

Name:.....

Student Number:

Teacher's Name:.....



Pymble Ladies' College

Mathematics Advanced HSC Trial Examination Term 3 2024

**General
Instructions**

- Reading time – 10 minutes
- Working time – 3 hours
- Write using non-erasable black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning or calculations

**Total marks
100****Section 1 – 10 marks** (pages 1-4)

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 6-33)

- Attempt Questions 11-35
- Allow about 2 hours and 45 minutes for this section

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Mathematics Advanced

HSC Trial Examination

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Section I – Multiple Choice Answer Sheet

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ○ B  C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

A  B  C  D 

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|----|-------------------------|-------------------------|-------------------------|-------------------------|
| 1 | A ☐ | B ☐ | C ☐ | D ☐ |
| 2 | A ☐ | B ☐ | C ☐ | D ☐ |
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| 10 | A ☐ | B ☐ | C ☐ | D ☐ |

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SECTION I

10 marks

Attempt Questions 1-10

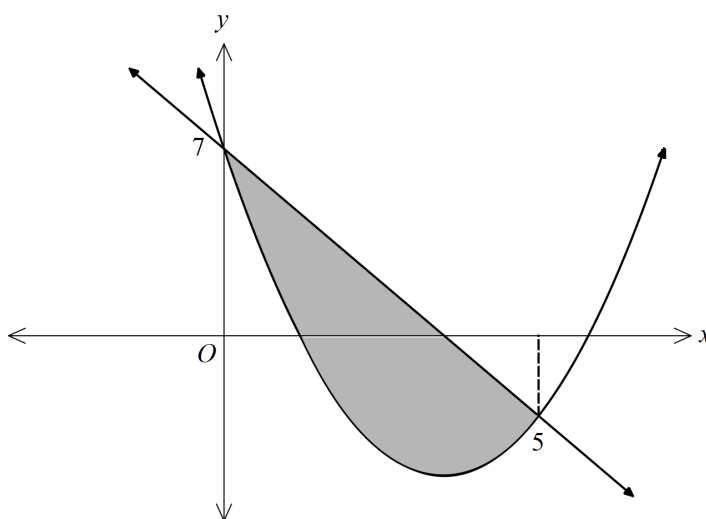
Allow about 15 minutes for this section

Use the Multiple-Choice Answer sheet for Questions 1-10

- 1 A circle has centre $(4, -5)$ and radius 6. What is the equation of the circle?

- (A) $(x-4)^2 + (y+5)^2 = 6$
(B) $(x+4)^2 + (y-5)^2 = 6$
(C) $(x-4)^2 + (y+5)^2 = 36$
(D) $(x+4)^2 + (y-5)^2 = 36$

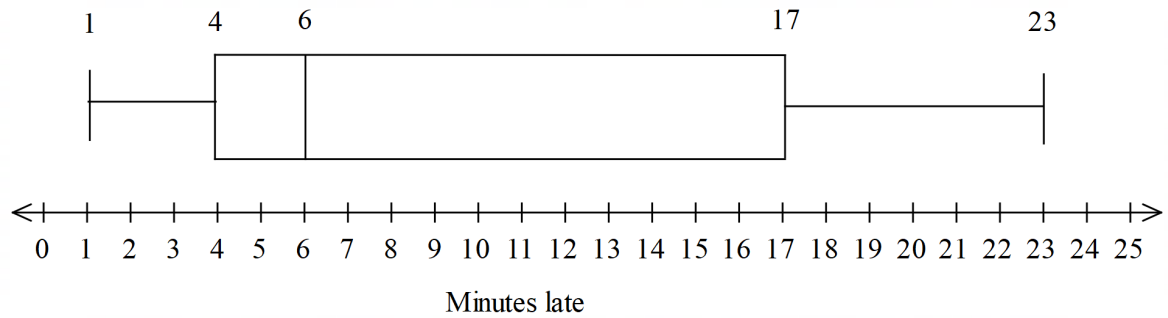
- 2 The shaded region, shown in the diagram below, is enclosed by the parabola $y = x^2 - 7x + 7$ and the line $y = 7 - 2x$.



Which of the following gives the area of the shaded region?

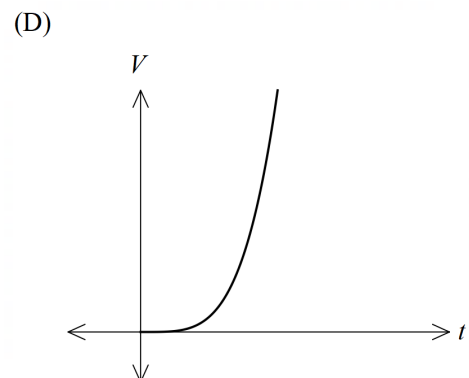
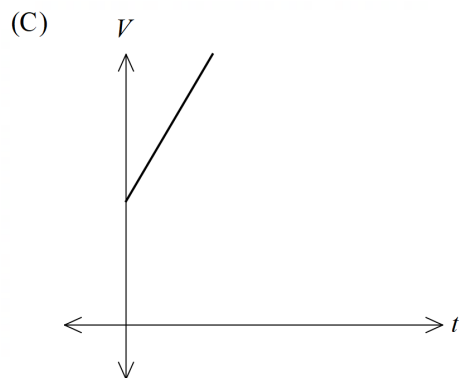
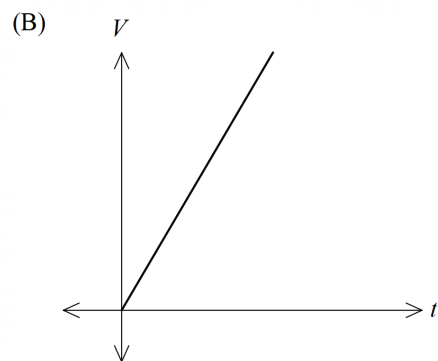
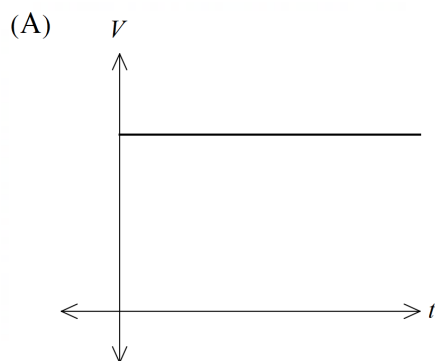
- (A) $\int (7 - 2x) dx - \int (x^2 - 7x + 7) dx$
(B) $\int_0^5 (x^2 - 5x) dx$
(C) $\int_0^5 (5x - x^2) dx$
(D) $\int_0^5 (x^2 - 9x + 14) dx$

- 3 The box plot below shows summary data for the number of minutes that buses arrived late at a rural bus stop.



Which term best describes the distribution of this data?

- (A) negatively skewed
 - (B) normal
 - (C) positively skewed
 - (D) symmetrical
- 4 A car, initially at rest, moves with constant acceleration along a straight horizontal road. Which of the graphs below shows how the car's velocity, V m/s, changes over time, t seconds?



- 5 Given that $\cos(\theta - 20^\circ) = \cos 60^\circ$, which of the following is a possible value for θ ?

(A) 40°
(B) 140°
(C) 280°
(D) 320°

- 6 A function f is given by the rule

$$f(x) = \begin{cases} (x-3)^2 + 2, & x \leq 3 \\ 3x - 7, & x > 3 \end{cases}$$

What is the value of $f(f(0))$?

(A) 11
(B) -7
(C) 26
(D) 66

- 7 The z -scores on a test are normally distributed, such that $P(-N < z < N) = p$ for some constants $N > 0$ and $0 < p < 1$. What is the value of $P(z < N)$?

(A) $\frac{1-p}{2}$
(B) $1 - \frac{p}{2}$
(C) $\frac{1+p}{2}$
(D) $\frac{1+2p}{3}$

8 What is $\int (\sin^2 x + \cos^2 x + \tan^2 x) dx$?

- (A) $\tan x + C$
- (B) $\cot x + C$
- (C) $x + \sec x + C$
- (D) $x - \ln(\cos^2 x) + C$

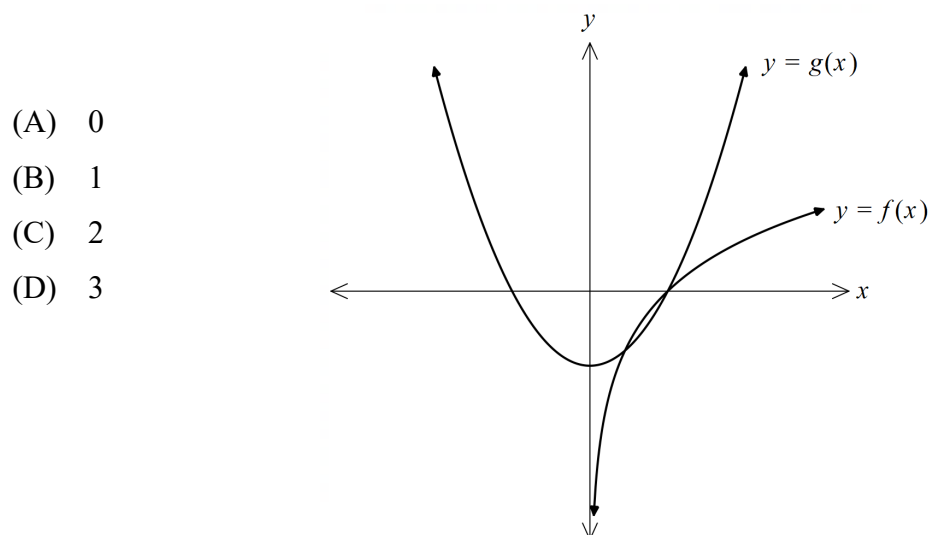
9 In a bag of n coloured discs, exactly 7 are red. Two discs are selected at random, without replacement. The probability that both discs are red is $\frac{21}{55}$.

Which quadratic equation could be used to find the value of n ?

- (A) $n^2 + n + 110 = 0$
- (B) $n^2 - n - 110 = 0$
- (C) $n^2 + n - 110 = 0$
- (D) $n^2 - n + 110 = 0$

10 The graph shows $y = f(x)$ and $y = g(x)$, where $f(x) = \ln x$ and $g(x) = x^2 - 1$.

How many solutions does the equation $[f(x)]^2 - [g(x)]^2 = 0$ have?



End of Section I

Name.....

Student
Number

Teacher's Name.....

Mathematics Advanced

Section II Answer Booklet

90 marks

Attempt Questions 11-35

Allow about 2 hours and 45 minutes for this section

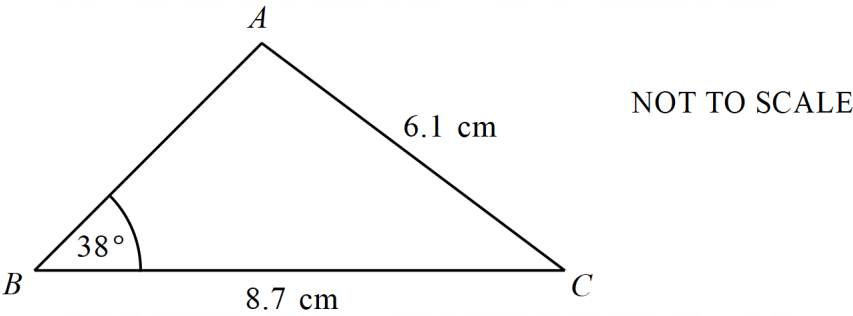
Instructions

- Answer in the spaces provided. These spaces provide guidance for the expected length of the answer.
- Extra writing space is provided at the end of the paper. If you use the additional space, clearly indicate which question you are answering.
- Your responses should include relevant mathematical reasoning and/or calculations.

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Question 11 (2 marks)

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The diagram shows a triangle ABC . AB is the shortest side. The lengths of AC and BC are 6.1 cm and 8.7 cm respectively. The size of angle ABC is 38° . Find the size of the largest angle, correct to the nearest degree.

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Question 12 (2 marks)

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Given that $3\log_2 x - 2\log_2 y = 9$, show that $x = Ay^p$ where A is an integer and p is a rational number.

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Question 13 (2 marks)

Find the anti-derivative of

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$$f(x)=\frac{1}{3x-4}.$$

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Question 14 (2 marks)

A normally distributed variable, X , has a mean of 2 and standard deviation of 0.5.
Use the empirical rule to find $P(0.5 \leq X \leq 3)$.

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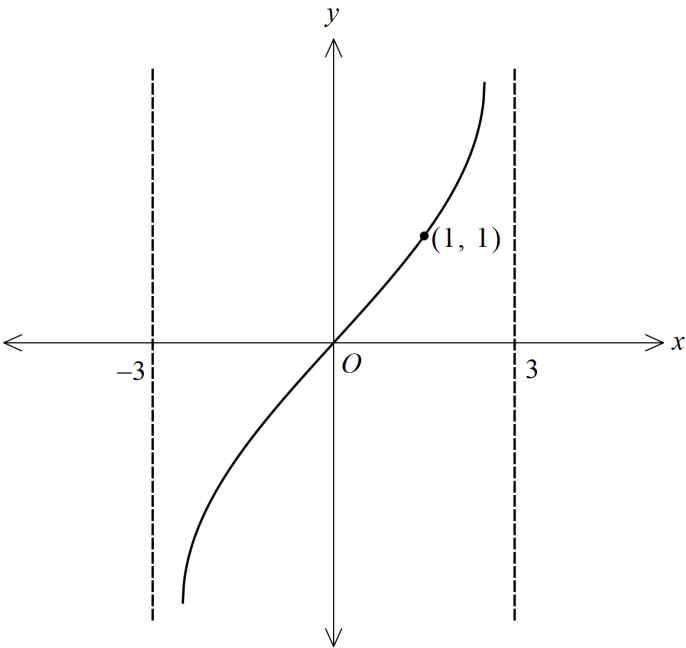
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Question 15 (2 marks)

The graph of $y = a \tan bx$ is shown below for $-3 < x < 3$. Find the values of a and b .

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Question 16 (5 marks)

Georgia buys a ticket in a lottery, where the first prize is \$10 000. Tickets cost \$20 each, with the probability of winning a prize shown in the discrete distribution table below.

Prize	0	20	100	500	5000	10 000
$P(X = x)$	0.75	M	0.03	0.002	0.001	0.0001

- (a) Find the value of **M**.1

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- (b) Show that Georgia is expected to lose \$5.66 every time she plays this lottery.2

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- (c) Find the variance of this discrete distribution.2

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Question 17 (2 marks)

Find the equation of the normal to the curve $y = 2x^3 - 3x$ at the point $(-1, 1)$.

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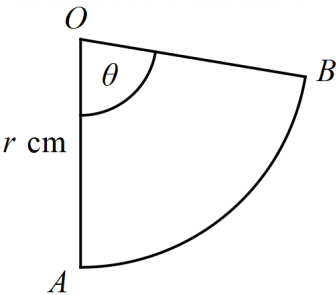
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Question 18 (3 marks)

In the diagram, AOB is a sector of a circle of radius r cm that contains an angle θ radians at the centre O of the circle. The area of the sector is 16 cm^2 and its perimeter is 5 times its arc length. Find the values of r and θ .

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Question 19 (3 marks)

Annabelle records the age in years and the price in dollars of 6 cars that are for sale.

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Age (A)	3	5	5	6	8	10
Price (P)	23 000	18 000	16 500	12 000	9700	7000

Find the equation of the least-squares regression line, giving each value to the nearest whole number, and use the equation to estimate the cost of a car which is 4 years old.

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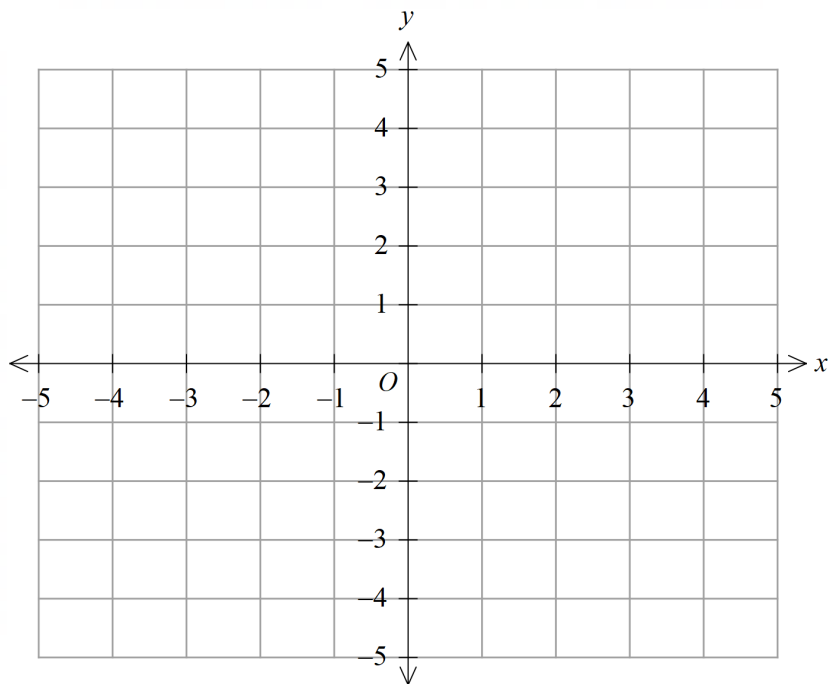
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Question 20 (4 marks)

- (a) Sketch the graphs of the functions $f(x) = 2x - 1$ and $g(x) = (1 - 2x)(3 + 4x)$. 2



- (b) Hence, solve the inequality $2x - 1 > (1 - 2x)(3 + 4x)$. 2

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Question 21 (10 marks)

A sample of 240 households were asked which, if any, of the following animals they own as pets.

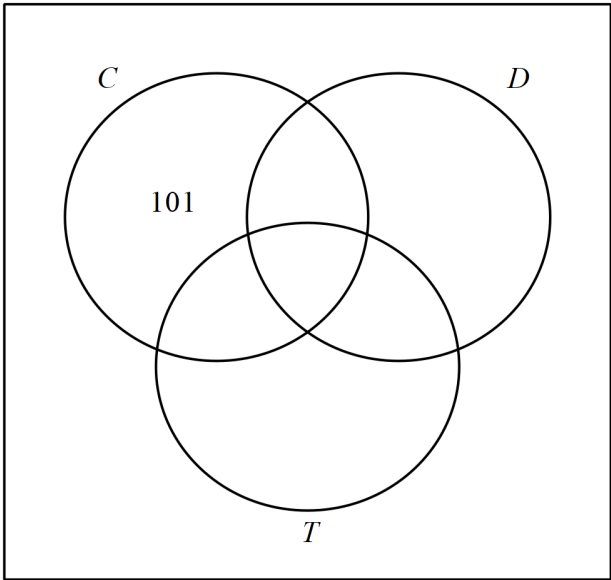
- cats (C)
- dogs (D)
- tortoises (T)

These results are shown in the table below.

Types of Pet	C	D	T	C and D	C and T	D and T	C, D and T
Number of households	153	70	45	48	21	32	17

(a) Represent this information by fully completing the Venn diagram below.

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A household is chosen at random from the sample.

(b) Find the probability that the household owns a cat only.

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Question 21 continues on next page

Question 21 continued.

- (c) Find the probability that the household owns at least two of the three types of pet. 2

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- (d) Find the probability that the household owns a cat, given that the household owns a tortoise. 2

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- (e) Use mathematical reasoning to determine whether a household owning a cat and a household owning a tortoise are independent of each other. 2

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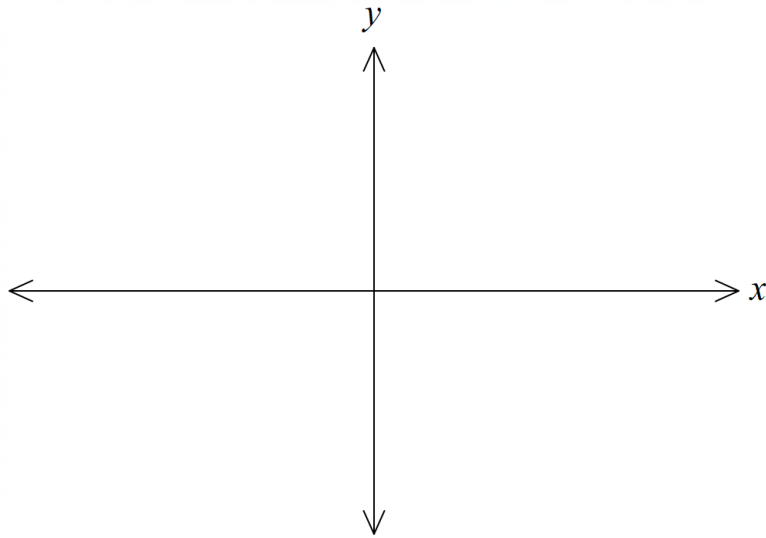
Question 22 (4 marks)

Consider the function $f(x) = (x-1)^2$. The following transformations were applied to the graph of $y = f(x)$ in the order shown below.

4

- Horizontal dilation by a scale factor of $\frac{1}{4}$.
- Vertical translation down 9 units.
- Reflection in the x -axis.

Find the equation of the transformed function and sketch its graph, showing the location of the vertex and its intercepts with the x and y axes.



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Question 23 (3 marks)

Find all values of θ , where $0^\circ \leq \theta \leq 360^\circ$, such that $\sin(\theta - 60^\circ) = -\frac{1}{2}$.

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Question 24 (5 marks)

Let $g(x) = e^{-x} \cos x$.

- (a) Find the coordinates of the stationary points of $g(x)$ for $0 \leq x \leq 2\pi$.

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You do **NOT** need to check the nature of the stationary points.

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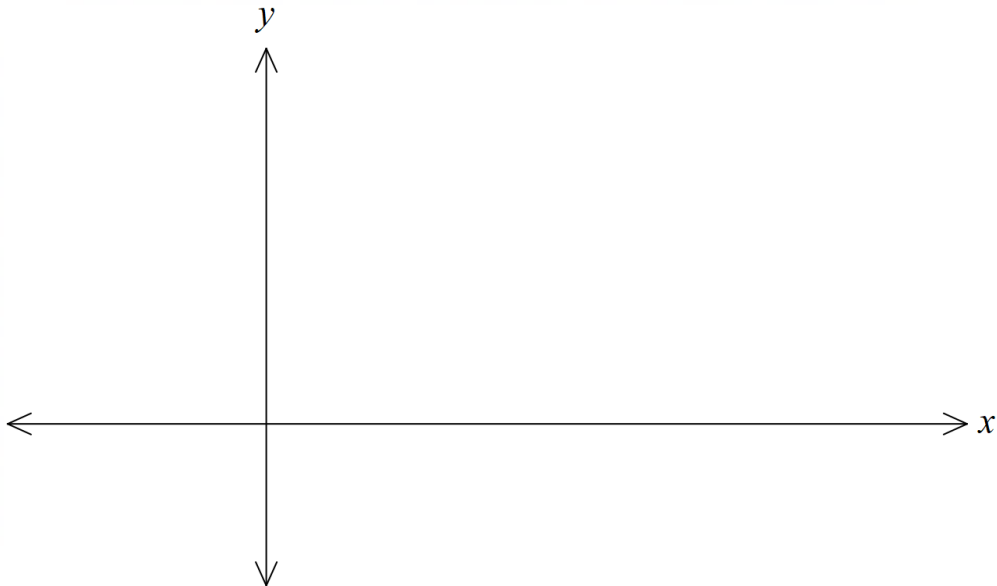
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Question 24 continues on next page

Question 24 continued.

- (b) Without using further calculus, sketch the graph of $y = g(x)$ for $0 \leq x \leq 2\pi$, showing stationary points and intercepts.

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Question 25 (3 marks)

The circle $2x^2 + 2y^2 - 12x + 16y - 22 = 0$ is translated 4 units to the left and 3 units up.
What is the centre and radius of the new circle?

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Question 26 (3 marks)

(a) Find the derivative of $y = e^{x^3}$.

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(b) Hence, find $\int_{-1}^0 x^2 e^{x^3} dx$.

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Question 27 (3 marks)

A geometric series has first term 1 and common ratio $\cos \theta$.

- 1**

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Question 28 (3 marks)

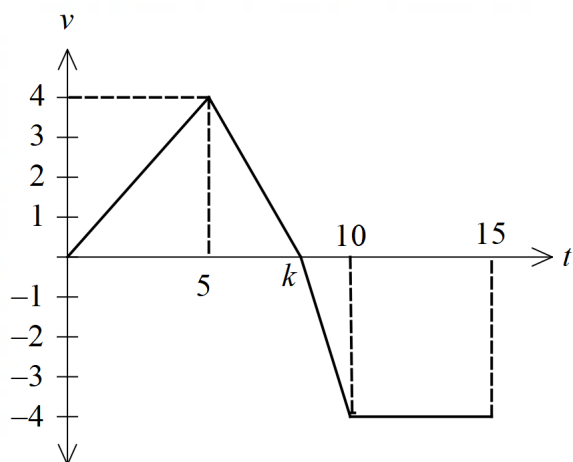
If $y = \frac{\cos x}{1 + \sin x}$ show that $\frac{d^2 y}{dx^2} + y \frac{dy}{dx} = 0$.

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This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for letter height. The entire page is otherwise blank, with no margins, text, or other markings.

3

The graph below shows how the car's velocity $v \text{ ms}^{-1}$ changes with time, t , seconds.



Initially the car has velocity 0 ms^{-1} .

Find the next time when the velocity of the car is 0 ms^{-1} .

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Question 30 (5 marks)

Piya has a greenhouse containing 900 tomato plants. Piya notices that some of the tomato plants are damaged by insects. Initially, there are 25 damaged tomato plants.
The number of tomato plants damaged by insects is inceasing by 32% each day.

- (a)

The total number of plants damaged by insects, x , is modelled by $x = A \times B^t$ where A and B are constants and t is the number of days after Piya first noticed the damaged plants.
Use this model to find the total number of plants damaged by insects 5 days after Piya noticed.

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- (b)

Explain, using mathematical reasoning, why this model is not realistic in the long term.

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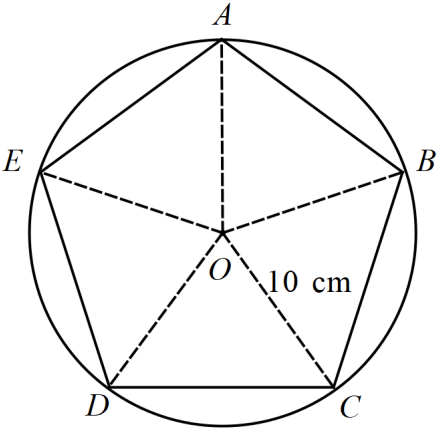
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Question 31 (4 marks)



NOT TO SCALE

In the diagram, $ABCDE$ is a regular pentagon inscribed in a circle with centre O and radius 10 cm.
Find the area of $\triangle ABE$, correct to 3 significant figures.

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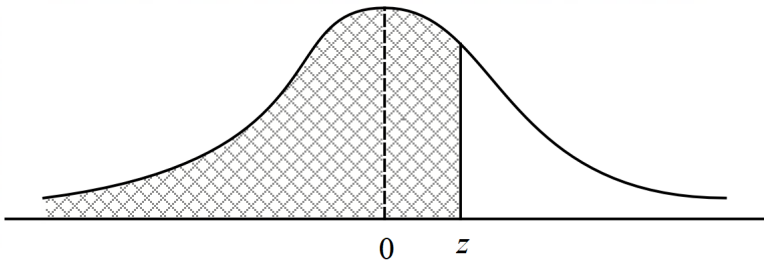
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Question 32 (3 marks)

A random variable is normally distributed with mean 0 and standard deviation 1. The table below gives the probability that this random variable lies below z for different values of z .

z	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.500	0.540	0.579	0.618	0.655	0.691	0.726	0.758	0.788	0.816
1	0.841	0.864	0.885	0.903	0.919	0.933	0.945	0.955	0.964	0.971

The probability values given in the table for different values of z are represented by the shaded area in the following diagram.



- (a) Calculate the probability that the random variable lies above -1.2 . 1

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Question 32 continues on next page.

Question 32 continued.

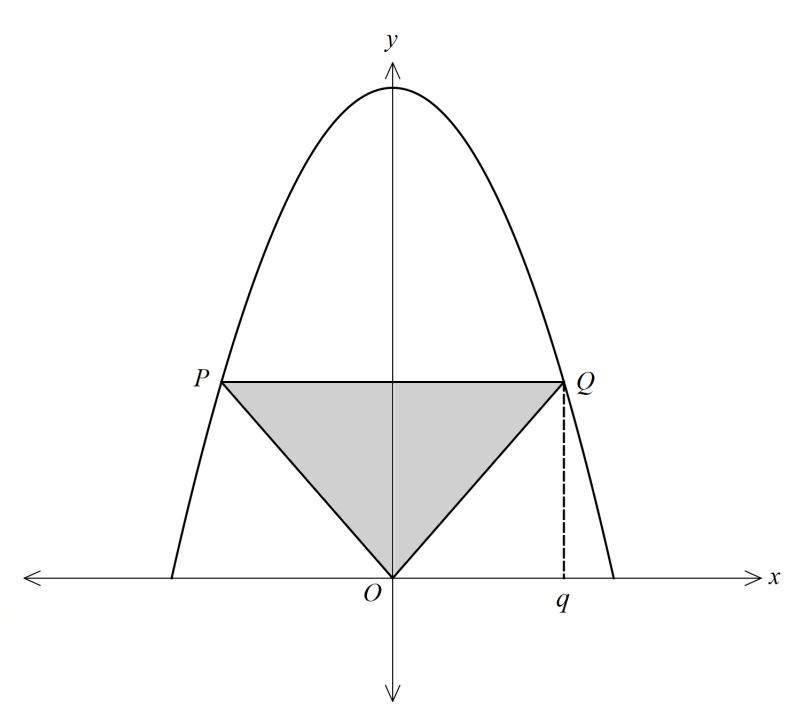
- (b) Celine fills bottles with water. The actual amount of water in each bottle is normally distributed with a mean of 602 mL and a standard deviation of 2.5 mL.

Using the values in the table, determine the percentage of bottles that contain between 599 mL and 604 mL of water.

[illegible]

Question 33 (5 marks)

The curve $y = 15 - x^2$ and the isosceles triangle OPQ , are shown on the diagram below.



Vertices P and Q lie on the curve such that Q lies vertically above the point $(q, 0)$.
The line PQ is parallel to the x -axis.

- (a) Show that the area, A , of the triangle OPQ , is given by $A = 15q - q^3$. **2**

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Question 33 continues on next page

Question 33 continued.

(b) Find the maximum area of $\triangle OPQ$.

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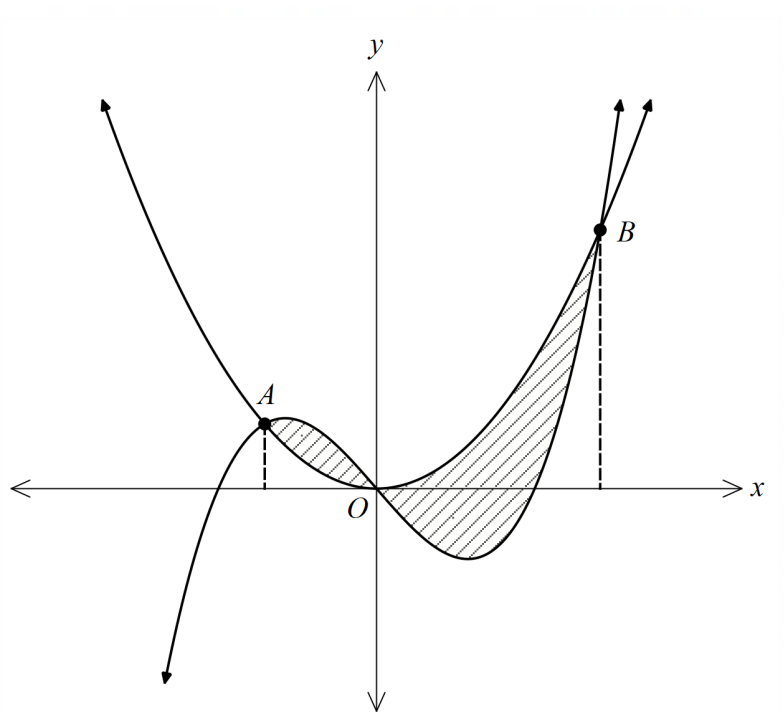
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Question 34 (4 marks)

The diagram shows the graphs of $y = x^3 - 2x$ and $y = x^2$.



The graphs intersect at A , B and the origin O .

- (a) Find the x -coordinates of A and B .

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Question 34 continues on next page

Question 34 continued.

(b) Find the area of the shaded region.

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Question 35 (5 marks)

The time taken, t minutes, for a machine to produce car parts can be modelled by a continuous random variable with probability density function given by

$$f(t) = \begin{cases} k(t-1)^2, & 1 \leq t \leq 2 \\ k\left(2 - \frac{t}{2}\right), & 2 < t \leq 4 \\ 0, & \text{otherwise} \end{cases}$$

- (a) Show that the value of k is $\frac{3}{4}$.

2

[illegible]

Question 35 continues on next page

Question 35 continued.

- (b) Let $F(t)$ be the cumulative distribution function of $f(t)$, where $F(2.16) = 0.3652$. Find the least amount of time taken to produce 88% of the parts.

3

This image shows a full page of a worksheet designed for handwriting practice. It features approximately 20 evenly spaced horizontal dotted lines across the entire width of the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

END OF PAPER

Name: Solutions

Student Number:

Teacher's Name:



Pymble Ladies' College

Mathematics Advanced HSC Trial Examination Term 3 2024

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Mathematics Advanced

HSC Trial Examination

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Section I – Multiple Choice Answer Sheet

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A ○ B ● C ○ D ○

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A  B  C  D 

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|----|------------------------------------|------------------------------------|------------------------------------|------------------------------------|
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SECTION I

10 marks

Attempt Questions 1-10

Allow about 15 minutes for this section

Use the Multiple-Choice Answer sheet for Questions 1-10

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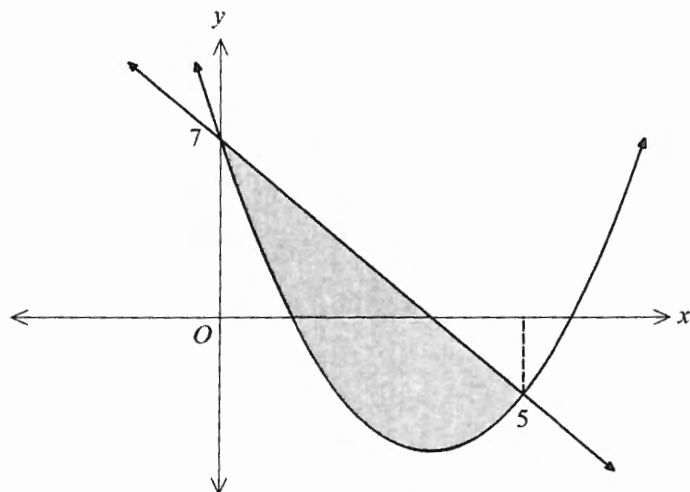
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(C) $(x-4)^2 + (y+5)^2 = 36$

(D) $(x+4)^2 + (y-5)^2 = 36$

- 2 The shaded region, shown in the diagram below, is enclosed by the parabola $y = x^2 - 7x + 7$ and the line $y = 7 - 2x$.



Which of the following gives the area of the shaded region?

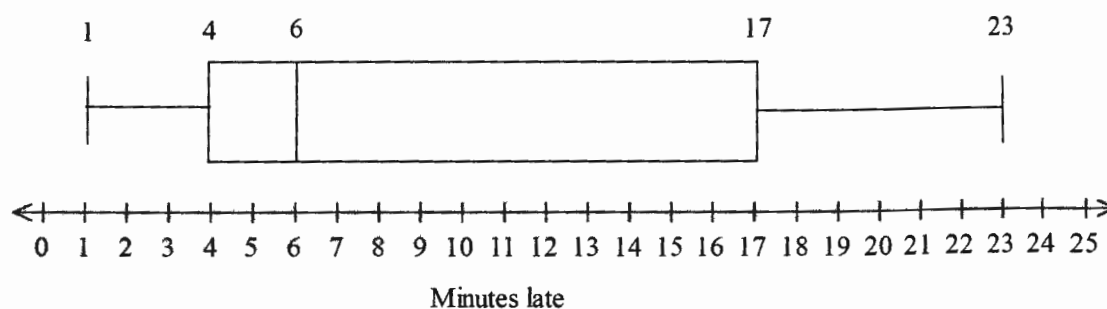
(A) $\int (7-2x) dx - \int (x^2 - 7x + 7) dx$

(B) $\int_0^5 (x^2 - 5x) dx$

(C) $\int_0^5 (5x - x^2) dx$

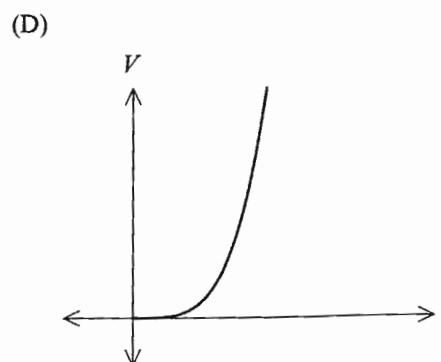
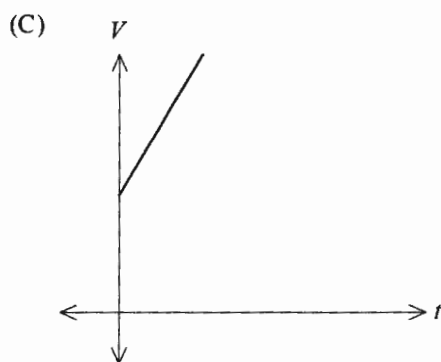
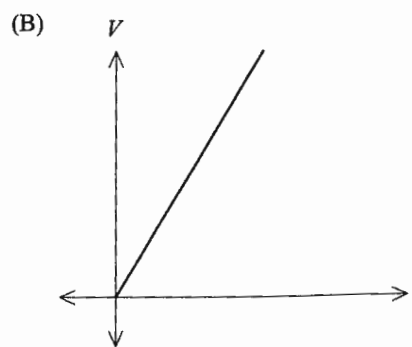
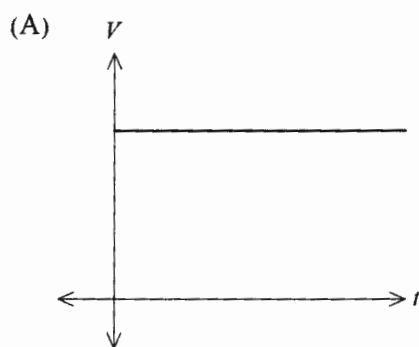
(D) $\int_0^5 (x^2 - 9x + 14) dx$

- 3 The box plot below shows summary data for the number of minutes that buses arrived late at a rural bus stop.



Which term best describes the distribution of this data?

- (A) negatively skewed
 - (B) normal
 - (C) positively skewed
 - (D) symmetrical
- 4 A car, initially at rest, moves with constant acceleration along a straight horizontal road. Which of the graphs below shows how the car's velocity, V m/s, changes over time, t seconds?



- 5 Given that $\cos(\theta - 20^\circ) = \cos 60^\circ$, which of the following is a possible value for θ ?

(A) 40°
(B) 140°
(C) 280°
(D) 320°

- 6 A function f is given by the rule

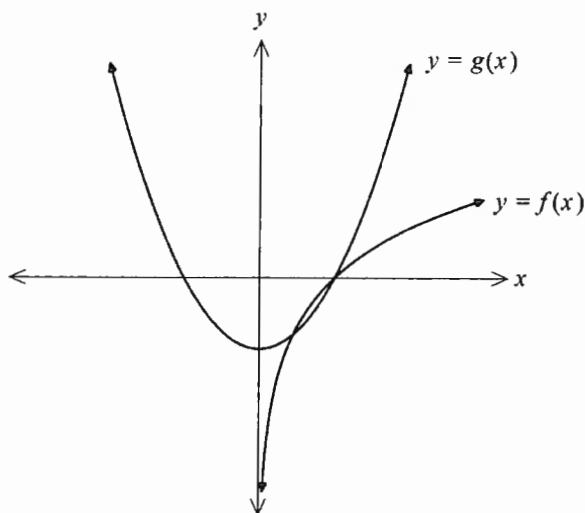
$$f(x) = \begin{cases} (x-3)^2 + 2, & x \leq 3 \\ 3x - 7, & x > 3 \end{cases}$$

What is the value of $f(f(0))$?

- (A) 11
(B) -7
(C) 26
(D) 66
- 7 The z -scores on a test are normally distributed, such that $P(-N < z < N) = p$ for some constants $N > 0$ and $0 < p < 1$. What is the value of $P(z < N)$?

(A) $\frac{1-p}{2}$
(B) $1 - \frac{p}{2}$
(C) $\frac{1+p}{2}$
(D) $\frac{1+2p}{3}$

- 8 What is $\int (\sin^2 x + \cos^2 x + \tan^2 x) dx$?
- (A) $\tan x + C$
 (B) $\cot x + C$
 (C) $x + \sec x + C$
 (D) $x - \ln(\cos^2 x) + C$
- 9 In a bag of n coloured discs, exactly 7 are red. Two discs are selected at random, without replacement. The probability that both discs are red is $\frac{21}{55}$.
 Which quadratic equation could be used to find the value of n ?
- (A) $n^2 + n + 110 = 0$
 (B) $n^2 - n - 110 = 0$
 (C) $n^2 + n - 110 = 0$
 (D) $n^2 - n + 110 = 0$
- 10 The graph shows $y = f(x)$ and $y = g(x)$, where $f(x) = \ln x$ and $g(x) = x^2 - 1$.
 How many solutions does the equation $[f(x)]^2 - [g(x)]^2 = 0$ have?



- (A) 0
 (B) 1
 (C) 2
 (D) 3

End of Section I

Name.....

Student
Number

Teacher's Name.....

Mathematics Advanced

Section II Answer Booklet

90 marks

Attempt Questions 11-35

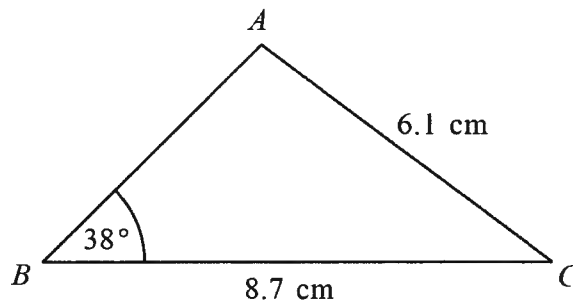
Allow about 2 hours and 45 minutes for this section

Instructions

- Answer in the spaces provided. These spaces provide guidance for the expected length of the answer.
- Extra writing space is provided at the end of the paper. If you use the additional space, clearly indicate which question you are answering.
- Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (2 marks)

2



NOT TO SCALE

The diagram shows a triangle ABC . AB is the shortest side. The lengths of AC and BC are 6.1 cm and 8.7 cm respectively. The size of angle ABC is 38° . Find the size of the largest angle, correct to the nearest degree.

$\frac{\sin A}{8.7} = \frac{\sin 38^\circ}{6.1}$	2	Correct
$\sin A = \frac{8.7 \sin 38^\circ}{6.1}$	1	Correct substitution into
$A = 61.41096428, 118.5890357$		sine rule to get
$\hat{=} 61^\circ, 119^\circ$		$\theta = 61^\circ$
$\therefore$ largest angle is 119°		

Question 12 (2 marks)

Given that $3 \log_2 x - 2 \log_2 y = 9$, show that $x = Ay^p$ where A is an integer and p is a rational number.

2

$\log_2 x^3 - \log_2 y^2 = 9$	2	Correct
$\log_2 \frac{x^3}{y^2} = 9$	1	Correct to *
$\therefore \frac{x^3}{y^2} = 2^9$ (*)		[or one correct]
$x^3 = 512 y^2$		
$x = 8 y^{2/3}$ As req'd		

Question 13 (2 marks)

Find the anti-derivative of

$$f(x) = \frac{1}{3x-4}$$

$$F(x) = \frac{1}{3} \ln |3x-4| + C$$

2

2 | Solution contains absolute value and plus C

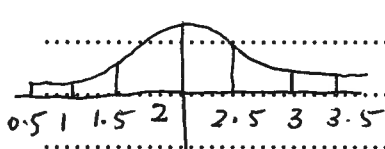
1 | missing absolute or C

2

Question 14 (1 mark)

A normally distributed variable, X , has a mean of 2 and standard deviation of 0.5. Use the empirical rule to find $P(0.5 \leq X \leq 3)$.

2



$$\frac{99.7}{2} = 49.85$$

$$\frac{95}{2} = 47.5$$

$$\begin{aligned} P(0.5 \leq X \leq 3) &= 49.85 + 47.5 \\ &= 97.35\% \\ &= 0.9735 \end{aligned}$$

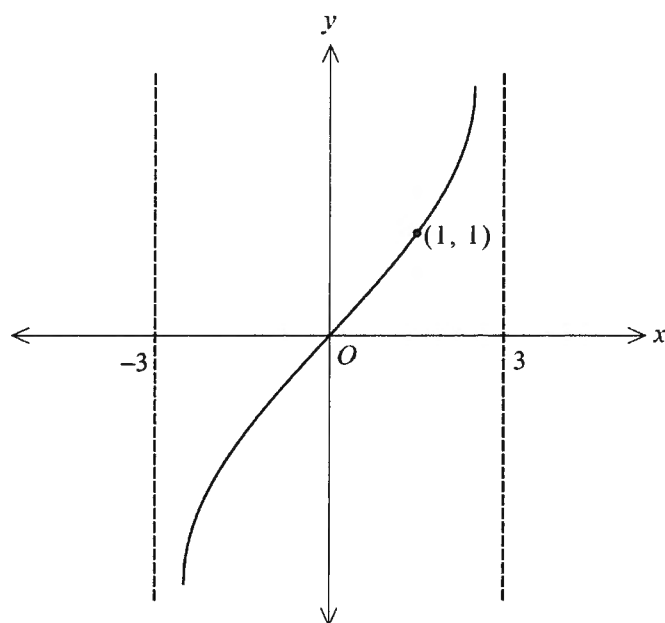
2 | Correct

1 | clearly shows either 49.85 or 47.5

Question 15 (2 marks)

The graph of $y = a \tan bx$ is shown below for $-3 < x < 3$. Find the values of a and b .

2



2	correct values of a and b
1	Mistakenly got $b = \pi/3$ but correct substitution to get $a = \frac{1}{\sqrt{3}}$

$$a \tan b = 1 \quad \text{--- ①} \quad \frac{\pi}{b} = 6 \quad \text{--- ②}$$

$$b = \frac{\pi}{6}$$

Sub ② into ①

$$\therefore y = \sqrt{3} \tan \frac{\pi}{6} x$$

$$a \tan \frac{\pi}{6} = 1$$

$$a \times \frac{1}{\sqrt{3}} = 1$$

$$a = \sqrt{3}$$

Question 16 (5 marks)

Georgia buys a ticket in a lottery, where the first prize is \$10 000. Tickets cost \$20 each, with the probability of winning a prize shown in the discrete distribution table below.

Prize	0	20	100	500	5000	10 000
$P(X=x)$	0.75	M	0.03	0.002	0.001	0.0001

Bold.

- (a) Find the value of M.

1

$$0.75 + M + 0.03 + 0.002 + 0.001 + 0.0001 = 1$$

$$M = 0.2169$$

1 r/w

- (b) Show that Georgia is expected to lose \$5.66 every time she plays this lottery.

2

$$E(X) = 0 \times 0.75 + 20 \times 0.2169 + 100 \times 0.03 + 500 \times 0.002$$

$$+ 5000 \times 0.001 + 10000 \times 0.0001$$

$$= 14.338$$

$$\text{Ticket cost} = 20$$

$$\therefore 14.338 - 20 = -5.66$$

$\therefore$ Georgia loses \$5.66 per game.

2	correct
1	correct
	$E(x)$

- (c) Find the variance of this discrete distribution.

2

$$\text{Var}(X) = E(X^2) - (E(X))^2$$

$$E(X^2) = 0^2 \times 0.75 + 20^2 \times 0.2169 + 100^2 \times 0.03 + 500^2 \times 0.002$$

$$+ 5000^2 \times 0.001 + 10000^2 \times 0.0001$$

$$= 35886.76$$

$$\text{Var}(X) = 35886.76 - (14.338)^2$$

$$= 35681.18176$$

2	correct
1	partially correct

Question 17 (2 marks)

Find the equation of the normal to the curve $y = 2x^3 - 3x$ at the point $(-1, 1)$.

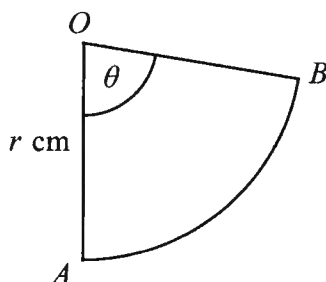
2

$\frac{dy}{dx} = 6x^2 - 3$		
when $x = -1$ $\frac{dy}{dx} = 6 - 3$	2	Correct
$\frac{dy}{dx} = 3$	1	Differentiating finding tangent gradient.
Gradient of normal $= -\frac{1}{3}$		
Eq ⁿ of normal: $y - 1 = -\frac{1}{3}(x + 1)$		
$3y - 3 = -x - 1$		
$x + 3y - 2 = 0$		

Question 18 (3 marks)

In the diagram, AOB is a sector of a circle of radius r cm that contains an angle θ radians at the centre O of the circle. The area of the sector is 16 cm^2 and its perimeter is 5 times its arc length. Find the values of r and θ .

3



Perimeter $= 2r + r\theta$	$\frac{1}{2} r^2 \theta = 16$ *	3	Correct
$2r + r\theta = 5r\theta$ *	$\frac{1}{2} r^2 \times \frac{1}{2} = 16$	2	Correct perimeter, area and progress to achieve one variable
$2r = 4r\theta$	$r^2 = 64$	1	Just * and *
$\theta = \frac{1}{2}$	$r = 8; r > 0$		

Question 19 (3 marks)

Annabelle records the age in years and the price in dollars of 6 cars that are for sale.

3

Age (A)	3	5	5	6	8	10
Price (P)	23 000	18 000	16 500	12 000	9700	7000

Find the equation of the least-squares regression line, giving each value to the nearest whole number, and use the equation to estimate the cost of a car which is 4 years old.

$$A = 28460 \quad P = 28460 - 2285A$$

$$B = -2285.405405 \quad \text{when } A = 4$$

$$\hat{=} -2285 \quad P = 28460 - 2285 \times 4$$

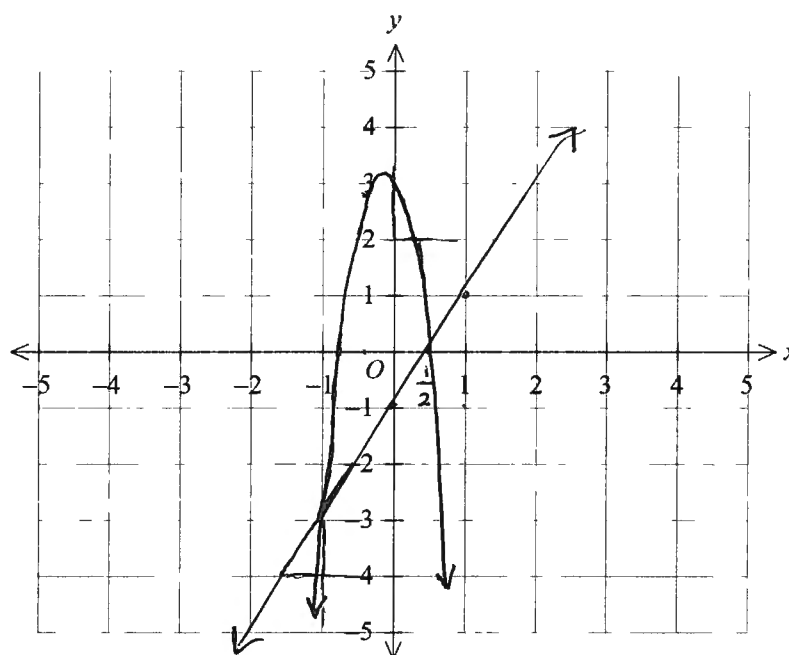
$$= \$19320$$

3	Correct (ignore rounding) \$19318.38 accepted.
2	Correct A and B and eq ⁿ for P
1	Correct A and B.

Question 20 (4 marks)

- (a) Sketch the graphs of the functions $f(x) = 2x - 1$ and $g(x) = (1 - 2x)(3 + 4x)$.

2



2	Both graphs
1	one correct.

- (b) Hence, solve the inequality $2x - 1 > (1 - 2x)(3 + 4x)$.

2

..... $x < -1$ or $x > 1/2$	2	Correct
.....	1	one part
.....		Correct
.....		

Question 21 (10 marks)

A sample of 240 households were asked which, if any, of the following animals they own as pets.

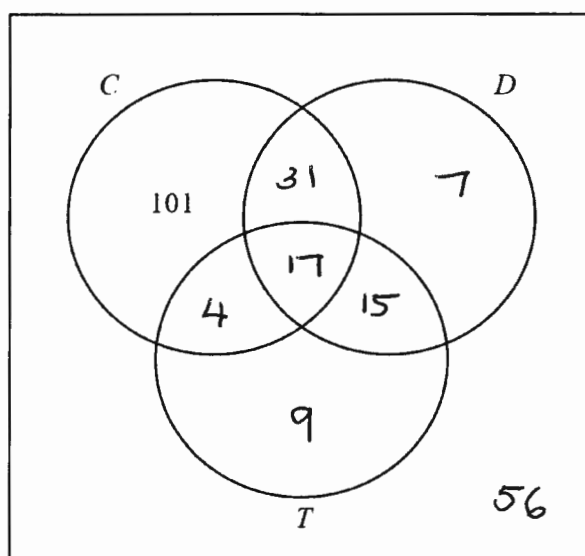
- cats (C)
- dogs (D)
- tortoises (T)

These results are shown in the table below.

Types of Pet	C	D	T	C and D	C and T	D and T	C, D and T
Number of households	153	70	45	48	21	32	17

(a) Represent this information by fully completing the Venn diagram below.

3



3 | correct, including
56 on outside
2 | Numbers in
2 of the 3 circles correct
1 | Number in one
of the circles
correct.

A household is chosen at random from the sample.

(b) Find the probability that the household owns a cat only.

1

$$P(\text{cat only}) = \frac{101}{240}$$

1 r/w

Question 21 continues on next page

Question 21 continued.

- (c) Find the probability that the household owns at least two of the three types of pet. 2

$$P(C \text{ and } D) + P(C \text{ and } T) + P(D \text{ and } T) + P(C \text{ and } D \text{ and } T)$$

$$= \frac{31 + 4 + 15 + 17}{240}$$

$$= \frac{67}{240}$$

1	3 of 4 - 1
2	Correct

- (d) Find the probability that the household owns a cat, given that the household owns a tortoise. 2

$$P(C|T) = \frac{P(C \cap T)}{P(T)}$$

$$= \frac{4 + 17}{45} \checkmark \text{ 1 }$$

$$= \frac{7}{15} \checkmark$$

1	1 $\frac{4}{45}$ or $\frac{17}{45}$ or just num/den. correct
2	Correct

- (e) Use mathematical reasoning to determine whether a household owning a cat and a household owning a tortoise are independent of each other. 2

METHOD 1:

$$P(C \cap T) = \frac{21}{240}$$

$$= \frac{7}{80} \text{ 1 }$$

$$P(C) \times P(T) = \frac{153}{240} \times \frac{45}{240}$$

$$= \frac{153}{1280}$$

$$P(C \cap T) \neq P(C) \times P(T)$$

$\therefore$ not independent

METHOD 2:

$$P(C|T) = \frac{7}{15} \text{ [from (d)]}$$

$$P(C) = \frac{153}{240}$$

$$= \frac{51}{80} \text{ 2 }$$

$$P(C|T) \neq P(C)$$

$\therefore$ not independent

1	1 of 2 correct in either 1 or 2
2	Correct

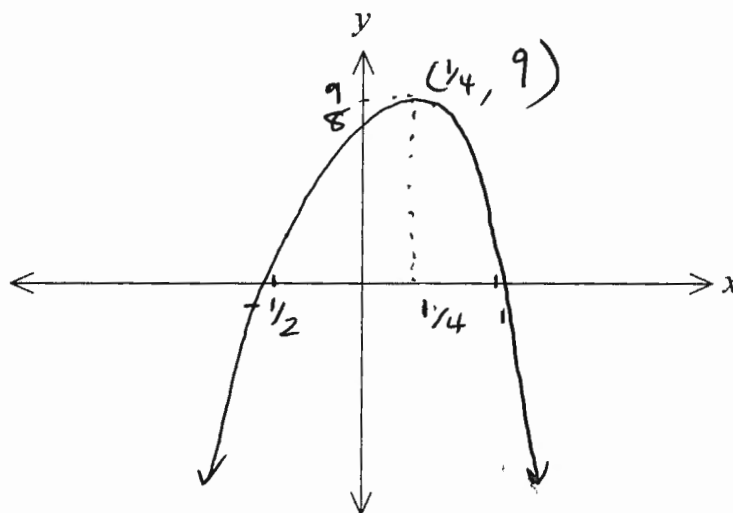
Question 22 (4 marks)

Consider the function $f(x) = (x-1)^2$. The following transformations were applied to the graph of $y = f(x)$ in the order shown below.

4

- Horizontal dilation by a factor of $\frac{1}{4}$.
- Vertical translation down 9 units.
- Reflection in the x-axis.

Find the equation of the transformed function and sketch its graph, showing the location of the vertex and its intercepts with the x and y axes.



4	correct
3	Correct, but requested detail missing; Vertex and intercepts sh
2	Correct eq ⁿ but graph incorrect of correct graph from
1	Two incorrect transformations correct

$$f(x) = (x-1)^2 \rightarrow (4x-1)^2 \rightarrow (4x-1)^2 - 9$$

$$\rightarrow -(4x-1)^2 + 9$$

$$\text{Let } g(x) = -(4x-1)^2 + 9$$

$$\text{When } g(x) = 0 \quad (4x-1)^2 = 9$$

$$4x-1 = 3 \text{ or } 4x-1 = -3$$

$$x = 1 \text{ or } x = -1/2$$

$$g(0) = 8$$

$$g(1/4) = 9$$

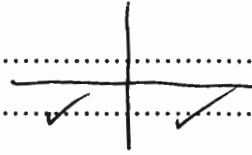
Question 23 (3 marks)

Find all values of θ , where $0^\circ \leq \theta \leq 360^\circ$, such that $\sin(\theta - 60^\circ) = -\frac{1}{2}$.

3

$$\sin(\theta - 60^\circ) = -\frac{1}{2}$$

Related angle 30°



$$\theta - 60^\circ = 210^\circ, -30^\circ$$

$$\theta = 270^\circ, 30^\circ$$

3	Correct
2	One correct value
	Correct quadrants
	Correct related angle
1	Quadrants and related angle

Question 24 (5 marks)

Let $g(x) = e^{-x} \cos x$.

- (a) Find the coordinates of the stationary points of $g(x)$ for $0 \leq x \leq 2\pi$.

3

You do **NOT** need to check the nature of the stationary points.

$$g'(x) = e^{-x} x - \sin x + \cos x x - e^{-x}$$

$$= -e^{-x} (\sin x + \cos x)$$

When $g'(x) = 0$ $-e^{-x} = 0$ or $\sin x + \cos x = 0$
 No solⁿs $\sin x = -\cos x$

Related $\tan x = -1$
 angle = $\frac{\pi}{4}$ $x = \frac{3\pi}{4}, \frac{7\pi}{4}$

$$g\left(\frac{3\pi}{4}\right) = \frac{-e^{-\frac{3\pi}{4}}}{\sqrt{2}} \quad g\left(\frac{7\pi}{4}\right) = \frac{e^{-\frac{7\pi}{4}}}{\sqrt{2}}$$

$\therefore$ Stationary points are $\left(\frac{3\pi}{4}, \frac{-e^{-\frac{3\pi}{4}}}{\sqrt{2}}\right)$

and $\left(\frac{7\pi}{4}, \frac{e^{-\frac{7\pi}{4}}}{\sqrt{2}}\right)$

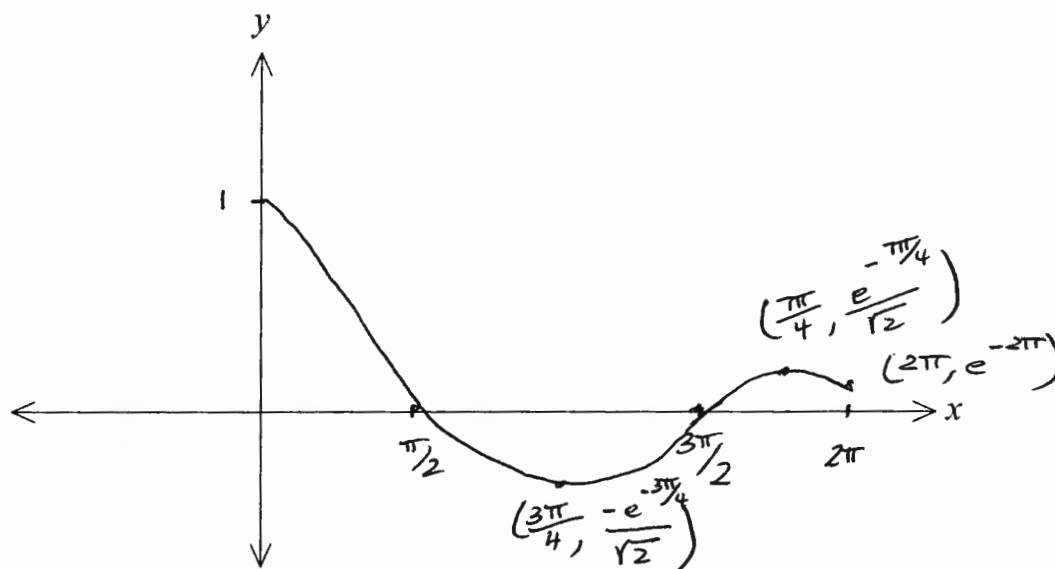
Question 24 continues on next page

3	Correct
2	Correct derivative Correct x values error in y values or one correct point
1	Correct derivative

Question 24 continued.

- (b) Without using further calculus, sketch the graph of $y = g(x)$ for $0 \leq x \leq 2\pi$, showing stationary points and intercepts.

2



$$\begin{aligned}
 g(0) &= e^0 \cos 0 = 1 \\
 g(2\pi) &= e^{-2\pi} \cos 2\pi = e^{-2\pi} \approx 0.00186
 \end{aligned}$$

$e^{-x} \cos x = 0$
 $e^{-x} = 0$ or $\cos x = 0$
 No solutions $x = \pi/2, 3\pi/2$

2	Correct
1	Partially correct
	Shape correct
	Endpoints correct
	Intercepts

Question 25 (3 marks)

The circle $2x^2 + 2y^2 - 12x + 16y - 22 = 0$ is translated 4 units to the left and 3 units up.
What is the centre and radius of the new circle?

3

$x^2 + y^2 - 6x + 8y - 11 = 0$	3	Correct
$(x-3)^2 + (y+4)^2 = 11 + 9 + 16$	2	Old centre Correct
$(x-3)^2 + (y+4)^2 = 36$		Radius correct
Centre $(3, -4)$ Radius 6		
New Centre $(-1, -1)$ Radius 6	1	Some progress to complete the square.

Question 26 (3 marks)

- (a) Find the derivative of $y = e^{x^3}$.

1

$$\frac{dy}{dx} = 3x^2 e^{x^3}$$

1 r/w

- (b) Hence, find $\int_{-1}^0 x^2 e^{x^3} dx$.

2

$$\begin{aligned} & \frac{1}{3} \int_{-1}^0 3x^2 e^{x^3} dx \\ &= \frac{1}{3} \left[e^{x^3} \right]_{-1}^0 \\ &= \frac{1}{3} \left\{ e^0 - e^{-1} \right\} \\ &= \frac{1}{3} \left\{ 1 - \frac{1}{e} \right\} \end{aligned}$$

2 Correct
1 one error in process

Question 27 (3 marks)

A geometric series has first term 1 and common ratio $\cos \theta$.

- (a) Find the sum to infinity of the series.

1

$$S_{\infty} = \frac{1}{1 - \cos \theta}$$

1 r/w

- (b) Find the smallest positive exact value of θ , in radians, which satisfies the equation

2

$$S_{\infty} = 2 - \sqrt{2}.$$

$$\frac{1}{1 - \cos \theta} = 2 - \sqrt{2}$$

$$1 - \cos \theta = \frac{1}{2 - \sqrt{2}}$$

$$\cos \theta = 1 - \frac{1}{2 - \sqrt{2}}$$

$$= \frac{2 - \sqrt{2} - 1}{2 - \sqrt{2}}$$

$$= \frac{1 - \sqrt{2}}{2 - \sqrt{2}} \times \frac{2 + \sqrt{2}}{2 + \sqrt{2}}$$

$$= \frac{2 + \sqrt{2} - 2\sqrt{2} - 2}{4 - 2}$$

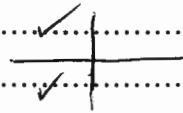
$$= \frac{-\sqrt{2}}{2}$$

$$\cos \theta = -\frac{1}{\sqrt{2}}$$

$$\theta = \frac{3\pi}{4}$$

Related angle
 $\frac{\pi}{4}$

2 correct
1 Attempt at simplifying surd to get to $-\frac{1}{\sqrt{2}}$
or isolating correctly $\cos \theta$



Question 28 (3 marks)

If $y = \frac{\cos x}{1 + \sin x}$ show that $\frac{d^2 y}{dx^2} + y \frac{dy}{dx} = 0$.

3

$$\frac{dy}{dx} = \frac{(1 + \sin x)(-\sin x) - \cos x(\cos x)}{(1 + \sin x)^2}$$

$$= \frac{-\sin x - \sin^2 x - \cos^2 x}{(1 + \sin x)^2}$$

$$= \frac{-\sin x - 1}{(1 + \sin x)^2} \quad ; \quad \sin^2 x + \cos^2 x = 1$$

$$= \frac{-(1 + \sin x)}{(1 + \sin x)^2}$$

$$= \frac{-1}{1 + \sin x} = -(1 + \sin x)^{-1}$$

$$\frac{d^2 y}{dx^2} = (1 + \sin x)^{-2} \times \cos x$$

$$= \frac{\cos x}{(1 + \sin x)^2}$$

$$\frac{d^2 y}{dx^2} + y \frac{dy}{dx} = \frac{\cos x}{(1 + \sin x)^2} + \frac{\cos x}{1 + \sin x} \times \frac{-1}{1 + \sin x}$$

$$= \frac{\cos x}{(1 + \sin x)^2} - \frac{\cos x}{(1 + \sin x)^2}$$

$$= 0$$

$$= \text{RHS} \quad \text{As req'd}$$

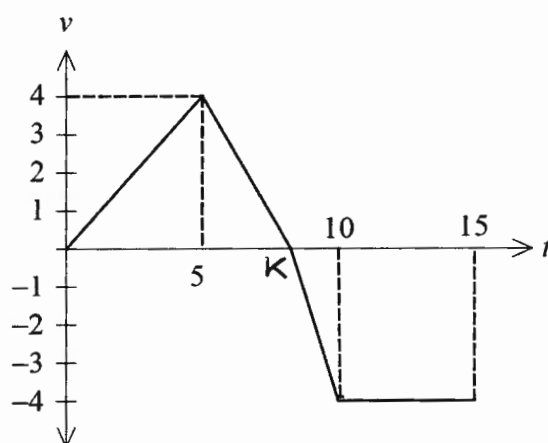
3 Correct show
2 correct applicat
of rule to
differentiate
 $\frac{dy}{dx} \rightarrow \frac{d^2 y}{dx^2}$
1 Applies
quotient
rule correctly
for $\frac{dy}{dx}$

Question 29 (3 marks)

A car is moving in a straight line along a horizontal road.

3

The graph below shows how the car's velocity $v \text{ ms}^{-1}$ changes with time, t , seconds.



3	correctly solved
2	Finding areas and creating eq ⁿ = -7
1	Finding one triangular area correctly

Over the period $0 \leq t \leq 15$ the car has a total displacement of -7 metres.

Initially the car has velocity 0 ms^{-1} .

Find the next time when the velocity of the car is 0 ms^{-1} .

Let the x-intercept be k as shown above

$$\begin{aligned} \text{Area above} &= \frac{1}{2} k \times 4 \\ &= 2k \end{aligned} \quad \begin{aligned} \text{Area below} &= \frac{1}{2} \times 4 (15 - k + 5) \\ &= 2(20 - k) \end{aligned}$$

$$2k - 2(20 - k) = -7$$

$$4k - 40 = -7$$

$$4k = 33$$

$$k = \frac{33}{4} \text{ seconds}$$

$\therefore$ Velocity is 0 m/s after $\frac{33}{4}$ seconds

Question 30 (5 marks)

Piya has a greenhouse containing 900 tomato plants. Piya notices that some of the tomato plants are damaged by insects. Initially, there are 25 damaged tomato plants. The number of tomato plants damaged by insects is increasing by 32% each day.

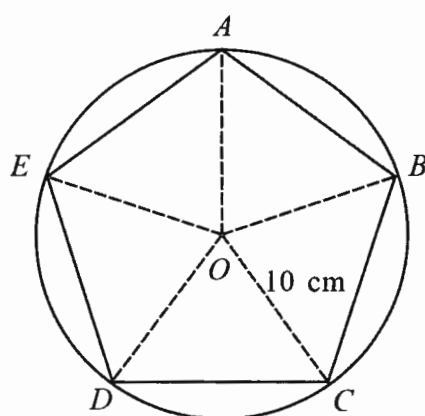
- (a) The total number of plants damaged by insects, x , is modelled by $x = A \times B^t$ where A and B are constants and t is the number of days after Piya first noticed the damaged plants. Use this model to find the total number of plants damaged by insects 5 days after Piya noticed.

When $t=0$, $x=25$	When $t=1$, $x=25 \times 1.32$	
	$= 33$	
$x = A \times B^t$	$25 = A \times B^0$	3
	$\therefore A = 25$	correct
$x = 25 \times B^t$		2
$33 = 25 \times B$	$\therefore x = 25 \times \left(\frac{33}{25}\right)^t$	Correct value of $B = 1.32$
$B = \frac{33}{25}$	When $t=5$	
	$x = 25 \times \left(\frac{33}{25}\right)^5$	1
	$= 100.1866 \dots$ plants	Finding A and attempting to find B

- (b) Explain, using mathematical reasoning, why this model is not realistic in the long term.

As $t \rightarrow \infty$, $x \rightarrow \infty$ since $\frac{33}{25} > 1$	2	explanation WITH mention of $t \leq 12$ only or $t \rightarrow \infty$ $x \rightarrow \infty$
But there are only 900 plants		
$\therefore$ they will eventually all be eaten	1	explanation regarding limit of 900 plants
OR The model is invalid beyond $t=12$ as $x = 25(1.32)^{13} = 923.42 \dots$		
which is not possible as there are only 900 plants.		

Question 32 (4 marks)



NOT TO SCALE

In the diagram, $ABCDE$ is a regular pentagon inscribed in a circle with centre O and radius 10 cm.

4

Find the area of $\triangle ABE$, correct to 3 significant figures.

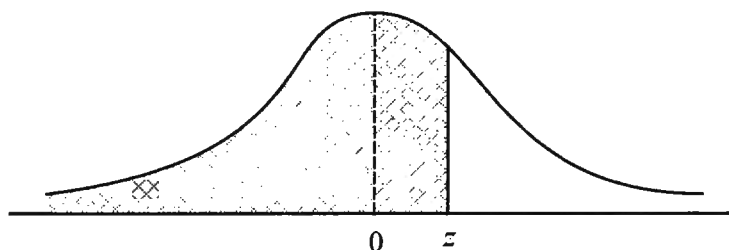
$\angle AOB = \frac{2\pi}{5}$	$\angle OAB = \pi - \frac{2\pi}{5}$	$\therefore \angle EAB = 2 \times \frac{3\pi}{10}$	
	$= \frac{3\pi}{5}$		
	$= \frac{3\pi}{10}$		
		4	Correct
$AB = AE$ (Regular Pentagon)		3	One error
$AB = \sqrt{10^2 + 10^2 - 2 \times 10 \times 10 \cos \frac{2\pi}{5}}$		2	Getting the length of AB OR getting the area of $\triangle AOB$
$\text{Area } \triangle ABE = \frac{1}{2} \times AB \times AE \sin \frac{3\pi}{5}$			
$= \frac{1}{2} (200 - 200 \cos \frac{2\pi}{5}) \sin \frac{3\pi}{5}$		1	Working out $\angle AOB = 72^\circ$ OR $\frac{2\pi}{5}$
$= 65.71638902$			
$= 65.7 \text{ cm}^2$ (3 sig. fig.)			
OR $\angle AOB = \frac{2\pi}{5}$	$\angle EOB = \frac{4\pi}{5}$		
Area of $\triangle AOB = \frac{1}{2} \times 10 \times 10 \times \sin \frac{2\pi}{5} = 50 \sin \frac{2\pi}{5}$			
Area of quadrilateral $ABOE = 2 \times 50 \sin \frac{2\pi}{5}$			
Area of $\triangle BOE = \frac{1}{2} \times 10 \times 10 \times \sin \frac{4\pi}{5} = 50 \sin \frac{4\pi}{5}$			
Area of $\triangle ABE = 100 \sin \frac{2\pi}{5} - 50 \sin \frac{4\pi}{5}$			
$=$			

Question 31 (3 marks)

A random variable is normally distributed with mean 0 and standard deviation 1. The table below gives the probability that this random variable lies below z for different values of z .

z	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.500	0.540	0.579	0.618	0.655	0.691	0.726	0.758	0.788	0.816
1	0.841	0.864	0.885	0.903	0.919	0.933	0.945	0.955	0.964	0.971

The probability values given in the table for different values of z are represented by the shaded area in the following diagram.



- (a) Calculate the probability that the random variable lies above -1.2 .

1

$$\begin{aligned}
 P(Z < 1.2) &= P(Z > -1.2) \\
 &= 0.885 \text{ from the table}
 \end{aligned}$$

1 r/w

Question 31 continues on next page.

Question 31 continued.

- (b) Celine fills bottles with water. The actual amount of water in each bottle is normally distributed with a mean of 602 mL and a standard deviation of 2.5 mL.

2

Using the values in the table, determine the percentage of bottles that contain between 599 mL and 604 mL of water.

$$z_1 = \frac{599 - 602}{2.5}$$

$$= -1.2$$

$$z_2 = \frac{604 - 602}{2.5}$$

$$= 0.8$$

$$P(-1.2 < Z < 0.8) = 0.788 - (1 - 0.885)$$

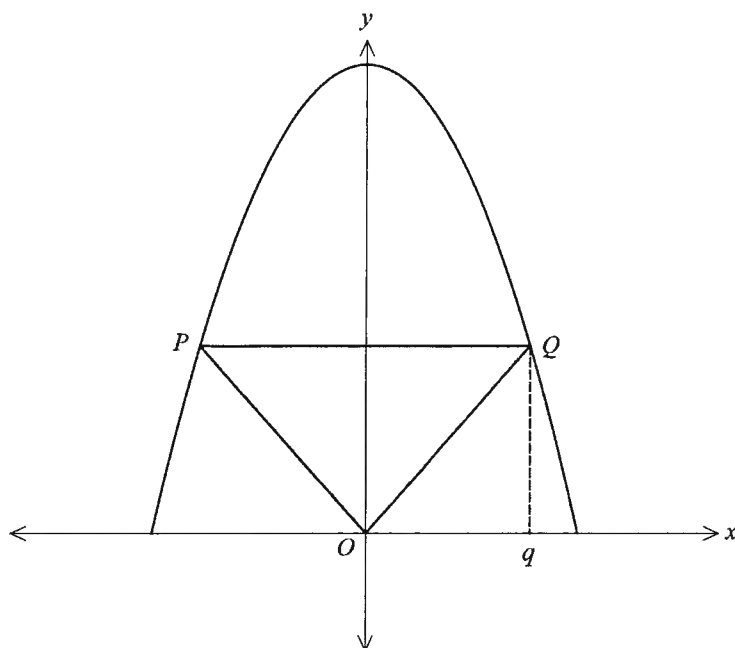
$$= 0.673$$

$$= 67.3\%$$

2	Correct
1	Working out z-scores.

Question 33 (5 marks)

The curve $y = 15 - x^2$ and the isosceles triangle OPQ , are shown on the diagram below.



Vertices P and Q lie on the curve such that Q lies vertically above the point $(q, 0)$.

The line PQ is parallel to the x -axis.

- (a) Show that the area, A , of the triangle OPQ , is given by $A = 15q - q^3$.

2

By symmetry, P has x -co-ordinate $-q$
and y coordinate of both P and $Q = 15 - q^2$

$\therefore PQ = 2q$

Height $\triangle POQ = 15 - q^2$

Area $= \frac{1}{2} \times 2q \times (15 - q^2)$

$= q(15 - q^2) = 15q - q^3$ As req'd

2	Correct
1	Finding the y coordinate of Q

Question 33 continues on next page

Question 33 continued.

(b) Find the maximum area of $\triangle OPQ$.

3

$$\frac{dA}{dq} = 15 - 3q^2 \qquad \frac{d^2A}{dq^2} = -6q$$

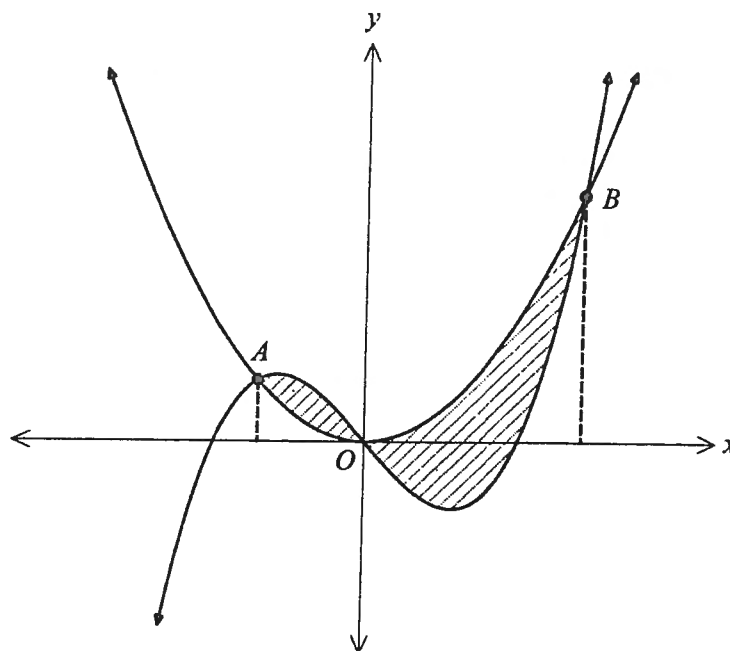
$$\begin{aligned} \text{When } \frac{dA}{dq} = 0 \quad 3q^2 &= 15 & \text{When } q = \sqrt{5} \\ q^2 &= 5 & \frac{d^2A}{dq^2} = -6\sqrt{5} < 0 \\ q &= \pm\sqrt{5} & \therefore q = \sqrt{5} \text{ will produce} \\ & & \text{a maximum area} \end{aligned}$$

$$\begin{aligned} \text{Maximum area} &= 15\sqrt{5} - 5\sqrt{5} \\ &= 10\sqrt{5} \text{ units}^2 \end{aligned}$$

3	Correct
2	Value for q , justified to find the maximum area
1	Just the derivative and letting it be zero

Question 34(4 marks)

The diagram shows the graphs of $y = x^3 - 2x$ and $y = x^2$.



The graphs intersect at A , B and the origin O .

- (a) Find the x -coordinates of A and B .

1

$$\text{so } x^3 - 2x = x^2$$

$$\therefore x^3 - x^2 - 2x = 0$$

$$x(x^2 - x - 2) = 0$$

$$x(x - 2)(x + 1) = 0$$

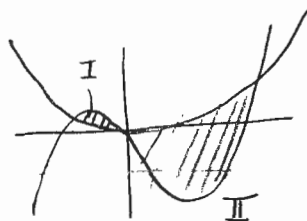
$$\therefore x = 0 \quad \text{or } x = 2 \quad \text{or } x = -1$$

$$\text{so } \boxed{\begin{array}{l} A: x = -1 \\ B: x = 2 \end{array}}$$

r or w

Question 35 continued.

(b) Find the area of the shaded region.



3

$$\begin{aligned}
 \text{I} &= \int_{-1}^0 \overset{\text{cubic} - \text{parab}}{x^3 - 2x - x^2} dx \\
 &= \left[\frac{x^4}{4} - x^2 - \frac{x^3}{3} \right]_{-1}^0 \\
 &= 0 - 0 - 0 - \left(\frac{(-1)^4}{4} - (-1)^2 - \frac{(-1)^3}{3} \right) \\
 &= \frac{5}{12}
 \end{aligned}$$

$$\begin{aligned}
 \text{II} &= \int_0^2 \overset{\text{parab} - \text{cubic}}{x^2 - x^3 + 2x} dx \\
 &= \left[\frac{x^3}{3} - \frac{x^4}{4} + x^2 \right]_0^2 \\
 &= \frac{2^3}{3} - \frac{2^4}{4} + 2^2 - (0 - 0 + 0) \\
 &= \frac{8}{3}
 \end{aligned}$$

$$\begin{aligned}
 \text{so total shaded} &= \frac{5}{12} + \frac{8}{3} \\
 &= \frac{37}{12} \text{ u}^2
 \end{aligned}$$

1
one region
correct bounds,
integrand
correctly
integrated
(2)

Incorrect integrand
but correctly
integrated

2 ECF uses
incorrect A, B value
but integrates
correctly.

3 correct
soln

Question 35 (5 marks)

The time taken, t minutes, for a machine to produce car parts can be modelled by a continuous random variable with probability density function given by

$$f(t) = \begin{cases} k(t-1)^2, & 1 \leq t \leq 2 \\ k\left(2 - \frac{t}{2}\right), & 2 < t \leq 4 \\ 0, & \text{otherwise} \end{cases}$$

- (a) Show that the value of k is $\frac{3}{4}$.

2

$$\int_1^2 k(t-1)^2 dt + \int_2^4 k\left(2 - \frac{t}{2}\right) dt = 1$$

$$\left[\frac{(t-1)^3}{3}\right]_1^2 + k\left[2t - \frac{t^2}{4}\right]_2^4 = 1$$

$$-0 + 2(4) - \frac{16}{4} - \left(4 - \frac{4}{4}\right) = 1$$

$$k\left(\frac{4}{3}\right) = 1$$

$$\therefore \boxed{k = \frac{3}{4}} \text{ as req'd}$$

1 mark

• show integr of $\int_1^2 k(t-1)^2 dt$

$\int_2^4 k\left(2 - \frac{t}{2}\right) dt$

• integral(s) = 1

2 marks

correct soln

- (b) Let $F(t)$ be the cumulative distribution function of $f(t)$, where $F(2.16) = 0.3652$. 3
Find the least amount of time taken to produce 88% of the parts.

$$\underline{1 \leq t \leq 2}: F_1(t) = \frac{3}{4} \left[\frac{(x-1)^3}{3} \right]_1^t \\ = \frac{1}{4} (t-1)^3$$

$$F(2) = \frac{1}{4}$$

$$\underline{2 \leq t \leq 4}: F_2(t) = \frac{3}{4} \left[2x - \frac{x^2}{4} \right]_2^t \\ = \frac{3}{4} \left(2t - \frac{t^2}{4} - 4 + 1 \right) \\ = \frac{3}{4} \left(2t - \frac{t^2}{4} \right) - \frac{9}{4}$$

$$\text{but } F(2) = \frac{1}{4} \text{ so } = \frac{3}{4} \left(2t - \frac{t^2}{4} \right) - 2$$

$$\text{so } F(t) = \begin{cases} \frac{1}{4} (t-1)^3 & 1 \leq t \leq 2 \\ \frac{3}{4} \left(2t - \frac{t^2}{4} \right) - 2 & 2 \leq t \leq 4 \\ 0 & \end{cases}$$

given $2.16 > 2$

$$\therefore F(t) = 0.88 \text{ is } 2 \leq t \leq 4$$

$$\text{so } \frac{3}{4} \left(2t - \frac{t^2}{4} \right) - 2 = 0.88$$

$$\frac{3}{4} \left(2t - \frac{t^2}{4} \right) = 2.88$$

$$2t - \frac{t^2}{4} = 3.84$$

$$t^2 - 8t + 15.36 = 0$$

$$t = \frac{8 \pm \sqrt{64 - 4(1)(15.36)}}{2}$$

$$t = 4.8 \text{ or } t = 3.2$$

but $2 \leq t \leq 4$

$$\text{so } \boxed{t = 3.2 \text{ sec}}$$

1. integrates $F_1(t)$ correctly

$$F(t) = 0.88$$

$$F_1(t) = \frac{1}{4} (t-1)^3$$

$$F_1(t) = \frac{1}{4} t^3 - \frac{3}{4} t^2 + \frac{3}{4} t - \frac{1}{4}$$

$$F_2 = \frac{3}{4} \left(2t - \frac{t^2}{4} - 3 \right)$$

$$F_2 = \frac{3}{4} t - \frac{3}{16} t^2 - \frac{9}{4}$$

2. attempts to solve

$$F_2(t) = 0.88$$

$F_1(t)$ correct AND:

$$F_2(t) = \frac{3}{4} \left(2t - \frac{t^2}{4} \right) - 2$$

$$\text{or } \frac{3}{4} \left(2t - \frac{t^2}{4} - 3 \right) + \frac{1}{4}$$

$$\text{or } \frac{3}{2} t - \frac{3}{16} t^2 - 2$$

$$\text{or } -\frac{3}{4} \left(2 - \frac{t}{2} \right)^2 + 1$$

3. equivalent correct quadratic soln

$$t = 3.2$$

