Student's Name:____________________________ Teacher's Code : NHM PPC MXF FEY JPN

Saint Ignatius' College, Riverview
Mathematics Assessment Task
2023

Year 12
Mathematics
Task 4
Trial HSC Examination
Date : 17th August 2023

General Instructions:

- **Reading time : 10 minutes**
- **Time Allowed: 3 hours**
- Write using blue or black pen only
- Board approved calculators may be used
- Attempt all questions in the space provided on the paper
- Write **your name** and **your teacher's code** in the positions indicated
- Marks may not be awarded for missing or carelessly arranged working.

Teachers :

- Mr N Mushan **NHM**
- Mr P Collins **PPC**
- Dr M Furtado **MXF**
- Mrs F Yates **FEY**
- Mr J Newey **JPN**

Topics Examined:

Section A **10 Marks**
Multiple Choice

Section B
Short Answer

Question 11-34 **90 Marks**

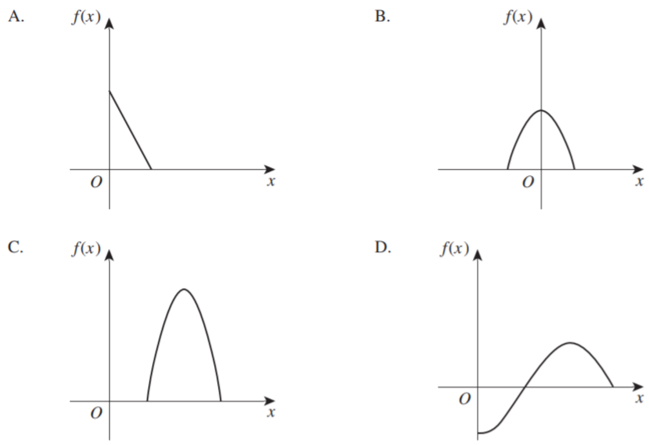
Total **100 Marks**

Multiple Choice

Questions 1 – 10 are multiple choice.

Select the best response and circle your answer on the corresponding answer sheet provided.

1. Which of the following graphs does NOT represent a probability density function?



2. The table shows the future value of a $1 annuity for varying interest rates over different time periods.

Time period	*Interest rate*				
	1%	*2%*	*3%*	*4%*	*5%*
1	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500
3	3.0301	3.0604	3.0909	3.1216	3.1525
4	4.0604	4.1216	4.1836	4.2465	4.3101
5	5.1010	5.2040	5.3091	5.4163	5.5256
6	6.1520	6.3081	6.4684	6.6330	6.8019
7	7.2135	7.4343	7.6625	7.8983	8.1420
8	8.2857	8.5830	8.8923	9.2142	9.5491

What is the present value of an annuity, correct to the nearest dollar, that would provide a future value of $52 200 after 4 years at 2% per annum, compounded half-yearly?

A. *$6082.50* **B.** *$6300.01*

C. *$12292.50* **D.** *$12664.50*

3. The amount of water that Elaine uses to wash her car is normally distributed with a mean of 20 litres and a standard deviation of 4 litres. On what percentage of occasions would Elaine expect to use between 12 litres and 24 litres of water to wash her car?

A. 34% **B.** 51%

C. 68% **D.** 81.5%

4. If $P(A)=0.4$, $P(B)=0.5$ and $P\left(A|B\right)=0.2$, what is the value of $P\left(A\cup B\right)$?

A. 0.6 **B.** 0.7

C. 0.8 **D.** 0.9

5. What is the domain and range of a circle with the equation $x^2+6x+y^2=7$

A. domain = $[-3-\sqrt{7},-3+\sqrt{7}]$; range = $[-\sqrt{7},\sqrt{7}]$

B. domain = $[3-\sqrt{7},3+\sqrt{7}]$; range = $[-\sqrt{7},\sqrt{7}]$

C. domain = $[-1,7]$; range = $[-4,4]$

D. domain = $[-7,1]$; range = $[-4,4]$

6. What is the maximum value of $y=3\sin\left(\frac{x}{2}\right)-1$ and the value of x for which this occurs in the domain $[0,2\pi]$?

A. the maximum value of $y=3\sin\left(\frac{x}{2}\right)-1$ is 2 occurring at $x=\frac{\pi}{2}$.

B. the maximum value of $y=3\sin\left(\frac{x}{2}\right)-1$ is 3 occurring at $x=\frac{\pi}{2}$.

C. the maximum value of $y=3\sin\left(\frac{x}{2}\right)-1$ is 2 occurring at $x=\pi$.

D. the maximum value of $y=3\sin\left(\frac{x}{2}\right)-1$ is 3 occurring at $x=\pi$.

7. What is the value of a if $\int_a^1 (4-x)\,dx=8$?

A. -2 **B.** -1

C. 0 **D**. $\frac{1}{2}$

8. What is the value of $\int_0^{\ln 5} -e^{-x}\,dx$?

A. $-\ln 5$ **B.** $e^{-\ln 5}$

C. $-\dfrac{4}{5}$ **D.** $-e^{-\ln 5}$

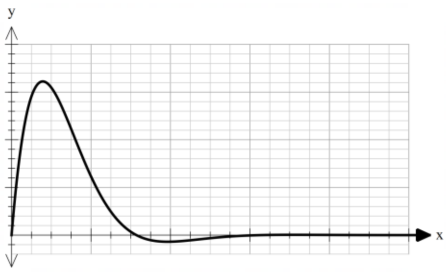
9. The function $y = f(x)$ has a stationary point at (2,1).

Which one of the following is definitely a stationary point of $y = -f(\frac{x}{3}) + 4$

A. $\left(\dfrac{2}{3}, -3\right)$ **B.** $\left(\dfrac{2}{3}, 3\right)$

C. $(6,3)$ **D.** $(6,-3)$

10. The graph of $y = e^{-x}\sin x$ in the domain $[0, \infty)$



The x-coordinate of the minimum turning point is:

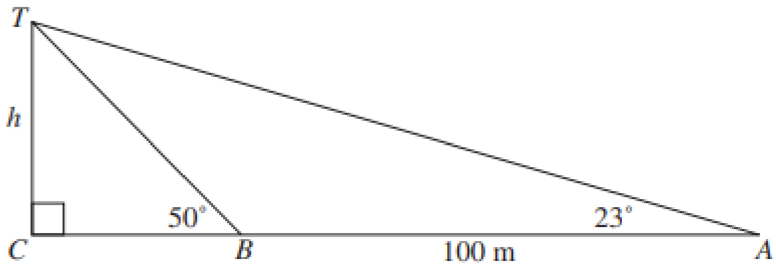
A. $\dfrac{\pi}{4}$ **B.** $\dfrac{3\pi}{4}$

C. $\dfrac{5\pi}{4}$ **D.** $\dfrac{7\pi}{4}$

END OF SECTION A

QUESTION 11 (4 marks)

The elevation of a tower from point *A* is 23°.
From a point *B*, 100 metres closer to the tower and on the same horizontal level as *A* the elevation is 50°.



(a) Show that $\angle BTA = 27^\circ$. **(1)**

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(b) Show that $BT = \dfrac{100\sin 23^\circ}{\sin 27^\circ}$ **(1)**

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(c) Find the height (h) of the tower **(2)**
[Give your answer correct to one decimal place]

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QUESTION 12 (3 marks)

The table below shows the marks out of 40 for nine students on two tests.
The tests cover similar content, but on one test the students **are allowed** to use a calculator, while on the other test, calculators are **not allowed**.

Student	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*
Calculator allowed (x)	8	10	14	14	16	22	28	32	36
Non – calculator (y)	12	14	6	4	28	16	24	22	18

(a) Use the marks for students A to I to find the value of the **Pearson correlation coefficient** between the marks x on the test with the calculator allowed and the marks y on the non-calculator test, and the **equation of the least squares regression line** of y on x **(2)**

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(b) Student **J** scored 25 on the test with calculators allowed but missed the non-calculator test.

Use the least squares regression line of y on x to estimate the mark of student **J** on the non-calculator test. **(1)**

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QUESTION 13 (8 marks)

(a) Find the nature of the stationary points of the curve $y = 2x^3 - 3x^2 - 12x + 2$ **(3)**

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(b) Draw a neat sketch of $y = 2x^3 - 3x^2 - 12x + 2$ showing the stationary points and the point of inflection for the domain $-2 \le x \le 3$. **(3)**

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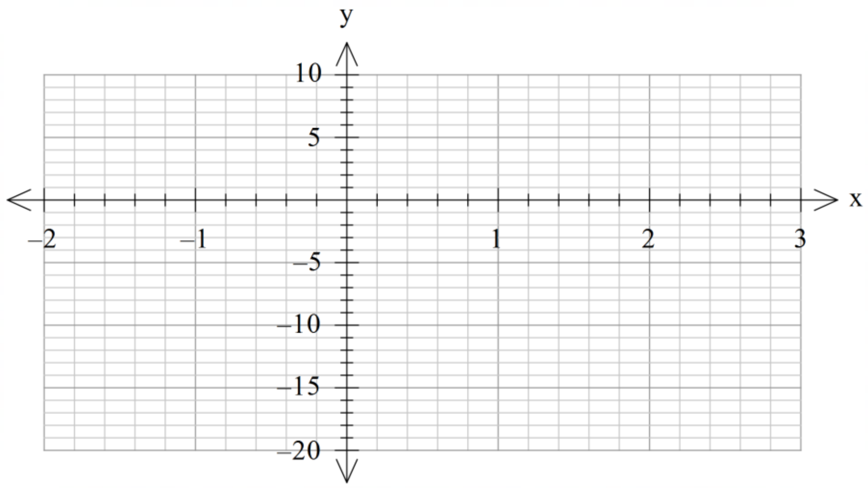
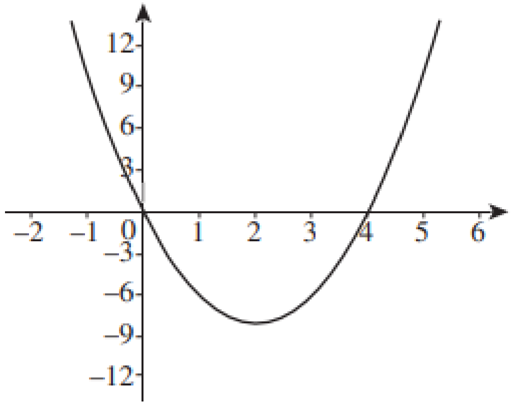


(c) Given the graph of $\frac{dy}{dx}$ below, on the same graph, draw a sketch of **(2)**

a possibility for the graph of *y*.



QUESTION 14 (3 marks)

(a) Find the equation of the tangent to the curve $y=\sqrt{25-x^2}$ at the point (3, 4). **(3)**

(Write your answer in general form)

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Student's Name:____________________________ Teacher's Code : NHM PPC MXF FEY JPN

QUESTION 15 (4 marks)

(a) Differentiate $y = (\ln x)^{-1}$ with respect to x. **(2)**

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(b) Differentiate $y = (x^2 + 4)\ \tan 3x$ with respect to x. **(2)**

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QUESTION 16 (2 marks)

Evaluate $\int_0^{\frac{\pi}{2}} 1 - \cos x\ dx.$ **(2)**

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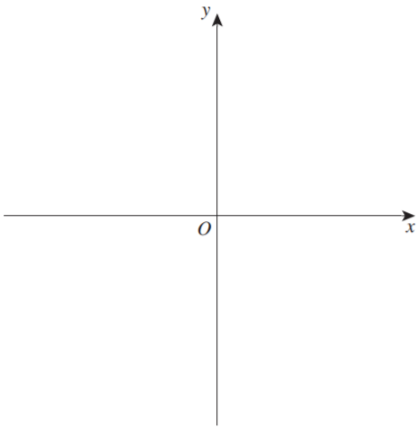
QUESTION 17 (2 marks)

The function $y = f(x)$ is continuous for all values of x. **(2)**
The following is known of the function's properties.

- When $x < -1$, $f'(x) > 0$ and $f''(x) < 0$.
- When $x > -1$, $f'(x) > 0$ and $f''(x) > 0$.
- $f(-1) = f'(-1) = f''(-1) = 0$

On the axes below, sketch the graph of $y = f(x)$, labelling any x-intercepts.



QUESTION 18 (3 marks)

Solve the pair of simultaneous equations: **(3)**

$\log_2 x + \log_2 y = 3 \quad ...(1)$
$\log_x y = 2 \quad ...(2)$

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QUESTION 19 (2 marks)

An arithmetic sequence has first term 29 and a common difference of -*4*. **(2)**
Find the sum of the first *n* terms of this sequence, calculate the least value of *n* for which the sum is negative.

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QUESTION 20 (3 marks)

Shirley uses her car every day. Sometimes, she needs a few attempts to start her car.
Let the discrete random variable X represent the number of attempts needed.
The probability function for the random variable X is given by:

$$P(X=x)=\begin{cases} a & x=1 \\ \frac{1}{2}P(X=x-1) & x=2,\ 3,\ 4,\ 5 \\ 0 & \text{for all other values of } x \end{cases}$$

(a) Show that $a=\dfrac{16}{31}$ **(2)**

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(b) The probability distribution table for the discrete random variable X is shown. **(1)**

x	1	2	3	4	5
$P(X=x)$	$\frac{16}{31}$	$\frac{8}{31}$	$\frac{4}{31}$	$\frac{2}{31}$	$\frac{1}{31}$

Shirley claims that after an entire year, she needs an average of two attempts to successfully start her car. Calculate the expected value (correct to 2 decimal places).

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QUESTION 21 (5 marks)

Complete the table of $y = 2^x$.

The table gives the values of $y = 2^x$ for *x*.

x	0	2	4	6
y	1	4	16	

(a) Complete the table above. **(1)**

(b) Hence, use the trapezoidal rule to find an approximation for $\int_0^6 2^x \, dx$. **(2)**

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(c) Use integration to evaluate $\int_0^6 2^x \, dx$ (write your answer correct to 2 decimal places) **(2)**

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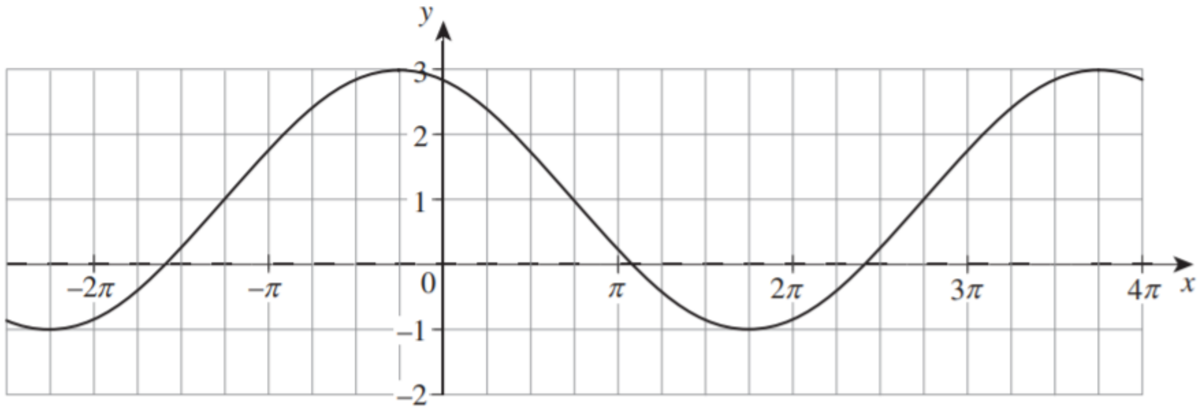
QUESTION 22 (3 marks)

The graph is a function of the form $y = 2\cos(a(x+b)) + c$.



Consider the equation of the function to determine the values of a, b and c **(3)**

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QUESTION 23 (3 marks)

At time t hours after 12:00 am, the height h metres of the deck of a boat **(3)**

above the level of the jetty is given by $h = 2\cos\left(\frac{4\pi}{25}t\right) + 1$.

Find correct to the nearest minute the first time after 12:00 am the boat **is level** with the jetty.

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QUESTION 24 (4 marks)

The students in a school were surveyed on the number of hours of sleep per week.
The results were normally distributed.
The survey indicated that 95% of students had between 42 and 54 hours of sleep per week.

(a) Show that the mean number of hours of sleep per week is 48. **(1)**

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(b) What was the standard deviation? **(1)**

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(c) How many hours of sleep per week would a student have who recorded a *z*-score of -2.5? **(1)**

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(d) What percentage of students would have indicated they had between 51 and 57 hours of sleep per week? **(1)**

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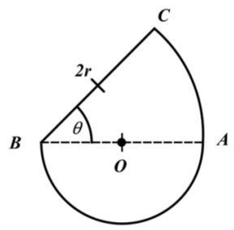
QUESTION 25 (5 marks)



The diagram shows the cross-section of a cam (a part in a car engine), consisting of a semi-circle diameter AB, centre O and radius r units; and a sector ABC with centre B, radius $2r$ units and angle θ. The area of the cam ABC is $\frac{1}{2}$ units^2.

(a) Show that $\theta = \frac{1-\pi r^2}{4r^2}$ **(2)**

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(b) Find the value of r (correct to 1 decimal place) which gives the least perimeter of the cam. **(3)**

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QUESTION 26 (3 marks)

The continuous random variable, X, has the probability density function

$$f(x) = \frac{1}{2}\sin x \text{ for } 0 \le x \le \pi .$$

(a) Find the cumulative distribution function. **(2)**

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(b) Find the first quartile of the distribution. **(1)**

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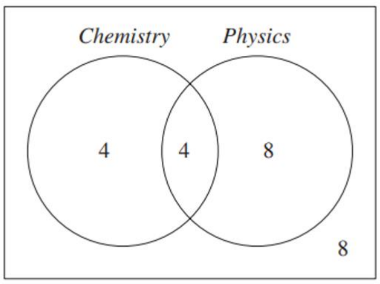
Student's Name:___________________________ Teacher's Code : NHM PPC MXF FEY JPN

QUESTION 27 (4 marks)

A teacher surveyed his class of 24 students about the subjects that they have chosen to study in the following year.

The results indicate that:
• 8 students chose to study neither Physics nor Chemistry
• 8 students chose to study Chemistry
• 12 students chose to study Physics.

A Venn diagram showing this information has been drawn below.



(a) A student choosing to study Chemistry is event 1. **(2)**
A student choosing to study Physics is event 2.
If a student is chosen at random,
determine whether these events are independent of each other.

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QUESTION 27 (continued)

(b) The entire year group is given the same survey. **(2)**

The results indicate that:

- the probability that a student chose to study Chemistry is $\frac{1}{3}$
- the probability that a student chose to study Physics is $\frac{2}{5}$
- given that a student chose to study Chemistry, the probability that they chose to study Physics is $\frac{3}{7}$

Calculate the probability that a student picked at random has chosen to study Chemistry or Physics.

QUESTION 28 (6 marks)

A particle is moving in a straight line with a velocity given by $\frac{dx}{dt} = -6\sin(2t)$,
where x is the displacement from the origin in metres and t is time measured in seconds.

The particle is initially at rest 4 m to the right of the origin.

(a) Show that the displacement of the particle is given by $x = 1 + 3\cos(2t)$ **(2)**

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(b) Show that the particle comes to rest again when it is 2 m to the **left** of the origin and $t = \frac{\pi}{2}$ seconds. **(2)**

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(c) Find a time when the particle reaches its maximum speed. **(2)**
What is this maximum speed?

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QUESTION 29 (4 marks)

(a) Show that $\frac{d}{dx}\left(\frac{\ln x}{x}\right)=\frac{1-\ln x}{x^2}$ **(2)**

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(b) Use the result in part (a), to evaluate $\int_e^{e^2} \frac{1-\ln x}{x\ln x}\,dx$ (leave your answer in exact form) **(2)**

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Student's Name:___________________________ Teacher's Code : NHM PPC MXF FEY JPN

QUESTION 30 (6 marks)

A person borrows \$5000 at 1.5% per month compound interest and repays the loan (including interest) by equal monthly payments over 3 years.

(a) Show that the Amount owing after 3 months is:

$A_3 = 5000(1.015)^3 - M(1 + 1.015 + 1.015^2)$ where M is the monthly payment **(1)**

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(b) Find the value of each equal monthly payment M. **(3)**

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(c) How long would the loan have been repaid if the monthly payment was \$200 per month (answer to the nearest month). **(2)**

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QUESTION 31 (4 marks)

At time t years after it was purchased the value $\$V$ of a car is given by $V = 25000\,e^{-0.5t}$

(a) Find the loss in value of the car during the third year. **(2)**

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(b) Find the year in which the car is losing value at a rate of \$100 per year. **(2)**

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QUESTION 32 (3 marks)

If $y = \tan^2 x$, find the values of the constants a and b such that $\dfrac{d^2y}{dx^2} = ay^2 + by + 2$ **(3)**

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QUESTION 33 (3 marks)

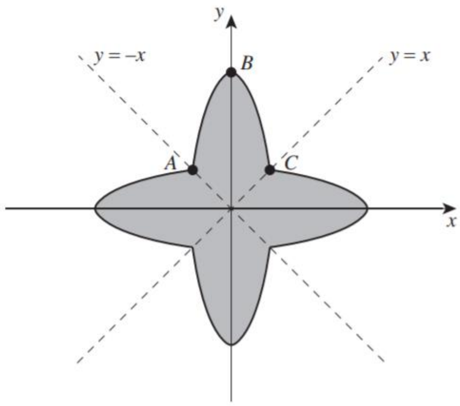
A company's logo is designed using the curve ABC, **(3)**

as shown by the shaded region in the diagram.

The curve ABC shows part of the function $y = 4 - 3x^2$.

This curve is reflected along the x-axis and the lines $y = x$ and $y = -x$

to form the rest of the logo.



Calculate the area of the company's logo.

QUESTION 34 (3 marks)

The discrete random variable X has the following probability distribution:

x	-1	0	1	a	$2a$
$P(X=x)$	$\frac{1}{10}$	a	b	b	$2b$

What is the **range** of possible values for the expected value, E(X)? **(3)**

End of Paper

Student's Name:______________________ Teacher's Code : NHM PPC MXF FEY JPN

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Topics Examined:

Section A Multiple Choice	**10 Marks**
Section B Short Answer	
Question 11-34	**90 Marks**
Total	**100 Marks**

SOLUTIONS

Solutions

Start your answer here.

* Students with incorrect answers noted in each Qu [ ]

MULTIPLE CHOICE

Q1 Probability Density Function on a closed interval $[a,b]$ needs to satisfy

- $f(x) \geq 0$ for $[a,b]$
- $\int_a^b f(x)\,dx = 1$

[28 students]

ANSWER D

Q2 2% p.a compounded half yearly

$r = \frac{2\%}{2} = 1\%$ (per half year)

$n = 4 \times 2$

$n = 8$

table interest factor 8.2857

$FV = PV \times \text{interest factor}$

$PV = \frac{FV}{\text{interest factor}}$

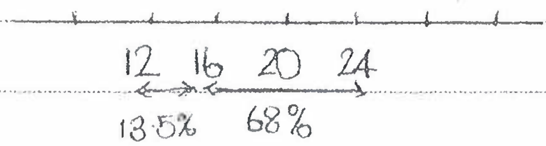
$= \frac{52200}{8.2857}$

[13 students]

$= \$6300.01$

ANSWER B

Q3



[3 students]

81.5%

ANSWER D

Q4 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Note : $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

$\therefore P(A \cap B) = P(A|B) \times P(B)$

[55 students]

$\therefore P(A \cup B) = P(A) + P(B) - P(A|B) . P(B)$

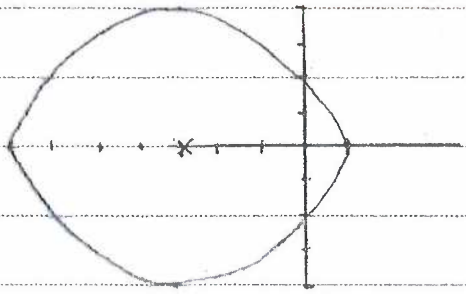
$= 0{\cdot}4 + 0{\cdot}5 - 0{\cdot}2 \times 0{\cdot}5$

$= 0{\cdot}8$ **ANSWER: C**

Q5 $x^2 + 6x + y^2 = 7$

$x^2 + 6x + 9 + y^2 = 7 + 9$

$(x+3)^2 + y^2 = 4^2$

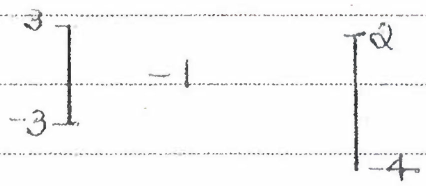


Domain : $= [-7, 1]$ [39 students]

Range $= [-4, 4]$

ANSWER D

Q6. $y = 3\sin\left(\frac{x}{2}\right) - 1$



Max occurs when $\frac{x}{2} = \frac{\pi}{2}$ [19 students]

$x = \pi$

ANSWER. C

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Start your answer here.

Q7 $\int_a^1 4 - x \, dx = 8$

$\left[4x - \frac{x^2}{2}\right]_a^1 = 8$

$\left[\left(4 - \frac{1}{2}\right) - \left(4a - \frac{a^2}{2}\right)\right] = 8$

$\frac{7}{2} - 4a + \frac{a^2}{2} = 8 \quad (\times 2)$

$a^2 - 8a + 7 = 16$

$a^2 - 8a - 9 = 0$

$(a-9)(a+1) = 0$ [9 students]

$a = -1$ or $a = 9$

ANSWER B

Q8 $\left[e^{-x}\right]_0^{\ln 5}$

$= e^{-\ln 5} - e^0$

$= e^{\ln 5^{-1}} - 1$

$= \frac{1}{5} - 1$ [17 students]

$= -\frac{4}{5}$

ANSWER C

Q9 $\frac{x}{3}$ horizontal dilation of 3 x-coordinate becomes 6 $(6, 1)$

$-f\left(\frac{x}{3}\right)$ reflection in the x-axis y coordinate becomes -1 $(6, -1)$

$+4$ vertical translation of 4 y coordinate becomes 3 $(6, 3)$

ANSWER C

[43 students]

Q10. $y = e^{-x}\sin x$

$$u = e^{-x}$$
$$u' = -e^{-x}$$
$$v = \sin x$$
$$v' = \cos x$$

$\frac{dy}{dx}$ = inside + outside

$= -e^{-x}\sin x + e^{-x}\cos x$

$= e^{-x}[\cos x - \sin x]$

Min T.P when $\cos x - \sin x = 0$ $\quad \left[\frac{dy}{dx} = 0\right]$

$$\frac{\cos x}{\cos x} = \frac{\sin x}{\cos x}$$

$$1 = \tan x$$

$$x = \frac{\pi}{4}, \frac{5\pi}{4}$$

test:

x	0.7	$\frac{\pi}{4}$,	0.8	
$\frac{dy}{dx}$	0.05	0	−0.009	Max T.P
x	3.9	$\frac{5\pi}{4}$,	4.0	
$\frac{dy}{dx}$	−0.0008	0	0.001	Min T.P

$\therefore$ Min T.P $x = \frac{5\pi}{4}$

ANSWER: C

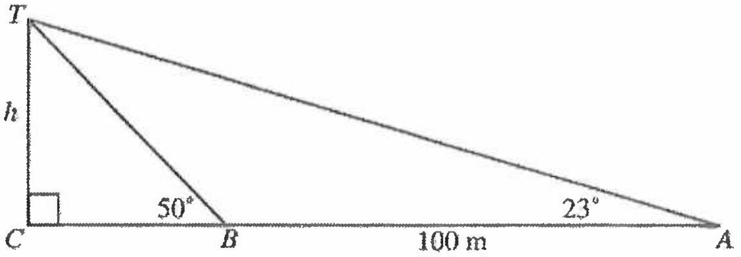
[60 Students]

Additional writing space on back page.

Student's Name: MARKER'S Comments. Teacher's Code : NHM PPC MXF FEY JPN

QUESTION 11 (4 marks)

The elevation of a tower from point A is 23°.
From a point B, 100 metres closer to the tower and on the same horizontal level as A the elevation is 50°.



(a) Show that $\angle BTA = 27^\circ$. **(1)**

In $\triangle TBA$: $\angle TBA = 130^\circ$ Angles in a st. line

$\therefore \angle BTA = 180^\circ - 130^\circ - 23^\circ = 27^\circ$ Angle sum of a $\triangle$.

(b) Show that $BT = \dfrac{100 \sin 23^\circ}{\sin 27^\circ}$ **(1)**

By Sine Rule $\dfrac{BT}{\sin 23} = \dfrac{100}{\sin 27}$

$\therefore BT = \dfrac{\sin 23}{\sin 27} \times 100$

$= 86.07$

(c) Find the height (h) of the tower **(2)**
[Give your answer correct to one decimal place]

$\sin 50 = \dfrac{h}{BT}$

$h = BT \sin 50 = 86.07 \times \sin 50$

$= 65.9$ (1dp).

* This Question was well answered by most students.

QUESTION 12 (3 marks)

The table below shows the marks out of 40 for nine students on two tests.
The tests cover similar content, but on one test the students **are allowed** to use a calculator, while on the other test, calculators are **not allowed**.

Student	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*
Calculator allowed (x)	8	10	14	14	16	22	28	32	36
Non−calculator (y)	12	14	6	4	28	16	24	22	18

(a) Use the marks for students A to I to find the value of the **Pearson correlation coefficient** between the marks x on the test with the calculator allowed and the marks y on the non-calculator test, and the **equation of the least squares regression line** of y on x **(2)**

All Calculations to be done by Calculator.

1. Pearson Correlation Coefficient = 0·05 ✓

2. Eqn. of Least Squares regression

$y = 0{\cdot}4x + 8$ ✓

(b) Student **J** scored 25 on the test with calculators allowed but missed the non-calculator test.

Use the least squares regression line of y on x to estimate the mark of student **J** on the non-calculator test. **(1)**

$x = 25$ $\quad y = 0{\cdot}4 \times 25 + 8$

$= 18$ ✓

* Some not aware of the use of calculator for this question.

QUESTION 13 (8 marks)

(a) Find the nature of the stationary points of the curve $y = 2x^3 - 3x^2 - 12x + 2$ (3)

$y = 2x^3 - 3x^2 - 12x + 2$

$y' = 6x^2 - 6x - 12$

for stationary pts. $y' = 0$

$6(x^2 - x - 2) = 0$

$(x-2)(x+1) = 0$

$x = 2, \quad x = -1$

$y = \underline{-18}, \quad y = \underline{9}$

Nature:

$y'' = 18 > 0 \qquad y'' = -18 < 0$

$\therefore$ MIN $\qquad \therefore$ MAX.

$(2, -18)$ MIN. $\quad (-1, 9)$ MAX.

Nature:

$y'' = 12x - 6$

@ $x = 2$, $y'' = 12(2) - 6$

$= 18 > 0$ MAX

@ $x = -1$ $y'' = 12(-1) - 6$

$= -16 < 0$ MIN.

* Just can't say max, min you need to work it out for full marks.

(b) Draw a neat sketch of $y = 2x^3 - 3x^2 - 12x + 2$ showing the stationary points and the point of inflection for the domain $-2 \le x \le 3$. (3)

* Since this curve is bounded by a domain, the y-values at these two pts. must be obtained:

$(-2, -2)$, $(3, -7)$, Intercept $x = 0$, $y = 2$

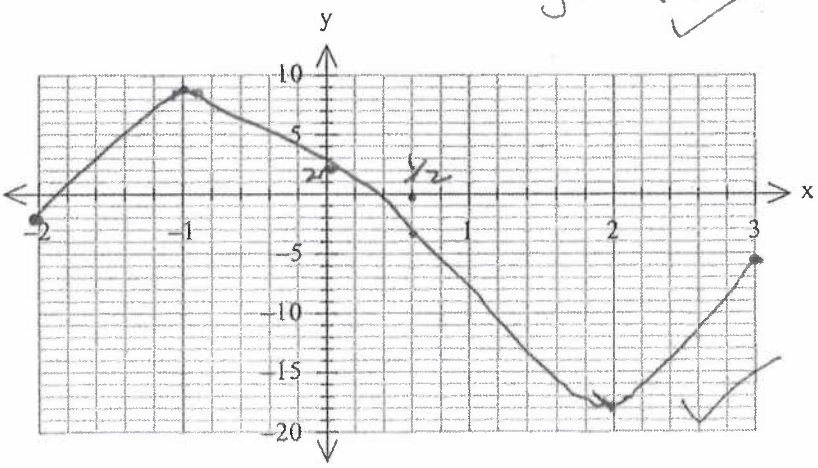
Infl: $y'' = 0 \quad 12x - 6 = 0$, $x = \frac{1}{2}$ $\quad$ * Check if POI

$y = -4.5$

x	0.4	1/2	0.6
y''	-1.2	0	1.2

Change in concavity $\therefore$ POI.

Many missed this



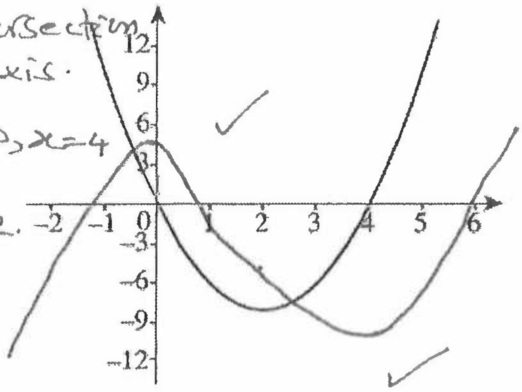
(c) Given the graph of $\frac{dy}{dx}$ below, on the same graph, draw a sketch of **(2)**

a possibility for the graph of y.

Looking for:

1. 3 pts. of intersection on the x-axis.
2. t.p. at $x=0, x=4$
3. Infl. at $x=2$.

for full marks



QUESTION 14 (3 marks)

(a) Find the equation of the tangent to the curve $y=\sqrt{25-x^2}$ at the point (3, 4). **(3)**

(Write your answer in general form)

$y = (25-x^2)^{1/2}$

$y' = \frac{1}{2}(25-x^2)^{-\frac{1}{2}} \times -2x$

$y' = \frac{-x}{\sqrt{25-x^2}}$ ✓

$m_t = \frac{-3}{\sqrt{25-9}} = -\frac{3}{4}$ ✓

@ $x=3$

$\therefore$ Eqn. of tgt: $y-y_1 = m_t(x-x_1)$

$y-4 = -\frac{3}{4}(x-3)$

$4y-16 = -3x+9$

$\therefore 3x+4y-25=0$ (In Gen. form) ✓

Solutions

With markers' comments!

Student's Name:______________________ Teacher's Code : NHM PPC MXF FEY JPN

QUESTION 15 (4 marks)

(a) Differentiate $y=(\ln x)^{-1}$ with respect to x. (2)

$\frac{dy}{dx} = -1(\ln x)^{-2} \times \frac{1}{x}$ ✓

$= \frac{-1}{x(\ln x)^2}$ ✓

Also accepted $\frac{-\frac{1}{x}}{(\ln x)^2}$

* well done

(b) Differentiate $y=(x^2+4)\tan 3x$ with respect to x. (2)

$\frac{dy}{dx} =$ inside + outside

$= 2x\tan 3x + 3(x^2+4)\sec^2 3x$ ✓ ✓

$u = x^2+4$

$u' = 2x$

$v = \tan 3x$

$v' = 3\sec^2 3x$

* well done

QUESTION 16 (2 marks)

Evaluate $\int_0^{\frac{\pi}{2}} 1-\cos x \, dx.$ (2)

$= \left[x - \sin x \right]_0^{\frac{\pi}{2}}$ ✓

$= \left[\left(\frac{\pi}{2} - \sin\frac{\pi}{2}\right) - (0 - \sin 0) \right]$

$= \frac{\pi}{2} - 1$ ✓

* carry through marks awarded as some made little errors integrating

* No +C's on this! It's a definite integral!

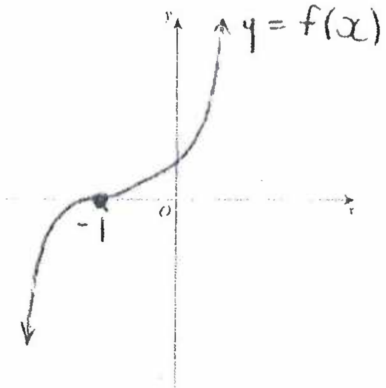
* No units either!!

QUESTION 17 (2 marks)

The function $y = f(x)$ is continuous for all values of x. **(2)**
The following is known of the function's properties.

- When $x < -1$, $f'(x) > 0$ and $f''(x) < 0$.
- When $x > -1$, $f'(x) > 0$ and $f''(x) > 0$.
- $f(-1) = f'(-1) = f''(-1) = 0$

On the axes below, sketch the graph of $y = f(x)$, labelling any x-intercepts.



✓ shape

✓ stationary 'flat' curve at $x = -1$ $(-1, 0)$

QUESTION 18 (3 marks)

Solve the pair of simultaneous equations: **(3)**

$\log_2 x + \log_2 y = 3$ ...(1)
$\log_x y = 2$ ...(2)

* many struggled

$\log_2 xy = 3$ (1) Rewrite (1)

$\log_x y = 2$ (2) Rewrite $y = x^2$ ✓ – 1 mark awarded early

$\therefore \log_2 x.x^2 = 3$
$\log_2 x^3 = 3$
$x^3 = 2^3$
$x^3 = 8$
$x = 2$ ✓

when $x = 2$
$y = x^2$
$y = 2^2$
$y = 4$ ✓

QUESTION 19 (2 marks)

An arithmetic sequence has first term 29 and a common difference of -4. **(2)**
Find the sum of the first n terms of this sequence, calculate the least value of n for which the sum is negative.

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}[58 + (n-1) - 4]$

$= \frac{n}{2}[58 - 4n + 4]$ $\qquad S_n < 0$

$= \frac{n}{2}[62 - 4n]$ $\qquad$ when $n = 16$ ✓

$= 31n - 2n^2$ ✓ had to get this far for first mark

$= n(31 - 2n)$

$n = 0, \frac{31}{2} = (15\frac{1}{2})$

* needed $n = 16$ for second mark $15\frac{1}{2}$ not good enough!

QUESTION 20 (3 marks)

Shirley uses her car every day. Sometimes, she needs a few attempts to start her car.
Let the discrete random variable X represent the number of attempts needed.
The probability function for the random variable X is given by:

$$P(X = x) = \begin{cases} a & x = 1 \\ \frac{1}{2}P(X = x-1) & x = 2, 3, 4, 5 \\ 0 & \text{for all other values of } x \end{cases}$$

(a) Show that $a = \frac{16}{31}$ **(2)**

$a + \frac{a}{2} + \frac{a}{4} + \frac{a}{8} + \frac{a}{16} = 1$ ✓

* V. difficult Question for cohort

$\frac{31}{16}a = 1$ ✓

$a = {}^{16}/_{31}$ (as required)

* many 'fudged' this

(b) The probability distribution table for the discrete random variable X is shown. **(1)**

x	1	2	3	4	5
$P(X = x)$	$\frac{16}{31}$	$\frac{8}{31}$	$\frac{4}{31}$	$\frac{2}{31}$	$\frac{1}{31}$

* well done by most

Shirley claims that after an entire year, she needs an average of two attempts to successfully start her car. Calculate the expected value (correct to 2 decimal places).

$E(x) = \sum x\,p(x)$

$= (1 \times \frac{16}{31}) + (2 \times \frac{8}{31}) + (3 \times \frac{4}{31}) + (4 \times \frac{2}{31}) + (5 \times \frac{1}{31})$

$= 1 \cdot 84$ ✓

Don't forget you can use your calculator here!!

QUESTION 21 (5 marks)

Complete the table of $y = 2^x$.

The table gives the values of $y = 2^x$ for x.

x	0	2	4	6
y	1	4	16	64

(a) Complete the table above. **(1)**

(b) Hence, use the trapezoidal rule to find an approximation for $\int_0^6 2^x \, dx$. **(2)**

$\doteqdot \frac{2}{2}\left[1 + 64 + 2(4+16)\right]$ ✓

$1\left[1 + 64 + 40\right]$

$= 105$ ✓

If you only had what is in brackets correct still 1 mark!

* many failed to get $\frac{h}{2}$ correct

No Units!!

(b) Use integration to evaluate $\int_0^6 2^x \, dx$ (write your answer correct to 2 decimal places) **(2)**

Use formula sheet for these integrals!

$\int f'(x)\, a^{f(x)}\, dx = \frac{a^{f(x)}}{\ln a} + C$ NESA reference sheet

$\int 1\, 2^x = \left[\frac{2^x}{\ln 2}\right]_0^6$ ✓

Many made error integrating & wrote $\left[\ln 2 \; 2^x\right]$

$= \frac{1}{\ln 2}\left[2^x\right]_0^6$

$= \frac{1}{\ln 2}\left[2^6 - 2^0\right]$

$= \frac{63}{\ln 2}$

$= 90.89$ (2 d.p.) ✓

Those who wrote 43.67 as answer +1 ctm

Again NO UNITS

You're not finding an Area!

Student's Name:______________________________ Teacher's Code : NHM PPC MXF FEY JPN

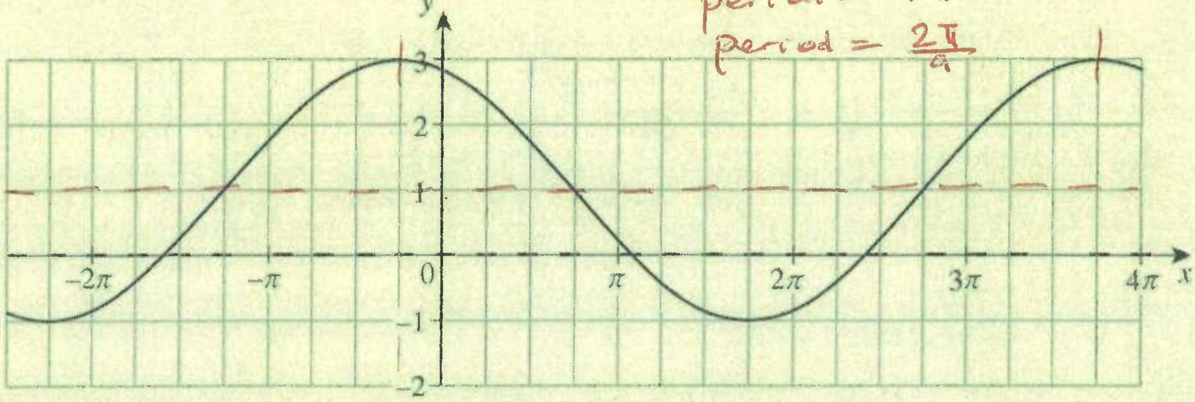
18

QUESTION 22 (3 marks)

The graph is a function of the form $y = 2\cos(a(x+b)) + c$.

period = 4π

period = $\frac{2\pi}{a}$



Consider the equation of the function to determine the values of a, b and c (3)

$4\pi = \frac{2\pi}{a}$ $\boxed{b = \frac{\pi}{4}}$ ✓ $\boxed{c = 1}$ ✓

$a = \frac{2\pi}{4\pi}$

$\boxed{a = \frac{1}{2}}$ ✓

QUESTION 23 (3 marks)

At time t hours after 12:00 am, the height h metres of the deck of a boat (3)

above the level of the jetty is given by $h = 2\cos\left(\frac{4\pi}{25}t\right) + 1$.

Find correct to the nearest minute the first time after 12:00 am the boat **is level** with the jetty.

$2\cos\frac{4\pi}{25}t + 1 = 0$ $\frac{25}{6}$ hours after 12:00 am

$2\cos\frac{4\pi}{25}t = -1$ ✓ $= 4$ h 10 mins after 12:00 am

$\cos\frac{4\pi}{25}t = -\frac{1}{2}$

$\frac{4\pi}{25}t = \frac{2\pi}{3}, \frac{4\pi}{3}, \ldots$ ∴ time $\boxed{4:10 \text{ am}}$ ✓

$t = \frac{25}{6}, \frac{25}{3}, \ldots$ ✓

4·16

4 hrs 10 min

250 mins.

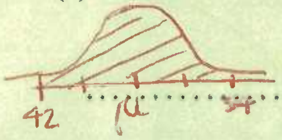
note $4°10'$ is not a time

it is an angle !!

QUESTION 24 (4 marks)

The students in a school were surveyed on the number of hours of sleep per week.
The results were normally distributed.
The survey indicated that 95% of students had between 42 and 54 hours of sleep per week.

(a) Show that the mean number of hours of sleep per week is 48. **(1)**



$\mu = \frac{42+54}{2} = \boxed{48}$ as required

or other reasonable logic.

(b) What was the standard deviation? **(1)**

$48 + 2\sigma = 54$

$2\sigma = 6$ $\boxed{\sigma = 3 \text{ hours}}$

(c) How many hours of sleep per week would a student have who recorded a z-score of -2.5? **(1)**

$z = \frac{x-\mu}{\sigma}$

$-2.5 = \frac{x-48}{3}$

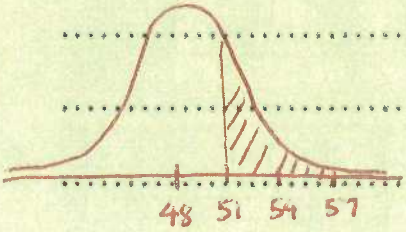
$-7.5 = x - 48$

$x = 40.5$ ie $\boxed{40.5 \text{ hours}}$

(d) What percentage of students would have indicated they had between 51 and 57 hours of sleep per week? **(1)**

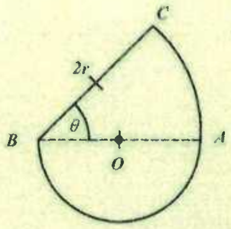
between 51 & 57 $= \frac{99.7\% - 68\%}{2}$

$= \boxed{15.85\%}$



%age between
z score of 1 and
z score of 3

QUESTION 25 (5 marks)



The diagram shows the cross-section of a cam (a part in a car engine), consisting of a semi-circle diameter AB, centre O and radius r units; and a sector ABC with centre B, radius $2r$ units and angle θ. The area of the cam ABC is $\frac{1}{2}$ units^2.

A = semi circle + sector

(a) Show that $\theta = \dfrac{1-\pi r^2}{4r^2}$ **(2)**

must show (not fudge)

$$A = \frac{\pi r^2}{2} + \frac{1}{2}(2r)^2\theta$$

$$\frac{1}{2} = \frac{1}{2}(\pi r^2 + 4r^2\theta)$$

$$1 = \pi r^2 + 4r^2\theta$$

$$1 - \pi r^2 = 4r^2\theta$$

$$\theta = \frac{1-\pi r^2}{4r^2}$$ as required.

(b) Find the value of r (correct to 1 decimal place) which gives the least perimeter of the cam. **(3)**

$$P = \tfrac{1}{2}\times 2\pi r + 2r + 2r\theta$$

$$= \pi r + 2r + \frac{2r}{1}\times\frac{1-\pi r^2}{4r^2}$$

$$= \pi r + 2r + \frac{1-\pi r^2}{2r}$$

$$= \pi r + 2r + \tfrac{1}{2}r^{-1} - \frac{\pi r}{2}$$

$$= \frac{\pi r}{2} + 2r + \tfrac{1}{2}r^{-1}$$

$$P' = \frac{\pi}{2} + 2 - \tfrac{1}{2}r^{-2}$$

$$= \frac{\pi}{2} + 2 - \frac{1}{2r^2}$$

for minim let $P' = 0$

$$\frac{1}{2r^2} = \frac{\pi}{2} + 2$$

$$\frac{1}{r^2} = \pi + 4$$

$$r^2 = \frac{1}{\pi + 4}$$

$$r = \pm 0.3741\ldots$$

$$P'' = r^{-3} = \frac{1}{r^3}$$

when $r = 0.3741\ldots$

$$P'' = \frac{1}{0.3741\ldots^3} = 19.0550\ldots$$

Since $P'' > 0$ ∴ concave up

∴ minimum occurs when

$$r = 0.3741\ldots = \boxed{0.4}\ (1\text{d.p})$$

must prove that minimum occurs.

QUESTION 26 (3 marks)

The continuous random variable, X, has the probability density function

$$f(x)=\frac{1}{2}\sin x \text{ for } 0\le x\le\pi .$$

(a) Find the cumulative distribution function. **(2)**

$$F(x) = \frac{1}{2}\int_0^x \sin x \, dx$$

$$= \frac{1}{2}\left[-\cos x\right]_0^x \quad \checkmark$$

$$= -\frac{1}{2}\cos x - -\frac{1}{2}\cos 0$$

$$F(x) = \frac{1}{2} - \frac{1}{2}\cos x$$

$$F(x) = \frac{1-\cos x}{2} \quad \checkmark$$

either or equivalent.

(b) Find the first quartile of the distribution. **(1)**

$$\frac{1-\cos x}{2} = 0.25$$

$$1-\cos x = 0.5$$

$$-\cos x = -0.5$$

$$\cos x = 0.5 \quad \checkmark$$

Correct answer.

$$\boxed{x = \frac{\pi}{3}}$$

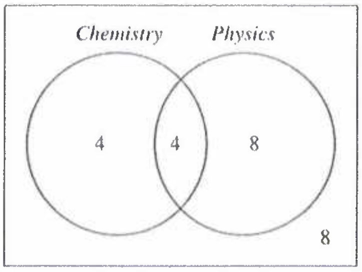
Student's Name:______________________________ Teacher's Code : NHM PPC MXF FEY JPN

QUESTION 27 (4 marks)

A teacher surveyed his class of 24 students about the subjects that they have chosen to study in the following year.

The results indicate that:
- 8 students chose to study neither Physics nor Chemistry
- 8 students chose to study Chemistry
- 12 students chose to study Physics.

A Venn diagram showing this information has been drawn below.



$n = 24$

(a) A student choosing to study Chemistry is event 1.
A student choosing to study Physics is event 2.
If a student is chosen at random,
determine whether these events are independent of each other. **(2)**

THIS IS FROM THE REF. SHEET → YOU NEED TO WRITE IT IN TERMS OF Q.

FOR INDEPENDENT EVENTS: $P(A \cap B) = P(A).P(B)$

So, $P(Ch \cap Ph) = p(Ch).\ p(Ph)$

$= \frac{8}{24} \times \frac{12}{24}$

OR $P(E_1 \cap E_2)$ $= \frac{1}{6}$

N.B. DO NOT WRITE ME A PHILOSOPHY THESIS ON THIS !!

Also from Venn diagram $P(Ch \cap Ph) = \frac{4}{24}$

$= \frac{1}{6}$

$\therefore$ Events are independent.

ALTERNATE METHOD

OR $P(Ch \mid Ph) = \frac{P(Ch \cap Ph)}{P(Ph)}$

$= \frac{4}{24} \div \frac{12}{24}$

$= \frac{1}{3}$

Also, $p(Ch) = \frac{8}{24}$

$= \frac{1}{3}$

$\therefore$ Ch independent of Ph

$P(Ph \mid Ch) = \frac{1}{2}$

$P(Ph) = \frac{1}{2}$

QUESTION 27 (continued)

(b) The entire year group is given the same survey. **(2)**

The results indicate that:

- the probability that a student chose to study Chemistry is $\frac{1}{3}$
- the probability that a student chose to study Physics is $\frac{2}{5}$
- given that a student chose to study Chemistry, the probability that they chose to study Physics is $\frac{3}{7}$

Calculate the probability that a student picked at random has chosen to study Chemistry or Physics.

$P(Ph|Ch) = \frac{3}{7}$

Now $P(Ph|Ch) = \frac{P(Ph \cap Ch)}{P(Ch)}$

$\therefore \frac{P(Ph \cap Ch)}{\frac{1}{3}} = \frac{3}{7}$

$\therefore P(Ph \cap Ch) = \frac{3}{7} \times \frac{1}{3}$

$= \frac{1}{7}$ ✓

$\therefore P(Ch \cup Ph) = P(Ch) + P(Ph) - P(Ph \cap Ch)$

$= \frac{2}{5} + \frac{1}{3} - \frac{1}{7}$ ✓

$= \frac{62}{105}$ [0.590] [59.0%]

HARDER PROBABILITY QUESTION MANY HAD TROUBLE WITH IT.

QUESTION 28 (6 marks)

A particle is moving in a straight line with a velocity given by $\frac{dx}{dt} = -6\sin(2t)$,

where x is the displacement from the origin in metres and t is time measured in seconds.

The particle is initially at rest 4 m to the right of the origin.

(a) Show that the displacement of the particle is given by $x = 1 + 3\cos(2t)$ **(2)**

$\frac{dx}{dt} = -6\sin(2t)$

$\therefore x = \int -6\sin(2t) + c$

HAD TO SHOW WHERE THE SUPPLIED FORMULA CAME FROM.

$\frac{6}{2}\cos(2t) + c$

$t = 0,\ x = +4$

$\therefore 4 = 3\cos(2(0)) + c$

$4 = 3 + c$

$\therefore c = 1$

$\therefore x = 3\cos(2t) + 1$

(b) Show that the particle comes to rest again when it is 2 m to the **left** of the origin and $t = \frac{\pi}{2}$ seconds. **(2)**

1 2

$v = \frac{dx}{dt} = 0 \quad \therefore -6\sin(2t) = 0$

two things: Speed = 0

location = -2

$\sin(2t) = 0$

$\therefore 2t = 0, \pi, 2\pi, \ldots$

$t = 0, \frac{\pi}{2}, \pi$

'Next' comes to rest $t = \frac{\pi}{2}$

$\therefore x = 1 + 3\cos(2 \times \frac{\pi}{2})$

$= 1 + 3(-1)$

$= -2$

i.e. 2 units to the left.

(c) Find a time when the particle reaches its maximum speed. **(2)**
What is this maximum speed?

TOO MANY EQUATED $\frac{dx}{dt}$ to 0.

$v = \frac{dx}{dt} = -6\sin(2t)$

$a = \frac{d^2x}{dt^2} = -12\cos(2t)$

Max speed when $a = 0$

$\therefore -12\cos(2t) = 0$

$\cos(2t) = 0$

$\therefore 2t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$

$t = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$

Speed $= |v|$

$= |-6\sin(2 \times \frac{\pi}{4})|$

$= |-6 \times 1|$

$= 6\ ms^{-1}$

QUESTION 29 (4 marks)

(a) Show that $\frac{d}{dx}\left(\frac{\ln x}{x}\right) = \frac{1-\ln x}{x^2}$ (2)

$u = \ln x \quad v = x$

$u' = \frac{1}{x} \quad v' = 1$ ✓

$\frac{d}{dx} = \frac{vu' - uv'}{v^2}$ ← THIS IS THE CORRECT FORMULA – BUT THE SUBSTITUTION IS WHAT PAYS

$= \frac{x \cdot \frac{1}{x} - \ln x \cdot 1}{x^2}$ ✓ ← SHOW ME THE MONEY! (MUST SEE THE CORRECT SUBSTITUTION INTO CORRECT FORMULA)

THIS IS A GIVEN! SO NO MARK FOR THIS LINE

$= \frac{1-\ln x}{x^2}$ as required

(b) Use the result in part (a), to evaluate $\int_e^{e^2} \frac{1-\ln x}{x \ln x}\,dx$ (leave your answer in exact form) (2)

$\int_e^{e^2} \frac{1-\ln x}{x \ln x}\,dx = \int_e^{e^2} \frac{1-\ln x}{x^2} \times \frac{x}{\ln x}\,dx$

$\left[\int \frac{f'(x)}{f(x)} = \ln f(x)\right]$

$= \int_e^{e^2} \frac{1-\ln x}{x^2} \div \frac{\ln x}{x}\,dx$

A HARD QUESTION "BAND 6".

$= \left[\ln\left(\frac{\ln x}{x}\right)\right]_e^{e^2}$ ✓

MANY DID NOT GET VERY FAR

$= \ln\left(\frac{\ln e^2}{e^2}\right) - \ln\left(\frac{\ln e}{e}\right)$

IT WAS CLEAR THAT SOME STUDENTS ACROSS SEVERAL CLASSES HAD SEEN THIS TYPE OF QUESTION BEFORE AND HANDLED IT WELL.

$= \ln\left(\frac{2}{e^2}\right) - \ln\left(\frac{1}{e}\right)$

$= \ln\left(\frac{2}{e^2} \times \frac{e}{1}\right)$

$= \ln\left(\frac{2}{e}\right)$

$\ln 2 - 1$ ✓

DO LOTS OF PAST PAPERS!!

30 a) $A_1 = 5000(1.015) - M$

$A_2 = A_1(1.015) - M$

$= 5000(1.015)^2 - M(1 + 1.015)$

$A_3 = A_2(1.015) - M$

1mk for clearly showing A_2.

b) $A_{36} = 5000(1.015)^{36} - M(1 + \ldots + 1.015^{35})$

$A_{36} = 0$

1mk

$\therefore 5000(1.015)^{36} - M(1 + \ldots + 1.015^{35}) = 0$

$M\left(\dfrac{1(1.015^{36} - 1)}{0.015}\right) = 5000(1.015^{36})$

$M = \$180.76$

1mk

Generally, well done!!

c) $5000(1.015)^n = \dfrac{200(1.015^n - 1)}{0.015}$

1mk

Solve for n with logs.

$75(1.015)^n = 200(1.015)^n - 200$

$125(1.015)^n = 200$

$(1.015)^n = \dfrac{8}{5}$

$n = \dfrac{\ln\left(\frac{8}{5}\right)}{\ln(1.015)}$

$= 31.57$ mths (32 mths)

1mk

Generally, well done!!

(31) a) $V = 25000e^{-0.5t}$

$t = 0 \quad V = 25000$

$t = 1 \quad V = 15163.26$

$t = 2 \quad V = 9196.99$

$t = 3 \quad V = 5578.25$

Many confused by the wording "during the third year"

1mk

$\therefore$ Loss during third year

$9196.99 - 5578.25$

Loss $= \$3618.74$

b) $V = 25000e^{-0.5t}$

$\frac{dV}{dt} = -12500e^{-0.5t}$

Many did not realise that "rate" meant $\frac{dV}{dt}$

Let $\frac{dV}{dt} = -100$

$-100 = -12500e^{-0.5t}$

$\frac{1}{125} = e^{-0.5t}$

$t = \frac{\ln\left(\frac{1}{125}\right)}{-0.5}$

1mk

$= 9.66$

or In 10^{th} year

32 $\quad y = \tan^2 x$

$y = (\tan x)^2$

$\frac{dy}{dx} = 2\tan x \cdot \sec^2 x$

Find $\frac{dy}{dx}$ for Easy first mk
1 mk

$= 2\tan x(1+\tan^2 x)$

$= 2\tan x + 2(\tan x)^3$

$\frac{d^2y}{dx^2} = 2\sec^2 x + 6\tan^2 x \cdot \sec^2 x$

$= 2(1+\tan^2 x) + 6(\tan^2 x)\cdot(1+\tan^2 x)$

$= 2 + 2\tan^2 x + 6\tan^2 x + 6\tan^4 x$

$= 2 + 8\tan^2 x + 6\tan^4 x$

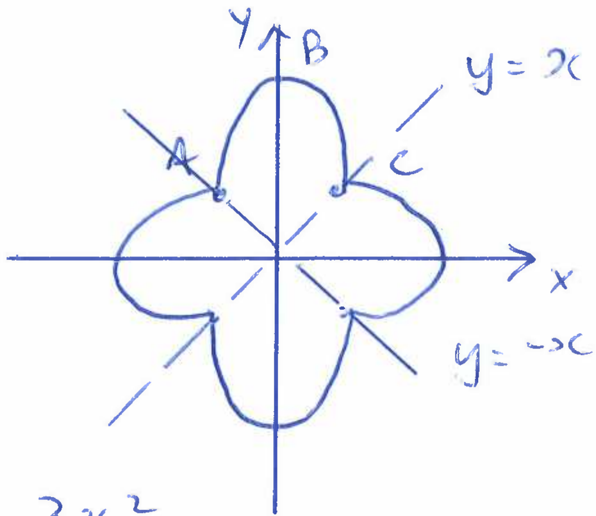
Get $\frac{d^2y}{dx^2}$ in terms of "tan"
1mk

$\frac{d^2y}{dx^2} = ay^2 + by + 2$

$\therefore a = 6$
$\quad b = 8$

1mk

(33)



$y = 4 - 3x^2$

(A) $y = -x$

$y = 4 - 3x^2$

$-x = 4 - 3x^2$

$3x^2 - x - 4 = 0$

$(3x-4)(x+1) = 0$

$\therefore x = \frac{4}{3} \quad x = -1$

$\therefore A(-1, 1)$

(B) Similarly

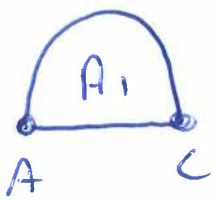
$y = x$

$y = 4 - 3x^2$

$B(1, 1)$

1mk for clearly finding points A and B

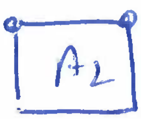
AREA Quite a few methods



$A_1 = 2\int_0^1 (4 - 3x^2)\,dx - 2\times 1$

$= 2\left[4x - x^3\right]_0^1 - 2$

$= 4\ u^2$



$A_2 = 2 \times 2$

$= 4\ u^2$

$LOGO = 4 \times A_1 + A_2$

$= 20u^2$

1mk for a significant effort in finding the area (many errors in what to do with "A2".

1mk for correct answer at $20u^2$

(35)

Sum of $P(X=x) = 1$

$$\frac{1}{10} + a + b + b + 2b = 1$$

$$\frac{1}{10} + a + 4b = 1$$

$$a = \frac{9}{10} - 4b$$

1mk

Many left page blank.

But with thought marks could have been gained.

Max occurs when $a = 0$

$\therefore b = \frac{9}{40}$ $\therefore$ Range of b

$0 \le b \le \frac{9}{40}$

$$E(X) = \sum x\,P(x)$$

$$= -\frac{1}{10} + 0 + b + ab + 4ab$$

$$= -\frac{1}{10} + b + 5ab$$

1mk

$$= -\frac{1}{10} + b + 5\left(\frac{9}{10} - 4b\right).b$$

$$= -\frac{1}{10} + \frac{11}{2}b - 20b^2$$

BTW: only two scores at 3mks were recorded.

Max $E(x)$ when $E'(x) = 0$

$$E'(X) = \frac{11}{2} - 40b$$

$$\frac{11}{2} - 40b = 0$$

$$b = \frac{11}{80}$$

when $b = 0$ $E(x) = -\frac{1}{10}$

$b = \frac{11}{80}$ $E(x) = \frac{89}{320}$

1mk.

$\therefore$ Range $-\frac{1}{10} \le E(x) \le \frac{89}{320}$

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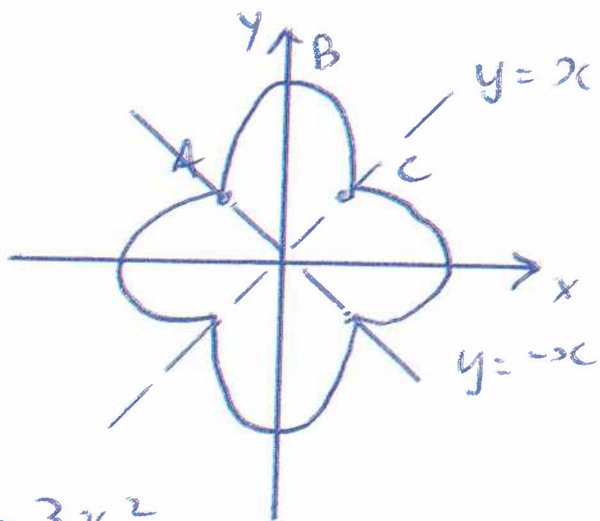

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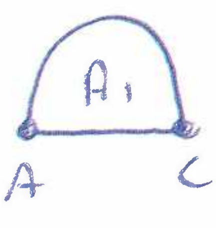

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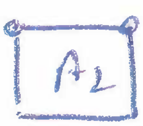

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