Student Number:

Ravenswood

2021 HSC Mathematics Advanced

Year 12 Trial Examination

Monday 23 August 2021

General instructions

- Write using a black or blue non–erasable pen
- Calculators approved by NESA may be used
- A Reference Sheet is provided.
- Show all necessary working
- Marks are shown in bold
- Use the Multiple Choice Answer sheet provided

Time Allowed – 2 hours and 10 minutes
Reading time – 10 minutes
Working time – 2 hours

Section I – Pages 2 to 4
- Multiple Choice
- Attempt Questions 1 - 5

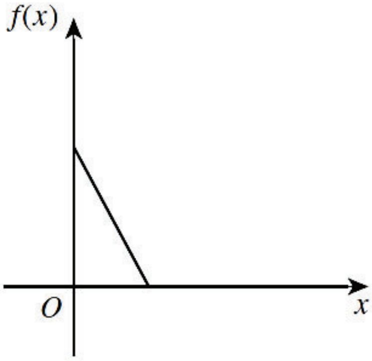
Section II – Pages 5 to 19
- Free Response
- Attempt Questions 6 - 19

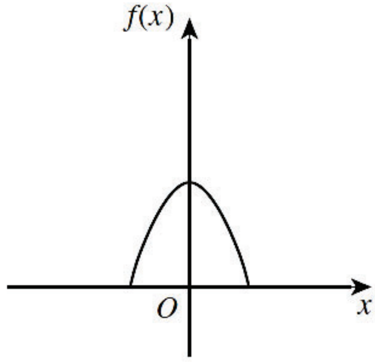
	Section I	Section II	Total
Functions	/1	/7	/8
Trigonometric Functions	/1	/7	/8
Calculus		/15	/15
Exponential and Logarithmic Functions		/3	/3
Statistical Analysis	/2	/17	/19
Series and Financial Mathematics	/1	/11	/12
			/65

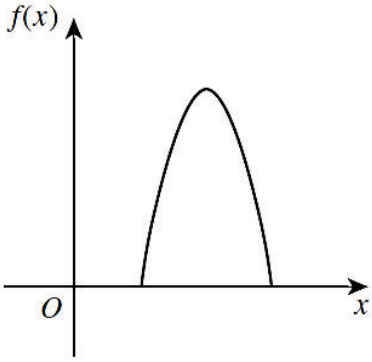
Attempt Questions 1 – 5

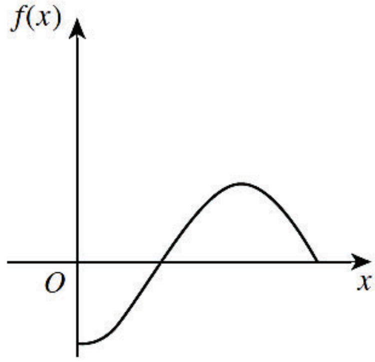
Use the separate multiple choice answer sheet for your answers.

1. Which of the following graphs does NOT represent a probability density function?

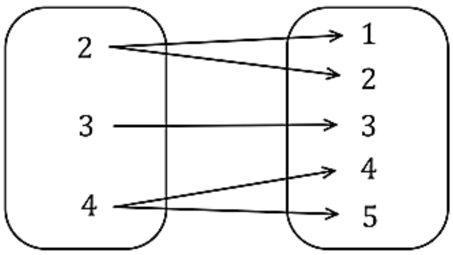
A. 

B. 

C. 

D. 

2. What type of relation is shown?



A. One – to – many

B. Many – to – many

C. One – to – one

D. Many – to – one

3. The table shows the future values of an annuity of $1 for different interest rates over 1 to 8 time periods. The contributions are made at the end of the year.

Time period	*Interest rate*				
	1%	*2%*	*3%*	*4%*	*5%*
1	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500
3	3.0301	3.0604	3.0909	3.1216	3.1525
4	4.0604	4.1216	4.1836	4.2465	4.3101
5	5.1010	5.2040	5.3091	5.4163	5.5256
6	6.1520	6.3081	6.4684	6.6330	6.8019
7	7.2135	7.4343	7.6625	7.8983	8.1420
8	8.2857	8.5830	8.8923	9.2142	9.5491

What is the value of each contribution to an annuity, correct to the nearest dollar, that would provide a future value of $34 800 after 4 years at 2% per annum, compounded half-yearly?

A. $4055

B. $4200

C. $8195

D. $8443

4. $\tan\theta = \frac{2}{3}$ and $\pi < \theta < \frac{3\pi}{2}$. What is the value of cosec θ

A. $-\frac{3}{\sqrt{13}}$

B. $-\frac{\sqrt{13}}{3}$

C. $-\frac{\sqrt{13}}{2}$

D. $-\frac{2}{\sqrt{13}}$

5. Breanna scored the following results in her subjects.

Subject	Mark	Class Mean	Standard Deviation
History	67	62	3
Maths	72	62	10
Art	82	62	18
Biology	68	62	5

In which subject did she perform the best?

A. History

B. Maths

C. Art

D. Biology

End of Section I

Section II **60 Marks**

Attempt Questions 6 – 19

Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.

Your responses should include relevant mathematical reasoning and/or calculations.

Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Question 6 **Marks**

Find $\dfrac{d}{dx}\left(\dfrac{\sin 3x}{e^x}\right)$ **2**

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Question 7 **2**

Evaluate $\displaystyle\int_1^2 \frac{2x+1}{x^2+x}\,dx$

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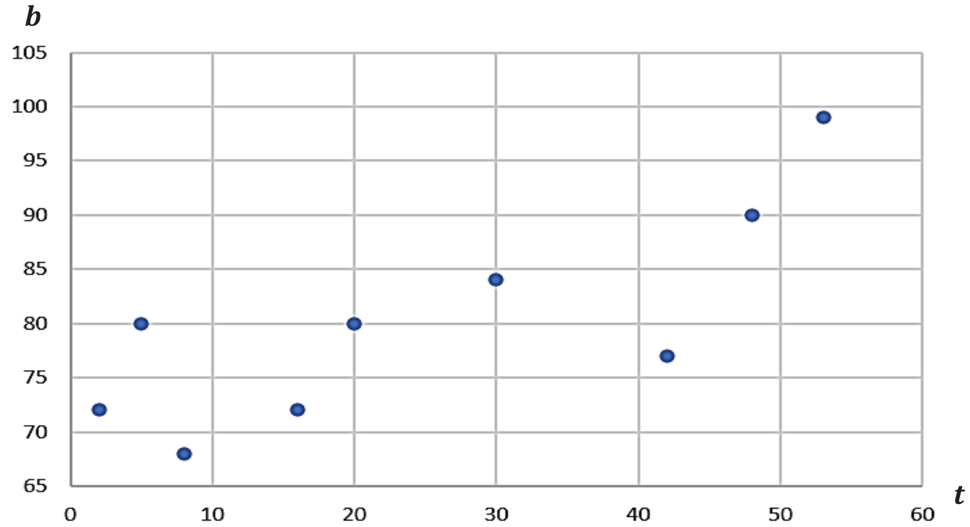
SECTION II

Question 8 **Marks**

Winnie and Adam are watching the semi-final of the Olympic games Men's Basketball match. It is a very exciting match. Winnie is worried that Adam will faint with excitement. She takes his pulse (heart rate) at various intervals during the game. The information is recorded in the table and graphed on the scatterplot below.

t = time since the match started (minutes)	2	5	8	16	20	30	42	48	53
b = heart rate (beats / minute)	72	80	68	72	80	84	77	90	99



a) Find the value of r, the correlation coefficient, correct to 2 decimal places. **1**

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b) Describe the association between the time since the match started and the heart rate. **1**

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c) Find the equation of the line of best fit, correct to 2 decimal places, using the variables given in the table. **2**

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d) Estimate the heart rate in beats/minute when the match had been playing for 35 minutes. **1**

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SECTION II

Question 9 **Marks**

a) Express $y = 2x^2 - 12x + 23$ in the form $y = 2(x - c)^2 + d$ **2**

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b) The graph of $y = x^2$ is transformed into the graph of $y = 2x^2 - 12x + 23$ by the transformations: **3**

- A vertical stretch with scale factor k followed by
- A horizontal translation of p units followed by
- A vertical translation of q units.

Write down the values of k, p and q.

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SECTION II

Question 10 **Marks**

In a school, 30 students take chemistry only, 20 take physics only, 10 take both chemistry and physics, and 60 take neither subject.

a) Find the probability that a student takes physics. **2**

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b) Find the probability that a student takes physics given that the student takes chemistry. **1**

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c) Are the events taking chemistry and taking physics independent? Justify your answer. **1**

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SECTION II

Question 11 **Marks**

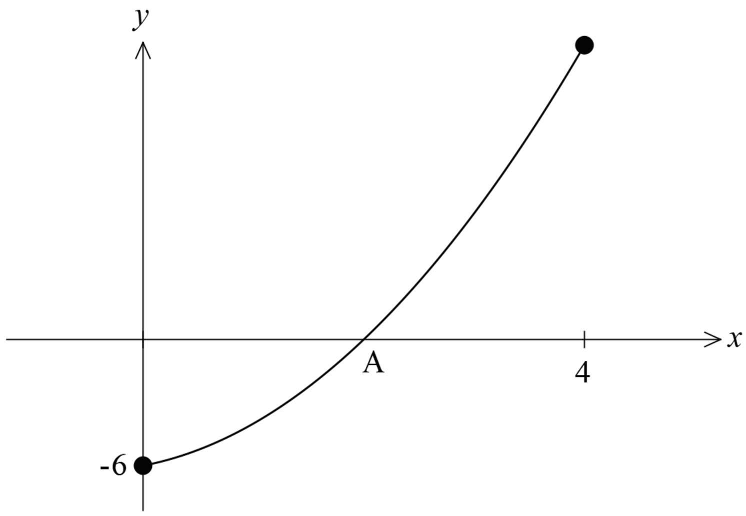
Solve $\log_5(4-x) - 2\log_5 x = 1$, find the value of x. **3**

SECTION II

Question 12 **Marks**

The diagram below shows the graph of $y = x^2 + x - 6$ for $0 \le x \le 4$.

NOT TO SCALE



a) Find the value of A. **1**

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b) Hence find the area bounded by the x axis and the curve for $0 \le x \le 4$. **3**

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SECTION II

Question 13 **Marks**

The heights of a group of students are normally distributed with a mean of 168cm and a standard deviation of 12 cm.

a) A student is chosen at random. Find the probability that the students height is greater than 180 cm. **2**

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b) In this group of students, 2.5% have heights less than d. Find the value of d. **2**

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Question 14

A city is suffering through a drought and adopts a plan to import water from another city. It is Given that the volume of water imported in the first year since the start of this plan is 1.5×10^7 m^3 and in subsequent years, the volume of water imported each year is 10% less than the volume of water imported in the previous year.

a) Show that there was 1.215×10^7 m^3 of water imported in the third year. **1**

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Question 14 continues on next page

Question 14 continued **Marks**

b) Find the total volume of water imported over the first 10 years. Leave your answer in scientific notation, correct to two significant figures. **2**

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c) Explain why the total volume of water imported since the start of the plan will never exceed $1.6 \times 10^8\ \text{m}^3$. **2**

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SECTION II

Question 15 **Marks**

Let $F(x)$ be a cumulative distribution function shown below.

$$F(x)=\begin{cases} 0 & x<1 \\ 0.25(x-1) & 1\le x<3 \\ 0.5+0.125(x-3) & 3\le x<7 \\ 1 & x\ge 7 \end{cases}$$

a) Sketch the graph of $F(x)$. **2**

b) Find $P(X \le 2)$ **1**

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c) Find a probability density function $f(x)$ that would have $F(x)$ as its cumulative distribution function. **1**

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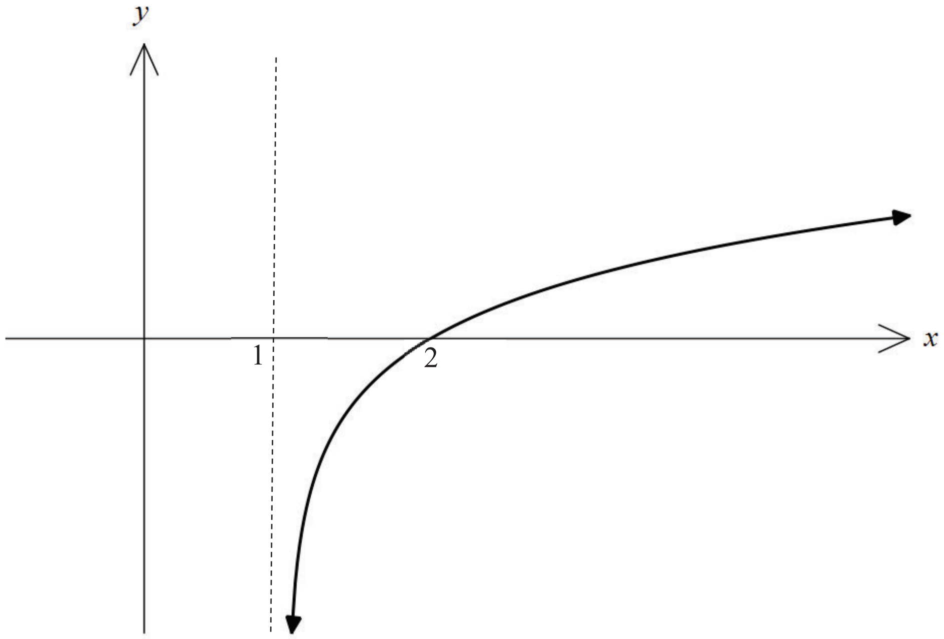
SECTION II

Question 16 **Marks**

The diagram below shows the graph of $y = f(x)$. $x = 1$ is an asymptote. **2**



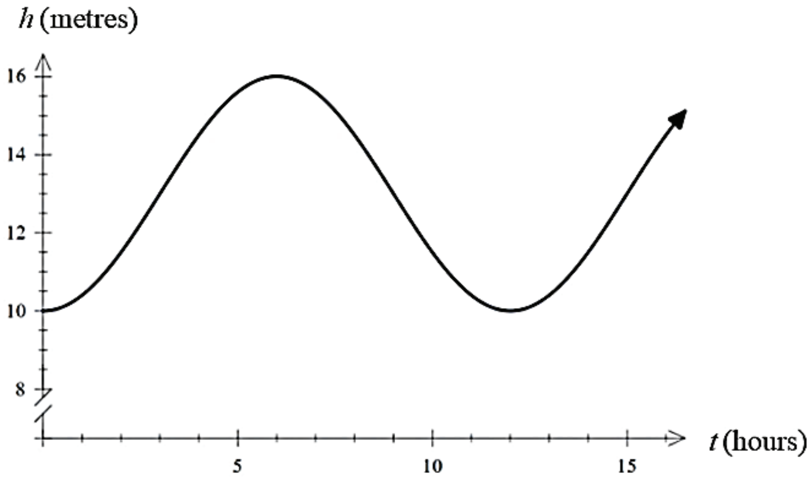
Sketch the graph of $y = f(1 - x)$.

SECTION II

Question 17 **Marks**

The rise and fall in sea level, due to tides, can be modelled by the cosine function below:

$$h(t) = A\cos(bt) + d$$



At 8:00am it is low tide, and the channel is 10m deep. At 2pm it is high tide, and the channel is 16m deep. A ship needs 11.5m of water to sail.

a) Find the values of A, b and d. **3**

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b) Between what times can the ship first sail on that day? **2**

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SECTION II

Question 18 **Marks**

Alex sets up an annuity account to fund his retirement.

- He makes a one-off payment
- The account will receive quarterly interest, and
- Alex will receive quarterly payments.

The annuity is modelled by the recurrence relation:

$$A_0 = P$$

$$A_n = 1.012 \times A_{n-1} - R$$

Alex sets up a spreadsheet according to the recurrence relation above:

	A	B	C	D	E
1	Quarter	Opening balance	Interest Payment	Annuity Payout	Closing Balance
2	1	$750,000	$9,000	17800	*B*
3	2		*A*	17800	
4	3				

a) State the values of P and R. **1**

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b) Calculate the values of A and B. **2**

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Question 18 continues on next page

SECTION II

Question 18 continued

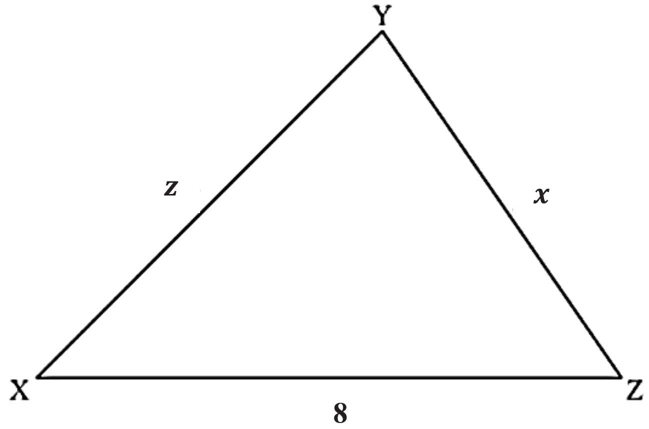
c) Determine how many years can Alex fund his retirement? Answer correct to one decimal place. **3**

SECTION II

Question 19 **Marks**

The triangle XYZ has $XZ = 8, YZ = x,\ XY = z$ as shown below.

The perimeter of the triangle XYZ is 22 units.



a) Show that $\cos Z = \frac{7x-33}{4x}$ **2**

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Question 19 continues on next page

Question 19 continued

b) Let the area of the triangle be A. Show that: **4**

$$A^2 = -33x^2 + 462x - 1089.$$

c) Hence, calculate the maximum area for the triangle, to the nearest square unit. **3**

End of assessment

SECTION II – Extra writing space

If you use this space, clearly indicate which question you are answering.

Student Number:

Ravenswood

2021 HSC Mathematics Advanced

Year 12 Trial Examination

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Exponential and Logarithmic Functions		/3	/3
Statistical Analysis	/2	/17	/19
Series and Financial Mathematics	/1	/11	/12
			/65

Section I **5 Marks**

Attempt Questions 1 – 5

Use the separate multiple choice answer sheet for your answers.

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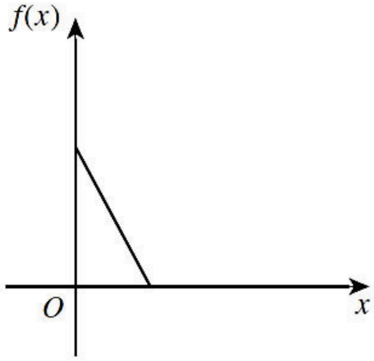
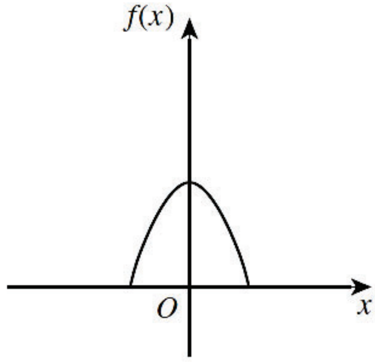
1. Which of the following graphs does NOT represent a probability density function?

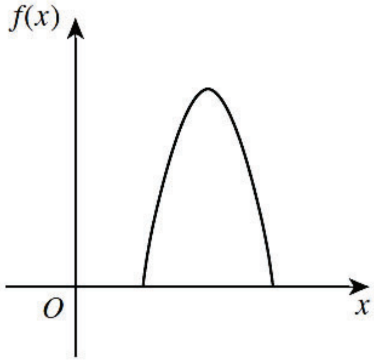
A.



B.



C.



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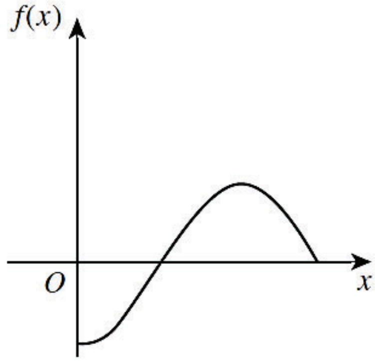
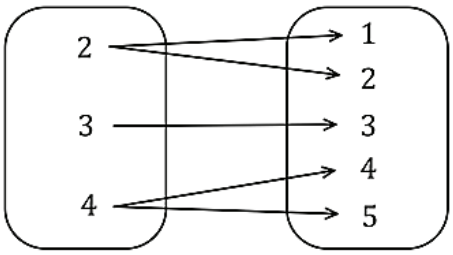


2. What type of relation is shown?



A. One – to – many

B. Many – to – many

C. One – to – one

D. Many – to – one

3. The table shows the future values of an annuity of $1 for different interest rates over 1 to 8 time periods. The contributions are made at the end of the year.

Time period	*Interest rate*				
	1%	*2%*	*3%*	*4%*	*5%*
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7	7.2135	7.4343	7.6625	7.8983	8.1420
8	8.2857	8.5830	8.8923	9.2142	9.5491

What is the value of each contribution to an annuity, correct to the nearest dollar, that would provide a future value of $34 800 after 4 years at 2% per annum, compounded half-yearly?

A. $4055

B. $4200

C. $8195

D. $8443

4. $\tan\theta = \frac{2}{3}$ and $\pi < \theta < \frac{3\pi}{2}$. What is the value of cosec θ

A. $-\frac{3}{\sqrt{13}}$

B. $-\frac{\sqrt{13}}{3}$

C. $-\frac{\sqrt{13}}{2}$

D. $-\frac{2}{\sqrt{13}}$

5. Breanna scored the following results in her subjects.

Subject	Mark	Class Mean	Standard Deviation
History	67	62	3
Maths	72	62	10
Art	82	62	18
Biology	68	62	5

In which subject did she perform the best?

A. History

B. Maths

C. Art

D. Biology

End of Section I

Section II 60 Marks

Attempt Questions 6 – 19

Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.

Your responses should include relevant mathematical reasoning and/or calculations.

Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Question 6 **Marks**

Find $\frac{d}{dx}\left(\frac{\sin 3x}{e^x}\right)$ **2**

$= \frac{e^x \times 3\cos 3x - \sin 3x \times e^x}{(e^x)^2}$ (1)

$= \frac{\cancel{e^x}^{1}(3\cos 3x - \sin 3x)}{e^{\cancel{2x}\, x}}$

$= \frac{3\cos 3x - \sin 3x}{e^x}$ (1)

Question 7 **2**

Evaluate $\int_1^2 \frac{2x+1}{x^2+x}\, dx$

$= \left[\ln(x^2+x)\right]_1^2$ (1)

$= \ln(2^2+2) - \ln(1^2+1)$

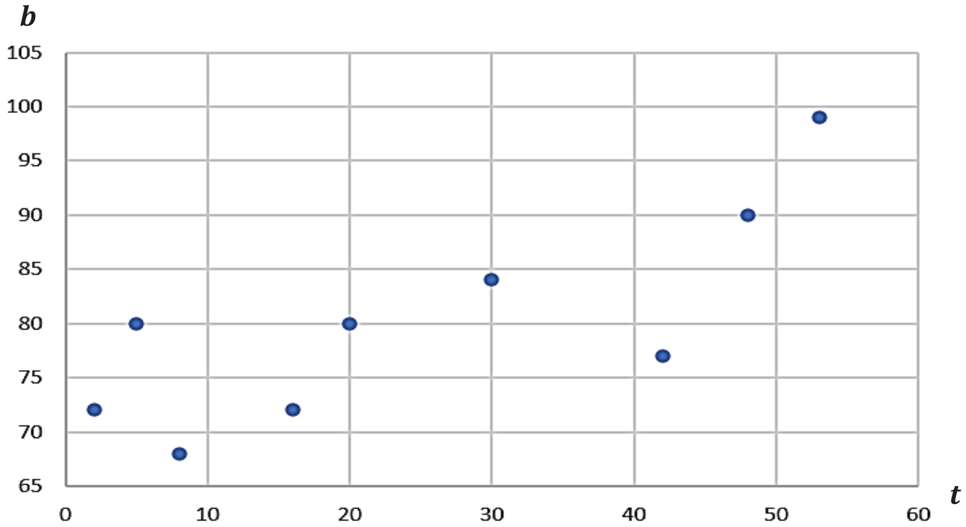
$= \ln 6 - \ln 2$

$= \ln 3$ (1)

Question 8 **Marks**

Winnie and Adam are watching the semi-final of the Olympic games Men's Basketball match. It is a very exciting match. Winnie is worried that Adam will faint with excitement. She takes his pulse (heart rate) at various intervals during the game. The information is recorded in the table and graphed on the scatterplot below.

t = time since the match started (minutes)	2	5	8	16	20	30	42	48	53
b = heart rate (beats / minute)	72	80	68	72	80	84	77	90	99



a) Find the value of r, the correlation coefficient, correct to 2 decimal places. **1**

$r = 0.7967$

$\doteqdot 0.80$ (1)

b) Describe the association between the time since the match started and the heart rate. **1**

strong, positive (1)

c) Find the equation of the line of best fit, correct to 2 decimal places, using the variables given in the table. **2**

$a = 70.175$ $b = 0.40368...$ $b = 70.18 + 0.40t$

$\doteqdot 70.18$ $\doteqdot 0.40$ (1)

(1)

d) Estimate the heart rate in beats/minute when the match had been playing for 35 minutes. **1**

$t = 35$ $b = 70.18 + 0.40 \times 35$

$= 84.18$ beats/minute (1)

SECTION II

Question 9 **Marks**

a) Express $y = 2x^2 - 12x + 23$ in the form $y = 2(x - c)^2 + d$ **2**

$y = 2(x^2 - 6x) + 23$

$= 2(x^2 - 6x + 9) + 23 - 18$ (1)

$= 2(x-3)^2 + 5$ (1)

b) The graph of $y = x^2$ is transformed into the graph of $y = 2x^2 - 12x + 23$ by the **3**

the transformations:

- A vertical stretch with scale factor k followed by
- A horizontal translation of p units followed by
- A vertical translation of q units.

Write down the values of k, p and q.

$y = 2(x-3)^2 + 5$

$k = 2$ (1) $p = 3$ (1) $q = 5$ (1)

Question 10 **Marks**

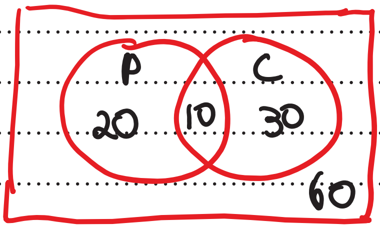
In a school, 30 students take chemistry only, 20 take physics only, 10 take both chemistry and physics, and 60 take neither subject.

a) Find the probability that a student takes physics. **2**

$30+20+10+60=120$ $P(\text{Physics}) = \frac{20+10}{120}$ (1)

$= \frac{1}{4}$ (1)

b) Find the probability that a student takes physics given that the student takes chemistry. **1**



$P(P|C) = \frac{10}{40}$

$= \frac{1}{4}$ (1)

c) Are the events taking chemistry and taking physics independent? Justify your answer. **1**

If independent then $P(P|C) = P(P)$ (1)

$P(P|C) = \frac{1}{4}$ $P(P) = \frac{1}{4}$

$\therefore$ taking chemistry and taking physics are independent

SECTION II

Question 11 **Marks**

Solve $\log_5(4 - x) - 2\log_5 x = 1$, find the value of x. **3**

$\log_5\left(\frac{4-x}{x^2}\right) = 1$ (1)

$5^1 = \frac{4-x}{x^2}$

$5x^2 + x - 4 = 0$

$(5x-4)(x+1) = 0$

$\therefore x = \frac{4}{5}$ OR $x = -1$ (1)

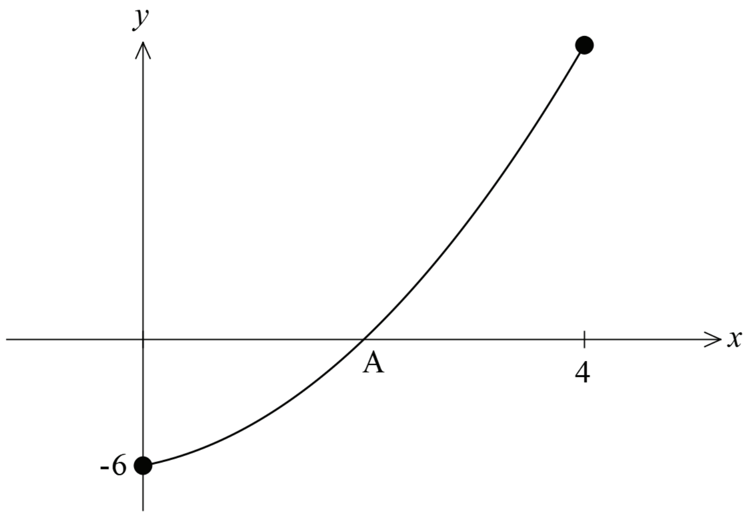
But $x > 0$ $\therefore x = \frac{4}{5}$ (1)

SECTION II

Question 12 **Marks**

The diagram below shows the graph of $y = x^2 + x - 6$ for $0 \leq x \leq 4$.

NOT TO SCALE



a) Find the value of A. **1**

$y = x^2 + x - 6$

$= (x+3)(x-2)$

if $y=0$ $x = -3$ OR $x=2$

as $A > 0$ then $A = 2$ (1)

b) Hence find the area bounded by the x axis and the curve for $0 \leq x \leq 4$. **3**

Area $= \left| \int_0^2 x^2+x-6 \, dx \right| + \int_2^4 x^2+x-6 \, dx$ (1)

$= \left| \left[\frac{x^3}{3} + \frac{x^2}{2} - 6x \right]_0^2 \right| + \left[\frac{x^3}{3} + \frac{x^2}{2} - 6x \right]_2^4$ (1)

$= \left| \frac{8}{3} + 2 - 12 - 0 \right| + \left(\frac{64}{3} + 8 - 24 \right) - \left(\frac{8}{3} + 2 - 12 \right)$

$= \frac{22}{3} + \frac{16}{3} - - \frac{22}{3}$

$= \frac{60}{3}$

$= 20$ units^2 (1)

SECTION II

Question 13 **Marks**

The heights of a group of students are normally distributed with a mean of 168cm and a standard deviation of 12 cm.

a) A student is chosen at random. Find the probability that the students height is greater than 180 cm. **2**

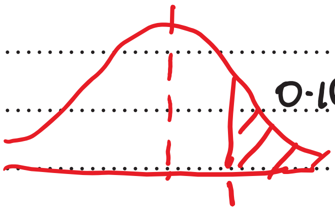
$z = \frac{x-\mu}{\sigma}$

$= \frac{180-168}{12}$

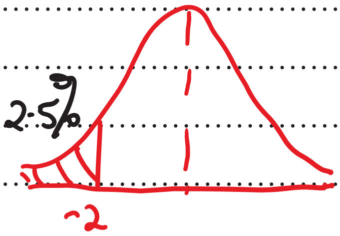
$= 1$ (1)

$\therefore P(X > 180) = P(Z > 1)$

$= 0.16$ (1)



b) In this group of students, 2.5% have heights less than d. Find the value of d. **2**



$z = -2$ (1)

$-2 = \frac{d-168}{12}$

$-24 + 168 = d$

$d = 144$ (1)

Question 14

A city is suffering through a drought and adopts a plan to import water from another city. It is Given that the volume of water imported in the first year since the start of this plan is 1.5×10^7 m^3 and in subsequent years, the volume of water imported each year is 10% less than the volume of water imported in the previous year.

a) Show that there was 1.215×10^7 m^3 of water imported in the third year. **1**

$A_1 = 1.5 \times 10^7$

$A_2 = 0.9 \times 1.5 \times 10^7$

$A_3 = 0.9 \times 0.9 \times 1.5 \times 10^7$ (1)

$= 1.215 \times 10^7$

$\therefore$ in 3rd year 1.215×10^7 m^3 is imported

Question 14 continues on next page

Question 14 continued **Marks**

b) Find the total volume of water imported over the first 10 years. Leave your answer in scientific notation, correct to two significant figures. **2**

$n=10 \quad a=1.5\times10^7 \quad r=0.9$

$S_{10} = \frac{1.5\times10^7(1-0.9^{10})}{1-0.9}$ (1)

$= \frac{1.5(1-0.9^{10})\times10^7}{0.1}$

$= 9.76898\ldots \times 10^7$

$\doteqdot 9.8\times10^7\ m^3$ (1)

$\therefore$ Total volume over 10 years $\doteqdot 9.8\times10^7\ m^3$

(−1) rounding penalty

c) Explain why the total volume of water imported since the start of the plan will never exceed 1.6×10^8 m^3. **2**

$S_\infty = \frac{1.5\times10^7}{1-0.9}$

$= \frac{1.5\times10^7}{0.1}$

$= 1.5\times10^8$ (1)

As $1.5\times10^8 < 1.6\times10^8$ then the total volume will never exceed 1.6×10^8 (1)

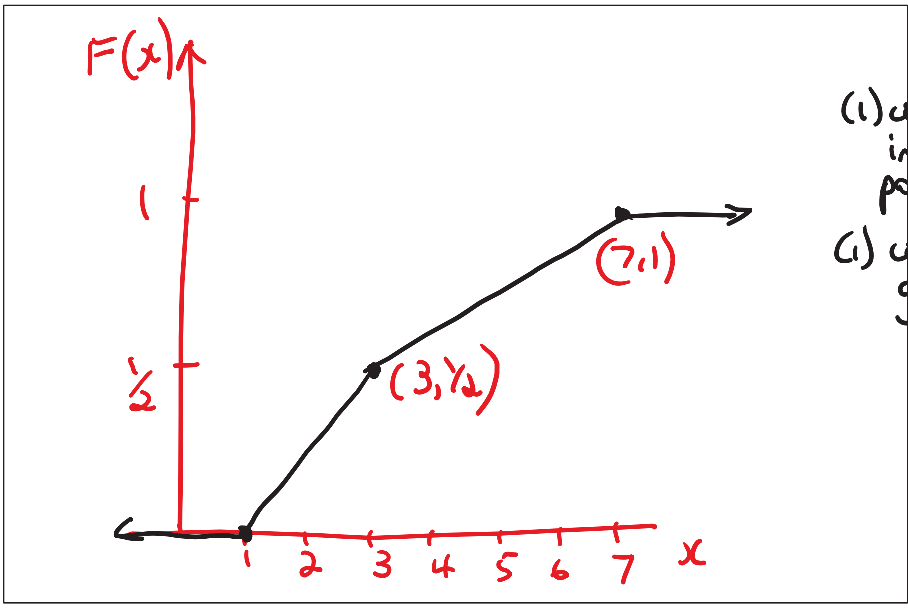
SECTION II

Question 15 **Marks**

Let $F(x)$ be a cumulative distribution function shown below.

$$F(x) = \begin{cases} 0 & x < 1 \\ 0.25(x-1) & 1 \le x < 3 \\ 0.5 + 0.125(x-3) & 3 \le x < 7 \\ 1 & x \ge 7 \end{cases}$$

a) Sketch the graph of $F(x)$. 2



(1) correct intersection points
(1) correct graph

b) Find $P(X \le 2)$ 1

$P(X \le 2) = F(2)$
$= 0.25(2-1)$
$= 0.25$ (1)

c) Find a probability density function $f(x)$ that would have $F(x)$ as its cumulative distribution function. 1

$f(x) = F'(x)$

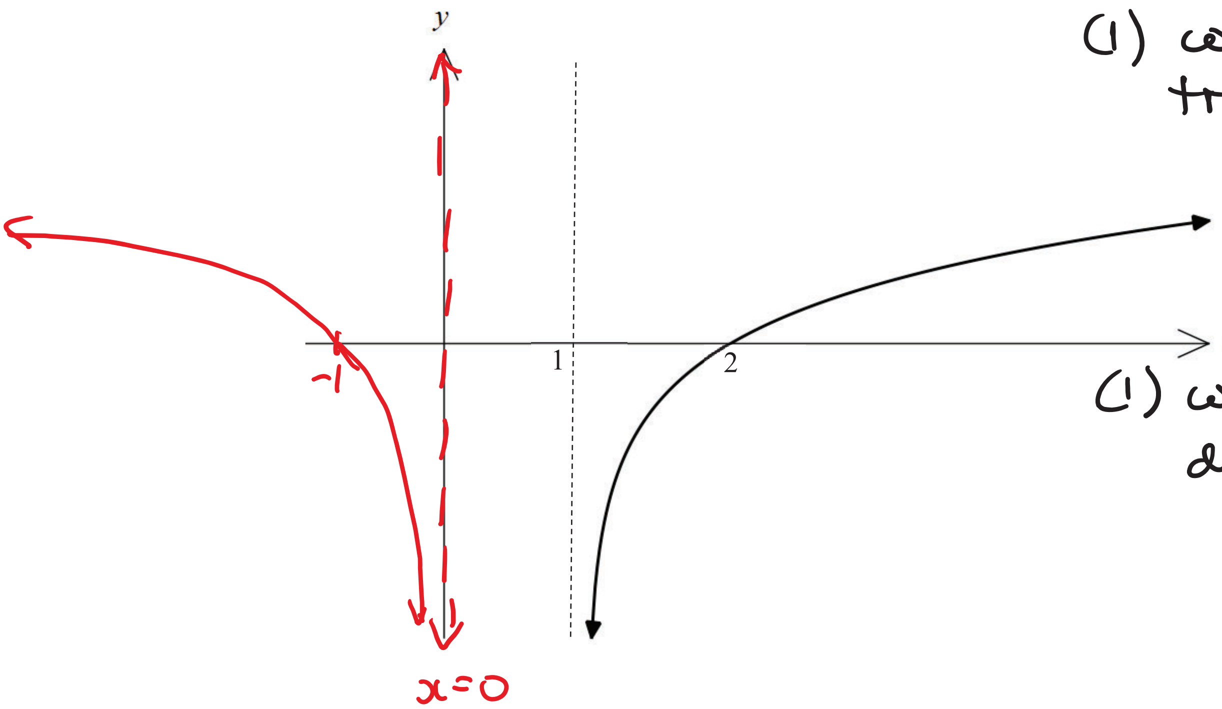
$F(x) = 0 \quad x<0$
$= \frac{1}{4}x - \frac{1}{4} \quad 1 \le x < 3$
$= \frac{1}{2} + \frac{1}{8}x - \frac{3}{8} \quad 3 \le x < 7$
$= 1 \quad x \ge 7$

$\therefore f(x) = \frac{1}{4} \quad 1 \le x < 3$
$= \frac{1}{8} \quad 3 \le x < 7$
$= 0 \quad$ elsewhere (1)

SECTION II

Question 16 **Marks**

The diagram below shows the graph of $y = f(x)$. $x = 1$ is an asymptote. **2**



(1) correct translation

(1) correct dilation

Sketch the graph of $y = f(1 - x)$.

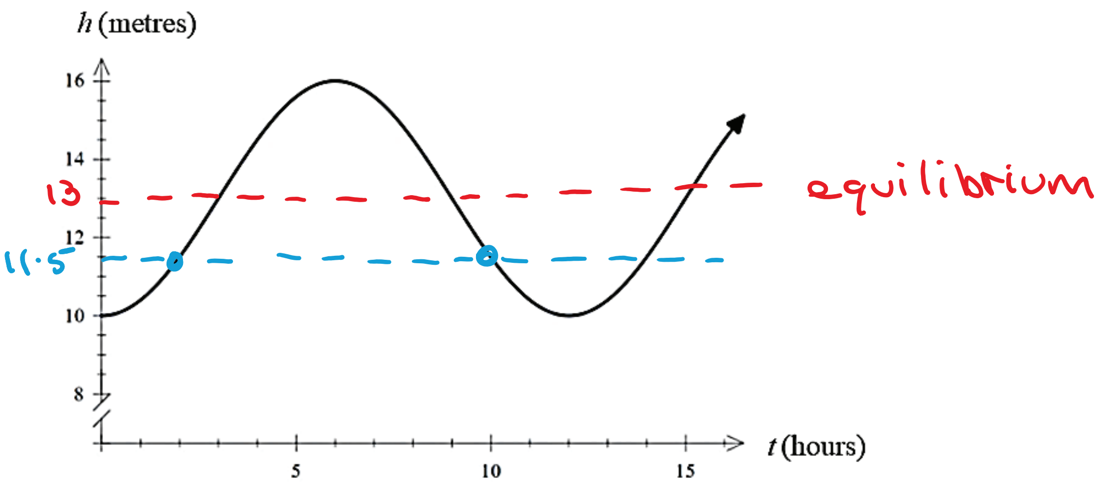
$y = f(-x+1)$

SECTION II

Question 17 **Marks**

The rise and fall in sea level, due to tides, can be modelled by the cosine function below:

$$h(t) = A\cos(bt) + d$$



At 8:00am it is low tide, and the channel is 10m deep. At 2pm it is high tide, and the channel is 16m deep. A ship needs 11.5m of water to sail.

a) Find the values of A, b and d. **3**

$d = 13$ (1) $\quad \frac{2\pi}{b} = 12$

$A = -3$ (1)

$b = \frac{2\pi}{12}$

$= \frac{\pi}{6}$ (1)

$\therefore h(t) = -3\cos\frac{\pi}{6}t + 13$

b) Between what times can the ship first sail on that day? **2**

$h(t) = 11.5 \qquad 11.5 = -3\cos\frac{\pi}{6}t + 13$

$-1.5 = -3\cos\frac{\pi}{6}t$

$\cos\frac{\pi}{6}t = \frac{1}{2} \qquad$ 1 4 $\frac{\pi}{3}$

$\therefore \frac{\pi}{6}t = \frac{\pi}{3}, \frac{5\pi}{3}$ (1)

$\therefore t = 2, 10$

$\therefore$ between $t = 2$ & $t = 10$, ie between 10am & 6pm (1)

$t = 0$ 8am

$t = 2$ 10am

$t = 10$ 6pm

Question 18 **Marks**

Alex sets up an annuity account to fund his retirement.

- He makes a one-off payment
- The account will receive quarterly interest, and
- Alex will receive quarterly payments.

The annuity is modelled by the recurrence relation:

$$A_0 = P$$

$$A_n = 1.012 \times A_{n-1} - R$$

Alex sets up a spreadsheet according to the recurrence relation above:

	A	B	C	D	E
1	Quarter	Opening balance	Interest Payment	Annuity Payout	Closing Balance
2	1	\$750,000	\$9,000	17800	*B*
3	2	741 200	*A*	17800	
4	3		8894.40		

741200

a) State the values of P and R. **1**

P= 750000 R= 17800 (1)

R/W

b) Calculate the values of A and B. **2**

B= 750000 + 9000 − 17800

= 741 200 (1)

A = 741200 × 0.012

= 8894.40 (1)

Question 18 continues on next page

Question 18 continued

c) Determine how many years can Alex fund his retirement? Answer correct to one decimal place. **3**

$A_1 = 1.012 \times 750000 - 17800$

$A_2 = (1.012 \times 750000 - 17800)1.012 - 17800$

$= 750000(1.012)^2 - 17800(1 + 1.012)$

$A_3 = 750000(1.012)^3 - 17800(1 + 1.012 + 1.012^2)$

$\vdots$

(1) $A_n = 7500000(1.012)^n - 17800\underbrace{(1 + 1.012 + \dots + 1.012^{n-1})}_{G.P}$

$A_n = 0$

$0 = 750000(1.012)^n - 17800\left[\frac{1(1.012^n - 1)}{1.012 - 1}\right]$

(1)

$= 750000(1.012)^n - \frac{17800}{0.012}\left[1.012^n - 1\right]$

$= 750000(1.012)^n - \frac{17800}{0.012}1.012^n + \frac{17800}{0.012}$

$\frac{17800}{0.012} = 1.012^n\left[\frac{17800}{0.012} - 750000\right]$

$1.012^n = \frac{17800}{0.012} \div \left[\frac{17800}{0.012} - 750000\right]$

$= 2.02272\dots$

$n \ln 1.012 = \ln 2.02272\dots$

$n = \frac{\ln 2.02272\dots}{\ln 1.012}$

$n = 59.055\dots$

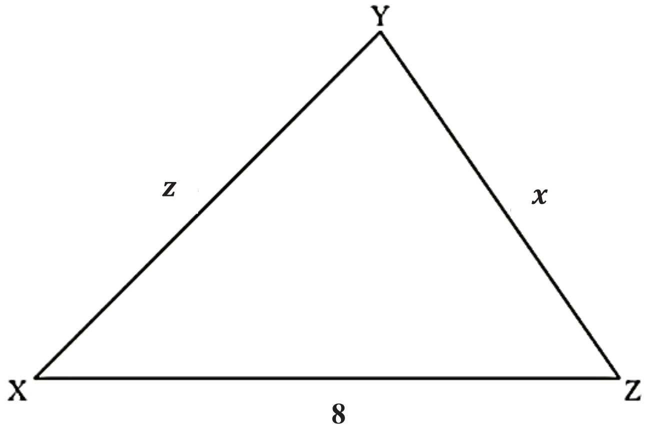
(1) $\therefore$ it takes $59.055\dots$ quarters or $14.7637\dots$ years or approx 14.8 years

SECTION II

Question 19 **Marks**

The triangle **XYZ** has $\mathbf{XZ = 8, YZ = x,\ XY = z}$ as shown below.

The perimeter of the triangle **XYZ** is 22 units.



a) Show that $\mathbf{\cos Z} = \dfrac{\mathbf{7x-33}}{\mathbf{4x}}$ **2**

$$\cos Z = \frac{8^2 + x^2 - (14-x)^2}{2 \times 8 \times x} \quad (1)$$

$x + z + 8 = 22$

$x + z = 14$

$z = 14 - x$

$$= \frac{64 + x^2 - (196 - 28x + x^2)}{16x}$$

$$= \frac{64 + x^2 - 196 + 28x - x^2}{16x}$$

$$= \frac{28x - 132}{16x} \quad (1)$$

$$= \frac{7x - 33}{4x}$$

Question 19 continues on next page

Question 19 continued

b) Let the area of the triangle be $\boldsymbol{A}$. Show that: **4**

$$A^2 = -33x^2 + 462x - 1089.$$

$A = \frac{1}{2} \times 8 \times x \times \sin Z$

$A^2 = 16x^2 \sin^2 Z$ (1)

$= 16x^2 (1 - \cos^2 Z)$

$= 16x^2 \left(1 - \left(\frac{7x - 33}{4x}\right)^2\right)$ (1)

$= 16x^2 \left[1 - \frac{(49x^2 - 462x + 1089)}{16x^2}\right]$ (1)

$= 16x^2 \left[\frac{16x^2 - 49x^2 + 462x - 1089}{16x^2}\right]$

$= -33x^2 + 462x - 1089$ (1)

c) Hence, calculate the maximum area for the triangle, to the nearest square unit. **3**

$\frac{d}{dx}(A^2) = -66x + 462$

Let $\frac{d}{dx}(A^2) = 0$ $\quad -66x + 462 = 0$

$x = 7$ (1)

$\frac{d^2}{dx^2}(A^2) = -66 < 0$ test (1)

$\therefore$ max value of A^2 occurs when $x = 7$

$x = 7$ $\quad A^2 = -33(7)^2 + 462(7) - 1089$

$= 528$

$\therefore A = \sqrt{528}$

$= 22.978...$ (1)

$\doteqdot 23$ units^2

End of assessment