

Teacher Name:

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NESA number

2025

Penrith High School
HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION



Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- In Questions 11 – 27 show relevant mathematical reasoning and/or calculations
- Write your NESA ID above and on the cover of booklets 1, 2, and 3

**Total Marks:
101**

Section I – 10 marks (pages 3 – 7)

- Attempt questions 1 – 10
- Allow about 15 minutes for this section

Section II – 91 marks (pages 9 – 35)

- Attempt questions 11 – 27
- Allow about 2 hours and 45 minutes for this section

Multiple Choice	Functions	Trigonometric Functions	Statistical Analysis	Financial Mathematics	Calculus
	/3		/3	/2	/2
	1, 2, 5		3, 4, 6	8, 10	7, 9
Total	Functions	Trigonometric Functions	Statistical Analysis	Financial Mathematics	Calculus
	/11	/16	/18	/11	/45
Total:				/101	

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Section I

10 marks

Attempt questions 1 – 10

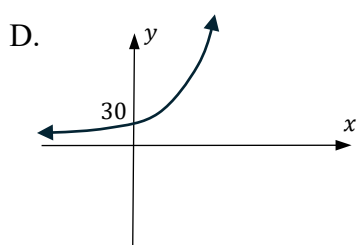
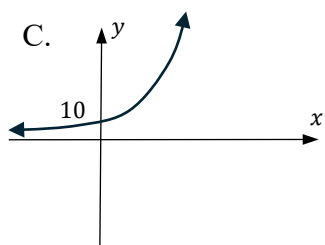
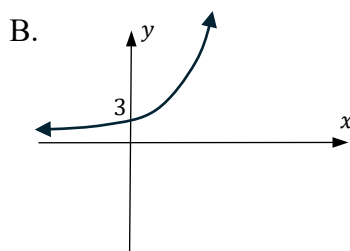
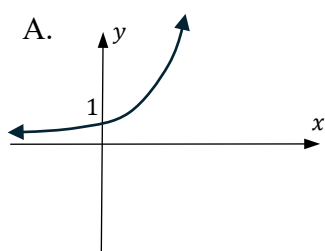
Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1 – 10

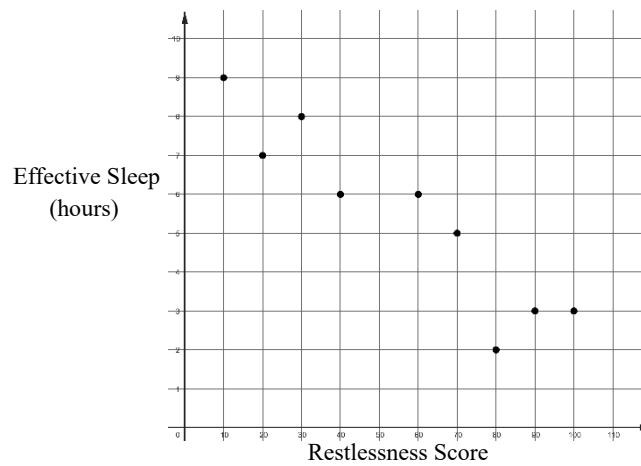
- 1 The parabola $g(x) = 2(x - 1)(x - 3)$ is transformed into a concave up parabola with vertex is at the origin.

Which of the following transformations could have achieved this?

- A. Horizontal translation of 2 units to the left followed by a vertical translation of 2 units up
 - B. Reflection in the x axis followed by a translation 1 unit to the left
 - C. Translation of 3 units to the right followed by a reflection in the x axis
 - D. Vertical dilation by a factor of $-\frac{1}{2}$ followed by a horizontal translation of 2 units to the right
- 2 Which of the following best represents the graph of $y = 10(3^x)$?

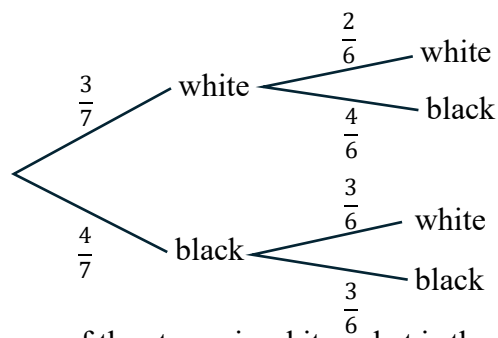


- 3 An app tracks the “restlessness score” and the number of hours of “effective sleep” for a user. The user can use the app to produce a scatter plot of these variables, an example of which is shown below:



Which Pearson’s correlation coefficient best describes the data?

- A. 0.8
 B. 0.2
 C. -0.2
 D. -0.8
- 4 A bag contains 3 white and 4 black stones. Two stones are drawn out of the bag with the probability tree below showing the possibilities:



If it is known that one of the stones is white, what is the probability that the stones are different colours?

- A. $\frac{1}{2}$
 B. $\frac{1}{3}$
 C. $\frac{4}{5}$
 D. $\frac{6}{7}$

5 Which of the following is equivalent to $\log_b \sqrt{c}$?

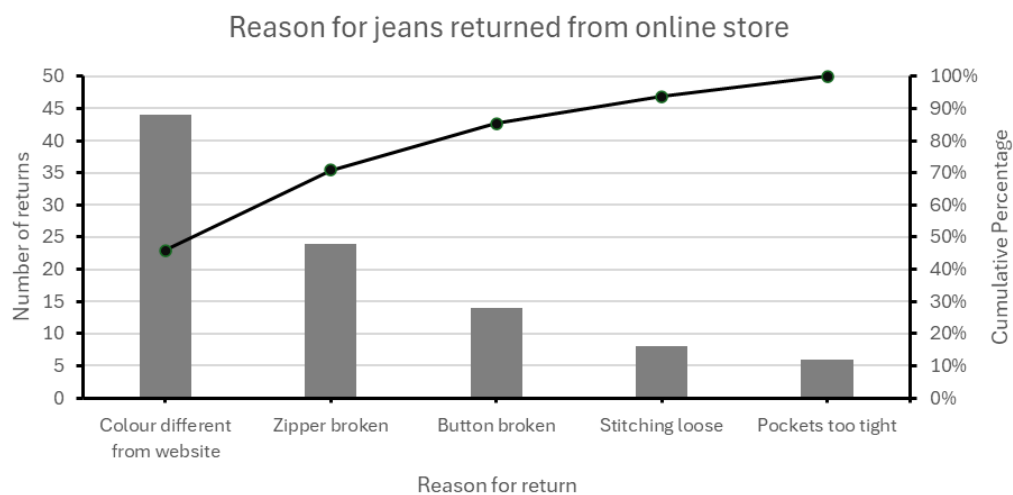
A. $\frac{2}{\log_c b}$

B. $\frac{\log_c b}{2}$

C. $\frac{1}{\log_c b^2}$

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6 The Pareto chart below shows the reasons that customers returned jeans purchased online during July 2025.



The manager of online business wishes to address at least 80% of returns by making the fewest changes to company processes.

Which reasons for return should the manager address?

- A. Colour different from website, Zipper broken, Button broken
- B. Colour different from website, Button broken, Stitching loose, Pockets too tight
- C. Colour different from website
- D. Button broken

- 7 Consider the function $y = 8x^3 - 15x^2 + 5$ with a domain $x \in [-1, 2]$.

Which of the following is true for the function?

- A. The maximum value of the function is 0 and the minimum value is $\frac{5}{4}$
 - B. The minimum value is $\frac{5}{4}$ and the maximum value is 0
 - C. The minimum value is $-\frac{45}{16}$ and the maximum value is 5
 - D. The minimum value is -18 and the maximum value is 9
- 8 The following table gives the contribution per period for an annuity with a future value of \$1 at different interest rates and different time periods.

Contribution per period for an annuity with a future value of \$1

<i>Number of periods</i>	<i>Interest rate (% per period)</i>					
	0.25%	0.5%	0.75%	1%	1.25%	1.5%
6	0.1656	0.1646	0.1636	0.1625	0.1615	0.1605
12	0.0822	0.0811	0.0800	0.0788	0.0778	0.0767
18	0.0544	0.0532	0.0521	0.0510	0.0499	0.0488
24	0.0405	0.0393	0.0382	0.0371	0.0360	0.0349
30	0.0321	0.0310	0.0298	0.0287	0.0277	0.0266
36	0.0266	0.0254	0.0243	0.0232	0.0222	0.0212

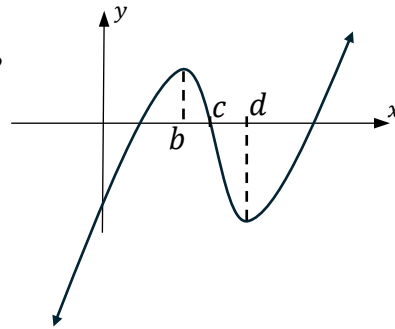
Gi-hun wants to have \$48,000 in 6 years time to pay off his university fees. He plans to make contributions every 3 months into an account with an interest rate of 3% pa. How much will Gi-hun need to deposit each quarter?

- A. \$1,833.60
- B. \$7,852.80
- C. \$7,900.80
- D. \$1,886.40

- 9 Consider the graph of $y = f(x)$.

At what value of x is $f''(x) > f'(x) > f(x)$?

- A. 0
- B. b
- C. c
- D. d



- 10 The formula for the n^{th} term of a series is given by

$$T_n = 400n - 2^n$$

What is the sum of the first 10 terms?

- A. 2 976
- B. 407 154
- C. -135 718
- D. 19 954

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Teacher Name:

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2025

Penrith High School
HSC TRIAL
EXAMINATION

NESA Number

Mathematics Advanced
Section II Answer Booklet 1

Section II

91 marks

Attempt questions 11 – 27

Allow about 2 hours and 45 minutes for this section

Booklet 1 – Attempt questions 11 to 17 (32 marks)

Booklet 2 – Attempt questions 18 to 22 (29 marks)

Booklet 3 – Attempt questions 23 to 27 (30 marks)

Instructions

- Answer the questions in the space provided. Sufficient spaces are provided for typical responses
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	Functions	Trigonometric Functions	Statistical Analysis	Financial Mathematics	Calculus
Booklet 1		/5	/5	/9	/13
		12	14	13, 16	11, 15, 17

Please turn over

Question 11 (3 marks)

Evaluate

3

$$\int_2^4 \frac{2x^2 - 1}{x} dx$$

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Question 12 (5 marks)

(a) Show that

3

$$\frac{1}{1 - \sin x} + \frac{1}{1 + \sin x} = 2\sec^2 x$$

(b) Hence, solve

2

$$\frac{1}{1 - \sin x} + \frac{1}{1 + \sin x} = 8, \quad \text{for } 0^\circ \leq x \leq 360^\circ$$

Question 13 (3 marks)

Find the sum of the terms in the arithmetic series:

3

$$71 + 64 + 57 + \cdots + (-48)$$

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Question 14 (5 marks)

The table below shows the distribution of a discrete random variable.

x	0	1	2	3
$P(X = x)$	0.22	0.66	$-0.1k$	k^2

- (a) Explain why the value of k is only -0.3 . 2

- (b) Find the expected value $E(X)$. 1

- (c) Find the standard deviation, correct to one decimal place. 2

A particle moves along the x axis with a displacement at time t seconds given by

(a) Where is the particle initially? 1

[illegible]

(b) Show that the particle comes to rest when $t = 3$. 2

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- 14 -

- (c) Find the total distance the particle travels in the first 5 seconds. Answer correct to one decimal place.

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Question 16 (6 marks)

Elsa borrows \$720,000 from the Bank of Arendelle at an interest rate of 6% per annum. She makes a monthly repayment of \$ M . Let the amount of money owing directly after her n^{th} repayment be denoted by A_n .

- (a) Show that after her second repayment Elsa owes

1

$$A_2 = 720\,000(1.005)^2 - M(1.005 + 1)$$

- (b) If Elsa wants to repay the loan after 30 years find the value of her monthly repayment M to the nearest cent.

3

Question 16 continues on page 17

- (c) Elsa instead decides to repay \$4,500 per month. How many months faster will she repay her loan? Answer to the nearest month. 2

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Question 17 (5 marks)

- (a) Differentiate $x^2 \ln x$

2

[illegible]

- (b) Hence, evaluate

3

$$\int_1^e x \ln x \, dx$$

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Section II extra writing space

If you use this space clearly indicate which question you are answering.

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Teacher Name:

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2025

Penrith High School
**HSC TRIAL
EXAMINATION**

NESA Number

**Mathematics Advanced
Section II Answer Booklet 2**

Booklet 2 – Attempt questions 18 to 22

Instructions

- Answer the questions in the space provided. Sufficient spaces are provided for typical responses
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Booklet 2	Functions	Trigonometric Functions	Statistical Analysis	Financial Mathematics	Calculus
	/4	/11	/10		/4
	20ab	19, 21	18,22		20c

Please turn over

Question 18 (6 marks)

A group of 40 adults were surveyed as to whether they paid a subscription fee to watch sport or television series online.

The following answers were given:

- 12 adults only pay for sport
- 6 adults pay for both
- 8 adults pay for neither

- (a) By drawing a Venn diagram below find the probability that an adult surveyed pays for television shows. **2**

- (b) An adult is chosen at random. If it is known that they pay for television series, what is the probability that they also pay for sport? **1**

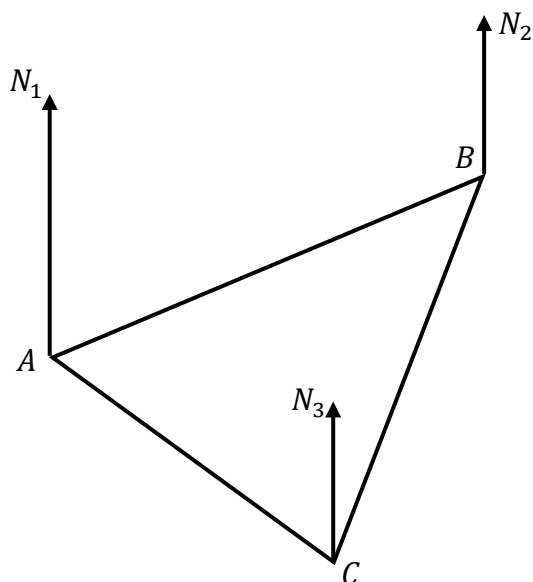
Question 18 continues on page 23

Question 19 (6 marks)

An expedition leaves camp A and walks 10km on a bearing of $050^\circ T$ to camp B . They then walk 8km on a bearing of $200^\circ T$ until they reach camp C .

- (a) Add the information from the question to the diagram below.

2



- (b) How far to the nearest km do they need to walk from camp C to camp A ?

2

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Question 19 continues on page 25

- (c) By using the cosine rule to first find $\angle ACB$, find the bearing they need to walk on from camp C to reach camp A . 2

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Question 20 (8 marks)

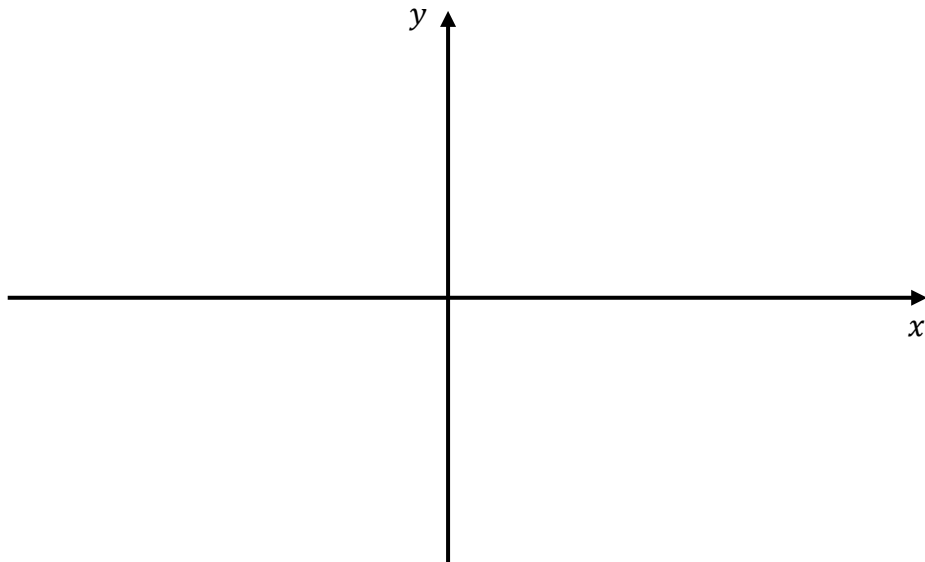
(a) Solve $|x| = \frac{1}{2}x + 3$

2

[illegible]

(b) On the axes below draw a neat sketch showing the graphs of $y = |x|$ and $y = \frac{1}{2}x + 3$. Show the x and y intercepts and any points of intersection.

2



Question 20 continues on page 27

- (c) An area between $y = |x|$ and $y = \frac{1}{2}x + 3$ is bounded by $x = -2$ and $x = k$, where $k > 0$. 4

Find the value of k such that the area is 10 square units.

[illegible]

Question 21 (5 marks)

The water depth d in Penrith harbour t hours after the first low tide of the day can be modelled with the equation:

$$d = 5.2 - 0.8 \cos\left(\frac{4\pi}{25}t\right)$$

- (a) What is the depth of the water at low tide? 1

- (b) If the first low tide of the day occurred at 6am, what time does the next low tide occur at? 2

- (c) Draw a neat sketch of the depth of the water in the harbour from the first low tide until the second low tide of the day. 2



Question 22 (4 marks)

Gout is training for the big 100m race at the end of the year. He is keeping track of the total number of hours that he has trained for and the times in seconds for his 100m, and recorded this data in the following table:

Hours (h)	3.7	5.6	8.2	10.1	11.1	14.8	17.6	20.8	22.5
Time (t)	14.2	13.9	13.7	13.3	13.4	12.9	12.8	12.8	12.4

- (a) Find the equation of the regression line for Time t as a function of Hours h . 2

- (b) Using the equation in (a), how many hours of training would Gout need to complete to run the 100m in 11 seconds? 1

- (c) Why is your answer to part (b) not reliable? 1

Section II extra writing space

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Teacher Name:

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2025

Penrith High School
HSC TRIAL
EXAMINATION

NESA Number

**Mathematics Advanced
Section II Answer Booklet 3**

Booklet 3 – Attempt questions 23 to 27

Instructions

- Answer the questions in the space provided. Sufficient spaces are provided for typical responses
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	Functions	Trigonometric functions	Statistical Analysis	Financial Mathematics	Calculus
Booklet 3	/4				/26
	23				24,25,26,27

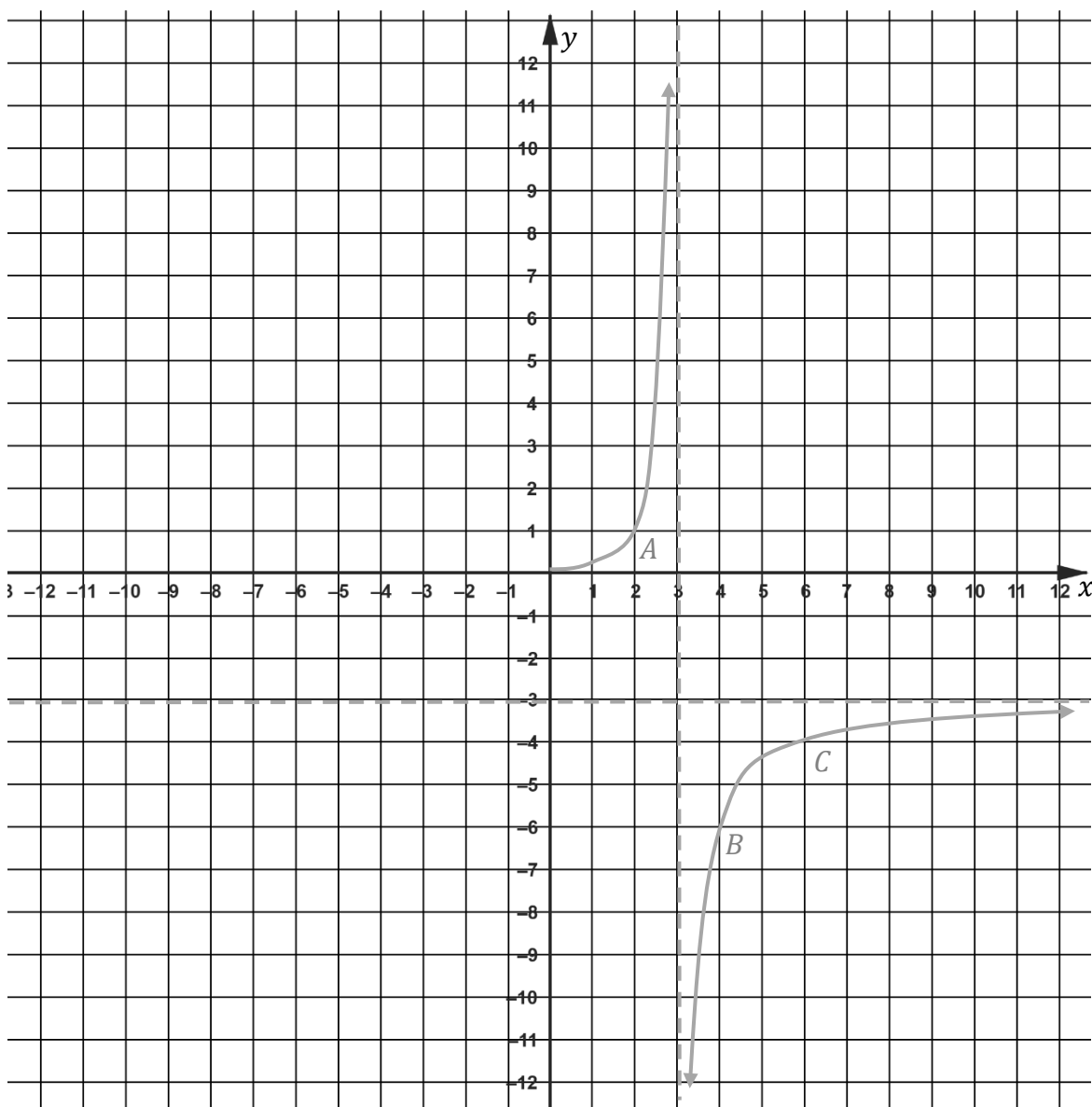
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Question 23 (4 marks)

The part of the graph of the odd function $y = f(x)$ where $x \geq 0$ has been revealed on the graph below. The graph of $y = f(x)$ passes through $A(2,1)$, $B(4,-6)$ and $C(6,-4)$. 4

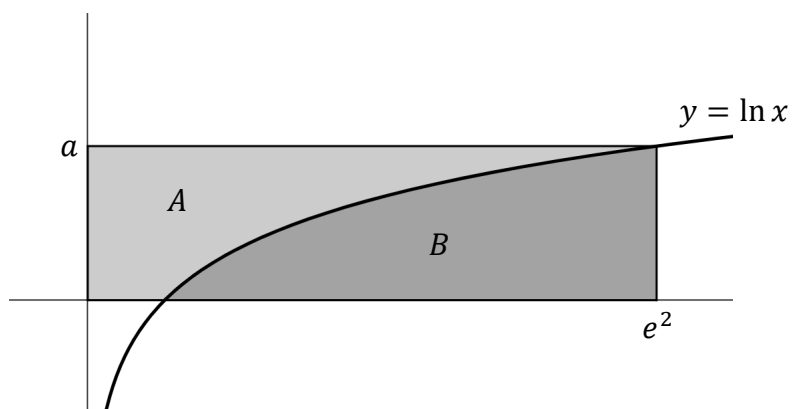
The function h is defined as $h(x) = f\left(\frac{x}{2}\right) + 2$ with $f(x)$ defined for $-\infty \leq x \leq \infty$.

Draw the complete graph of $y = h(x)$ on the same axes over its entire domain, clearly showing the new locations of the points A , B and C as well as any asymptotes.



Question 24 (7 marks)

Aneeka and Brian are investigating areas A and B respectively on either side of the graph of $y = \ln x$ as shown on the diagram below:



- (a) Find the value of a .

1

- (b) Brian uses the trapezoidal rule with 3 function values to estimate the area B . He approximates all of the values used in his working out to 2 decimal places where necessary.

2

What is his estimate for the area of B ?

Question 24 continues on page 34

- (c) Aneeka evaluates the area A by using an integral against the y axis. What exact value does she get for the area A ? 2

[illegible]

Question 24 continues on page 35

- (d) Brian considers the area $A + B$ and uses his estimate of the area B from part (b) and Aneeka's answer in part (c) to find an approximation for the value of e . 2

What is his approximation to 2 decimal places?

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- (e) Why is Brian's approximate value for e an underestimate? 1

Question 25 (7 marks)

- (a) Draw a neat sketch of $y = x^3 - 6x^2 + 9x - 3$ on page 37 showing the y intercept, all turning points and all points of inflexion. You need to determine the nature of any stationary points that you find. You do not need to find the x intercepts. 6

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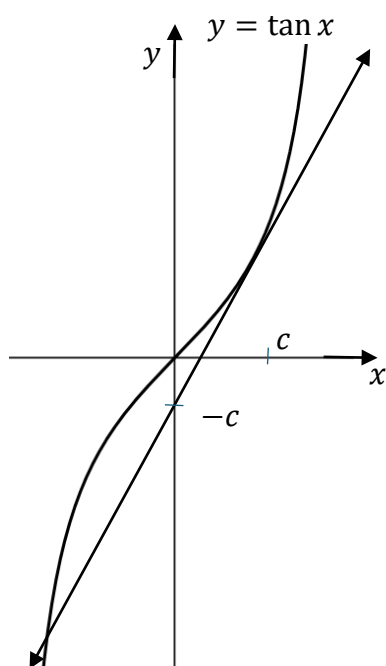
Question 25 continues on page 37

(b) For what values of k does $x^3 - 6x^2 + 9x - 3 = k$ have 3 solutions?

1

Question 26 (4 marks)

The tangent to $y = \tan x$ at the point where $x = c$, $c \in \left(0, \frac{\pi}{2}\right)$ is drawn on the diagram below. The value c is chosen so that the y intercept of the tangent is $-c$.



- (a) Show that the equation of the tangent is
 $y = x \sec^2 c - c \sec^2 c + \tan c$

3

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Question 26 continues on page 39

(b) Show that the gradient of this tangent can be simplified to $\frac{c^2+1}{c^2}$.

1

[illegible]

Question 27 (7 marks)

The population P of bin chickens on the main oval t days after school starts for the year

$$P(t) = 200 - \frac{44}{1 + 10e^{-0.2t}}$$

- (a) Show that the rate of growth of the size of the population is 2

$$P'(t) = \frac{-88e^{-0.2t}}{(1 + 10e^{-0.2t})^2}$$

- (b) Find the range of $P(t)$ by considering the value of P as $t \rightarrow \infty$. 2

- (c) Show that the rate of change of the population can be written as 2

$$P'(t) = \frac{1}{220}(P - 156)(P - 200)$$

Question 27 continues on page 41

Handwriting practice lines consisting of 20 horizontal dotted lines.

(d) Hence find the population of bin chickens when the population is decreasing at its fastest rate. 1

Handwriting practice lines consisting of 8 horizontal dotted lines.

End of paper 😊

Teacher Name: **Solution & Feedback**

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Section I

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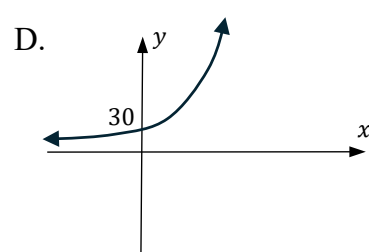
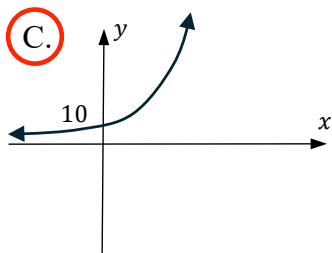
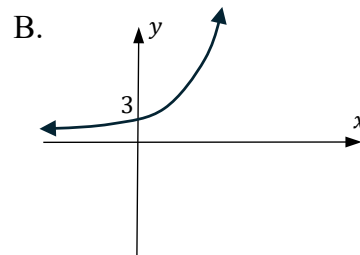
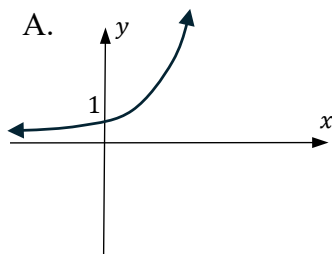
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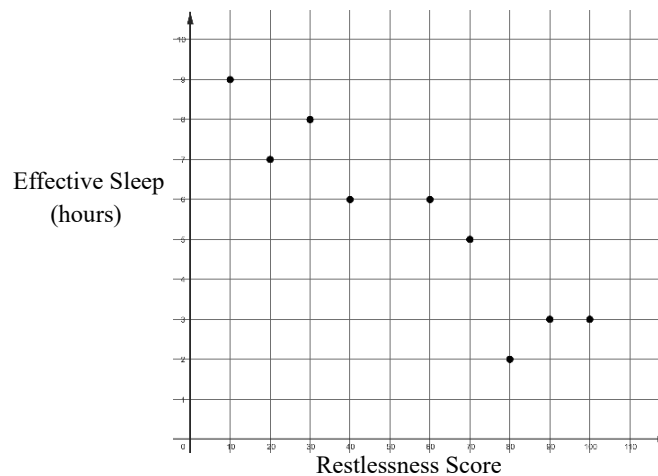
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Which of the following transformations could have achieved this?

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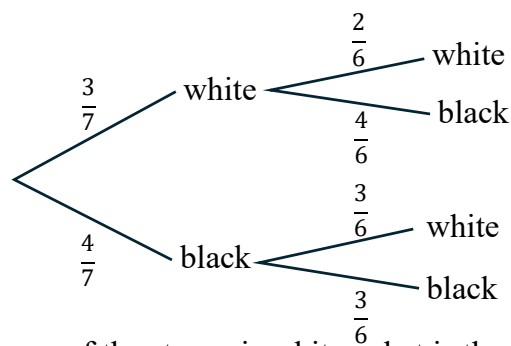


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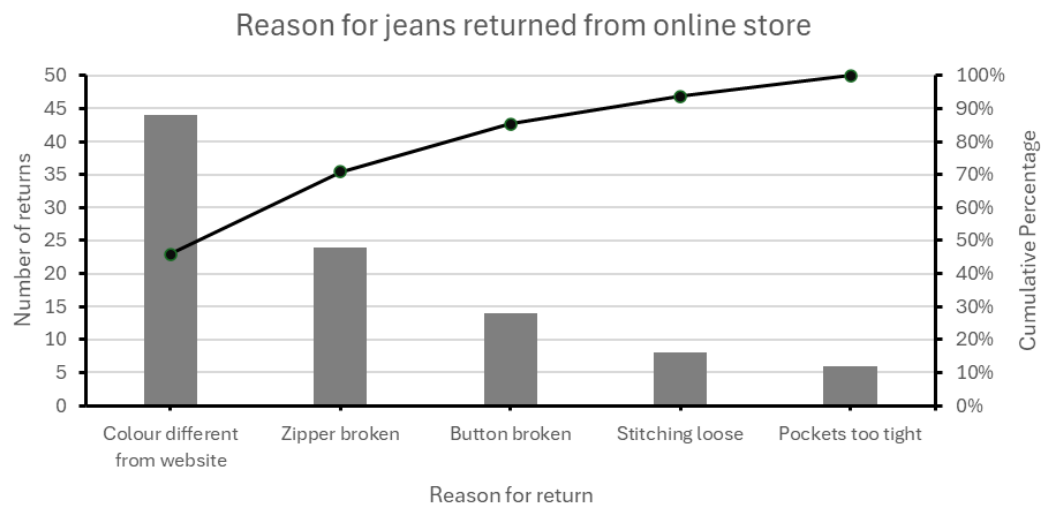
A. $\frac{2}{\log_c b}$

B. $\frac{\log_c b}{2}$

☒ C. $\frac{1}{\log_c b^2}$

D. $\frac{1}{\log_c \sqrt{b}}$

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The manager of online business wishes to address at least 80% of returns by making the fewest changes to company processes.

Which reasons for return should the manager address?

☒ A. Colour different from website, Zipper broken, Button broken

B. Colour different from website, Button broken, Stitching loose, Pockets too tight

C. Colour different from website

D. Button broken

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Which of the following is true for the function?

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Contribution per period for an annuity with a future value of \$1

Number of periods	Interest rate (% per period)					
	0.25%	0.5%	0.75%	1%	1.25%	1.5%
6	0.1656	0.1646	0.1636	0.1625	0.1615	0.1605
12	0.0822	0.0811	0.0800	0.0788	0.0778	0.0767
18	0.0544	0.0532	0.0521	0.0510	0.0499	0.0488
24	0.0405	0.0393	0.0382	0.0371	0.0360	0.0349
30	0.0321	0.0310	0.0298	0.0287	0.0277	0.0266
36	0.0266	0.0254	0.0243	0.0232	0.0222	0.0212

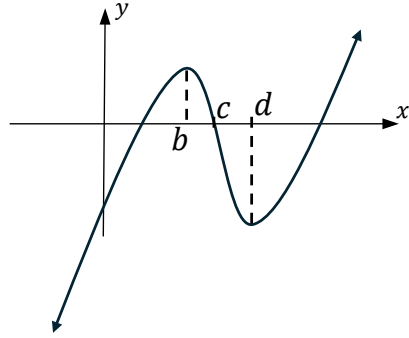
Gi-hun wants to have \$48,000 in 6 years time to pay off his university fees. He plans to make contributions every 3 months into an account with an interest rate of 3% pa. How much will Gi-hun need to deposit each quarter?

- ☒ A. \$1,833.60
- B. \$7,852.80
- C. \$7,900.80
- D. \$1,886.40

- 9 Consider the graph of $y = f(x)$.

At what value of x is $f''(x) > f'(x) > f(x)$?

- A. 0
- B. b
- C. c
- ☒ D. d



- 10 The formula for the n^{th} term of a series is given by

$$T_n = 400n - 2^n$$

What is the sum of the first 10 terms?

- A. 2 976
- B. 407 154
- C. -135 718
- ☒ D. 19 954

MARKING

Teacher Name: ...GUIDELINES.....

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2025

Penrith High School
**HSC TRIAL
EXAMINATION**

NESA Number

**Mathematics Advanced
Section II Answer Booklet 1**

Section II

90 marks

Attempt questions 11 – 27

Allow about 2 hours and 45 minutes for this section

Booklet 1 – Attempt questions 11 to 17 (32 marks)

Booklet 2 – Attempt questions 18 to 22 (29 marks)

Booklet 3 – Attempt questions 23 to 27 (29 marks)

Instructions

- Answer the questions in the space provided. Sufficient spaces are provided for typical responses
- Your responses should include relevant mathematical reasoning and/or calculations
- Extra writing space is provided at the back of the booklet. If you use this space, clearly indicate what question you are answering
- Write your NESA number above

	Functions	Trigonometric Functions	Statistical Analysis	Financial Mathematics	Calculus
Booklet 1		/5	/5	/9	/13
		12	14	13, 16	11, 15, 17

Please turn over

Question 11 (3 marks)

Evaluate

3

$$\int_2^4 \frac{2x^2 - 1}{x} dx$$

$$= \int_2^4 \frac{2x^2}{x} - \frac{1}{x} dx$$

$$= \int_2^4 2x - \frac{1}{x} dx \quad \checkmark \text{ split}$$

$$= \left[x^2 - \ln|x| \right]_2^4 \quad \checkmark \text{ antiderivative}$$

$$= (4^2 - \ln|4|) - (2^2 - \ln|2|)$$

$$= 12 - \ln 4 + \ln 2$$

$$= 12 - 2\ln 2 + \ln 2$$

$$= 12 - \ln 2$$

← mark paid
for unsimplified
logs in answer

~~unsimplified~~ simplified

answer

with single logarithm

(Allow $12 + \ln \frac{1}{2}$)

Question 12 (5 marks)

(a) Show that

3

$$\frac{1}{1 - \sin x} + \frac{1}{1 + \sin x} = 2\sec^2 x$$

$$\begin{aligned} \text{LHS} &= \frac{1 + \cancel{\sin x} + 1 - \cancel{\sin x}}{(1 - \sin x)(1 + \sin x)} && \checkmark \text{ common denominator} \\ &= \frac{2}{1 - \sin^2 x} && \checkmark \text{ simplify} \\ &= \frac{2}{\cos^2 x} && \checkmark \text{ Pythagoras} \\ &= 2\sec^2 x && \text{ \& finish} \\ &= \text{RHS} \quad \square && \text{ No penalty for LHS = RHS setting out.} \end{aligned}$$

(b) Hence, solve

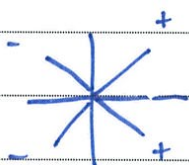
2

$$\frac{1}{1 - \sin x} + \frac{1}{1 + \sin x} = 8, \quad \text{for } 0^\circ \leq x \leq 360^\circ$$

$$\begin{aligned} 2\sec^2 x &= 8 && \text{from (a)} \\ \sec^2 x &= 4 && \checkmark \text{ using a to form equation} \\ \frac{1}{\cos^2 x} &= 4 \end{aligned}$$

$$\cos^2 x = \frac{1}{4}$$

$$\cos x = \pm \frac{1}{2}$$



$$x = 60^\circ, 120^\circ, 240^\circ, 300^\circ$$

$\checkmark$ 4 solutions correct

Question 13 (3 marks)

Find the sum of the terms in the arithmetic series:

3

$$71 + 64 + 57 + \dots + (-48)$$

$$a = 71$$

$$T_n = 71 + (n-1)(-7)$$

$$d = 64 - 71$$

$$= -7 \quad \checkmark \text{ find } d$$

$$-48 = 71 - 7n + 7$$

$$7n = 126$$

$$n = 18 \quad \checkmark \text{ Find } n$$

$$\text{i.e. } -48 = T_{18}$$

$$S_{18} = \frac{18}{2} (71 + (-48))$$

$$S_{18} = 207 \quad \checkmark \text{ answer}$$

Question 14 (5 marks)

The table below shows the distribution of a discrete random variable.

x	0	1	2	3
$P(X = x)$	0.22	0.66	$-0.1k$	k^2

- (a) Explain why the value of k is only -0.3 .

2

$$k^2 - 0.1k + 0.66 + 0.22 = 1 \quad \checkmark \quad (\text{Sum of probabilities is 1})$$

$$k^2 - 0.1k - 0.12 = 0$$

$$(k - 0.4)(k + 0.3) = 0$$

$$k = 0.4 \text{ or } k = -0.3$$

$$\text{but } P(X=2) \geq 0 \quad \text{i.e. } -0.1k \geq 0 \quad \downarrow \text{ explain}$$

$$k \leq 0$$

$$\therefore k = -0.3 \text{ only.}$$

- (b) Find the expected value $E(X)$.

1

$$E(X) = 0 \times 0.22 + 1 \times 0.66 + 2 \times 0.03 + 3 \times (-0.3)^2$$

$$= 0.99 \quad \checkmark$$

- (c) Find the standard deviation, correct to one decimal place.

2

$$\begin{array}{ccccc} x^2 & 0 & 1 & 4 & 9 \\ P(X^2) & 0.22 & 0.66 & 0.03 & 0.09 \end{array}$$

$$E(X^2) = 0 \times 0.22 + 1 \times 0.66 + 4 \times 0.03 + 9 \times 0.09 \quad \checkmark \text{ Find } E(X^2) \text{ or } E[(X-\mu)^2]$$

$$= 1.59$$

$$\text{Var}(X) = E(X^2) - [E(X)]^2$$

$$= 1.59 - (0.99)^2$$

$$= 0.6099$$

$\therefore$ standard deviation

$$\sigma_n(X) = \sqrt{0.6099}$$

is 0.8 (1 dec pl)

$$= 0.78096094... \quad \checkmark$$

Question 15 (5 marks)

A particle moves along the x axis with a displacement at time t seconds given by

$$x = \frac{32}{(t+1)^2} + t - 12$$

- (a) Where is the particle initially?

1

↓
 $t=0 \Rightarrow x = \frac{32}{(0+1)^2} + 0 - 12$

$$x = 20$$

$\therefore$ Initially 20m to right of origin.

- (b) Show that the particle comes to rest when $t = 3$.

2

$$v = \frac{dx}{dt} = (-2)(32)(1)(t+1)^{-3} + 1$$

✓ differentiate x

$$v = \frac{-64}{(t+1)^3} + 1$$

when at rest $v=0$

$$0 = \frac{-64}{(t+1)^3} + 1$$

$$(t+1)^3 = 64$$

$$t+1 = 4$$

$$t = 3$$

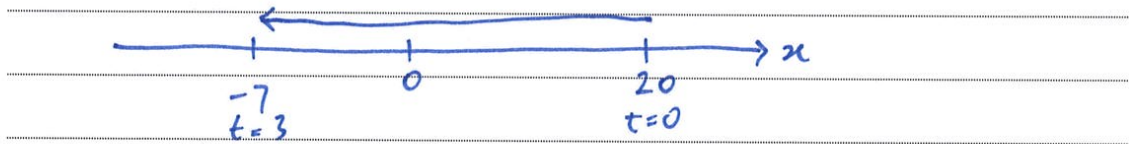
solve equal to zero
or sub in $t=3$
and show $v=0$

$\therefore$ Particle comes to rest when $t=3$.

Question 15 continues on page 15

- (c) Find the total distance the particle travels in the first 5 seconds.
Answer correct to one decimal place.

2



$$\text{At } t=3: x = \frac{32}{(3+1)^2} + 3 - 12 \quad a = \frac{dv}{dt} = -64(-3)(1)(t+1)^{-4}$$

$$x = -7$$

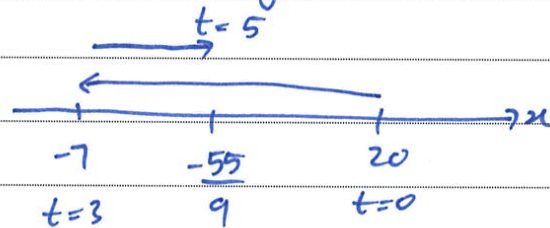
✓ location
at $t=3$ or $t=5$

$$\text{At } t=3: a = \frac{192}{4^4} > 0$$

$$\text{At } t=5: x = \frac{32}{(5+1)^2} + 5 - 12$$

$$= \frac{-55}{9}$$

∴ Particle moves to the right after $t=3$.



$$\therefore \text{Distance travelled} = 20 + 7 + \left(7 - \frac{55}{9}\right)$$

$$= 27.9 \text{ m (1 dec pl)} \quad \checkmark \text{ answer}$$

Many students
would benefit from
understanding the particle is
moving on the number line.

Question 16 (6 marks)

Elsa borrows \$720,000 from the Bank of Arendelle at an interest rate of 6% per annum. She makes a monthly repayment of \$ M . Let the amount of money owing directly after her n^{th} repayment be denoted by A_n .

- (a) Show that after her second repayment Elsa owes

1

$$A_2 = 720\,000(1.005)^2 - M(1.005 + 1)$$

$$\frac{6\%}{12} = 0.005$$

$$A_1 = 720\,000(1.005) - M$$

$$A_2 = A_1(1.005) - M$$

$$= (720\,000(1.005) - M)(1.005) - M$$

$$A_2 = 720\,000(1.005)^2 - M(1.005 + 1)$$

these steps
crucial to

show

- (b) If Elsa wants to repay the loan after 30 years find the value of her monthly repayment M to the nearest cent.

3

$$A_n = 720\,000(1.005)^n - M(1.005^{n-1} + \dots + 1.005 + 1)$$

Repaid after $30 \times 12 = 360$ months

$$\therefore A_{360} = 0$$

$$0 = 720\,000(1.005)^{360} - M \left(1 + \frac{(1.005)^{360} - 1}{1.005 - 1} \right)$$

$$M \left(\frac{1.005^{360} - 1}{0.005} \right) = 720\,000(1.005)^{360}$$

$$M = \frac{720\,000(1.005)^{360}(0.005)}{1.005^{360} - 1}$$

$$M = \$4,316.76 \quad (\text{nearest cent})$$

many students
used 30

✓ GP sum
formula

many students
used 359 here

Question 16 continues on page 17

- (c) Elsa instead decides to repay \$4,500 per month. How many months faster will she repay her loan? Answer to the nearest month. 2

Need to find n such that $A_n = 0$:

$$0 = 720\,000 (1.005)^n - 4500 \left(\frac{1.005^n - 1}{0.005} \right)$$

✓ forms equation

$$0 = 720\,000 (1.005)^n - 900\,000 (1.005^n - 1)$$

$$0 = 720\,000 (1.005)^n - 900\,000 (1.005)^n + 900\,000$$

$$180\,000 (1.005)^n = 900\,000$$

$$1.005^n = 5$$

$$\ln(1.005^n) = \ln 5$$

$$n \ln(1.005) = \ln 5$$

$$n = \frac{\ln 5}{\ln 1.005}$$

$$n = 322.691632...$$

awarded here

$$n = 322.691632...$$

She will pay off loan after 323 months (nearest month)

$$360 - 323 = 37 \text{ months faster}$$

✓ answer

Question 17 (5 marks)

(a) Differentiate $x^2 \ln x$

2

$$\frac{d}{dx} x^2 \ln x = x^2 \cdot \frac{1}{x} + 2x \ln x$$

$$= x + 2x \ln x$$

or
✓ for some attempt to use product rule

(b) Hence, evaluate

3

$$\int_1^e x \ln x \, dx$$

Many students did not understand to manipulate question to use part (a)

$$= \frac{1}{2} \int_1^e 2x \ln x \, dx$$

✓ one correct manipulation ($\frac{1}{2}$ or $-x$)

$$= \frac{1}{2} \int_1^e (2x \ln x + x) - x \, dx$$

$$= \frac{1}{2} \left[x^2 \ln x - \frac{x^2}{2} \right]_1^e \text{ from (a) } \checkmark \text{ used (a) as antiderivative}$$

$$= \frac{1}{2} \left[(e^2 \ln e - \frac{e^2}{2}) - (1 \ln 1 - \frac{1}{2}) \right]$$

$$= \frac{1}{2} \left[\frac{e^2}{2} + \frac{1}{2} \right] \checkmark$$

$$= \frac{e^2 + 1}{4}$$

Teacher Name:

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2025

Penrith High School
**HSC TRIAL
EXAMINATION**

NESA Number

**Mathematics Advanced
Section II Answer Booklet 2**

Booklet 2 – Attempt questions 18 to 22

Instructions

- Answer the questions in the space provided. Sufficient spaces are provided for typical responses
- Your responses should include relevant mathematical reasoning and/or calculations
- Extra writing space is provided at the back of the booklet. If you use this space, clearly indicate what question you are answering
- Write your NESA number above

Booklet 2	Functions	Trigonometric Functions	Statistical Analysis	Financial Mathematics	Calculus
	/4	/11	/10		/4
	20ab	19, 21	18,22		20c

Please turn over

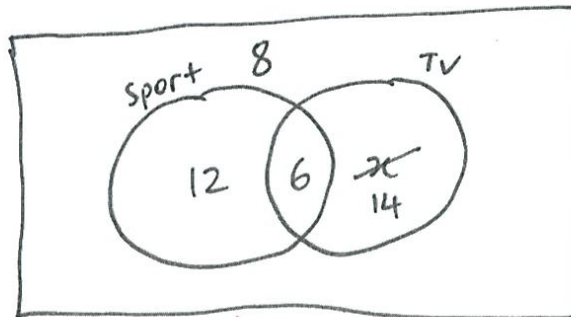
Question 18 (6 marks)

A group of 40 adults were surveyed as to whether they paid a subscription fee to watch sport or television series online.

The following answers were given:

- 12 adults only pay for sport
- 6 adults pay for both
- 8 adults pay for neither

- (a) By drawing a Venn diagram below find the probability that an adult surveyed pays for television shows. 2



1 mark

Some students did not draw the rectangle

$$x + 6 + 8 + 12 = 40$$

$$x = 14$$

$$P(\text{pay for TV}) = \frac{20}{40} = \frac{1}{2}$$

1 mark

- (b) An adult is chosen at random. If it is known that they pay for television series, what is the probability that they also pay for sport? 1

$$P(\text{sport} | \text{TV}) = \frac{6}{20}$$

$$= \frac{3}{10}$$

1 mark

Question 18 continues on page 23

This question was generally done well.

- (c) Two adults are chosen at random. What is the probability that they both pay the same types of subscription fees? Do not include the people that pay for neither. 3

$$P(\text{both TV only}) = \frac{14}{40} \times \frac{13}{39} = \frac{182}{1560}$$

$$P(\text{both sport only}) = \frac{12}{40} \times \frac{11}{39} = \frac{132}{1560}$$

$$P(\text{both TV and sport}) = \frac{6}{40} \times \frac{5}{39} = \frac{30}{1560}$$

$$P(\text{both pay same}) = \frac{182 + 132 + 30}{1560}$$

$$= \frac{43}{195}$$

— 1 mark

2 marks - correct
1 mark - some progress

- Some students forget to include people who pay neither in the total and

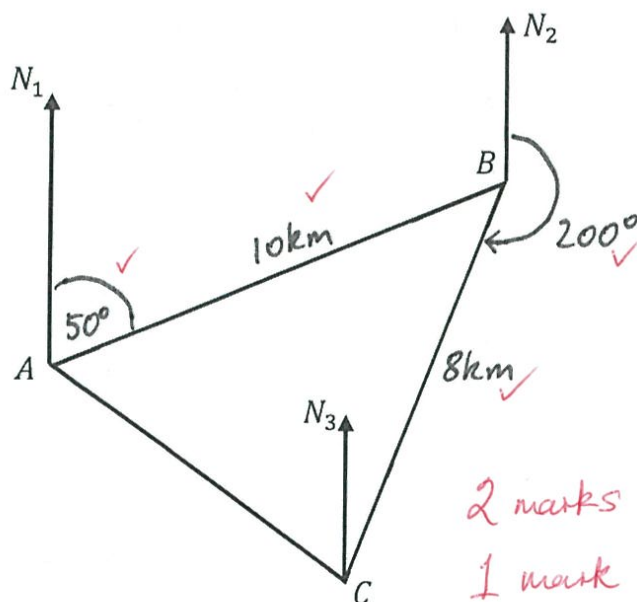
did $P(\text{both TV only}) = \frac{14}{32} \times \frac{13}{31}$, for example.

Question 19 (6 marks)

An expedition leaves camp A and walks 10km on a bearing of $050^\circ T$ to camp B. They then walk 8km on a bearing of $200^\circ T$ until they reach camp C.

- (a) Add the information from the question to the diagram below.

2



2 marks $\rightarrow$ 3 or 4 correct.
1 mark $\rightarrow$ 1 or 2 correct.

- (b) How far to the nearest km do they need to walk from camp C to camp A?

2

$$\angle ABN_2 = 180^\circ - 50^\circ = 130^\circ$$

$$\angle ABC = 360^\circ - 200^\circ - 130^\circ = 30^\circ$$

$$AC^2 = 10^2 + 8^2 - 2(10)(8) \cos 30^\circ \frac{\sqrt{3}}{2}$$

$$AC = \sqrt{164 - 80\sqrt{3}} \quad (\because AC > 0)$$

$$AC = 5 \text{ km (nearest km)}$$

2 marks $\rightarrow$ Correct answer rounded to nearest km.

1 mark $\rightarrow$ correctly substituting into the cosine rule.

Question 19 continues on page 25

Generally done well.

- (c) By using the cosine rule to first find $\angle ACB$, find the bearing they need to walk on from camp C to reach camp A. 2

If using $AC = 5$:

If using $AC = \sqrt{164 - 80\sqrt{3}}$

$$\cos \angle ACB = \frac{5^2 + 8^2 - 10^2}{2(5)(8)}$$

$$\cos \angle ACB = \frac{164 - 80\sqrt{3} + 8^2 - 10^2}{2(\sqrt{164 - 80\sqrt{3}})(8)}$$

$$\angle ACB = 97.9032077...^\circ$$

$$\angle ACB = 97.5224315...^\circ$$

$$\angle ACB = 97^\circ 54' \text{ (nearest min)}$$

$$\angle ACB = 97^\circ 31' \text{ (nearest min)}$$

$$\angle CBN_2 = 360^\circ - 200^\circ = 160^\circ$$

$$\angle BCN_3 = 180^\circ - 160^\circ = 20^\circ$$

$$\angle ACN_3 = \angle ACB - 20^\circ$$

$$\begin{aligned} \therefore \text{Bearing is } 360^\circ - \angle ACN_3 \\ = 360^\circ - (\angle ACB - 20^\circ) \\ = 380^\circ - \angle ACB \end{aligned}$$

$$= 282^\circ 6' \text{ T or } 282^\circ \text{ T or } 282^\circ 29' \text{ T}$$

2 marks $\rightarrow$ correct Answer.

1 mark $\rightarrow$ Some minor errors.

Question 20 (8 marks)

(a) Solve $|x| = \frac{1}{2}x + 3$

2

$$x = \frac{1}{2}x + 3 \quad \text{or} \quad -x = \frac{1}{2}x + 3$$

$$2x = x + 6$$

$$-2x = x + 6$$

$$x = 6$$

$$-3x = 6$$

$$x = -2$$

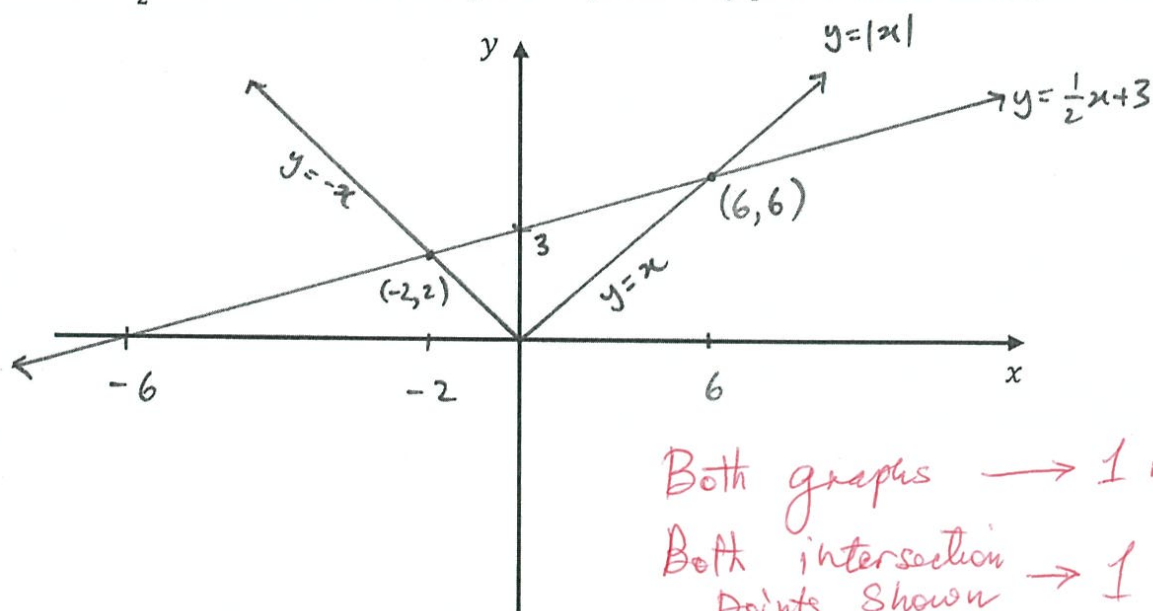
$$\therefore x = -2, 6$$

1 mark for each correct solution.

Generally done well.

- (b) On the axes below draw a neat sketch showing the graphs of $y = |x|$ and $y = \frac{1}{2}x + 3$. Show the x and y intercepts and any points of intersection.

2



Both graphs $\rightarrow$ 1 marks.
Both intersection points shown $\rightarrow$ 1 mark.

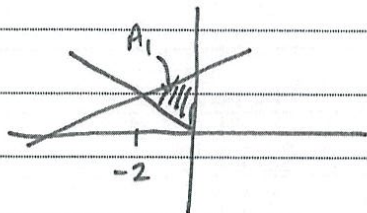
Question 20 continues on page 27

Some students drew the graph but did not extend it enough so that they intersected

- (c) An area between $y = |x|$ and $y = \frac{1}{2}x + 3$ is bounded by $x = -2$ and $x = k$, where $k > 0$.

4

Find the value of k such that the area is 10 square units.



$$A_1 = \frac{1}{2} \times b \times h$$

$$= \frac{1}{2} \times 3 \times 2$$

$$= 3$$

OR

$$A_1 = \int_{-2}^0 \left(\frac{1}{2}x + 3 - (-x) \right) dx$$

$$= \int_{-2}^0 \left(\frac{3}{2}x + 3 \right) dx$$

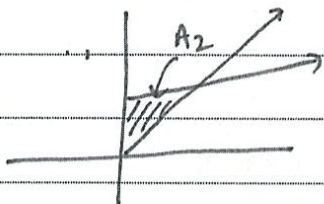
$$= \left[\frac{3x^2}{4} + 3x \right]_{-2}^0$$

$$= 0 - \left(\frac{3(-2)^2}{4} + 3(-2) \right)$$

$$= 3$$

1 mark.

$\therefore$ Need $7u^2$ more.



$$A_2 = \frac{1}{2} \times b \times h$$

$$= \frac{1}{2} \times 3 \times 6$$

$$= 9$$

$$\therefore k < 6$$

$$7 = \int_0^k \left(\frac{1}{2}x + 3 - x \right) dx$$

1 mark.

$$7 = \int_0^k \left(-\frac{1}{2}x + 3 \right) dx$$

But $k < 6$

$$7 = \left[-\frac{1}{4}x^2 + 3x \right]_0^k$$

$$7 = -\frac{1}{4}k^2 + 3k - 0$$

$$k^2 - 12k + 28 = 0$$

1 mark.

$$k^2 - 12k + 36 = 8$$

$$(k-6)^2 = 8$$

$$k-6 = \pm 2\sqrt{2}$$

$$k = 6 \pm 2\sqrt{2}$$

$\therefore k = 6 - 2\sqrt{2}$ only 1 mark.

Poorly done. Many students tried to do this question as a single integral

$$\int_{-2}^k \left(\frac{1}{2}x + 3 - |x| \right) dx.$$

Question 21 (5 marks)

The water depth d in Penrith harbour t hours after the first low tide of the day can be modelled with the equation:

$$d = 5.2 - 0.8 \cos\left(\frac{4\pi}{25}t\right)$$

- (a) What is the depth of the water at low tide?

1

if $t=0$ $d = 5.2 - 0.8$
 (first low tide) $= 4.4 \text{ m}$

- (b) If the first low tide of the day occurred at 6am, what time does the next low tide occur at?

2

Period = $\frac{2\pi}{\frac{4\pi}{25}} = 12.5$

$\therefore$ Next low tide at 6:30 pm

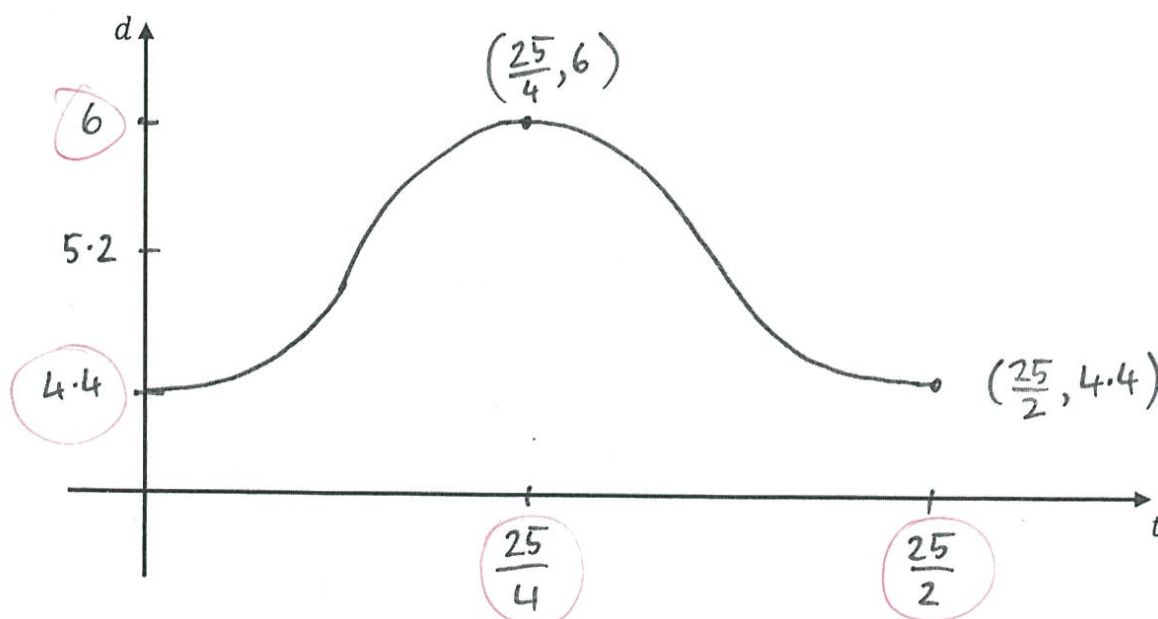
OR

$4.4 = 5.2 - 0.8 \cos \frac{4\pi t}{25}$
 $-0.8 = -0.8 \cos \frac{4\pi t}{25}$
 $\cos \frac{4\pi t}{25} = 1$
 $\frac{4\pi t}{25} = 0, 2\pi$
 $t = 0, 12.5$
 6:30 pm

2 marks - Correct
 1 mark - Minor errors

- (c) Draw a neat sketch of the depth of the water in the harbour from the first low tide until the second low tide of the day.

2



2 marks — All points clearly shown
 1 mark — minor errors
 Many students had the correct points but the graph wasn't very neat.

Question 22 (4 marks)

Gout is training for the big 100m race at the end of the year. He is keeping track of the total number of hours that he has trained for and the times in seconds for his 100m, and recorded this data in the following table:

Hours (h)	3.7	5.6	8.2	10.1	11.1	14.8	17.6	20.8	22.5
Time (t)	14.2	13.9	13.7	13.3	13.4	12.9	12.8	12.8	12.4

- (a) Find the equation of the regression line for Time t as a function of Hours h . 2

$$t = 14.37 - 0.09h \quad (\text{values to 2 dec pl})$$

2 marks $\rightarrow$ Correct answer.

1 mark $\rightarrow$ Some errors.

- (b) Using the equation in (a), how many hours of training would Gout need to complete to run the 100m in 11 seconds? 1

$$t = 11 \Rightarrow 11 = 14.37 - 0.09h$$

$$0.09h = 3.37$$

$$h = 37.444 \dots$$

$$h = 37.4 \text{ (one dec. pl.)}$$

generally well done.

- (c) Why is your answer to part (b) not reliable? 1

A time of 100m is an extrapolation from the dataset. It is not guaranteed that the relationship identified in (a) will still apply to values $t < 12.4$.

A number of other explanations were provided by students but mark was only awarded for the above explanation (involving extrapolation).

Teacher Name:

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2025

Penrith High School
**HSC TRIAL
EXAMINATION**

NESA Number

**Mathematics Advanced
Section II Answer Booklet 3**

Booklet 3 – Attempt questions 23 to 27

Instructions

- Answer the questions in the space provided. Sufficient spaces are provided for typical responses
- Your responses should include relevant mathematical reasoning and/or calculations
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- Write your NESA number above

	Functions	Trigonometric functions	Statistical Analysis	Financial Mathematics	Calculus
Booklet 3	/4				/26
	23				24,25,26,27

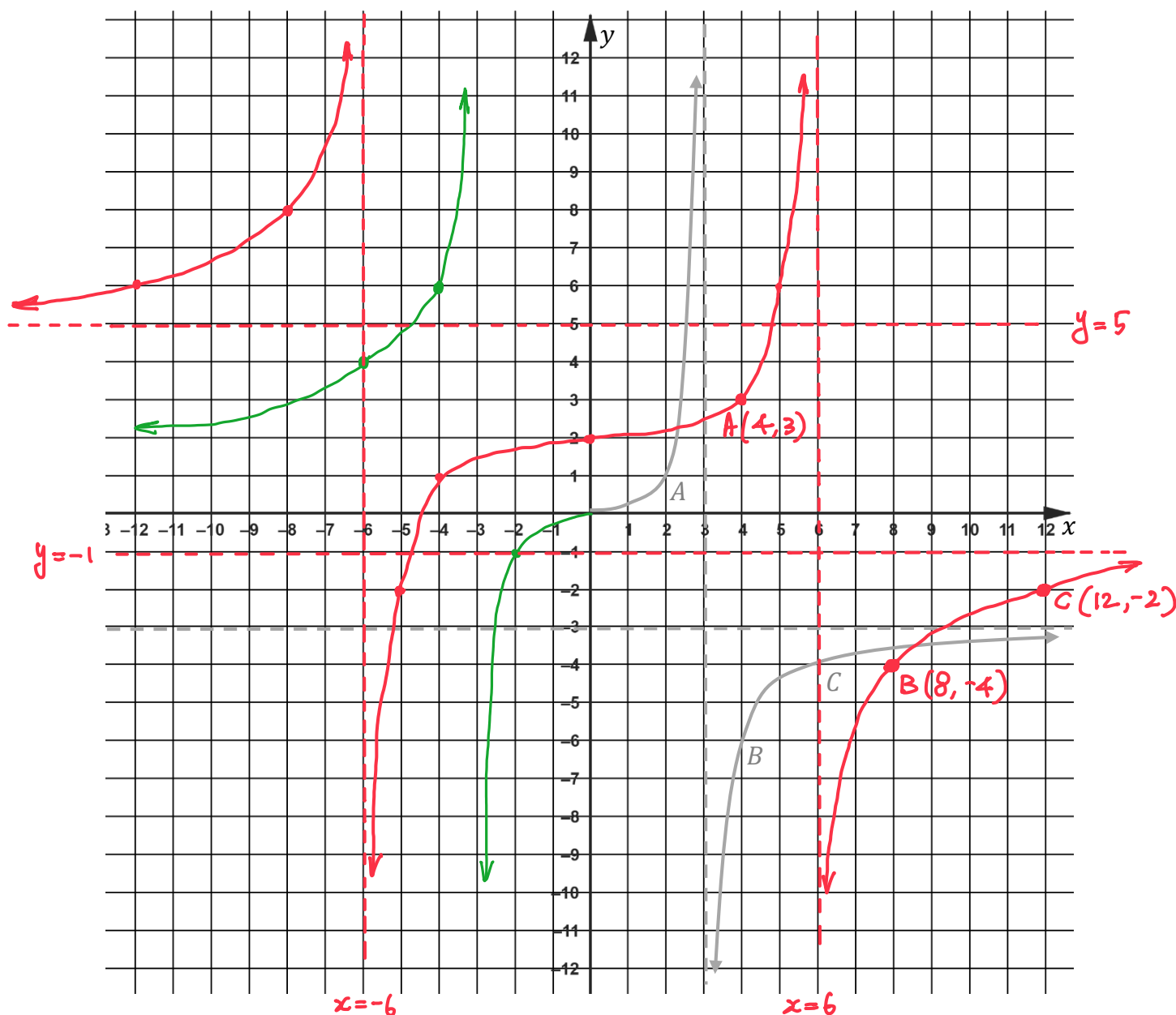
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Question 23 (4 marks)

The part of the graph of the odd function $y = f(x)$ where $x \geq 0$ has been revealed on the graph below. The graph of $y = f(x)$ passes through $A(2,1)$, $B(4,-6)$ and $C(6,-4)$. 4

The function h is defined as $h(x) = f\left(\frac{x}{2}\right) + 2$ with $f(x)$ defined for $-\infty \leq x \leq \infty$.

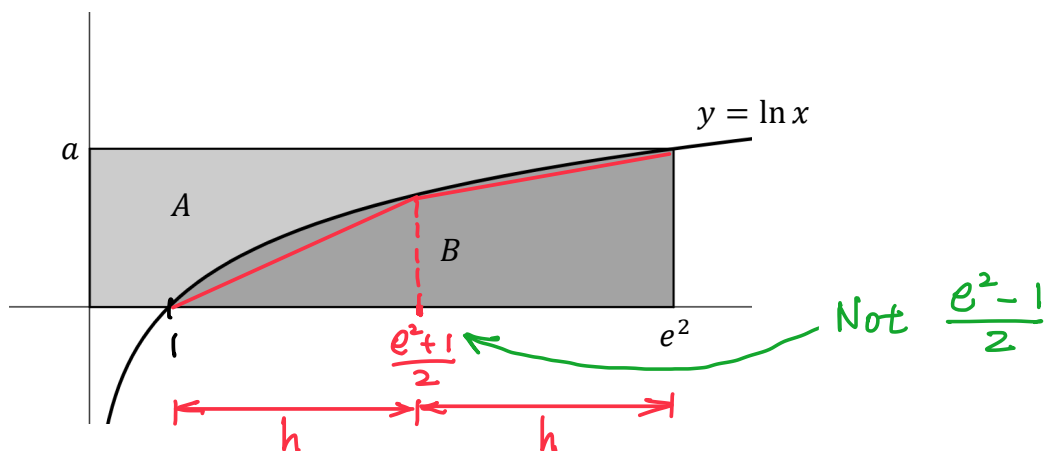
Draw the complete graph of $y = h(x)$ on the same axes over its entire domain, clearly showing the new locations of the points A , B and C as well as any asymptotes.



* A large number of student graphed only on the right half of the number plane, leaving the other side blank.

Question 24 (7 marks)

Aneeka and Brian are investigating areas A and B respectively on either side of the graph of $y = \ln x$ as shown on the diagram below:



- (a) Find the value of a .

1

$$a = \ln e^2$$

$$= 2$$

Poorly done

- (b) Brian uses the trapezoidal rule with 3 function values to estimate the area B . He approximates all of the values used in his working out to 2 decimal places where necessary.

$\rightarrow n = 2$ (Two trapeziums), not 3.

2

What is his estimate for the area of B ?

$$h = \frac{e^2 - 1}{2}$$

x	1	$\frac{e^2 + 1}{2}$	e^2
y	0	1.43	2

$$\text{Area} \doteq \frac{e^2 - 1}{2(2)} (0 + 1.43) + \frac{e^2 - 1}{2(2)} (1.43 + 2) \quad \textcircled{1}$$

$$= 7.77 \text{ (2 d.p.)} \quad \textcircled{1}$$

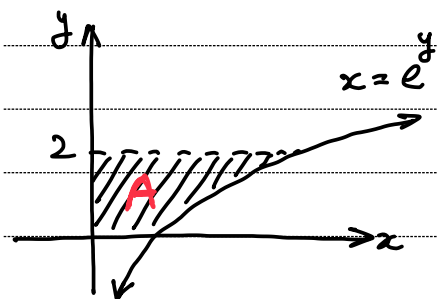
Alternative:

$$\text{Area} \doteq \frac{e^2 - 1}{2(2)} (0 + 2 \times 1.43 + 2)$$

$$= 7.77 \text{ (2 d.p.)}$$

Question 24 continues on page 34

- (c) Aneeka evaluates the area A by using an integral against the y axis. What exact value does she get for the area A ? 2



Common mistakes

* $\int_0^2 e^x dx = [e^x]_0^2$

* $e^2 - e^0 = e^2$

$$\text{Area} = \int_0^2 e^y dy$$

$$= [e^y]_0^2$$

$$= e^2 - e^0$$

$$= e^2 - 1$$

Question 24 continues on page 35

Done
Well

- (d) Brian considers the area $A + B$ and uses his estimate of the area B from part (b) and Aneeka's answer in part (c) to find an approximation for the value of e . 2

What is his approximation to 2 decimal places?

$$\text{Rectangle} \doteq A + B$$

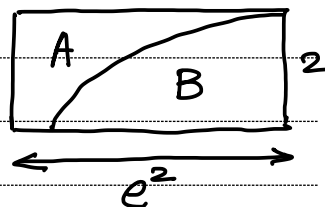
$$e^2 \times 2 \doteq (e^2 - 1) + 7.77$$

$$2e^2 \doteq e^2 - 1 + 7.77$$

$$e^2 \doteq 6.77$$

$$e \doteq \sqrt{6.77} \quad (\text{since } e > 0)$$

$$e \doteq 2.60$$



$$* 2e^2 \doteq e^2 - 1 + 7.77$$

$$2e^2 \doteq 14.159$$

$\vdots$

No calculator for e .

Done
Well

- (e) Why is Brian's approximate value for e an underestimate? 1

Since the graph of $y = \ln x$ is concave down, the trapezoidal rule gives an underestimate of the area below the curve.

Hence, the estimation of e is an underestimation.

Question 25 (7 marks)

- (a) Draw a neat sketch of $y = x^3 - 6x^2 + 9x - 3$ on page 37 showing the y intercept, all turning points and all points of inflexion. You need to determine the nature of any stationary points that you find. You do not need to find the x intercepts. 6

For y -int, Sub $x=0$,

$$y = -3$$

$$y' = 3x^2 - 12x + 9$$

For stationary points, sub $y' = 0$,

$$3x^2 - 12x + 9 = 0$$

$$x^2 - 4x + 3 = 0$$

$$(x-3)(x-1) = 0$$

$$x = 3 \text{ or } x = 1$$

$$y = -3$$

$$y = 1$$

(2)

$$y'' = 6x - 12$$

* Not 'local max' or 'max'

When $x = 3$, $y'' = 6(3) - 12 = 6 > 0$ $\cup$ Must show.

$\therefore$ Minimum turning point at $(3, -3)$

When $x = 1$, $y'' = 6(1) - 12 = -6 < 0$ $\cap$

$\therefore$ Maximum turning point at $(1, 1)$ (1)

For possible inflection points, sub. $y'' = 0$,

$$6x - 12 = 0$$

$$x = 2$$

$$y = -1$$

Check for concavity,

x	1	2	3
y''	-6	0	6
	-		+

Must show

$\therefore$ Point of inflection at $(2, -1)$ (1)

Question 25 continues on page 37

(b) For what values of k does $x^3 - 6x^2 + 9x - 3 = k$ have 3 solutions?

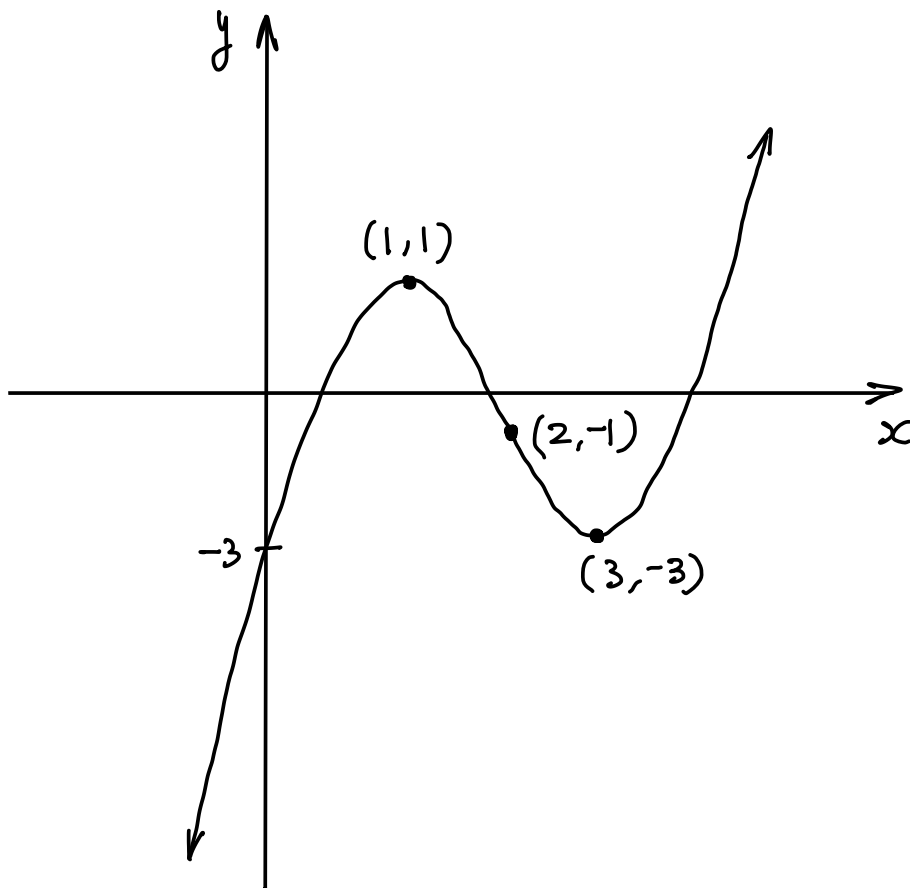
1

$$-3 < k < 1$$

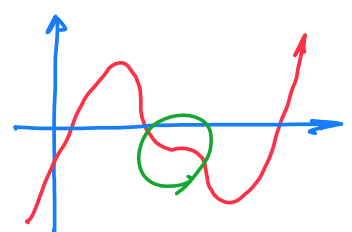
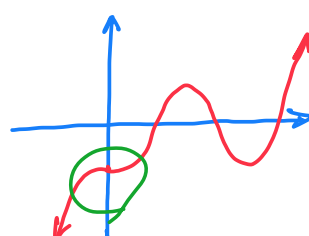
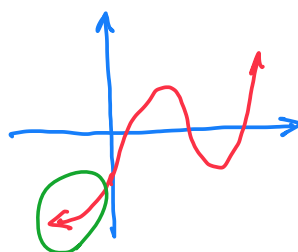
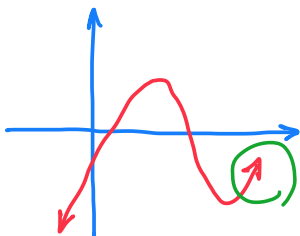
Common mistake

$$-3 \leq k \leq 1$$

Only two solution when $k = -3$ or 1 .

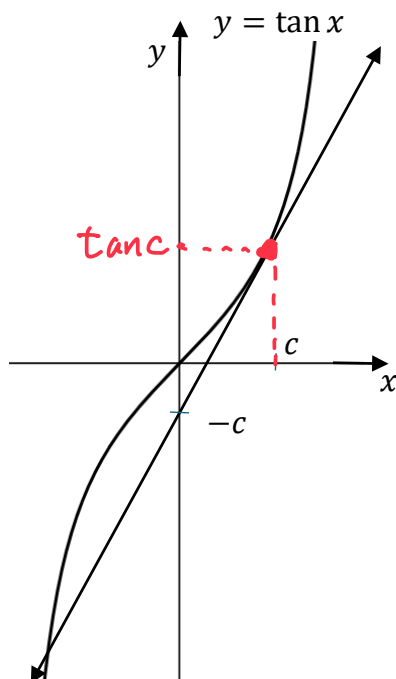


Common mistakes



Question 26 (4 marks)

The tangent to $y = \tan x$ at the point where $x = c$, $c \in \left(0, \frac{\pi}{2}\right)$ is drawn on the diagram below. The value c is chosen so that the y intercept of the tangent is $-c$.



Done
well

- (a) Show that the equation of the tangent is
 $y = x \sec^2 c - c \sec^2 c + \tan c$

3

$$y' = \sec^2 x$$

When $x=c$, $y = \tan c$, ①

$y' = \sec^2 c$ ① $m_T \neq \sec^2 x$

Equation of tangent,

$$y - y_1 = m(x - x_1)$$

$$y - \tan c = \sec^2 c (x - c) \quad \text{①}$$

$$y = x \sec^2 c - c \sec^2 c + \tan c$$

Question 26 continues on page 39

Poorly
done

(b) Show that the gradient of this tangent can be simplified to $\frac{c^2+1}{c^2}$.

1

Tangent passes through $(0, -c)$

$$y = x \sec^2 c - c \sec^2 c + \tan c$$

$$-c = -c \sec^2 c + \tan c$$

$$-c = -c(1 + \tan^2 c) + \tan c$$

$$-c = -c - c \tan^2 c + \tan c$$

$$c \tan^2 c - \tan c = 0$$

$$\tan c (c \tan c - 1) = 0$$

$$\tan c = 0 \quad \text{or} \quad c \tan c - 1 = 0$$

$$c = 0$$

Not possible

Since $c \in (0, \frac{\pi}{2})$

$$\tan c = \frac{1}{c} \quad [1]$$

$$\tan^2 c = \frac{1}{c^2}$$

$$1 + \tan^2 c = \frac{1}{c^2} + 1$$

$$\sec^2 c = \frac{1 + c^2}{c^2}$$

$$\therefore \text{Gradient} = \frac{c^2 + 1}{c^2}$$

Alternative 2:

$$m = \frac{\tan c + c}{c - 0}$$

$$\sec^2 c = \frac{\tan c + c}{c}$$

$$1 + \tan^2 c = \frac{\tan c + c}{c}$$

$$c + c \tan^2 c = \tan c + c$$

$$c \tan^2 c - \tan c = 0$$

$$\tan c (c \tan c - 1) = 0$$

∴

Alternative:

$(0, -c), (c, \tan c)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{\tan c + c}{c - 0}$$

$$= \frac{\frac{1}{c} + c}{c} \quad (\text{Since } \tan c = \frac{1}{c})$$

$$= \frac{1 + c^2}{c} \times \frac{1}{c}$$

$$= \frac{1 + c^2}{c^2}$$

Question 27 (7 marks)

The population P of bin chickens on the main oval t days after school starts for the year

$$P(t) = 200 - \frac{44}{1 + 10e^{-0.2t}}$$

- (a) Show that the rate of growth of the size of the population is

2

$$P'(t) = \frac{-88e^{-0.2t}}{(1 + 10e^{-0.2t})^2}$$

$$P(t) = 200 - 44(1 + 10e^{-0.2t})^{-1}$$

$$P'(t) = 44(-2e^{-0.2t})(1 + 10e^{-0.2t})^{-2} \quad (\text{chain rule})$$

$$= \frac{-88e^{-0.2t}}{(1 + 10e^{-0.2t})^2}$$

* A large number of students used the quotient rule.

- (b) Find the range of $P(t)$ by considering the value of P as $t \rightarrow \infty$.

2

Since $P'(t) < 0$ for all $t \geq 0$, $P(t)$ is decreasing function.
ie. $P(t)$ is max. when $t=0$.

$$P(0) = 200 - \frac{44}{1 + 10e^0} = 196 \quad \leftarrow \text{Show steps for } P=196$$

$$\text{As } t \rightarrow \infty, e^{-0.2t} \rightarrow 0 \quad \therefore P(t) \rightarrow 200 - 44 = 156$$

$$\therefore 156 < P(t) \leq 196$$

(1)

(1)

* $P(t) \neq 156$

* $[196, 156) ???$

- (c) Show that the rate of change of the population can be written as

2

$$P'(t) = \frac{1}{220}(P - 156)(P - 200)$$

$$P'(t) = \frac{-88e^{-0.2t}}{(1 + 10e^{-0.2t})^2} \quad (\text{from (a)})$$

$$= \frac{2e^{-0.2t}}{1 + 10e^{-0.2t}} \times \frac{-44}{1 + 10e^{-0.2t}}$$

$$= \frac{2e^{-0.2t}}{1 + 10e^{-0.2t}} \times (P - 200) \quad \left(\text{Since } P = 200 - \frac{44}{1 + 10e^{-0.2t}} \right)$$

$$= \frac{1}{220} \left(\frac{440e^{-0.2t}}{1 + 10e^{-0.2t}} \right) (P - 200)$$

$$\begin{aligned}
&= \frac{1}{220} (P-200) \left(\frac{44 + 440e^{-0.2t} - 44}{1 + 10e^{-0.2t}} \right) \\
&= \frac{1}{220} (P-200) \left(\frac{44(1+10e^{-0.2t})}{1+10e^{-0.2t}} - \frac{44}{1+10e^{-0.2t}} \right) \\
&= \frac{1}{220} (P-200) \left(44 - \frac{44}{1+10e^{-0.2t}} \right) \\
&= \frac{1}{220} (P-200) \left(200 - \frac{44}{1+10e^{-0.2t}} - 156 \right) \\
&= \frac{1}{220} (P-200) (P-156)
\end{aligned}$$

* If $P = 200 - \frac{44}{1+10e^{-0.2t}}$ is substituted into $P'(t) = \frac{1}{220} (P-156)(P-200)$, awarded 2 marks only if all steps are completely correct; otherwise, awarded zero.
Must not manipulate the equation that needs to be proven.

- (d) Hence find the population of bin chickens when the population is decreasing at its fastest rate. 1

$$P'(t) = \frac{1}{220} (P-156)(P-200)$$

This is a parabola with P -intercepts of $(156, 0)$ and $(200, 0)$.

Since it is concave up, the minimum value of $P'(t)$ occurs at $P = \frac{156 + 200}{2} = 178$.

$\therefore$ The population is decreasing at the fastest rate when $P = 178$.

End of paper 😊