Student Number: _____

Roseville COLLEGE

2022 **HIGHER SCHOOL CERTIFICATE TRIAL EXAMINATION**

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (pages 2 – 7)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 9 – 32)

- Attempt Questions 11 – 34
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10.

---

1 What is $25-(2x-3y)^2$ in fully factorised form?

(A) $(5-2x-3y)(5+2x+3y)$

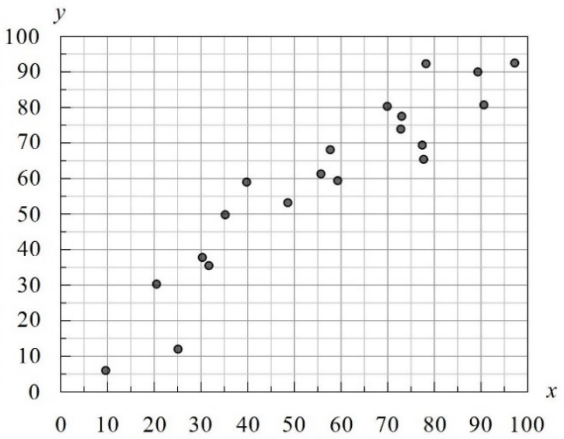
(B) $(5-2x-3y)(5+2x-3y)$

(C) $(5-2x+3y)(5+2x+3y)$

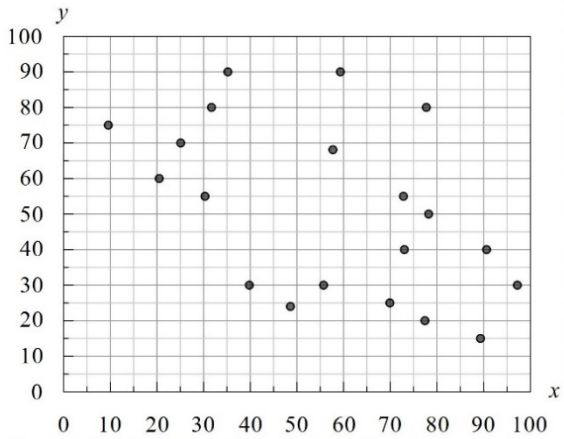
(D) $(5-2x+3y)(5+2x-3y)$

2 Scatterplots for two sets of bivariate data are shown below.

Data Set M



Data Set N



Which of the following is the best estimate for the values of the two correlation coefficients?

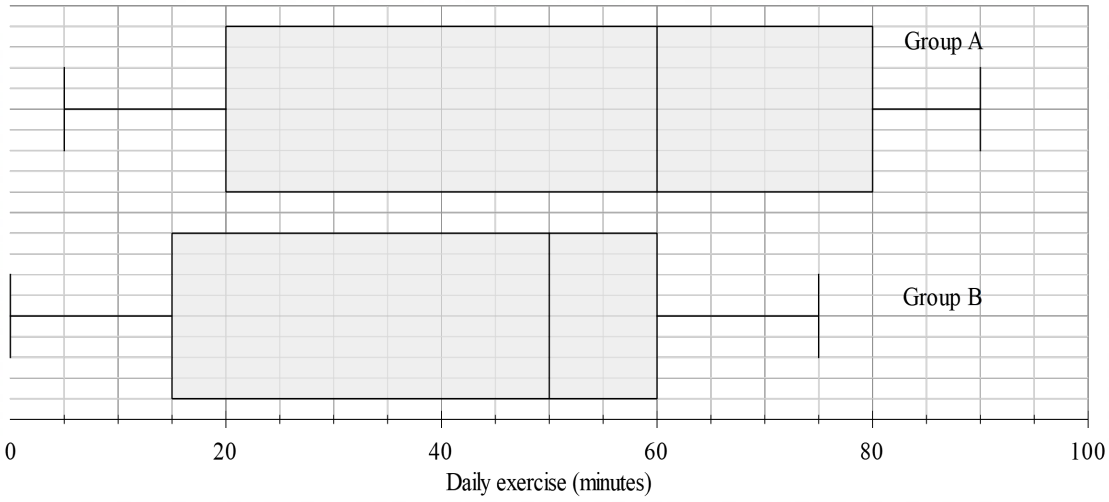
(A) Set M $r=-0.9$ Set N $r=0.9$

(B) Set M $r=-0.5$ Set N $r=0.5$

(C) Set M $r=0.9$ Set N $r=-0.5$

(D) Set M $r=0.5$ Set N $r=-0.9$

3 Employees in a workplace were divided into two groups for a fitness challenge over a week. Each employee tracked the number of minutes of exercise they completed each day and reported their daily average.



The parallel box plots above show the data collected for the two groups.

There were 72 employees in Group A.

The same number of employees in each group reported an average daily exercise time of more than 60 minutes.

How many employees in Group B reported an average daily exercise time of less than 15 minutes?

(A) 15

(B) 18

(C) 36

(D) 72

4 What is $\int 6x^2\left(4x^3-5\right)^3\ dx$?

(A) $\dfrac{\left(4x^3-5\right)^4}{8}+C$

(B) $12x\left(4x^3-5\right)^4+C$

(C) $2x^3\left(4x^3-5\right)^4+C$

(D) $\dfrac{2x^3\left(4x^3-5\right)^4}{8}+C$

5 John's marks, the class mean and standard deviation in three tests are given in the table.

	Mathematics	English	Science
John's mark	59	60	70
Class mean	55	50	72
Class standard deviation	4	5	2

Which of the following would be a correct conclusion?

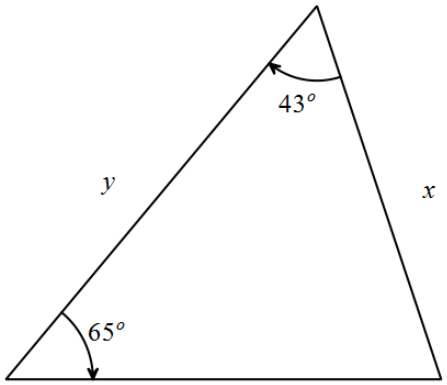
(A) He did better in Science compared to English.

(B) He did equally well in English and Science.

(C) He did better in Mathematics compared to English.

(D) He did better in English compared to Mathematics.

6 From the diagram, what is the value of y ?



NOT TO SCALE

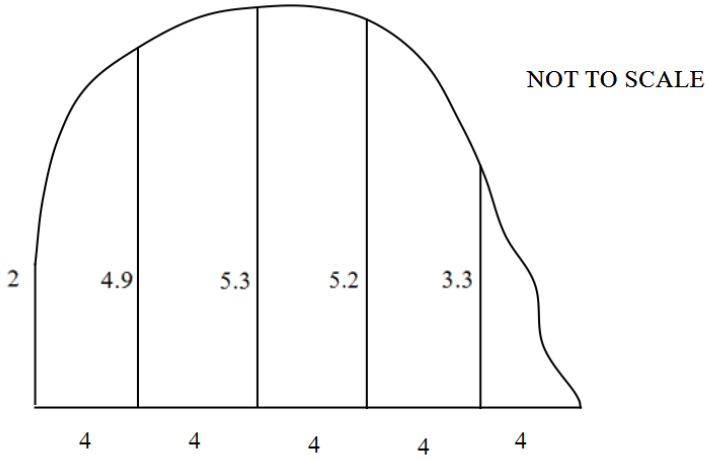
(A) $y = \dfrac{x \sin 72°}{\sin 65°}$

(B) $y = \dfrac{x \sin 43°}{\sin 65°}$

(C) $y = \dfrac{x \sin 65°}{\sin 72°}$

(D) $y = \dfrac{x \sin 72°}{\sin 43°}$

7 The diagram below shows a native garden. All measurements are in metres.



What is an approximate value for the area of the native garden using the trapezoidal rule with 5 intervals?

(A) 39 m^2

(B) 58 m^2

(C) 72 m^2

(D) 79 m^2

8 If $y = \log_2 4x$, what is $\frac{dy}{dx}$?

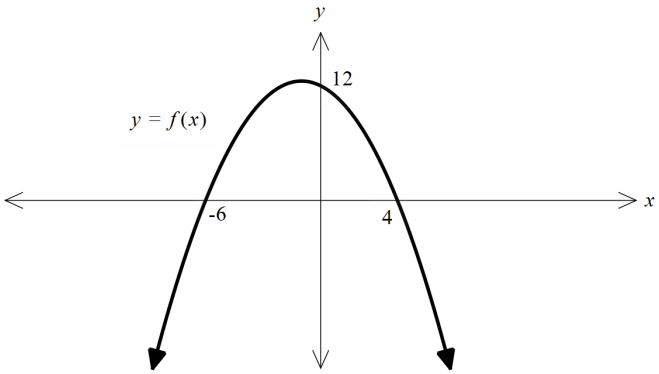
(A) $\frac{\log_e 4x}{\log_e 8}$

(B) $\frac{1}{x \log_e 2}$

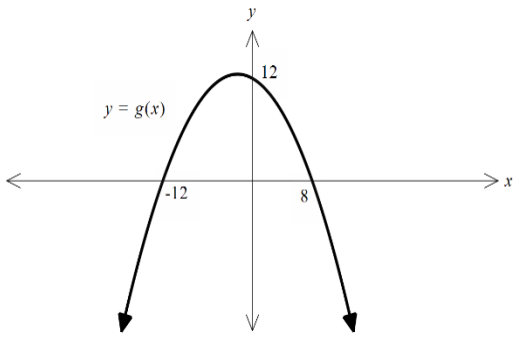
(C) $\frac{\log_e 2x}{\log_e 2}$

(D) $\frac{\log_e 4}{\log_e 2x}$

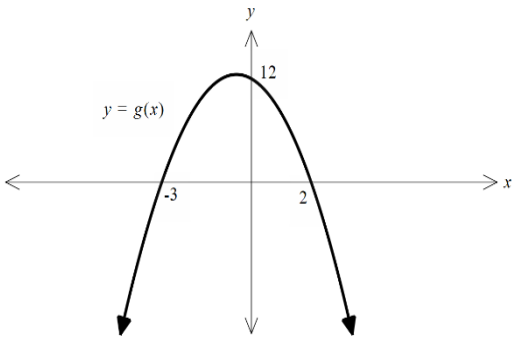
9 The figure below shows the graph of $y = f(x)$. If $g(x) = f\left(\frac{x}{2}\right)$, which of the following may represent the graph of $y = g(x)$?



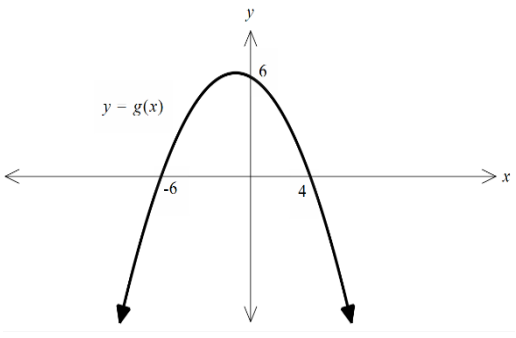
(A)



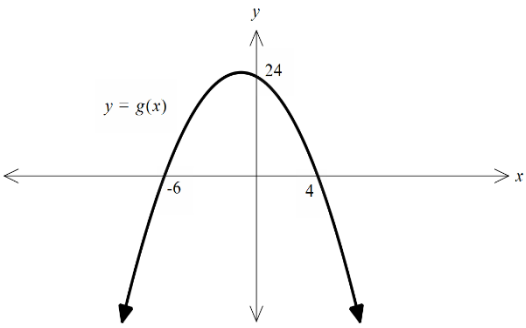
(B)



(C)



(D)



10 Consider the geometric series $1+\left(6-\sqrt{a}\right)+\left(6-\sqrt{a}\right)^2+\left(6-\sqrt{a}\right)^3+...$

If the above series is to have a limiting sum, which of the following statements is correct?

(A) $9 < a < 64$ and $a \neq 36$

(B) $36 < a < 49$

(C) $25 < a < 49$ and $a \neq 36$

(D) $9 < a < 81$

~ BLANK PAGE ~

2022

HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

Student Number: _____

Mathematics Advanced

Section II Answer Booklet

Section II

90 marks
Attempt Questions 11 – 34
Allow about 2 hours and 45 minutes for this section

Booklet 1 – Attempt Questions 11 – 29 (62 marks)
Booklet 2 – Attempt Questions 30 – 34 (28 marks)

Instructions

- Write your Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on page 24. If you use this space, clearly indicate which questions you are answering.

Please turn over

Question 11 (2 marks)

Express $\frac{2x}{x^2-4}-\frac{x+1}{x^2-x-2}$ as a single algebraic fraction, in simplest form. **2**

.....

.....

.....

.....

.....

.....

.....

Question 12 (2 marks)

Differentiate $\frac{2x}{3x^2+1}$. **2**

.....

.....

.....

.....

.....

.....

.....

Question 13 (3 marks)

Find the equation of the normal to the curve $y = x^4 - 3x^2 + 18x + 24$ at the point where $x = -2$. **3**

.....

.....

.....

.....

.....

.....

.....

.....

Question 14 (2 marks)

A discrete random variable X with mean 4 has the following probability distribution for some constants a and p. **2**

x	a	$2a$	$3a$	$4a$
$p(x)$	$4p$	$3p$	$2p$	p

Find the values of a and p.

.....

.....

.....

.....

.....

Question 15 (4 marks)

Find the following:

a) $\int \sqrt[3]{4x+7}\, dx$ **2**

.....

.....

.....

.....

.....

b) $\int_0^{\frac{\pi}{6}} \frac{dx}{\cos^2 x}$ **2**

.....

.....

.....

.....

.....

.....

.....

Question 16 (2 marks)

Show that the curve $y = x^2 - 8\ln x$ is concave up for all values of $x > 0$. **2**

.....

.....

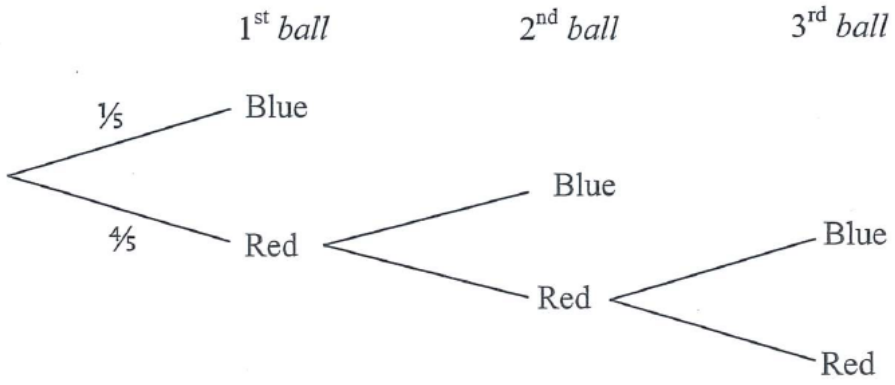
.....

.....

.....

Question 17 (4 marks)

A bag contains 1 blue and 4 red balls. A first ball is chosen at random without replacement from the bag. If this first ball is blue, no further balls are chosen. If the first ball is red, a second ball is chosen at random without replacement from the remaining balls. If this second ball is blue, no further balls are chosen. If the second ball is red, a third ball is chosen at random without replacement from the remaining balls.



a) Complete the probability tree diagram above. **1**

b) Find the probability that 3 red balls are chosen. **1**

.....

.....

c) Find the probability that at least one ball of each colour is chosen. **2**

.....

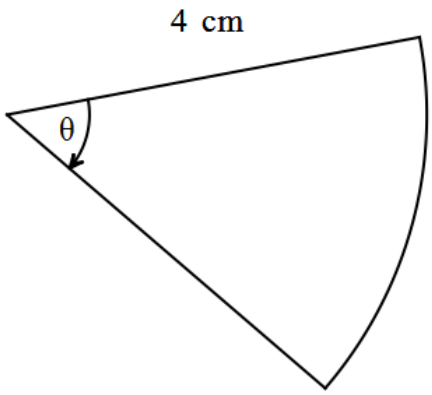
.....

.....

.....

Question 18 (2 marks)

The radius and the area of the sector below are 4 cm and $3\pi\ cm^2$ respectively.



Calculate the exact perimeter of the sector in terms of π. **2**

.....

.....

.....

.....

.....

.....

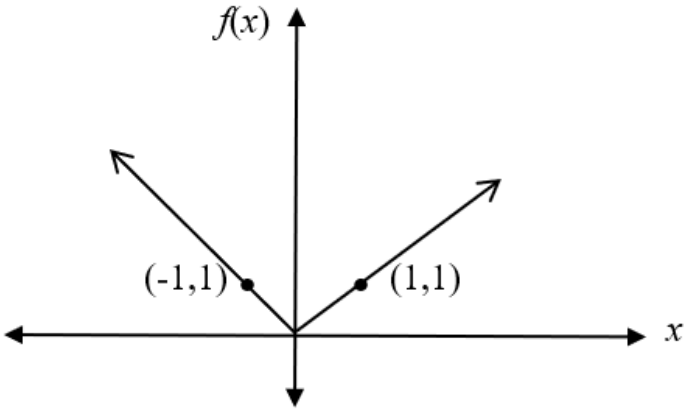
.....

.....

Question 19 (2 marks)

The curve $f(x) = |x|$ is shown below. **2**

On the same diagram, sketch $y = 2[f(x) + 2]$.



Question 20 (5 marks)

A manufacturer stores drums of chemicals. During storage, evaporation takes place. A random sample of 10 drums was taken and the time in storage, x weeks, and the evaporation loss, y mL, are shown in the table below.

x (weeks)	4	6	7	9	11	13	14	16	17	19
y (mL)	35	49	52	60	68	78	81	89	87	95

a) Find the correlation coefficient r, correct to 2 decimal places, and comment on the strength of the relationship. **2**

.....

.....

.....

b) Find the equation of the least squares' regression line. **1**

.....

.....

.....

c) Using your model, predict the amount of evaporation that would take place after 35 weeks. **1**

.....

.....

.....

d) Comment on the reliability of this prediction from your answer in part c). **1**

.....

.....

.....

Question 21 (3 marks)

The seats in a theatre are numbered in numerical order from the first row to the last row. The first row has 14 seats. Each successive row has 4 more seats than the previous one.

a) How many seats are there in the fourth row? **1**

.....

.....

.....

.....

b) The theatre has a maximum capacity of 1320 seats, what is the greatest number of rows of seats in the theatre? **2**

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Question 22 (4 marks)

Let $g(x) = \begin{cases} x-1 & x < 0 \\ x^2+1 & 0 \le x \le 3 \end{cases}$

a) Sketch $g(x)$ showing all essential features. **2**

b) State the range of $g(x)$. **2**

.....

.....

.....

.....

Question 23 (4 marks)

The table below shows the future values of an annuity of $1 for different rates of interest and for different numbers of compounding periods. The contributions are made at the end of each time period.

Future Value Interest Factors

Time Period	***Interest Rate***				
	1%	2%	3%	4%	5%
1	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500
3	3.0301	3.0604	3.0909	3.1216	3.1525
4	4.0604	4.1216	4.1836	4.2465	4.3101
5	5.1010	5.2040	5.3091	5.4163	5.5256
6	6.1520	6.3081	6.4684	6.6330	6.8019
7	7.2135	7.4343	7.6625	7.8983	8.1420
8	8.2857	8.5830	8.8923	9.2142	9.5491

a) An annuity account is to be opened in order to save $5000 in 2 years' time. Equal instalments are to be made at the end of each quarter for 2 years at an interest rate of 12% p.a. compounding quarterly. Find the amount of each quarterly instalment. **2**

.....

.....

.....

.....

.....

b) Find the amount in the annuity account after 1 year. **2**

.....

.....

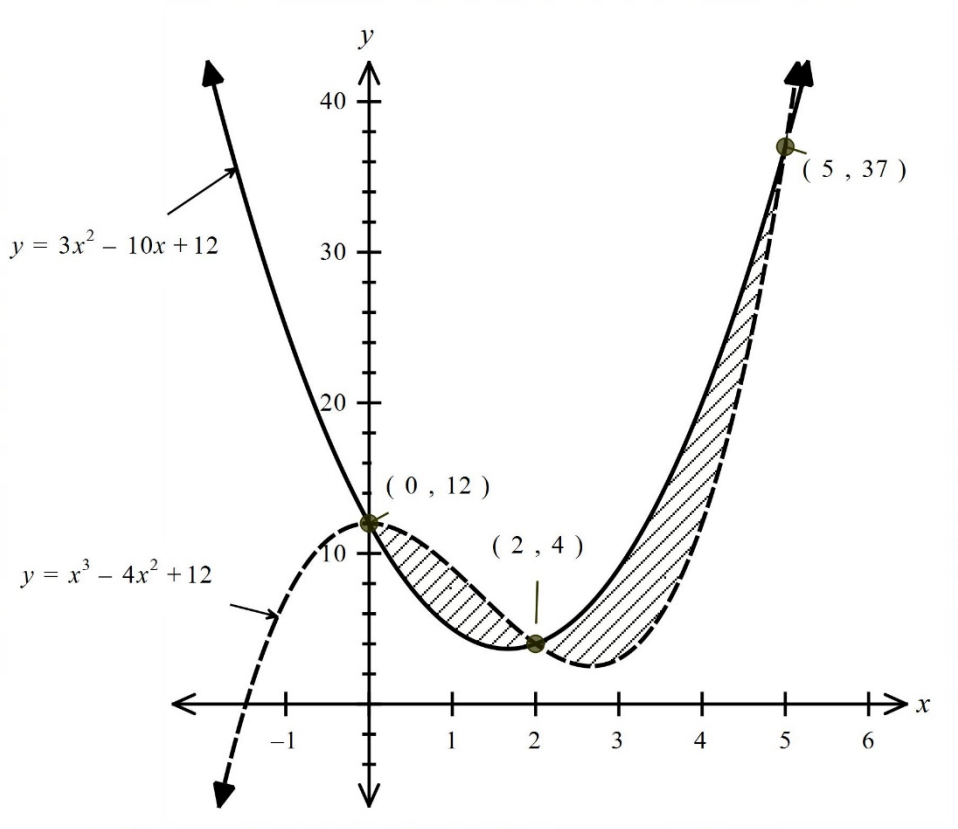
.....

.....

.....

Question 24 (4 marks)

The curves $y = x^3 - 4x^2 + 12$ and $y = 3x^2 - 10x + 12$ intersect at the points (0, 12), (2, 4) and (5, 37) as shown on the graph below.



Calculate the total shaded area enclosed between the two curves. **4**

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Question 25 (4 marks)

a) Show that $\frac{d}{dx}\left(3x^2e^{2x+1}\right) = 6xe^{2x+1}\left(x+1\right)$. **2**

.....

.....

.....

.....

.....

b) Hence, evaluate $\int_1^2 x\left(x+1\right)e^{2x+1}\ dx$. **2**

.....

.....

.....

.....

.....

Question 26 (2 marks)

Prove that $\sin^2 x + \cot^2 x = \text{cosec}^2 x - \cos^2 x$. **2**

.....

.....

.....

.....

.....

.....

.....

Questions 11 – 26 are worth 49 marks in total

Question 27 (4 marks)

Consider the curve $y = \dfrac{x^2}{x-2}$.

a) State the domain of the curve. **1**

.....

.....

b) Find the coordinates and nature of the stationary points on the curve. **3**

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Question 28 (4 marks)

a) Find all solutions of $\tan\left(2x-\frac{\pi}{3}\right)=\frac{1}{\sqrt{3}}$ for all x in the interval $[0,\ 2\pi]$. **3**

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

b) The graph of $y=\tan\left(2x-\frac{\pi}{3}\right)$ is plotted along with its vertical asymptotes. **1**

Identify any ONE of the vertical asymptotes.

.....

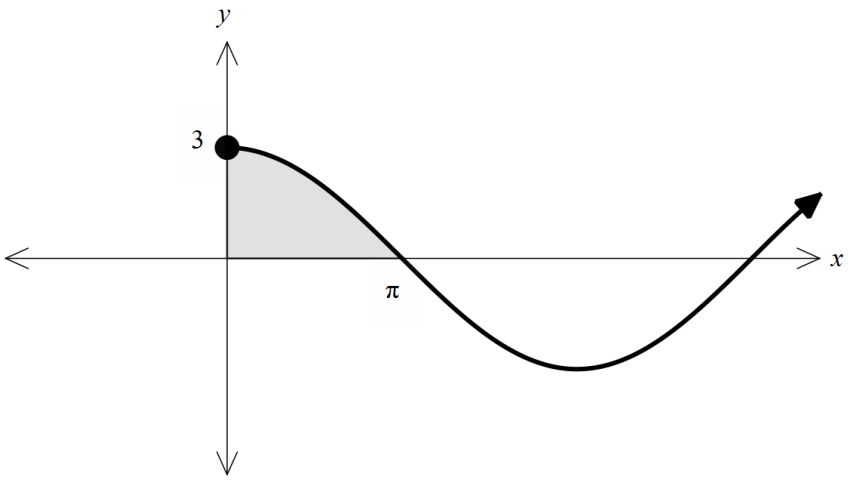
.....

.....

.....

Question 29 (5 marks)

The graph below represents the function $f(x) = p\cos(qx)$.



a) What type of function is $f(x)$? (One-to-one, many-to-one, one-to-many or many-to-many) **1**

.....

b) Find the value of p and q. **2**

.....

.....

.....

.....

c) Hence, evaluate the area of the shaded region. **2**

.....

.....

.....

.....

.....

.....

.....

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

2022

HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

Student Number: _____

Mathematics Advanced

Section II Answer Booklet

Booklet 2 – Attempt Questions 30 – 34 (28 marks)

Instructions

- Write your Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on page 32. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 30 (7 marks)

A continuous random variable X has probability density function $f(x)$ given by $f(x) = \frac{2}{x^2}$ for $1 \le x \le 2$ and $f(x) = 0$ for $x < 1$ or $x > 2$.

a) Show that $f(x)$ is a probability density function. **2**

.....

.....

.....

.....

b) Find the cumulative distribution function $F(x)$ of the random variable X. **2**

.....

.....

.....

.....

c) Given Q_1, Q_2, Q_3 are the quartiles of the distribution of X and $\frac{1}{Q_1}$, $\frac{1}{Q_2}$, $\frac{1}{Q_3}$ are consecutive terms in an arithmetic sequence, find d. **3**

.....

.....

.....

.....

.....

.....

.....

.....

.....

Question 31 (5 marks)

At the start of year 2000, a colony of 600 animals was established in a sanctuary. Each year the population size grows at a rate of 10% per year and at the end of each year 50 of the animals are released into the wild. The population size P_n at the end of the n^{th} year can be determined using the recurrence relation $P_n = 1.1 \times P_{n-1} - 50$ where $n = 1,\ 2,\ 3,\ \ldots$ and $P_0 = 600$.

a) Write down expressions for P_1, P_2, P_3 in terms of P_0 and hence show that $P_n = 100 \times (1.1)^n + 500$. **3**

.....

.....

.....

.....

.....

.....

.....

.....

.....

b) Find the year in which the growth in population size from the beginning of that year to the start of the next year first exceeds 100. **2**

.....

.....

.....

.....

.....

.....

.....

.....

.....

Question 32 (6 marks)

A particle is moving in a straight line. At time t seconds, it has displacement x metres from a fixed point O in the line and velocity $v = 3\sqrt{t+1} - 9$ metres per second. Initially, the particle is at O.

a) Show that $x = 2(t+1)\sqrt{t+1} - 9t - 2$. **2**

.....

.....

.....

.....

.....

.....

.....

b) Find when and where the particle is at rest. **2**

.....

.....

.....

.....

.....

.....

.....

c) Find the distance travelled by the particle in the first 24 seconds of its motion. **2**

.....

.....

.....

.....

Question 33 (4 marks)

Henry goes to the gym on Fridays. He uses only the swimming pool or the rowing machines.

On any given Friday, $P(R|S) = \frac{3}{8}$, $P(S|R) = \frac{3}{10}$ and $P(R \cup S) = \frac{7}{9}$

where R represents the event he uses the rowing machines and
S represents the event he uses the swimming pool.

Calculate the values of $P(R)$ and $P(S)$. **4**

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

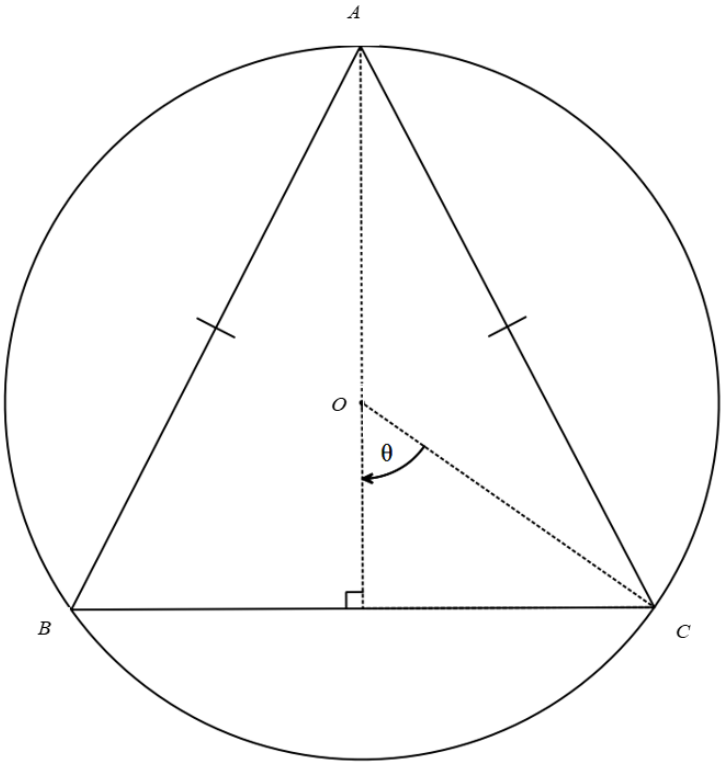
.....

.....

.....

Question 34 (6 marks)

An isosceles triangle ΔABC is inscribed inside a circle of fixed radius 2 and centre O. Let θ be defined as in the diagram below.



a) Show that the area of the triangle ΔABC is given by $A = 4\sin\theta(1+\cos\theta)$. **2**

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Question 34 continues on page 31

Question 34 (continued)

b) Find the value of θ that maximises the area of ΔABC. **4**

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

End of paper

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

2022 HSC Trial Examination
Mathematics Advanced

Section I Multiple Choice Answers

1. D
2. C
3. C
4. A
5. D
6. A
7. D
8. B
9. A
10. C

Section II

Question 11

Criteria	Marks
Correct result using any method	2
Some correct factorisation and manipulation	1

Answer

$$\frac{2x}{x^2-4}-\frac{x+1}{x^2-x-2}=\frac{2x}{(x-2)(x+2)}-\frac{x+1}{(x-2)(x+1)}$$

$$=\frac{2x}{(x-2)(x+2)}-\frac{1}{(x-2)}$$

$$=\frac{2x-(x+2)}{(x+2)(x-2)}$$

$$=\frac{2x-\mathrm{x}-2}{(x+2)(x-2)}$$

$$=\frac{x-2}{(x+2)(x-2)}$$

$$=\frac{1}{x+2}$$

Question 12

Criteria	Marks
Provides correct answer	2
Attempts to use product rule or equivalent merit	1

Answer

$$f(x)=\frac{2x}{3x^2+1}$$

$$\frac{d}{dx}\left(\frac{2x}{3x^2+1}\right)=\frac{\left(3x^2+1\right)(2)-(2x)(6x)}{\left(3x^2+1\right)^2}$$

$$=\frac{2-6x^2}{\left(3x^2+1\right)^2}$$

Question 13

Criteria	Marks
Correct equation in either form	4
Finding gradient of normal and point of contact	3
Finding gradient of tangent and point of contact	2
Finding gradient or tangent or point of contact	1

Answer

$y = x^4 - 3x^2 + 18x + 24$

when $x = -2$

$y = (-2)^4 - 3(-2)^2 + 18(-2) + 24 = -8$

$y' = 4x^3 - 6x + 18$

when $x = -2$

$y' = 4(-2)^3 - 6(-2) + 18$

$= -32 + 12 + 18 = -2$

Gradient of tangent $m_1 = -2$

Gradient of normal $m_2 = \frac{1}{2}$ so that $m_1 \, . \, m_2 = -1$

Equation of normal :

$y - y_1 = m_2(x - x_1)$ OR use $y = mx + b$

$y - (-8) = \frac{1}{2}(x - (-2))$ $\quad -8 = \frac{1}{2}(-2) + b$

$2y + 16 = x + 2$ $\quad -8 = -1 + b \Rightarrow b = -7$

$x - 2y - 14 = 0$ $\quad y = \frac{1}{2}x - 7$

Question 14

Criteria	Marks
Uses the values of the mean and probability sum to find the values of p, a	2
Substantial progress eg. writes one equation for a, p using the mean or probability sum	1

Answer

$\sum p(x) = 1 \Rightarrow 10p = 1 \quad \therefore p = \frac{1}{10}$

$E(X) = 4 \Rightarrow 4ap + 6ap + 6ap + 4ap = 4 \quad \therefore 20ap = 4 \quad \therefore a = 2$

Question 15

a)

Criteria	Marks
Provides correct solution	2
Progress towards solution	1

Answer

$$\int \sqrt[3]{4x+7}\ dx$$

$$= \int (4x+7)^{\frac{1}{3}}\ dx$$

$$= \frac{(4x+7)^{\frac{4}{3}}}{\frac{4}{3}} \times \frac{1}{4} + C$$

$$= \frac{3(4x+7)^{\frac{4}{3}}}{16} + C$$

b)

Criteria	Marks
Provides correct solution	2
Integrates primitive function correctly	1

Answer

$$\int_0^{\frac{\pi}{6}} \frac{dx}{\cos^2 x}$$

$$= \int_0^{\frac{\pi}{6}} \sec^2 x\, dx$$

$$= \tan x \Big|_0^{\frac{\pi}{6}}$$

$$= \tan \frac{\pi}{6} - \tan 0$$

$$= \frac{1}{\sqrt{3}}$$

Question 16

Criteria	Marks
Shows second derivative is positive for all $x > 0$	2
Substantial progress eg finds the first derivative	1

Answer

$$y = x^2 - 8\ln x$$

$$\frac{dy}{dx} = 2x - \frac{8}{x}$$

$$\frac{d^2y}{dx^2} = 2 + \frac{8}{x^2}$$

$$\therefore \frac{d^2y}{dx^2} > 0 \text{ for all } x > 0$$

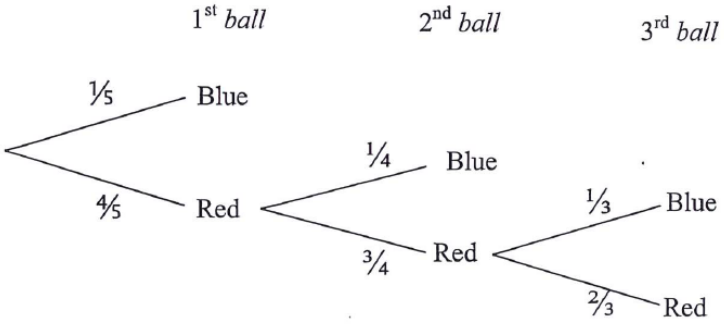
$\therefore$ curve is concave up for all $x > 0$

Question 17

a)

Criteria	Marks
Completes probability tree	1

Answer



b)

Criteria	Marks
Correct answer	1

Answer

$$P(R, R, R) = \tfrac{4}{5} \times \tfrac{3}{4} \times \tfrac{2}{3} = \tfrac{2}{5}$$

c)

Criteria	Marks
Calculates required probability	2
Substantial progress eg identifies one appropriate path through the tree with its probability	1

Answer

$$P(R, B) + P(R, R, B) = \tfrac{4}{5} \times \tfrac{1}{4} + \tfrac{4}{5} \times \tfrac{3}{4} \times \tfrac{1}{3} = \tfrac{2}{5}$$

Question 18

Criteria	Marks
Provides correct solution	2
Finds the value of θ or equivalent merit	1

Answer

$$\frac{1}{2}(4)^2\,\theta = 3\pi$$

$$\theta = \frac{3\pi}{8}$$

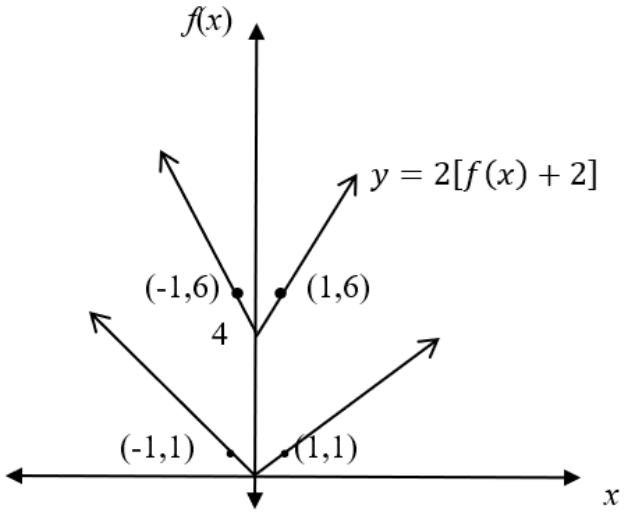
$$P = 4 + 4 + 4\left(\frac{3\pi}{8}\right)$$

$$P = 8 + \frac{3\pi}{2} cm$$

Question 19

Criteria	Marks
Provides the correct answer	2
Provides some relevant information	1

Answer



Question 20

a)

Criteria	Marks
Provides correct solution	2
Calculates r value or states relationship correctly	1

Answer

$r = 0.9901098193$

$r = 0.99\,(2d.p)$

$\therefore$ strong positive linear relationship

b)

Criteria	Marks
Provides correct solution	1

Answer

$y = A + Bx$

$A = 24.117338$

$B = 3.903677758$

$y = 24.12 + 3.90x$

c)

Criteria	Marks
Provides correct solution	1

Answer

$y = 24.12 + 3.9(35)$

$y = 160.62$

$y = 160$

d)

Criteria	Marks
Provides correct explanation	1

Answer

Although there is a strong relationship between the variables our x value of 35 is far away from our data set. i.e. we are extrapolating and the value is less reliable the farther away from the data set we used in our calculations.

Question 21

a)

Criteria	Marks
Provides correct solution	1

Answer

$a = 14, d = 4$

$T_4 = 14 + (4-1)4$

$T_4 = 26$

26 seats in row 4

b)

Criteria	Marks
Provides correct solution	2
Attempts to use formula or equivalent merit	1

Answer

$$1320 = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$1320 = \frac{n}{2}\left(28 + 4n - 4\right)$$

$$2640 = \left(24 + 4n\right)$$

$$4n^2 + 24n - 2640 = 0$$

$$\text{as } n > 0, n = \frac{-24 + \sqrt{24^2 - 4(4)(-2640)}}{2(4)}$$

$n \approx 22.86503431...$

$\therefore n = 22$

$\therefore$ maximum 22 rows

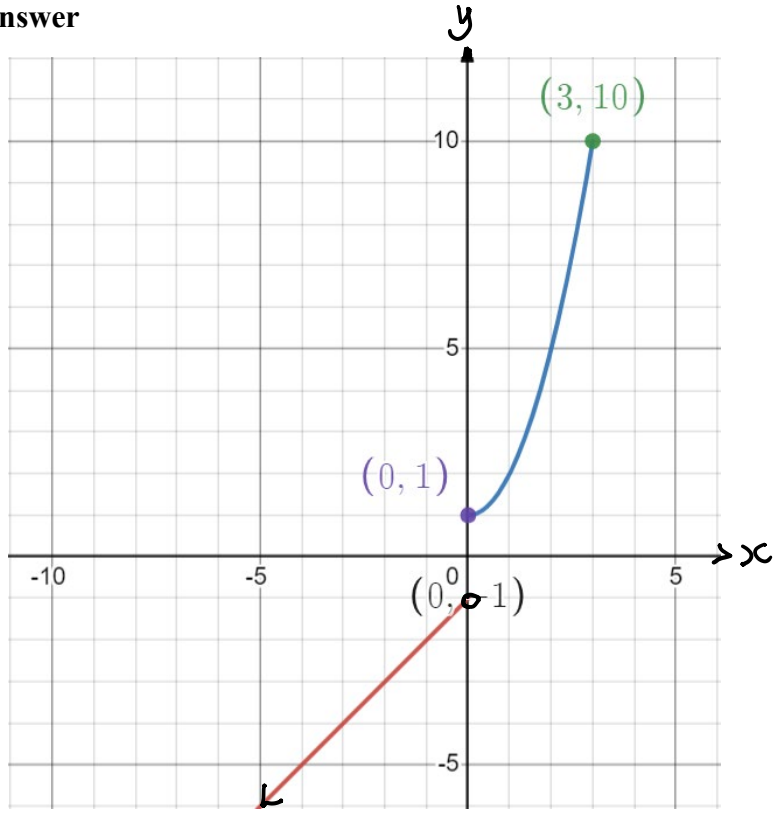
Question 22

a)

Criteria	Marks
Provides correct solution, showing all intercepts with relevant axes and key features such as vertex and endpoint	2
Correct graph shape but missing either intercepts, open/closed circle, or endpoint (3,10)	1

Answer

a)



b)

Criteria	Marks
Provides correct solution	2
States 1 correct section of range values	1

Answer

$R: y \leq -1 \cup 1 \leq y \leq 10$

Question 23

a)

Criteria	Marks
Uses appropriate table entry to calculate the quarterly instalment	2
Substantial progress eg correct table value selected, or correct procedure but error in table value	1

Answer

Future value of \$1 annuity for 8 quarters at 3% per quarter is \$8.8923.
Quarterly instalment \$$a$ is given by $8 \cdot 8923 \times a = 5000$

$a = 5000 \div 8 \cdot 8923$ *Ans.* \$562.28 quarterly

b)

Criteria	Marks
Uses appropriate table entry and answer from (a) to calculate the future value after 1 year	2
Substantial progress eg selects correct table value	1

Answer

Future value of annuity of \$562.28 quarterly for 4 quarters is $4 \cdot 1836 \times \$562.28 = \$2352.35$

Question 24

Criteria	Marks
Correct answer	4
If done without separating integrals, answer is then $10\ \frac{5}{12}$	3
Correct integration	2
Some relevant calculations relating to integrals	1

Answer

$$\text{Area} = \left|\int_0^2 x^3 - 4x^2 + 12\ dx - \int_0^2 3x^2 - 10x + 12\ dx\right|$$

$$+ \left|\int_2^5 3x^2 - 10x + 12\ dx - \int_2^5 x^3 - 4x^2 + 12\ dx\right|$$

$$= \left|\int_0^2 x^3 - 7x^2 + 10x\ dx\right|$$

$$+ \left|\int_2^5 -x^3 + 7x^2 - 10x\ dx\right|$$

$$= \left|\left[\frac{x^4}{4} - \frac{7x^3}{3} + 5x^2\right]_0^2\right| + \left|\left[-\frac{x^4}{4} + \frac{7x^3}{3} - 5x^2\right]_2^5\right|$$

$$= \left|\left(\frac{2^4}{4} - \frac{7(2)^3}{3} + 5(2)^2\right) - (0)\right|$$

$$+ \left|\left(-\frac{5^4}{4} + \frac{7(5)^3}{3} - 5(5)^2\right) - \left(-\frac{2^4}{4} + \frac{7(2)^3}{3} - 5(2)^2\right)\right|$$

$$= \left|\left(5\frac{1}{3}\right) - (0)\right| + \left|\left(10\ \frac{5}{12}\right) - \left(-5\ \frac{1}{3}\right)\right|$$

$$= \left|5\frac{1}{3}\right| + \left|15\ \frac{3}{4}\right|$$

$$= 21\frac{1}{12}\ \text{sq units}$$

Question 25

a)

Criteria	Marks
Correct answer	2
Correct application of product rule	1

Answer

$$\frac{d}{dx}(3x^2e^{2x+1}) = 3x^2 \,.\, 2e^{2x+1} + 6x.e^{2x+1}$$

$$= 6x^2\, e^{2x+1} + 6x.e^{2x+1}$$

$$= 6xe^{2x+1}\,(x+1)$$

$$= 6x(x+1)e^{2x+1}$$

b)

Criteria	Marks
Correct answer	2
Correct application of anti-derivative	1

Answer

$$\int_1^2 x(x+1)e^{2x+1}\,dx = \frac{1}{6}\int_1^2 6x(x+1)e^{2x+1}\,dx$$

$$= \frac{1}{6}[3x^2e^{2x+1}]_1^2$$

$$= \frac{1}{6}\left(3(2)^2e^{2(2)+1} - 3(1)^2e^{2(1)+1}\right)$$

$$= \frac{1}{6}\,(12e^5 - 3e^3)$$

$$= 2e^5 - \frac{1}{2}\,e^3$$

Question 26

Criteria	Marks
Complete and valid proof	2
Some relevant trig relations derived from the given statement	1

Answer

$$\text{LHS} = \sin^2 x + \cot^2 x$$

$$= \sin^2 x + \frac{\cos^2 x}{\sin^2 x}$$

$$= \frac{\sin^2 x . \sin^2 x + (\cos^2 x)}{\sin^2 x}$$

$$= \frac{\sin^2 x(1 - \cos^2 x) + (\cos^2 x)}{\sin^2 x}$$

$$= \frac{\sin^2 x - \sin^2 x \cos^2 x + \cos^2 x}{\sin^2 x}$$

$$= \frac{\sin^2 x + \cos^2 x - \sin^2 x \cos^2 x}{\sin^2 x}$$

$$= \frac{1 - \sin^2 x \cos^2 x}{\sin^2 x}$$

$$= \frac{1}{\sin^2 x} - \frac{\sin^2 x \cos^2 x}{\sin^2 x}$$

$$= \text{cosec}^2 x - \cos^2 x = \text{RHS}$$

Question 27

Criteria	Marks
Differentiates to find both stationary points and determines their nature	4
Substantial progress eg finds the stationary points and the nature of one of them	3
Moderate progress eg finds the stationary points	2
Some progress eg applies the quotient rule for differentiation	1

Answer

$$y=\frac{x^2}{x-2}$$

$$\frac{dy}{dx}=\frac{2x(x-2)-x^2}{(x-2)^2}$$

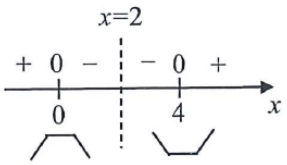
$$=\frac{x^2-4x}{(x-2)^2}$$

$$\frac{dy}{dx}=\frac{x(x-4)}{(x-2)^2}$$

$$\frac{dy}{dx}=0 \Rightarrow \begin{cases} x=0,\ y=0 \\ \quad or \\ x=4,\ y=8 \end{cases}$$

Stationary points are $(0\,,0)$ and $(4\,,8)$

Sign of $\frac{dy}{dx}$



$(0\,,0)$ is a maximum turning point. $(4\,,8)$ is a minimum turning point

Question 28

a)

Criteria	Marks
Finds all FOUR solutions for x	3
Finds all FOUR solutions for $2x-\frac{\pi}{3}$ OR finds TWO solutions for x	2
Correct domain adjustment OR finds solutions for $\tan x=\frac{1}{\sqrt{3}}$	1

Answer

$0\le x\le 2\pi \Rightarrow 0\le 2x\le 4\pi$

$$\Rightarrow -\frac{\pi}{3}\le 2x-\frac{\pi}{3}\le \frac{11\pi}{3}$$

The solutions to $\tan\left(2x-\frac{\pi}{3}\right)=\frac{1}{\sqrt{3}}$ in the interval $\left[-\frac{\pi}{3},\frac{11\pi}{3}\right]$ are in the first, third, fifth and seventh quadrants:

$$2x-\frac{\pi}{3}=\frac{\pi}{6},\pi+\frac{\pi}{6},2\pi+\frac{\pi}{6},3\pi+\frac{\pi}{6}$$

$$=\frac{\pi}{6},\frac{7\pi}{6},\frac{13\pi}{6},\frac{19\pi}{6}$$

$$2x=\frac{\pi}{2},\frac{3\pi}{2},\frac{5\pi}{2},\frac{7\pi}{2}$$

$$x=\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}$$

b)

Criteria	Marks
Gives ANY correct solution	1

Answer

Consider the asymptote corresponding to $y=\tan x$ at $x=\frac{\pi}{2}$.

$$2x-\frac{\pi}{3}=\frac{\pi}{2}$$

$$2x=\frac{5\pi}{6}$$

$$x=\frac{5\pi}{12}$$

Question 29

a)

Criteria	Marks
Provides correct solution	1

Answer

Many-to-one

b)

Criteria	Marks
Provides correct solution	2
Finds one correct value	1

Answer

$p = 3, q = \frac{1}{2}$

c)

Criteria	Marks
Provides correct solution	2
Correct primitive function or equivalent merit	1

Answer

$$A = \int_0^{\pi} 3\cos\left(\frac{x}{2}\right)$$

$$A = \left[6\sin\left(\frac{x}{2}\right)\right]_0^{\pi}$$

$$A = 6\sin\frac{\pi}{2} - 6\sin 0$$

$$A = 6u^2$$

Question 30

a)

Criteria	Marks
Verifies both conditions are satisfied for $f(x)$ to be a probability density function	2
Substantial progress eg. finds and evaluates the anti-derivative	1

Answer

$$f(x)=\frac{2}{x^2},\quad 1\le x\le 2 \qquad \int_1^2 f(x)\,dx=\left[-\frac{2}{x}\right]_1^2$$

$$\therefore f(x)>0 \qquad =-1+2$$

$$=1$$

Hence $f(x)$ is a probability density function.

b)

Criteria	Marks
Uses integration to find the distribution function	2
Substantial progress eg correct procedure but one error or omission	1

Answer

$$F(x)=\int_1^x \frac{2}{x^2}dx$$

$$=\left[-\frac{2}{x}\right]_1^x$$

$$=-\frac{2}{x}+2$$

$$\therefore F(x)=\begin{cases} 0 & , \quad x<1 \\ 2\left(1-\frac{1}{x}\right) & , \quad 1\le x\le 2 \\ 1 & , \quad x>2 \end{cases}$$

c)

Criteria	Marks
Finds the reciprocals of the quartiles and correct d	3
Substantial progress eg. correct procedure but an error in execution	2
Some progress eg. finds one of the quartiles or its reciprocal correctly	1

Answer

$$F(x)=\tfrac{1}{4} \Rightarrow 2\left(1-\frac{1}{x}\right)=\tfrac{1}{4} \qquad F(x)=\tfrac{1}{2} \Rightarrow 2\left(1-\frac{1}{x}\right)=\tfrac{1}{2} \qquad F(x)=\tfrac{3}{4} \Rightarrow 2\left(1-\frac{1}{x}\right)=\tfrac{3}{4}$$

$$\frac{1}{x}=\tfrac{7}{8} \qquad \frac{1}{x}=\tfrac{3}{4} \qquad \frac{1}{x}=\tfrac{5}{8}$$

$$\therefore \frac{1}{Q_1}=\tfrac{7}{8} \qquad \therefore \frac{1}{Q_2}=\tfrac{3}{4} \qquad \therefore \frac{1}{Q_3}=\tfrac{5}{8}$$

$$\therefore \frac{1}{Q_2}-\frac{1}{Q_1}=-\tfrac{1}{8} \text{ and } \frac{1}{Q_3}-\frac{1}{Q_2}=-\tfrac{1}{8}\ .$$

$$\therefore d=-\frac{1}{8}$$

Question 31

a)

Criteria	Marks
Generalises expressions for P_1, P_2, P_3 to find expression P_n then uses sum of GP to simplify	3
Substantial progress eg process correct, but one error made, or simplification not completed	2
Some progress eg finds expressions for P_1, P_2, P_3	1

Answer

$$P_n = 1{\cdot}1 \times P_{n-1} - 50$$

$$P_1 = 1{\cdot}1 \times P_0 - 50$$

$$P_2 = 1{\cdot}1 \times P_1 - 50 = 1{\cdot}1 \times \left(1{\cdot}1 \times P_0 - 50\right) - 50 = \left(1{\cdot}1\right)^2 P_0 - 50\left(1{\cdot}1 + 1\right)$$

$$P_3 = 1{\cdot}1 \times P_2 - 50 = 1{\cdot}1 \times \left\{\left(1{\cdot}1\right)^2 P_0 - 50\left(1{\cdot}1 + 1\right)\right\} - 50 = \left(1{\cdot}1\right)^3 P_0 - 50\left\{\left(1{\cdot}1\right)^2 + 1{\cdot}1 + 1\right\}$$

Generalising: $P_n = \left(1{\cdot}1\right)^n P_0 - 50\left\{1 + 1{\cdot}1 + \left(1.1\right)^2 + \ldots + \left(1.1\right)^{n-1}\right\}$ where $\{\ldots\}$ is the sum of n terms of a GP with $a = 1,\ r = 1{\cdot}1$

Given $P_0 = 600$,

$$P_n = \left(1{\cdot}1\right)^n \times 600 - 50\frac{\left(1{\cdot}1\right)^n - 1}{1{\cdot}1 - 1}$$

$$= \left(1{\cdot}1\right)^n \times 600 - 500\left\{\left(1{\cdot}1\right)^n - 1\right\}$$

$$\therefore P_n = 100 \times \left(1{\cdot}1\right)^n + 500$$

b)

Criteria	Marks
Solves $P_n - P_{n-1} > 100$ to find the required year	2
Substantial progress eg correct procedure but one error in calculation	1

Answer

Require least integer value of n such that $P_n - P_{n-1} > 100$

$$100\left\{\left(1{\cdot}1\right)^n - \left(1{\cdot}1\right)^{n-1}\right\} > 100$$

$$\left(1{\cdot}1\right)^{n-1}\left\{1{\cdot}1 - 1\right\} > 1$$

$$\left(1{\cdot}1\right)^{n-1} > 10$$

$$\left(n-1\right)\log_{10}\left(1{\cdot}1\right) > 1$$

$$n - 1 > \frac{1}{\log_{10}\left(1{\cdot}1\right)} \approx 24{\cdot}16$$

$n - 1 = 25,\ 26,\ 27, \ldots$ Required year is 2026.

Question 32

a)

Criteria	Marks
Uses integration and initial conditions to obtain required result	2
Substantial progress eg integrates with respect to t	1

Answer

$$\frac{dx}{dt} = 3(t+1)^{\frac{1}{2}} - 9$$

$$x = 2(t+1)^{\frac{3}{2}} - 9t + c$$

$$t = 0,\ x = 0 \Rightarrow c = -2$$

$$\therefore\ x = 2(t+1)\sqrt{t+1} - 9t - 2$$

b)

Criteria	Marks
Finds time and position when particle at rest	2
Substantial progress eg finds t when $v = 0$	1

Answer

$$v = 0 \Rightarrow \sqrt{t+1} = 3$$

$$t = 8,\quad x = -20$$

Also when $t = 0$, $v = -3$

Particle is at rest at time 8 seconds when 20 m from O in initial direction of travel .

c)

Criteria	Marks
Finds distance travelled	2
Substantial progress eg finds x when $t = 24$	1

Answer

$t = 24 \Rightarrow x = 32$ Particle travels from $x = 0$ to $x = -20$ then to $x = 32$ in first 24 seconds.

Hence distance travelled is $20 + 20 + 32 = 72$ m

Question 33

Criteria	Marks
Provides the correct answer	4
Majority of solution is correct	3
Makes reasonable progress	2
Shows some relevant information	1

Answer

$$P(R|S) = \frac{P(R \cap S)}{P(S)} = \frac{3}{8}$$

$$\therefore P(R \cap S) = \frac{3}{8}P(S)$$

$$P(S|R) = \frac{P(R \cap S)}{P(R)} = \frac{3}{10}$$

$$\therefore P(R \cap S) = \frac{3}{10}P(R)$$

$$\frac{3}{8}P(S) = \frac{3}{10}P(R)$$

$$\therefore P(S) = \frac{4}{5}P(R)$$

$$P(R \cup S) = P(R) + P(S) - P(R \cap S)$$

$$\frac{7}{9} = P(R) + \frac{4}{5}P(R) - \frac{3}{10}P(R)$$

$$\therefore P(R) = \frac{14}{27}$$

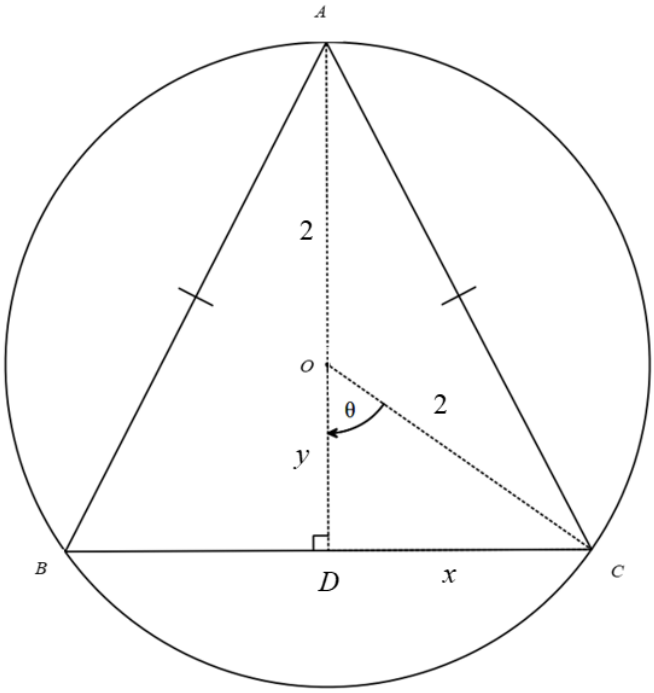
$$\therefore P(S) = \frac{56}{135}$$

Question 34

a)

Criteria	Marks
Provides correct solution	2
Progress towards solution	1

Answer



In ΔODC,

$\sin\theta = \frac{x}{2}$

$\cos\theta = \frac{y}{2}$

$x = 2\sin\theta$

$y = 2\cos\theta$

Area of $\Delta ABC = \frac{bh}{2}$

$b = 4\sin\theta$

$h = 2 + 2\cos\theta$

$A = \frac{8\sin\theta(1+\cos\theta)}{2}$

$A = 4\sin\theta(1+\cos\theta)$

b)

Criteria	Marks
Provides correct solution	4
Finds correct θ values, including testing nature	3
Attempts to solve $\frac{dA}{d\theta}=0$, using the identity $\sin^2\theta+\cos^2\theta=1$, or equivalent merit.	2
Differentiates correctly or equivalent merit	1

Answer

$A=4\sin\theta(1+\cos\theta)$

$\frac{dA}{d\theta}=4\cos\theta(1+\cos\theta)-4\sin^2\theta$

$\frac{dA}{d\theta}=4\cos\theta+4\cos^2\theta-4\sin^2\theta$

$\frac{dA}{d\theta}=4\cos\theta+4\cos^2\theta-4(1-\cos^2\theta)$

$\frac{dA}{d\theta}=8\cos^2\theta+4\cos\theta-4$

$\frac{dA}{d\theta}=4(2\cos^2\theta+\cos\theta-1)$

$\frac{dA}{d\theta}=0$

$\therefore(2\cos\theta-1)(\cos\theta+1)=0$

$\cos\theta=\frac{1}{2},\theta=\frac{\pi}{3}$

$\cos\theta\neq-1, 0<\theta<\pi$

x	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\frac{dA}{d\theta}$	>0	0	<0
	Increasing	Stationary	Decreasing

$\therefore\theta=\frac{\pi}{3}$ is a maximum S.P.

$A\left(\frac{\pi}{3}\right)=4\sin\frac{\pi}{3}\left(1+\cos\frac{\pi}{3}\right)$

$A=3\sqrt{3}$