) + C$

D. $\frac{1}{2}\ln(x^2 - 1) + C$

8 A group of university students were asked if they regularly drink green tea. The table below shows the results of the survey.

	Regularly drinks green tea	Doesn't drink green tea	Total
Male	6	84	
Female	33	75	
Total			

What is the probability that a randomly chosen female student will be a regular drinker of green tea?

A. $\frac{1}{6}$

B. $\frac{25}{66}$

C. $\frac{11}{36}$

D. $\frac{1}{18}$

9 The common ratio of a geometric sequence is 3 and the fifth term is 324.
Find the second term.

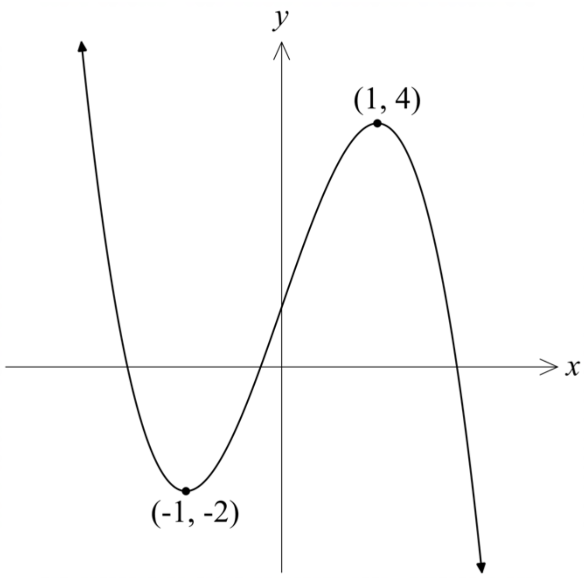
A. 12

B. 4

C. 36

D. 108

10 The graph of $y = f(x)$ is shown below. For what values of k does the equation $f(x) = k$ have 3 solutions?



A. $k = 4$ or -2

B. $k > 4$ or $k < -2$

C. $-1 < k < 1$

D. $-2 < k < 4$

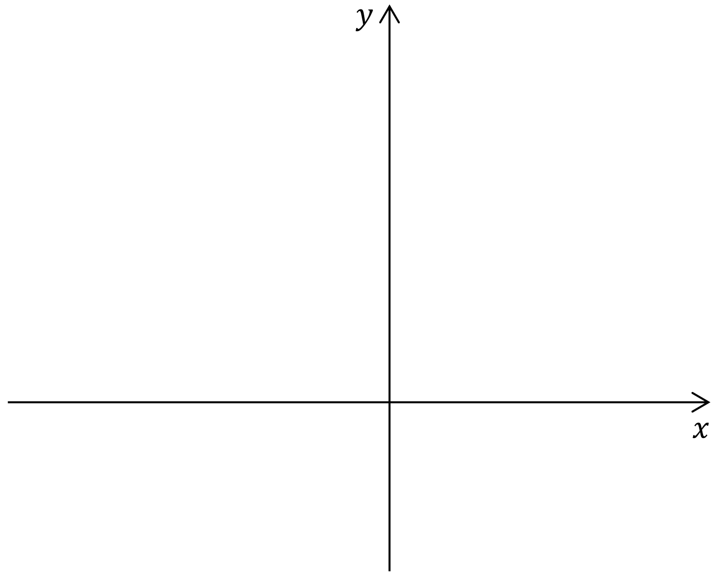
Section II

90 marks
Attempt Questions 11–33
Allow about 2 hours and 45 minutes for this section

- Answer the questions in the spaces provided.
- Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (4 marks)

(a) Sketch $y = x^2 + x - 2$ and $y = 1 - x$ on the same number plane, clearly showing their points of intersection. **3**



....

....

....

....

....

....

....

....

(b) Use the graphs to solve $x^2 + x - 2 > 1 - x$. **1**

....

....

Question 12 (3 marks)

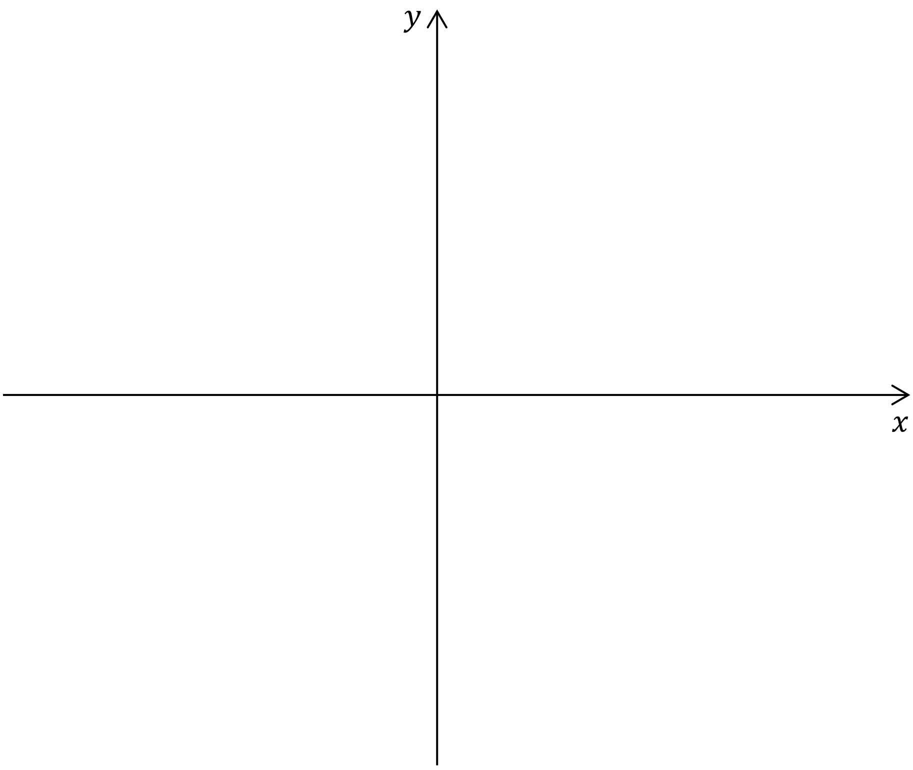
A farmer harvests peaches to supply to a local food manufacturer. Peaches that weigh under 155 grams are rejected and in this year 97.5% of the harvested peaches are accepted. Peaches that are greater than 182 grams are sold as premium peaches and 16% of this year's peaches are categorised as premium. If the weight of the peaches is normally distributed, find the mean and standard deviation of the weights of the peaches. **3**

Question 13 (4 marks)

Consider $y = \dfrac{1}{x-1}$.

(a) State the equation of the vertical and horizontal asymptotes. **2**

(b) Sketch the graph of $y = \dfrac{1}{x-1}$, showing the asymptotes and the y-intercept. **2**



Question 14 (2 marks)

Describe a suitable sequence of transformations that turns $f(x) = \dfrac{1}{x^2}$ into **2**

$g(x) = -\dfrac{1}{(x-1)^2} + 3$.

....

....

....

....

....

Question 15 (3 marks)

Consider the function $f(x) = x^2 - 8x$.

(a) Show that $\dfrac{f(x+h) - f(x)}{h} = 2x + h - 8$. **2**

....

....

....

....

....

....

....

....

....

....

(b) Hence differentiate $f(x) = x^2 - 8x$ from First Principles. **1**

....

....

....

....

....

Question 16 (5 marks)

Let X be a random variable with probability density function $f(x)$ given by

$$f(x) = \begin{cases} \dfrac{k}{\sqrt{x}}, & 1 \le x \le 9 \\ 0, & \text{otherwise} \end{cases}$$

(a) Show that $k = \dfrac{1}{4}$. **2**

(b) Show that the cumulative distribution function is $F(x) = \dfrac{1}{2}\left(\sqrt{x} - 1\right)$ **2**

(c) Hence, or otherwise, find the median value of X. **1**

Question 17 (3 marks)

The table below, gives the future value of an annuity of $1 at the given interest rates for different periods. The contributions are made at the end of each period.

Future value of an annuity of $1						
	Interest rate per period					
Period	1%	2%	3%	4%	5%	6%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500	2.0600
3	3.0301	3.0604	3.0909	3.1216	3.1525	3.1836
4	4.0604	4.1216	4.1836	4.2465	4.3101	4.3746
5	5.0101	5.2040	5.3091	5.4163	5.5256	5.6371

(a) Use the table to find the future value of an annuity of $3400 per year for 5 years at 3% per annum, compounding annually. **2**

....

....

....

....

(b) The interest rate increases to 8% per annum, compounding annually, for the 6^{th} year and the holder of the annuity account decides to extend the investment by one additional year. Calculate the balance of the annuity account after the 6^{th} contribution is made. **1**

....

....

....

....

Question 18 (6 marks)

Tides at a location vary over time and can be modelled by a cosine function of the form $h(t) = k\cos at + c$, where k, a and c are constants and t represents the number of hours after midnight. On a particular day, a low tide of 0.5 metres occurred at 6:00 am followed by a high tide of 3.5 metres at midday.

(a) Determine the value of k, a and c. Hence write the rule of $h(t)$. **3**

(b) Sketch the graph of $h(t)$ for the time span from midnight to midday on the day. **2**

(c) At what time does the second low tide occur? **1**

Question 19 (3 marks)

For $y = \ln\left(\dfrac{1-\cos x}{1+\cos x}\right)$, show that $\dfrac{dy}{dx} = 2\,\mathrm{cosec}\, x$. **3**

....

....

....

....

....

....

....

....

....

....

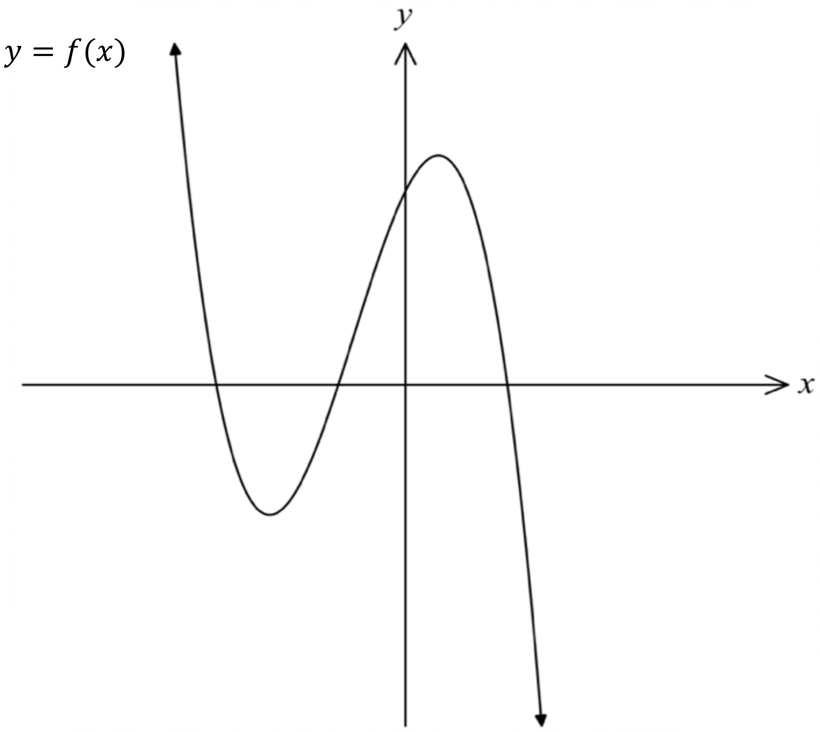
....

....

....

Question 20 (2 marks)

The graph of $y = f(x)$ is given below. On the same number plane, sketch the graph of its derivative $y = f'(x)$. **2**



Question 21 (8 marks)

A function is given by $y = 2x^3 + 3x^2 - 12x - 4$.

(a) Find the stationary points and determine their nature. **3**

Question 21 continues on page 15

Question 21 (continued)

(b) Find the point of inflection of the function $y = 2x^3 + 3x^2 - 12x - 4$. **2**

....

....

....

....

....

....

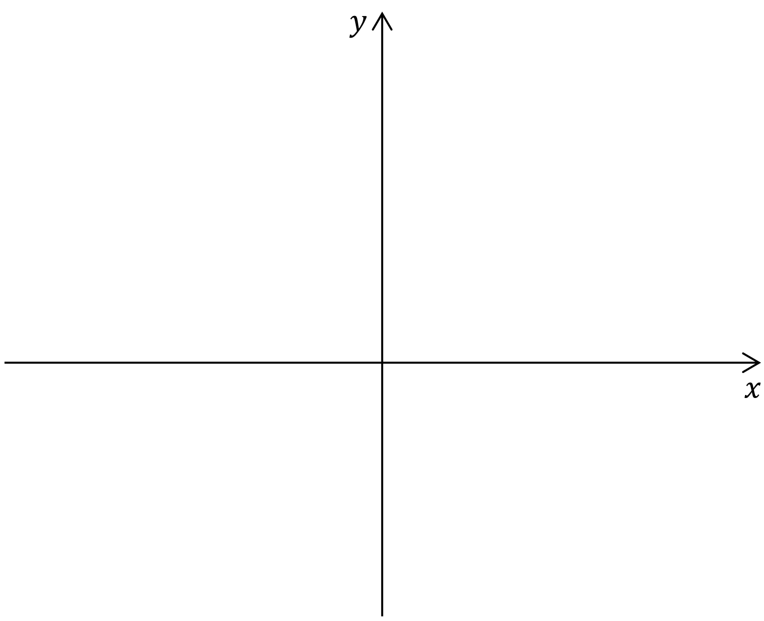
....

....

....

....

(c) Sketch the curve, showing the stationary points, point of inflection and the y-intercept. **2**



(d) Determine the global maximum of the function in the domain $[-4, 3]$. **1**

....

....

....

....

Question 22 (3 marks)

(a) Find $\dfrac{d}{dx}(x^2 - 9)^5$. **1**

....

....

....

....

(b) Hence, or otherwise, find $\displaystyle\int x(x^2 - 9)^4\,dx$ **2**

....

....

....

....

....

Question 23 (3 marks)

If $y = xe^x$, show that $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + y = 0$ **3**

....

....

....

....

....

....

....

....

....

....

....

Question 24 (6 marks)

The velocity of a particle moving along a straight line is given by $v = 4 - 4t$, where t is time in seconds. Initially the particle is 6 units to the right of the origin.

(a) Show that the particle is initially moving to the right? **1**

....

....

....

....

(b) Show that the displacement function is $x = 6 + 4t - 2t^2$. **2**

....

....

....

....

....

....

....

(c) When is the particle at the origin? **2**

....

....

....

....

....

....

....

....

Question 24 continues on page 18

Question 24 (continued)

(d) Explain why the particle continues to move to the left and never changes direction as $t \to \infty$. **1**

....

....

....

....

Question 25 (3 marks)

$x, 8x$ and $3x^2$ are three consecutive terms of an arithmetic sequence.

(a) Show that $3x^2 - 15x = 0$ **1**

....

....

....

....

(b) Find the value of the 4^{th} term. **2**

....

....

....

....

....

....

....

....

....

....

Question 26 (5 marks)

A group of young children were surveyed to find if there was a correlation between a child's age and height. The results are shown in the table.

Age in years (x)	8	5	2	6	12	11
Height in cm (h)	124	113	85	121	153	144

(a) Find the correlation coefficient (r) using your calculator, correct to 2 decimal places. **1**

....

....

(b) Using your calculator, find the equation of the least-squares regression line in the form $h = Bx + A$; where A and B are **integers**. **1**

....

....

(c) Use your equation to estimate the height of a child who is 7 years old. **1**

....

....

....

(d) Is this equation a good model to use to make a valid estimate of a 50 year-old person's height? Justify your answer. **2**

....

....

....

....

....

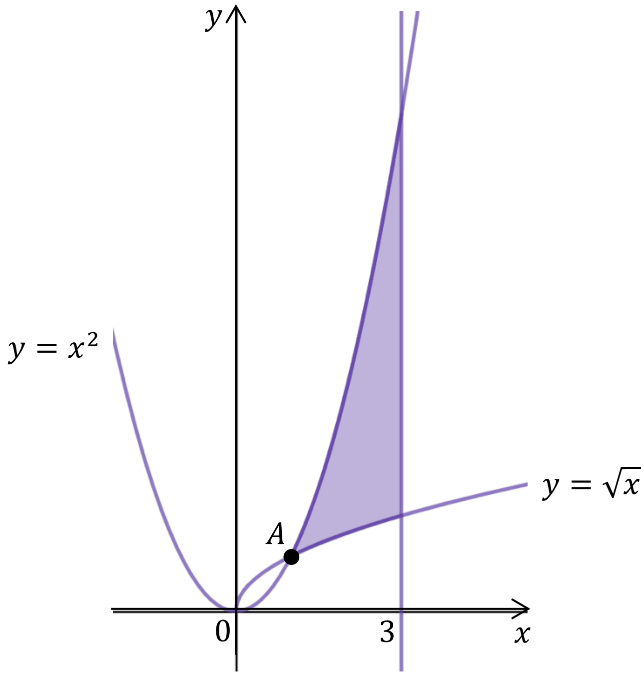
....

....

....

Question 27 (4 marks)

The diagram below shows the graph of $y = x^2$, $y = \sqrt{x}$ and $x = 3$.



(a) Show that the x-coordinate of the point A is 1. **1**

....

....

....

....

....

(b) Calculate the area of the shaded region, correct to one decimal place. **3**

....

....

....

....

....

....

....

....

....

Question 28 (5 marks)

Tim borrows \$700 000 at an interest rate of 3.12% per annum over 25 years, compounded monthly. The loan is to be repaid in equal monthly repayments.
Let M be the monthly repayment and A_n be the balance of the loan after n months.

(a) Show that $A_{300} = 700\ 000 \times 1.0026^{300} - \dfrac{M(1.0026^{300} - 1)}{0.0026}$. **3**

(b) Hence find the value of M to pay off the loan in 25 years. **2**

Question 29 (3 marks)

Solve $\tan(2x) = \sqrt{3}$ for $0 \le x \le 2\pi$. **3**

....

....

....

....

....

....

....

....

....

Question 30 (3 marks)

A student scored 71 in a Physics test and 88 in a Chemistry test. In the Physics test, the mean was 64 and the standard deviation was 4. In the Chemistry test, the mean was 78 and the standard deviation was 6.

(a) Find the z-scores for the student's Physics mark and Chemistry mark. **1**

....

....

....

....

....

....

(b) Hence determine in which test the student did better. Justify your answer. **2**

....

....

....

....

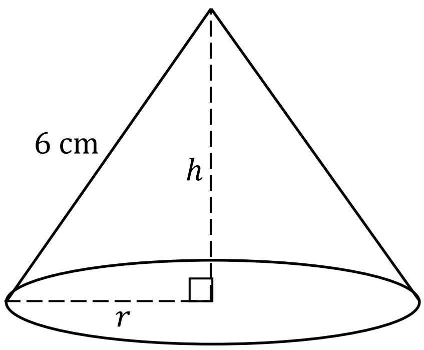
....

Question 31 (3 marks)

Use the trapezoidal rule with four subintervals to estimate $\int_1^3 \ln(x^2)\,dx$, correct to 2 decimal places. **3**

Question 32 (5 marks)

A cone has a slant height of 6 cm as shown in the diagram below.



(a) Show that the volume of the cone is $V = \frac{\pi}{3}(36h - h^3)$ **2**

Question 32 continues on page 24

Question 32 (continued)

(b) Find the maximum volume of the cone. **3**

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

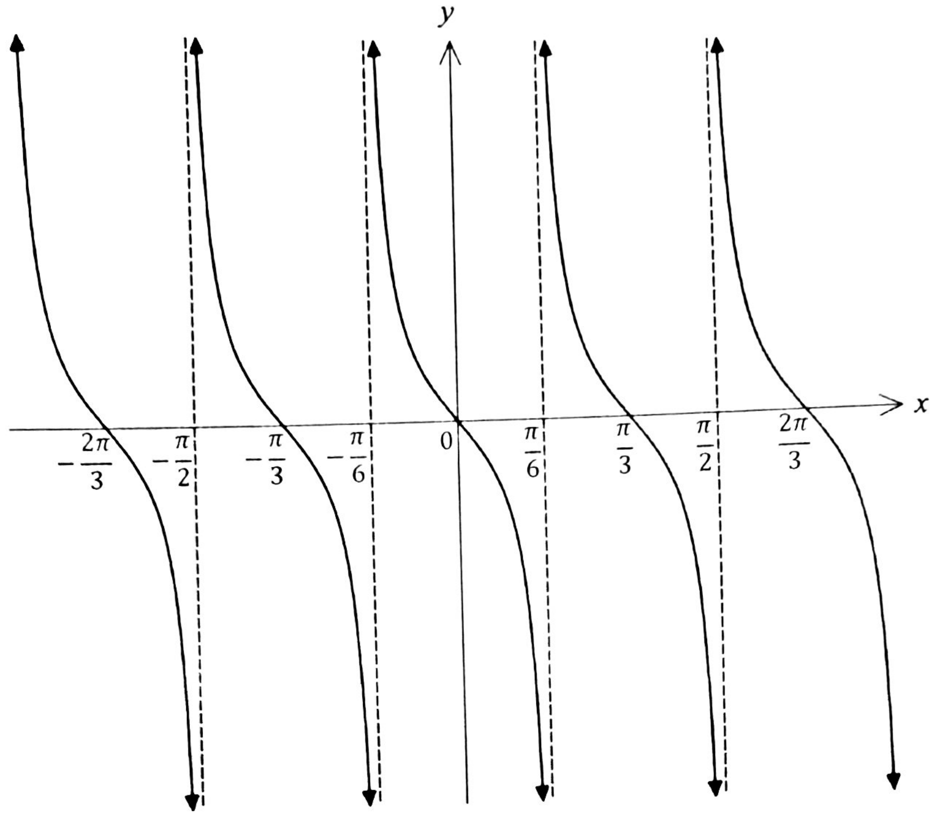
....

Question 33 (4 marks)

Let $f(x) = P\sin x + Q\cos x$, where P and Q are constants. If $f\left(\frac{\pi}{6}\right) = \frac{4 - 3\sqrt{3}}{2}$ and $f'\left(\frac{\pi}{6}\right) = \frac{4\sqrt{3} + 3}{2}$, find the value of P and Q. **4**

End of paper

4 Which of the following equations best describes the graph shown below?



A. $y = \tan\left(\frac{x}{6}\right)$

B. $y = -\tan\left(\frac{x}{3}\right)$

C. $y = \tan\left(x - \frac{\pi}{6}\right)$

D. $y = -\tan(3x)$

$\frac{\pi}{n} = \frac{\pi}{3}$

$n = 3.$

5 When the tap of a water tank is turned on, water flows out at a rate of $\frac{dV}{dt} = 24 + 10t - t^2$ litres per minute, where t is the time in minutes. When does the water stop flowing out of the tank?

A. 5 minutes

B. 6 minutes

C. 12 minutes

D. 24 minutes

$\frac{dv}{dt} = 0.$

$24 + 10t - t^2 = 0.$

$\begin{matrix} 12 & & -t \\ 2 & & t \end{matrix}$

$(12-t)(2+t) = 0$

$t = 12, -2.$

6 Which one of the following descriptions of a Pareto chart is incorrect?

A. A Pareto chart has two vertical axes. Frequencies on the left and cumulative percentage on the right.

B. A Pareto chart consists of three graphs drawn on the same chart.

C. A Pareto chart is useful for displaying categorical data from the most to the least important.

D. The columns of the frequency histogram in a Pareto chart are arranged in descending order.

7 Find $\int \frac{x}{x^2 - 1} dx$.

$\frac{1}{2}\int \frac{2x}{x^2-1} dx$

$= \frac{1}{2} \ln(x^2-1) + c$

A. $\ln(x^2 - 1) + C$

B. $2\ln(x^2 - 1) + C$

C. $\ln(2x) + C$

D. $\frac{1}{2}\ln(x^2 - 1) + C$

8 A group of university students were asked if they regularly drink green tea. The table below shows the results of the survey.

	Regularly drinks green tea	Doesn't drink green tea	Total
Male	6	84	
Female	33	75	108
Total			

What is the probability that a randomly chosen female student will be a regular drinker of green tea?

$\frac{33}{108} = \frac{11}{36}$

A. $\frac{1}{6}$

B. $\frac{25}{66}$

C. $\frac{11}{36}$

D. $\frac{1}{18}$

9 The common ratio of a geometric sequence is 3 and the fifth term is 324. Find the second term.

A. 12

B. 4

C. 36

D. 108

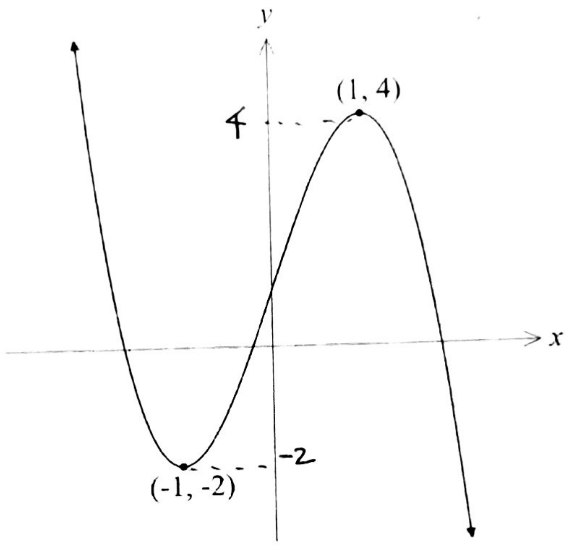
$324 = a \times 3^4$

$a = 4$

$T_2 = 4 \times 3$

$= 12$

10 The graph of $y = f(x)$ is shown below. For what values of k does the equation $f(x) = k$ have 3 solutions?



A. $k = 4$ or -2

B. $k > 4$ or $k < -2$

C. $-1 < k < 1$

D. $-2 < k < 4$

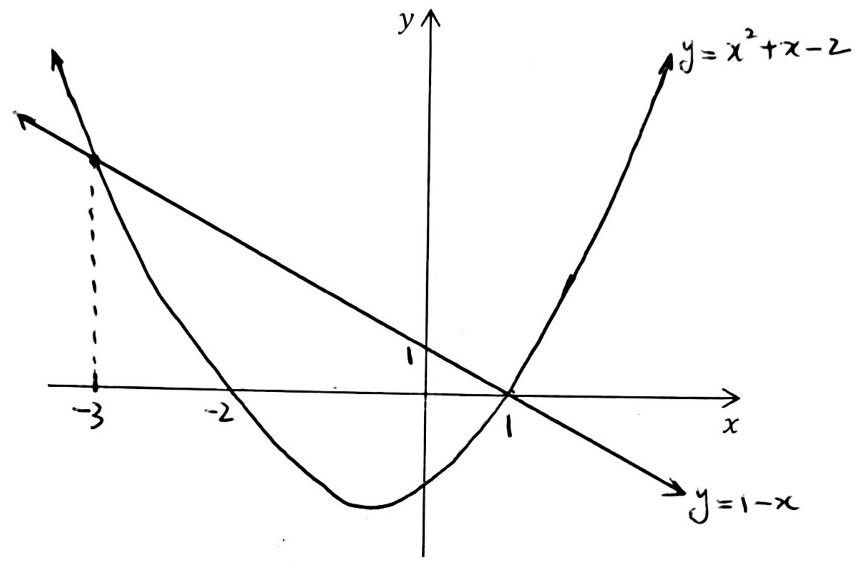
Section II

90 marks
Attempt Questions 11–33
Allow about 2 hours and 45 minutes for this section

- Answer the questions in the spaces provided.
- Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (4 marks)

(a) Sketch $y = x^2 + x - 2$ and $y = 1 - x$ on the same number plane, clearly showing their points of intersection. **3**



$0 = x^2 + x - 2$

$(x+2)(x-1) = 0$

$x = -2, 1$

$x^2 + x - 2 = 1 - x$

$x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

$x = -3, 1$

$y = 4, 0$

Point of intersections at $(-3, 4)$ and $(1, 0)$

(b) Use the graphs to solve $x^2 + x - 2 > 1 - x$. **1**

$x < -3$ or $x > 1$

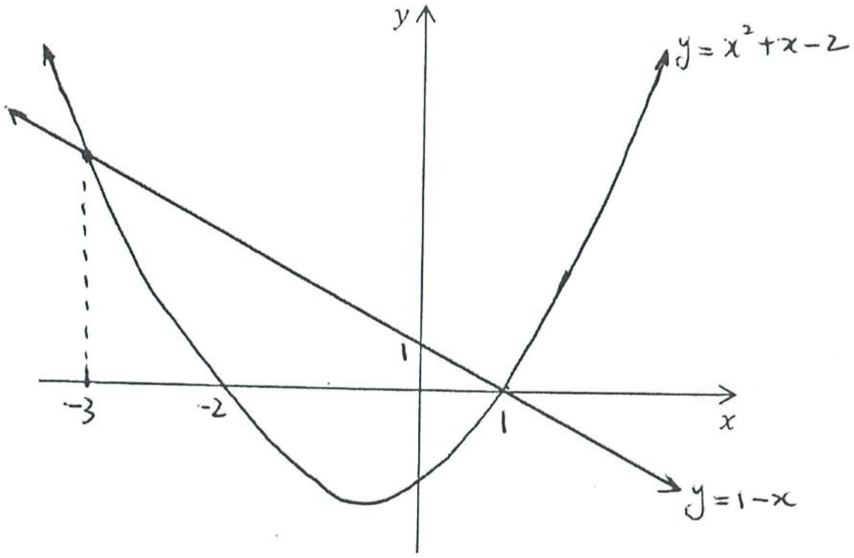
Section II

90 marks
Attempt Questions 11–33
Allow about 2 hours and 45 minutes for this section

- Answer the questions in the spaces provided.
- Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (4 marks)

(a) Sketch $y = x^2 + x - 2$ and $y = 1 - x$ on the same number plane, clearly showing their points of intersection. **3**



Lots of students incorrectly put the vertex on the y-axis

$0 = x^2 + x - 2$

$(x+2)(x-1) = 0$

$x = -2, 1$

$x^2 + x - 2 = 1 - x$

$x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

$x = -3, 1$

$y = 4, 0$

Point of intersections at $(-3, 4)$ and $(1, 0)$

(b) Use the graphs to solve $x^2 + x - 2 > 1 - x$. **1**

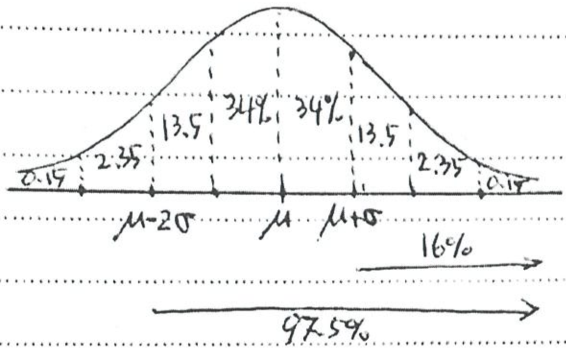
Where is the parabola greater than the straight line?

$x < -3$ or $x > 1$

This Q was done very poorly.

Question 12 (3 marks)

A farmer harvests peaches to supply to a local food manufacturer. Peaches that weigh under 155 grams are rejected and in this year 97.5% of the harvested peaches are accepted. Peaches that are greater than 182 grams are sold as premium peaches and 16% of this year's peaches are categorised as premium. If the weight of the peaches is normally distributed, find the mean and standard deviation of the weights of the peaches. 3



Drawing a picture made this question a lot easier.

$\mu+\sigma = 182$ [1]

$\mu-2\sigma = 155$ [2]

[1]−[2], $3\sigma = 27$

$\therefore \sigma = 9$

Sub $\sigma = 9$ into [1],

$\mu + 9 = 182$

$\therefore \mu = 173$

OR

$z = \frac{x-\mu}{\sigma}$

$1 = \frac{182-\mu}{\sigma} \rightarrow \sigma = 182-\mu$ — (1)

$-2 = \frac{155-\mu}{\sigma} \rightarrow -2\sigma = 155-\mu$ — (2)

(1) into (2)

$-2(182-\mu) = 155-\mu$

$-364+2\mu = 155-\mu$

$3\mu = 519$

$\mu = 173$

Sub into (1)

$\sigma = 182-173$

$= 9$

Question 13 (4 marks)

Consider $y = \dfrac{1}{x-1}$.

(a) State the equation of the vertical and horizontal asymptotes. **2**

$x - 1 \neq 0$

$x \neq 1$

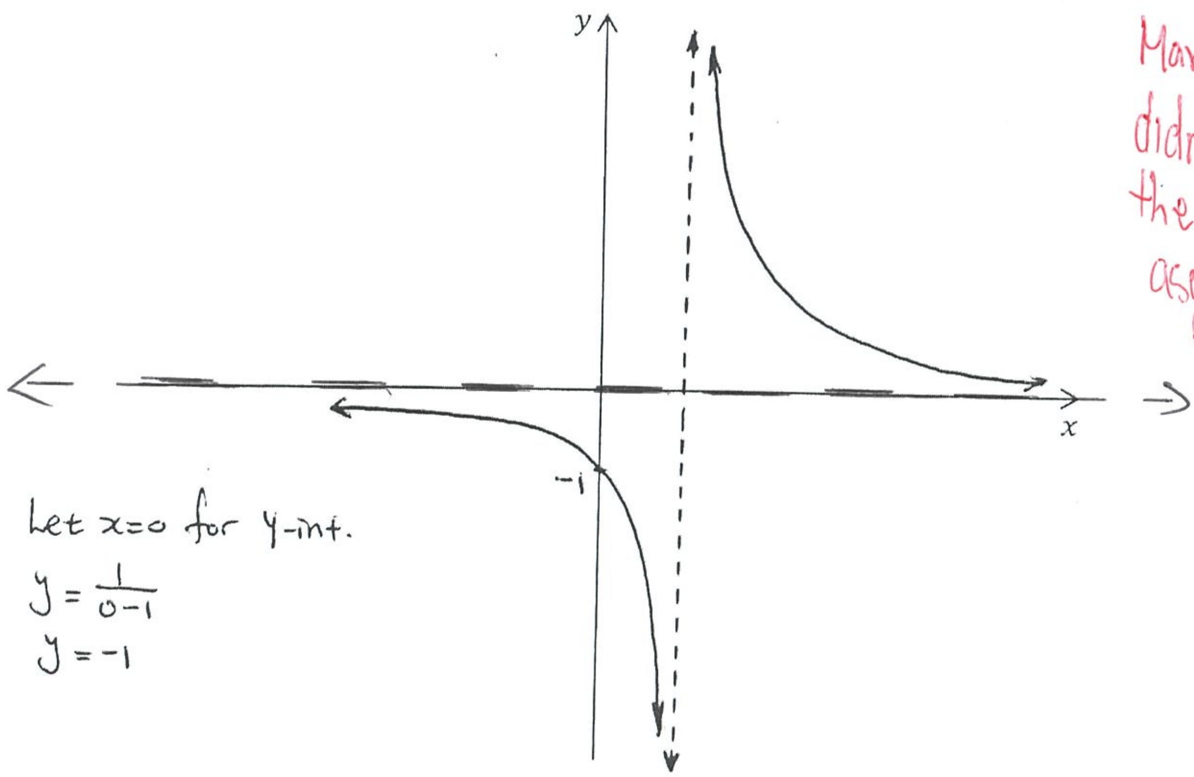
$\therefore$ The vertical asymptote is $x = 1$

As $x \to \infty$, $f'(x) \to 0$

$x \to -\infty$, $f'(x) \to 0$

$\therefore$ The horizontal asymptote is $y = 0$.

(b) Sketch the graph of $y = \dfrac{1}{x-1}$, showing the asymptotes and the y-intercept. **2**



Many students didn't show the horizontal asymptote.

Let $x=0$ for y-int.

$y = \frac{1}{0-1}$

$y = -1$

Question 14 (2 marks)

Describe a suitable sequence of transformations that turns $f(x) = \frac{1}{x^2}$ into $g(x) = -\frac{1}{(x-1)^2} + 3$. 2

Translate to the right by 1 unit,
reflect across the x-axis
translate upwards by 3 units

Question 15 (3 marks)

Consider the function $f(x) = x^2 - 8x$.

(a) Show that $\frac{f(x+h) - f(x)}{h} = 2x + h - 8$. 2

$$L.H.S = \frac{(x+h)^2 - 8(x+h) - (x^2 - 8x)}{h}$$

$$= \frac{\cancel{x^2} + 2hx + h^2 - 8x - 8h \cancel{-x^2} + 8x}{h}$$

$$= \frac{2hx + h^2 - 8h}{h}$$

$$= 2x + h - 8$$

(b) Hence differentiate $f(x) = x^2 - 8x$ from First Principles. 1

$$f'(x) = \lim_{h \to 0} (2x + h - 8)$$

$$= 2x - 8$$

Question 16 (5 marks)

Let X be a random variable with probability density function $f(x)$ given by

$$f(x) = \begin{cases} \dfrac{k}{\sqrt{x}}, & 1 \le x \le 9 \\ 0, & \text{otherwise} \end{cases}$$

(a) Show that $k = \dfrac{1}{4}$. **2**

$$\int_1^9 \frac{k}{\sqrt{x}}\,dx = 1$$

$$k\int_1^9 x^{-1}\,dx = 1$$

$$k\left[2x^{\frac{1}{2}}\right]_1^9 = 1$$

$$k\left(2\sqrt{9} - 2\sqrt{1}\right) = 1$$

$$4k = 1$$

$$k = \frac{1}{4}$$

(b) Show that the cumulative distribution function is $F(x) = \dfrac{1}{2}(\sqrt{x} - 1)$ **2**

$$F(x) = \int_1^x \frac{1}{4\sqrt{x}}\,dx$$

$$= \frac{1}{4}\int_1^x x^{-\frac{1}{2}}\,dx$$

$$= \frac{1}{4}\left[2\sqrt{x}\right]_1^x$$

$$= \frac{1}{4}\left(2\sqrt{x} - 2\right)$$

$$= \frac{1}{2}\left(\sqrt{x} - 1\right)$$

OR

$$F(x) = \int f(x)\,dx$$

$$= \int \frac{1}{4\sqrt{x}}\,dx$$

$$= \int \frac{1}{4}x^{-\frac{1}{2}}\,dx$$

$$= \frac{1}{2}\sqrt{x} + C$$

$$F(9) = 1$$

$$\frac{1}{2}\sqrt{9} + C = 1$$

$$C = -\frac{1}{2}$$

$$F(x) = \frac{1}{2}\sqrt{x} - \frac{1}{2}$$

$$= \frac{1}{2}\left(\sqrt{x} - 1\right)$$

(c) Hence, or otherwise, find the median value of X. **1**

$$F(x) = \frac{1}{2}$$

$$\frac{1}{2}\left(\sqrt{x} - 1\right) = \frac{1}{2}$$

$$\sqrt{x} - 1 = 1$$

$$\sqrt{x} = 2$$

$$x = 4$$

$\therefore$ Median $= 4$

Question 17 (3 marks)

The table below, gives the future value of an annuity of $1 at the given interest rates for different periods. The contributions are made at the end of each period.

Future value of an annuity of $1						
	Interest rate per period					
Period	1%	2%	3%	4%	5%	6%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500	2.0600
3	3.0301	3.0604	3.0909	3.1216	3.1525	3.1836
4	4.0604	4.1216	4.1836	4.2465	4.3101	4.3746
5	5.0101	5.2040	5.3091	5.4163	5.5256	5.6371

(a) Use the table to find the future value of an annuity of $3400 per year for 5 years at 3% per annum, compounding annually. 2

5.3091×3400

$= \$18050.94$

(b) The interest rate increases to 8% per annum, compounding annually, for the 6th year and the holder of the annuity account decides to extend the investment by one additional year. Calculate the balance of the annuity account after the 6th contribution is made. 1

$18050.94 \times 1.08 + 3400$

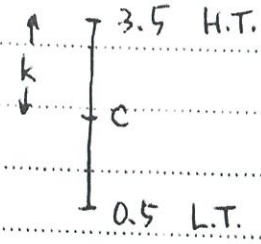
$= \$22895.02$

Read the question carefully! This was done very poorly.

Question 18 (6 marks)

Tides at a location vary over time and can be modelled by a cosine function of the form $h(t) = k\cos at + c$, where k, a and c are constants and t represents the number of hours after midnight. On a particular day, a low tide of 0.5 metres occurred at 6:00 am followed by a high tide of 3.5 metres at midday.

(a) Determine the value of k, a and c. Hence write the rule of $h(t)$. **3**



$k = \frac{3.5 - 0.5}{2}$, $c = \frac{3.5 + 0.5}{2}$

$\therefore k = 1.5$ $\therefore c = 2$

Period $= 6 \times 2$

$= 12$ hours

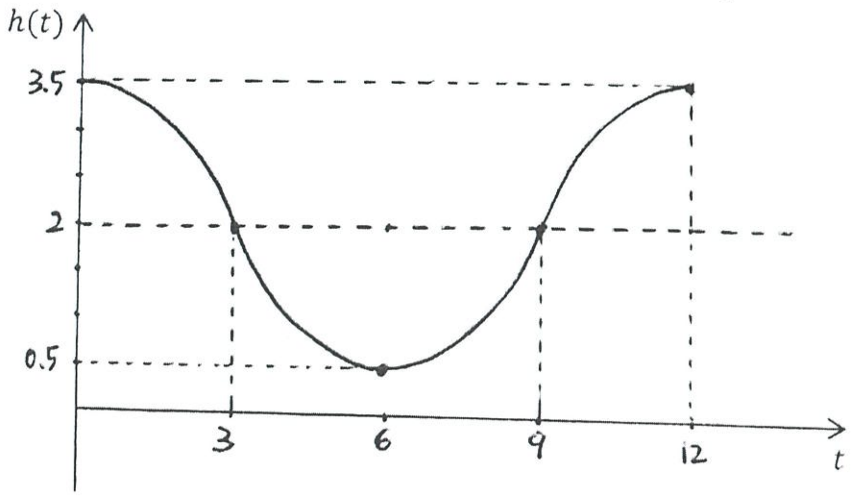
$\frac{2\pi}{a} = 12$

$\therefore a = \frac{\pi}{6}$

$\therefore h(t) = 1.5 \cos \frac{\pi}{6} t + 2$

Many students did not write the final equation for $h(t)$ Read the question.

(b) Sketch the graph of $h(t)$ for the time span from midnight to midday on the day. **2**



If you couldn't do part a then you still could plot points from the Q and joined them with a cosine graph.

(c) At what time does the second low tide occur? **1**

6 pm on the day

This answer could have been logically deduced from the Q.

This was done poorly.

Question 19 (3 marks)

For $y = \ln\left(\dfrac{1-\cos x}{1+\cos x}\right)$, show that $\dfrac{dy}{dx} = 2\,\mathrm{cosec}\,x$. **3**

① $y = \ln(1-\cos x) - \ln(1+\cos x)$

① $\dfrac{dy}{dx} = \dfrac{\sin x}{1-\cos x} + \dfrac{\sin x}{1+\cos x}$

$$= \frac{\sin x(1+\cos x) + \sin x(1-\cos x)}{(1-\cos x)(1+\cos x)}$$

① $$= \frac{\sin x + \cancel{\sin x\cos x} + \sin x - \cancel{\sin x \cos x}}{1-\cos^2 x}$$

$$= \frac{2\sin x}{\sin^2 x}$$

$$= \frac{2}{\sin x}$$

$$= 2\,\mathrm{cosec}\,x$$

* Most students used the alternative method.

Alternative : Quotient rule

① $$y' = \frac{\dfrac{\sin x(1+\cos x)+\sin x(1-\cos x)}{(1+\cos x)^2}}{\dfrac{1-\cos x}{1+\cos x}}$$

$$= \frac{\sin x + \cancel{\sin x\cos x} + \sin x - \cancel{\sin x\cos x}}{(1+\cos x)^2} \times \frac{1+\cos x}{1-\cos x}$$

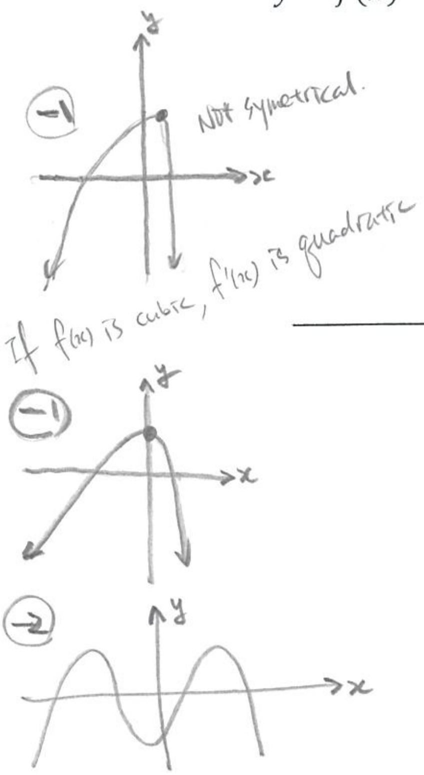
① $$= \frac{2\sin x}{(1+\cos x)(1-\cos x)}$$

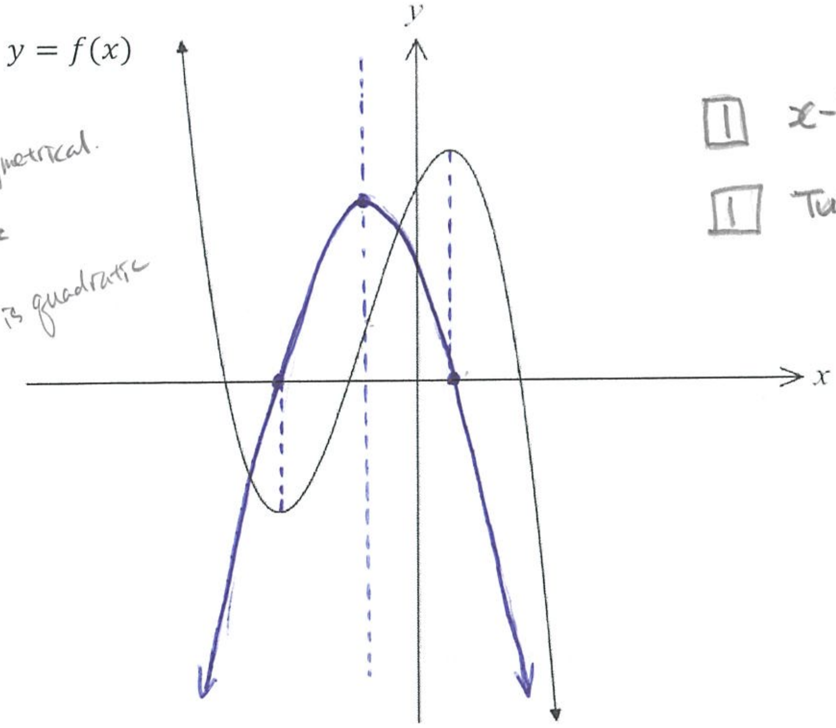
① $$= \frac{2\sin x}{1-\cos^2 x} = \frac{2\sin x}{\sin^2 x} = \frac{2}{\sin x}$$

$$= 2\,\mathrm{cosec}\,x$$

Question 20 (2 marks)

The graph of $y = f(x)$ is given below. On the same number plane, sketch the graph of its derivative $y = f'(x)$. **2**





[1] x-intercepts

[1] Turning point & concavity

Question 21 (8 marks)

A function is given by $y = 2x^3 + 3x^2 - 12x - 4$.

(a) Find the stationary points and determine their nature. **3**

$\frac{dy}{dx} = 6x^2 + 6x - 12$

Let $\frac{dy}{dx} = 0$ for stationary points.

$0 = 6x^2 + 6x - 12$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = -2, 1$ ①

$y = 16, -11$

$y'' = 12x + 6$

When $x = -2$, $y'' = -18 < 0$

$\therefore$ Maximum turning point at $(-2, 16)$ ①

When $x = 1$, $y'' = 18 > 0$

$\therefore$ Minimum turning point at $(1, -11)$ ①

(−1) Concave up when $x = 1$

Concave down when $x = -2$

$\boxed{\text{No penalty}}$ * maximum or minimum (Many students)

* max/min point * Local max/min

* max/min Stationary points, Max T.P

Need to use the full words.

(−1)

x	-3	-2	-1
y'	$+$	0	$-$

No numbers

Note: Don't need to use both of table of tangent & y''.

Question 21 continues on page 15

Question 21 (continued)

(b) Find the point of inflection of the function $y = 2x^3 + 3x^2 - 12x - 4$. **2**

Let $y'' = 0$ for inflection points.

$12x + 6 = 0$ — (−1) If the change of concavity is not checked

$x = -\frac{1}{2}$ (1)

$y = \frac{5}{2}$

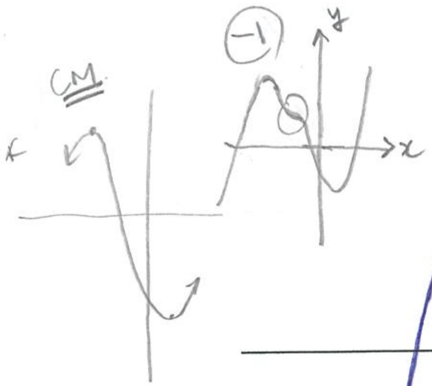
Check the concavity, — CM: Incorrect y-coordinate (Did not penalise)

x	-1	$-\frac{1}{2}$	0
y''	-6		6
	$\cap$		$\cup$

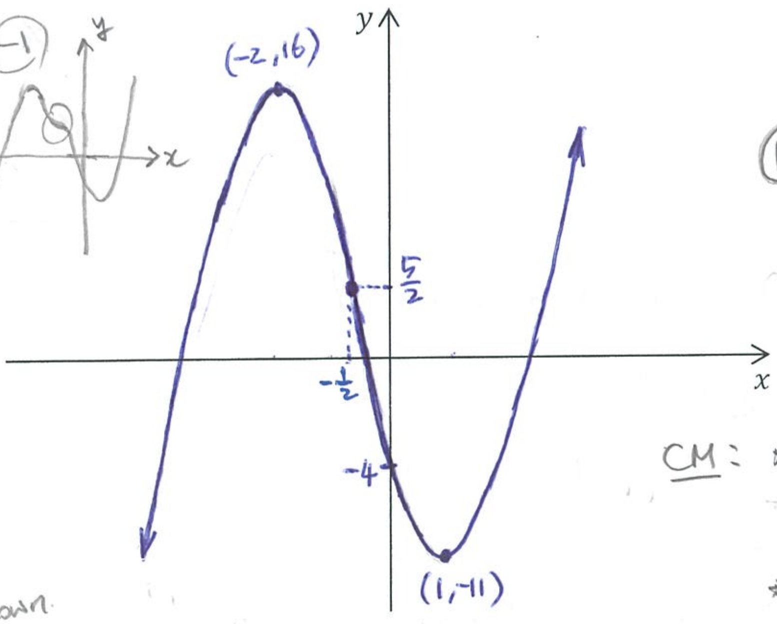
(1)

$\therefore$ Point of inflection at $\left(-\frac{1}{2}, \frac{5}{2}\right)$

(c) Sketch the curve, showing the stationary points, point of inflection and the y-intercept. **2**



* Messy
Use a pencil, then go over it with a pen
* Coordinates not shown.



(1) Stationary points

(1) Inflection point & y-int.

CM: * Messy (Use a pencil, then go over it with a pen)
* Did not show the coordinates of the turning points.

(d) Determine the global maximum of the function in the domain $[-4, 3]$. **1**

When $x = 3$, $y = 41$, — Max. turning point at $(-2, 16)$

When $x = -4$, $y = -36$

$\therefore$ Global maximum $= 41$

Question 22 (3 marks)

(a) Find $\dfrac{d}{dx}(x^2-9)^5$. 1

$= 5(x^2-9)^4 \cdot 2x$

$= 10x(x^2-9)^4$

(b) Hence, or otherwise, find $\int x(x^2-9)^4\,dx$ 2

$\int 10x(x^2-9)^4 dx = (x^2-9)^5$ (from (a))

$\therefore \int x(x^2-9)^4 dx = \frac{1}{10}(x^2-9)^5 + C$

1 1

CM: No C at the end.

(Did not penalise)

Question 23 (3 marks)

If $y = xe^x$, show that $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + y = 0$ 3

$\frac{dy}{dx} = e^x + xe^x$ (by product rule) 1

$\frac{d^2y}{dx^2} = e^x + e^x + xe^x$

$= 2e^x + xe^x$ 1

$\left.\begin{aligned} LHS &= 2e^x + xe^x - 2(e^x + xe^x) + xe^x \\ &= \cancel{2e^x} + xe^x - \cancel{2e^x} - 2xe^x + xe^x \end{aligned}\right\}$ 1

$= 0$

$= RHS$

[No marks] $y' = xe^x$

$y'' = xe^x$

$\therefore xe^x - 2(xe^x) + xe^x = 0.$

Question 24 (6 marks)

The velocity of a particle moving along a straight line is given by $v = 4 - 4t$, where t is time in seconds. Initially the particle is 6 units to the right of the origin.

(a) Show that the particle is initially moving to the right? **1**

When $t=0$, $V=4$
>0

$\therefore$ Start to move to the right.

(b) Show that the displacement function is $x = 6 + 4t - 2t^2$. **2**

$x = \int (4-4t)\,dt$

$x = 4t - 2t^2 + c$ (1)

Sub $x=6$, $t=0$ (-1) If no steps show for $C=6$

$\therefore C = 6$ (1)

$\therefore x = 6 + 4t - 2t^2$

(c) When is the particle at the origin? **2**

(-1) $0=(3-t)(1+t)$ $\hookrightarrow x=3$ with no explanation

$0 = 6 + 4t - 2t^2$

$0 = 3 + 2t - t^2$

$\begin{matrix} 3 & & -t \\ 1 & & t \end{matrix}$

$0 = (3-t)(1+t)$

(1) $t = 3$ or -1 (-1) If stopped here.

(1) $\therefore t = 3$ (Since $t \geq 0$)

$\therefore$ After 3 seconds.

Question 24 continues on page 18

Question 24 (continued)

(d) Explain why the particle continues to move to the left and never changes direction as $t \to \infty$. **1**

As $t \to \infty$, $v \to -\infty < 0$

$\therefore$ The particle continues to move to the left and never changes direction.

Alternative

As $t \to \infty$, $x \to -\infty$ or because $\ddot{x} = -4 < 0$

Question 25 (3 marks)

$x, 8x$ and $3x^2$ are three consecutive terms of an arithmetic sequence.

(a) Show that $3x^2 - 15x = 0$ **1**

$8x - x = 3x^2 - 8x$

$3x^2 - 15x = 0$

(b) Find the value of the 4^{th} term. **2**

$3x(x-5) = 0$

$x = 0, 5$

$\therefore x = 5$ (since $x > 0$) 1

$d = 7x$

$= 35$

$T_4 = 3x^2 + 35$

$= 3(5)^2 + 35$

$= 110$ 1

Question 26 (5 marks)

A group of young children were surveyed to find if there was a correlation between a child's age and height. The results are shown in the table.

Age in years (x)	8	5	2	6	12	11
Height in cm (h)	124	113	85	121	153	144

(a) Find the correlation coefficient (r) using your calculator, correct to 2 decimal places. 1

0.98

Well done

(b) Using your calculator, find the equation of the least-squares regression line in the form $h = Bx + A$; where A and B are **integers**. 1

whole number only.

$h = 6x + 77$

No marks for decimal / fraction.

(c) Use your equation to estimate the height of a child who is 7 years old. 1

$h = 6 \times 7 + 77$

$h = 119$ cm

CFE

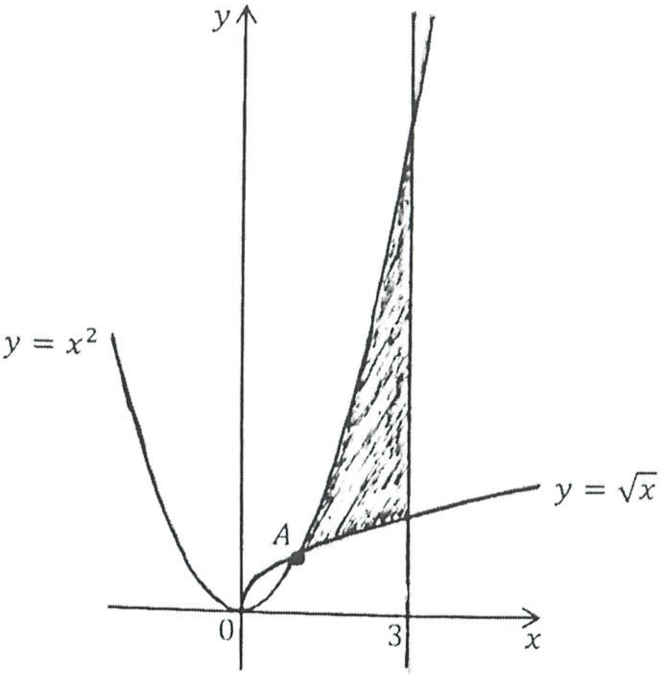
(d) Is this equation a good model to use to make a valid estimate of a 50 year-old person's height? Justify your answer. 2

1

The equation is not valid to estimate the height of a 50 year-old person, because the growth rate 1 slows down and gradually become zero as a person gets older. It is only valid for age between 8 and 12, inclusive.

Question 27 (4 marks)

The diagram below shows the graph of $y = x^2$, $y = \sqrt{x}$ and $x = 3$.



(a) Show that the x-coordinate of the point A is 1. **1**

$x^2 = \sqrt{x}$

$x^4 = x$

$x^4 - x = 0$

$x^3(x-1) = 0$

$x = 0, 1$

$\therefore x = 1$

Alternative

Sub $x=1$ into $y = x^2$

$y = 1$

Sub $x=1$ into $y = \sqrt{x}$

$y = 1$

$\therefore$ The x-coordinate of the point A is 1.

Well done.

(b) Calculate the area of the shaded region, $\boxed{\text{correct to one decimal place.}}$ **3**

penalise 1 mark.

$\int_1^3 (x^2 - \sqrt{x})\,dx$ — 1

$= \left[\frac{x^3}{3} - \frac{2x^{\frac{3}{2}}}{3}\right]_1^3$ — 2

$= (9 - 2\sqrt{3}) - (\frac{1}{3} - \frac{2}{3})$

$= 5.9$ unit^2

1 mark for writing the correct integral

1 mark for correctly integrating

1 mark for correct answer.

Question 28 (5 marks)

Tim borrows \$700 000 at an interest rate of 3.12% per annum over 25 years, compounded monthly. The loan is to be repaid in equal monthly repayments. Let M be the monthly repayment and A_n be the balance of the loan after n months.

This question is worth 3 marks.

(a) Show that $A_{300} = 700\ 000 \times 1.0026^{300} - \dfrac{M(1.0026^{300} - 1)}{0.0026}$. **3**

$r = 3.12\% \div 12 = 0.26\%$ per month

$A_1 = 700000 \times 1.0026 - M$

$A_2 = A_1 \times 1.0026 - M$

$= 700000 \times 1.0026^2 - M(1 + 1.0026)$

$A_3 = A_2 \times 1.0026 - M$

$= 700000 \times 1.0026^3 - M(1 + 1.0026 + 1.0026^2)$

2 marks for showing A_1, A_2, A_3

$\vdots$

$A_{300} = 700000 \times 1.0026^{300} - M\underbrace{(1 + 1.0026 + 1.0026^2 + \dots + 1.0026^{299})}_{\text{G.P. } a=1,\ r=1.0026,\ n=300}$

1 mark for showing this step.

$= 700000 \times 1.0026^{300} - M \cdot \dfrac{1(1.0026^{300} - 1)}{1.0026 - 1}$

Sum of geo serie

$= 700000 \times 1.0026^{300} - \dfrac{M(1.0026^{300} - 1)}{0.0026}$

* A lot of students just quoted a formula, which is not enough working, you need to derive A_{300}.

(b) Hence find the value of M to pay off the loan in 25 years. **2**

$0 = 700000 \times 1.0026^{300} - \dfrac{M(1.0026^{300} - 1)}{0.0026}$

$\dfrac{M(1.0026^{300} - 1)}{0.0026} = 700000 \times 1.0026^{300}$

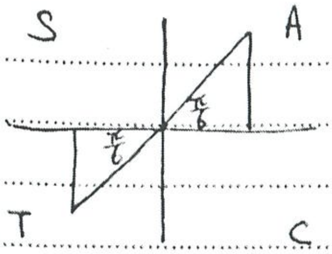
$M = \dfrac{0.0026(700000 \times 1.0026^{300})}{1.0026^{300} - 1}$

$\therefore M = \$3363.33$

Well done.

Question 29 (3 marks)

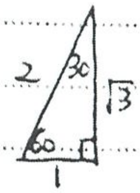
Solve $\tan(2x) = \sqrt{3}$ for $0 \le x \le 2\pi$. 3



$0 \le 2x \le 4\pi$

$2x = \frac{\pi}{3}, \frac{4\pi}{3}, \frac{7\pi}{3}, \frac{10\pi}{3}$

$\therefore x = \frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{6}, \frac{5\pi}{3}$



1 mark for $2x = \frac{\pi}{3}$

1 mark for $x = \frac{\pi}{6}$ and another angle

Full marks for getting all 4 angles.

Question 30 (3 marks)

A student scored 71 in a Physics test and 88 in a Chemistry test. In the Physics test, the mean was 64 and the standard deviation was 4. In the Chemistry test, the mean was 78 and the standard deviation was 6.

(a) Find the z-scores for the student's Physics mark and Chemistry mark. 1

For Physics, $\mu = 64$, $\sigma = 4$

$z = \frac{71-64}{4}$

$= 1.75$

For Chemistry, $\mu = 78$, $\sigma = 6$

$z = \frac{88-78}{6}$

$= 1.67$

If you only get one correct, there's no half mark.

Well done.

(b) Hence determine in which test the student did better. Justify your answer. 2

The student did better in (1) Physics, because a higher positive z-score means further above the mean.

(1)

Well done.

Question 31 (3 marks)

Use the trapezoidal rule with four subintervals to estimate $\int_1^3 \ln(x^2)\,dx$, correct to 2 decimal places. 3

$h = \frac{3-1}{4} = \frac{1}{2}$ 1

x	1	1.5	2	2.5	3
y	0	$\ln 2.25$	$\ln 4$	$\ln 6.25$	$\ln 9$

1

$\int_1^3 \ln(x^2)\,dx \approx \frac{0.5}{2}\left\{0 + 2(\ln 2.25 + \ln 4 + \ln 6.25) + \ln 9\right\}$

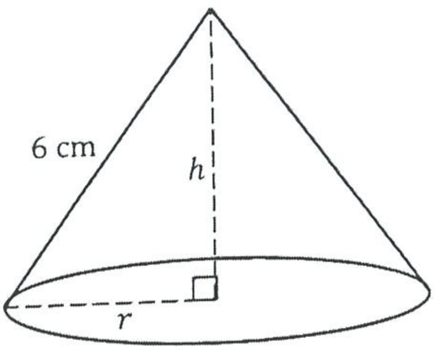
≈ 2.56 1

* Most students didn't draw the table, so carried forward error "CFE" marks could not be given

* Full marks given for correct answer with <u>reasonable</u> working out

Question 32 (5 marks)

A cone has a slant height of 6 cm as shown in the diagram below.



(a) Show that the volume of the cone is $V = \frac{\pi}{3}(36h - h^3)$ 2

$\boxed{r^2 + h^2 = 6^2 \text{ (by Pythagoras' theorem)}}$ — 1 mark.

$$\left.\begin{aligned} r^2 &= 36 - h^2 \\ V &= \tfrac{1}{3}\pi r^2 h \\ &= \tfrac{1}{3}\pi h(36 - h^2) \\ &= \tfrac{\pi}{3}(36h - h^3) \end{aligned}\right\}$$ 1 mark.

Question 32 continues on page 24

Question 32 (continued)

(b) Find the maximum volume of the cone. **3**

$\frac{dV}{dh} = \frac{\pi}{3}(36 - 3h^2)$

Let $\frac{dV}{dh} = 0$ for stationary points,

$0 = \frac{\pi}{3}(36 - 3h^2)$

$36 - 3h^2 = 0$

$h^2 = 12$

$h = 2\sqrt{3}$ (since $h > 0$) — ① mark for finding h

$V'' = \frac{\pi}{3}(-6h)$

$= -2\pi h$

— ① mark for testing for max.

When $h = 2\sqrt{3}$, $V'' = -2\pi(2\sqrt{3})$

$= -4\pi\sqrt{3}$

< 0 ∩

∴ The volume is maximum when $h = 2\sqrt{3}$

∴ Maximum Volume $= \frac{\pi}{3}\left(36 \times 2\sqrt{3} - (2\sqrt{3})^3\right)$

$= \frac{\pi}{3}(72\sqrt{3} - 24\sqrt{3})$

$= \frac{\pi}{3}(48\sqrt{3})$

$= 16\pi\sqrt{3}$ unit^3 — ① mark for correct volume.

* Alot of students lost a mark for not testing max/min. You cannot just make assumptions that $h = \sqrt{12}$ is max.

Question 33 (4 marks)

Let $f(x) = P\sin x + Q\cos x$, where P and Q are constants. If $f\left(\frac{\pi}{6}\right) = \frac{4-3\sqrt{3}}{2}$ and $f'\left(\frac{\pi}{6}\right) = \frac{4\sqrt{3}+3}{2}$, find the value of P and Q. **4**

$P\sin\frac{\pi}{6} + Q\cos\frac{\pi}{6} = \frac{4-3\sqrt{3}}{2}$

$\frac{P}{2} + \frac{\sqrt{3}Q}{2} = \frac{4-3\sqrt{3}}{2}$

$P + \sqrt{3}Q = 4 - 3\sqrt{3}$ [1] —— 1 mark

$f'(x) = P\cos x - Q\sin x$

$P\cos\frac{\pi}{6} - Q\sin\frac{\pi}{6} = \frac{4\sqrt{3}+3}{2}$

$\frac{\sqrt{3}P}{2} - \frac{Q}{2} = \frac{4\sqrt{3}+3}{2}$

$\sqrt{3}P - Q = 4\sqrt{3}+3$

$3P - \sqrt{3}Q = 12 + 3\sqrt{3}$ [2] —— 1 mark.

[1]+[2], $4P = 16$

$\therefore P = 4$

Sub. $P=4$ into [1],

$4 + \sqrt{3}Q = 4 - 3\sqrt{3}$

$\sqrt{3}Q = -3\sqrt{3}$

$\therefore Q = -3$

$\therefore \boxed{P = 4}, \boxed{Q = -3}$

1 mark.

1

* If you made an attempt and it is correct but did not finish the question I gave you 1 mark for trying.

End of paper