Roseville College

2023 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Approved calculators may be used
- A reference sheet is provided at the back of this paper
- In Questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks : 100

Section I – 10 marks (pages 2 – 8)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 9 – 35)

- Attempt Questions 11 – 34
- Allow about 2 hours and 45 minutes for this section

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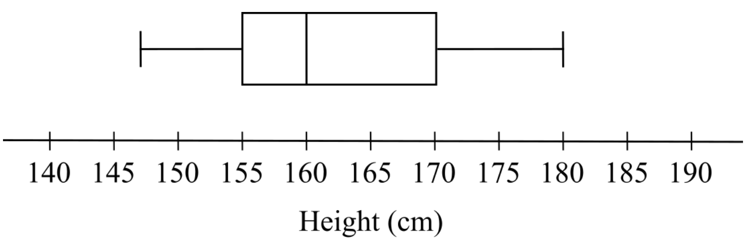
Section I - 10 marks

Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10

1. The heights of students in a class are represented in the boxplot below. A new student, who will become the tallest member, is joining the class.



What is the maximum height the new student can be so they are not classified as an outlier?

A. 187.5 cm

B. 192.5 cm

C. 197.5 cm

D. 202.5 cm

2. The table below shows the values of the functions $f(x)$ and $g(x)$ for various values of x.

x	1	2	3	4	5
$f(x)$	3	4	5	1	2
$g(x)$	5	3	2	1	4

What is the value of $f(g(3))$?

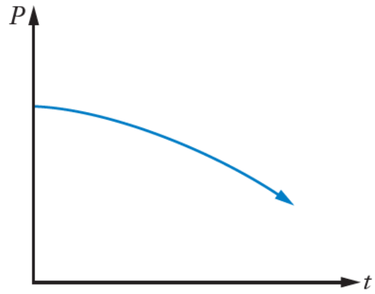
A. 2

B. 3

C. 4

D. 5

3. The population (P) of Marvel, a super villain's town is shown over time (t) by the graph below:



Which statement best describes the population of Marvel.

A. The population is increasing at an increasing rate.

B. The population is increasing at a decreasing rate.

C. The population is decreasing at an increasing rate.

D. The population is decreasing at a decreasing rate.

4. The curve $y = \ln x$ is translated to the left by π units and then dilated horizontally by a scale factor of 3.

What is the equation which describes this new curve?

A. $y = \ln\left(\dfrac{x}{3} + \pi\right)$

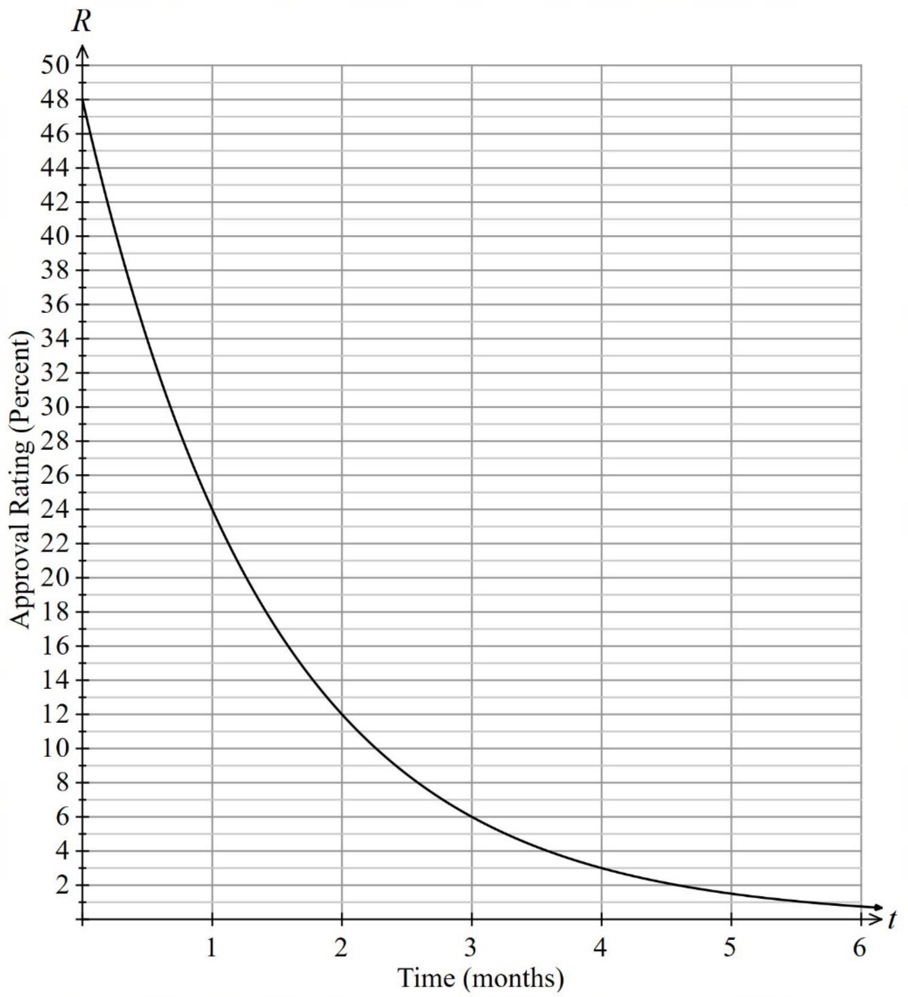
B. $y = \ln\left(\dfrac{x}{3} - \pi\right)$

C. $y = 3\ \ln\,(x + \pi)$

D. $y = \ln\left(\dfrac{x+\pi}{3}\right)$

5. The approval rating of a politician falls dramatically after an election. The rate of decrease can be modelled by the equation: $R = a \times b^{-t}$, where R is the percentage approval rating, t is the time in months since the election and a and b are positive constants.

The graph below shows the relationship.



Which equation could be used to model the graph shown?

A. $R = 48 \times 2^{-t}$

B. $R = 24 \times 2^{-t}$

C. $R = 2 \times 48^{-t}$

D. $R = 48 \times 4^{-t}$

6. Find the range of $y = \frac{1}{2} - \frac{3}{2}\sin\left(2x + \frac{\pi}{3}\right)$.

A. $\left[-\frac{3}{2}, \frac{3}{2}\right]$

B. $[-1, 2]$

C. $[-1, 1]$

D. $\left[-\frac{1}{2}, \frac{1}{2}\right]$

7. Differentiate $x\sin\left(\frac{1}{x}\right)$

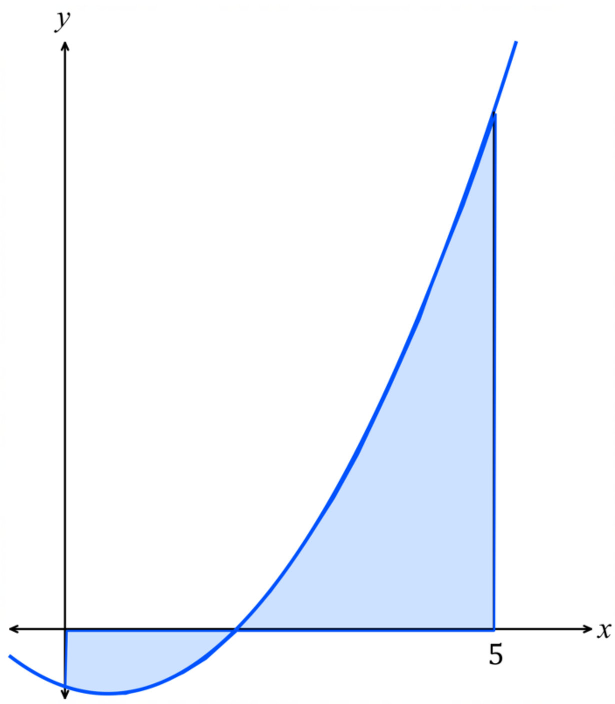
A. $\sin\left(\frac{1}{x}\right) - \frac{1}{x}\cos\left(\frac{1}{x}\right)$

B. $\cos\left(\frac{1}{x}\right) - \frac{1}{x}\sin\left(\frac{1}{x}\right)$

C. $\sin\left(\frac{1}{x}\right) + \frac{1}{x}\cos\left(\frac{1}{x}\right)$

D. $x\cos\left(\frac{1}{x}\right)$

8.



Which expression will give the area of the shaded region bounded by the curve $y = x^2 - x - 2$, the x-axis and the lines $x = 0$ and $x = 5$?

A. $A = \left|\int_0^1 x^2 - x - 2\ dx\right| + \int_1^5 x^2 - x - 2\ dx$

B. $A = \int_0^1 x^2 - x - 2\ dx + \left|\int_1^5 x^2 - x - 2\ dx\right|$

C. $A = \left|\int_0^2 x^2 - x - 2\ dx\right| + \int_2^5 x^2 - x - 2\ dx$

D. $A = \int_0^2 x^2 - x - 2\ dx + \left|\int_2^5 x^2 - x - 2\ dx\right|$

9. The probability density function for the continuous random variable X is:

$$f(x) = \begin{cases} 2xe^{-x^2} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

What is the mode of the continuous random variable X?

A. $\ln 2$

B. $\ln\sqrt{2}$

C. $\sqrt{2}$

D. $\dfrac{1}{\sqrt{2}}$

10. The z scores on a test are normally distributed such that $P(-N < x < N) = p$ for some constants $N > 0$ and $0 < p < 1$. What is the value of $P(z < N)$?

A. $\dfrac{1-p}{2}$

B. $1-\dfrac{p}{2}$

C. $\dfrac{1+p}{2}$

D. $\dfrac{1+2p}{3}$

END OF SECTION 1

Roseville College

2023 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Class and Teacher

Student Number

Student Name

Mathematics Advanced

Section II Answer Booklet

90 marks
Attempt Questions 11 - 34
Allow about 2 hours and 45 minutes for this section

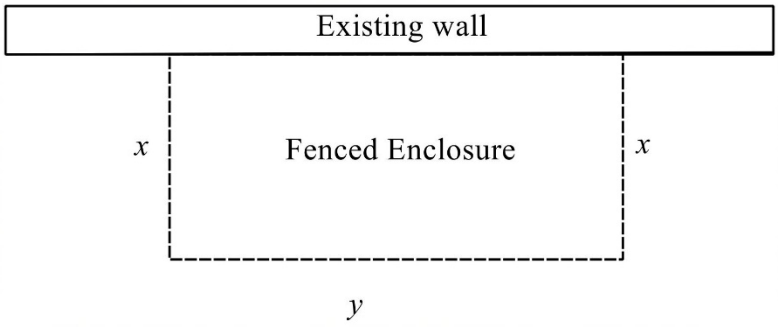
Instructions

- Write your Name and Class details in the spaces provided at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of the booklet.
  If you use this space, clearly indicate which question you are answering.

Please turn over

Question 11 (4 marks)

Chloe plans to build a rectangular enclosure for her new chickens against an existing wall, using 16 metres of wire fencing as shown below. She calls the dimensions of the enclosure x and y.



(a) She reasons that $2x + y = 16$. Write y in terms of x and hence, show that the area of the enclosure can be written as $A = 16x - 2x^2$. **1**

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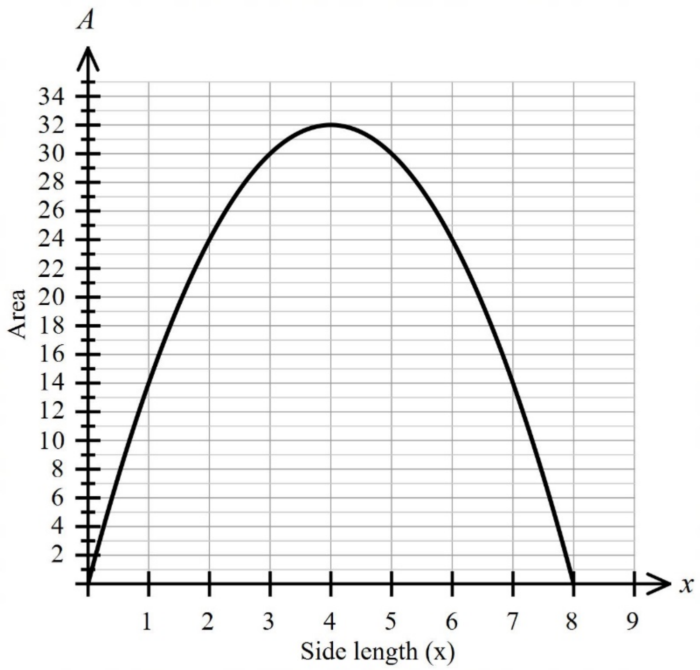
(b) The graph of $A = 16x - 2x^2$ is shown below. **2**



Use the graph to determine the value(s) of x which would give the enclosure an area of 24 m^2 and give the dimensions of the enclosure in each case.

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Question 11 continues on page 10.

Question 11 (continued)

(c) Determine the dimensions (both x and y) that will achieve the maximum possible area for the enclosure. **1**

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Question 12 (4 marks)

Differentiate the following with respect to x:

(a) $\dfrac{\log_e x}{x^2}$ **2**

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(b) $\sqrt{\sin x}$ **2**

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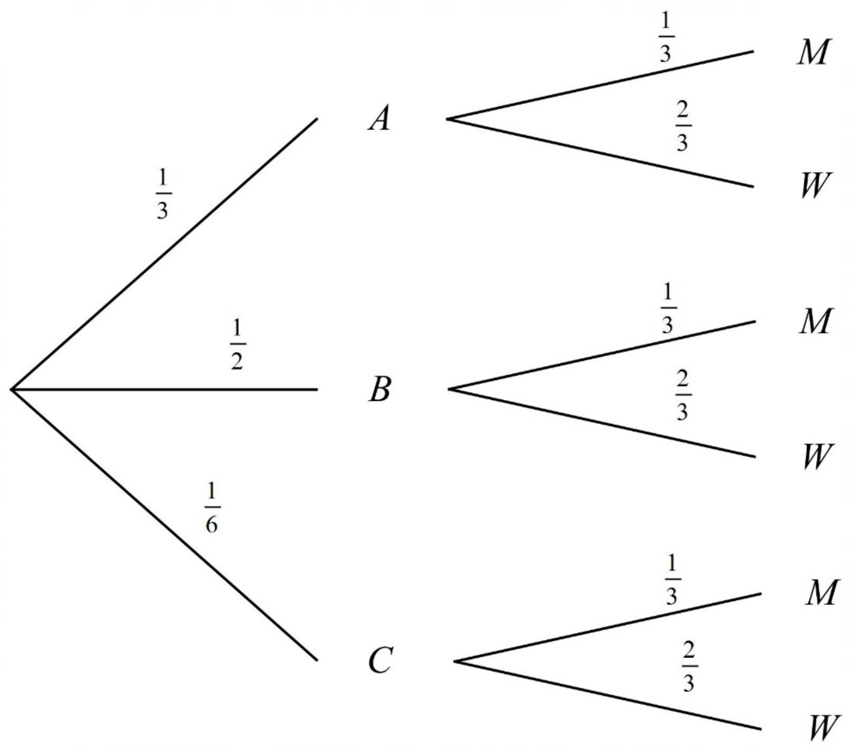
Question 13 (3 marks)

There are three candidates for team leader on a camp, Andie, Bec and Cat.

There are two candidates for team mascot at the camp, Miao and Woof.

Based on a survey of camp attendees, the probability tree shows the likelihood of the candidates being chosen for each position.



(a) What is the probability that Andie will be leader with Miao as mascot? **1**

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(b) What is the probability that Andie will be the leader or Miao will be the mascot, but not both? **2**

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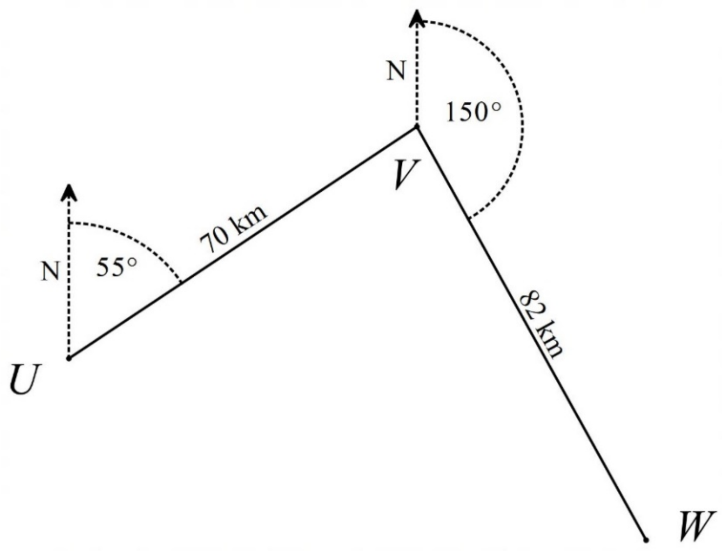
Question 14 (4 marks)

A helicopter leaves Underwood and flies 70 km on a bearing of 055° to Vanna Beach.
It then flies 82 km on a bearing of 150° to Weston.

The diagram below illustrates the journey.



(a) Calculate, to the nearest km, the distance that the helicopter will fly from Weston back to Underwood directly. **2**

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(b) Determine the bearing (from Weston) on which it should fly to return to Underwood. Give your answer to the nearest degree. **2**

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Question 15 (2 marks)

Find the domain for the circle $x^2 - 4x + y^2 + 6y - 3 = 0$ **2**

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Question 16 (4 marks)

Find the integrals.

(a) $\int 2^x dx$ **1**

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(b) $\int \frac{3x}{x^2+2} dx$ **1**

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(c) $\int_0^{\frac{\pi}{2}} \sin\frac{x}{2}\ dx$ **2**

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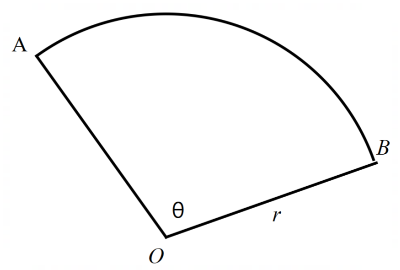
Question 17 (3 marks)



In the diagram AOB, is a sector of a circle of radius r cm that contains an angle θ radians at the centre O of the circle. The area of the sector is $9\ cm^2$ and its perimeter is 5 times its arc length. By solving simultaneous equations, find the values of r and θ. **3**

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Question 18 (3 marks)

(a) Show $\dfrac{3\sec^2 3x}{\tan 3x} = 3\sec 3x \ \operatorname{cosec} 3x$ **2**

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(b) Hence or otherwise find $\displaystyle\int 3\sec 3x \ \operatorname{cosec} 3x \, dx$ **1**

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Question 19 (4 marks)

The random variable X has the probability distribution given in the following table

X	1	2	3	4	5	6
$P(X = x)$	0.15	0.2	0.35	0.05	0.2	0.05

(a) Find E(X) **1**

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(b) Calculate the variance of X **1**

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(c) Find $P(X > 2 \mid X \leq 5)$ **2**

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Question 20 (3 marks)

The rate of change of volume V with respect to time (t) is $\dfrac{dV}{dt} = (2t-1)^2$. If $V = 5$ when $t = \dfrac{1}{2}$, find the volume when $t = 3$. **3**

Question 21 (4 marks)

For an arithmetic series the third term (T_3) is 32 and the sixth term (T_6) is 17.

(a) Find the common difference and the first term. **2**

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(b) Find the sum of the terms from T_{10} to T_{20} inclusive. **2**

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Question 22 (6 marks)

Consider the curve given by $y = 2x^3 - \frac{1}{2}x^4$

(a) Find the x intercepts **1**

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(b) Find the stationary points and determine their nature. **3**

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Question 22 continued on the next page

Question 22 (continued)

(c) Sketch the curve, showing essential features. **2**

Question 23 (5 marks)

At 7 pm on a Wednesday evening after a storm, Jack's water tank was full. The capacity of the tank was 3 000 litres. The tap on the tank, however, was leaking such that the change in volume at any time (t) hours was proportional to the volume (V) of the tank and can be modelled by the equation $V = V_0 e^{-kt}$.

(a) Find V_0, the initial volume of the tank. **1**

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(b) Given that the volume of the tank after 3 hours is 1 900 litres, find the value of k correct to 4 decimal places. **2**

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(c) By the time Jack discovered that the tank was leaking, there were only 250 litres of water remaining. At what time and on which day did Jack discover the leak (correct to the nearest minute)? **2**

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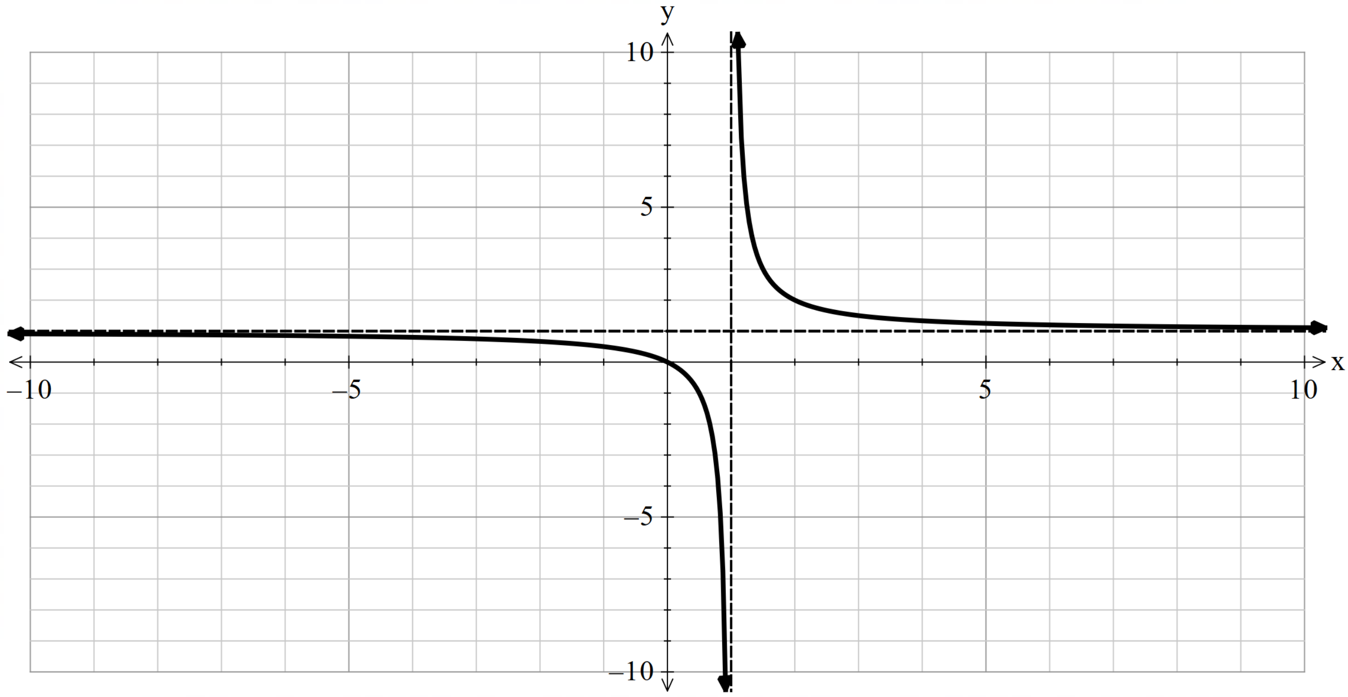
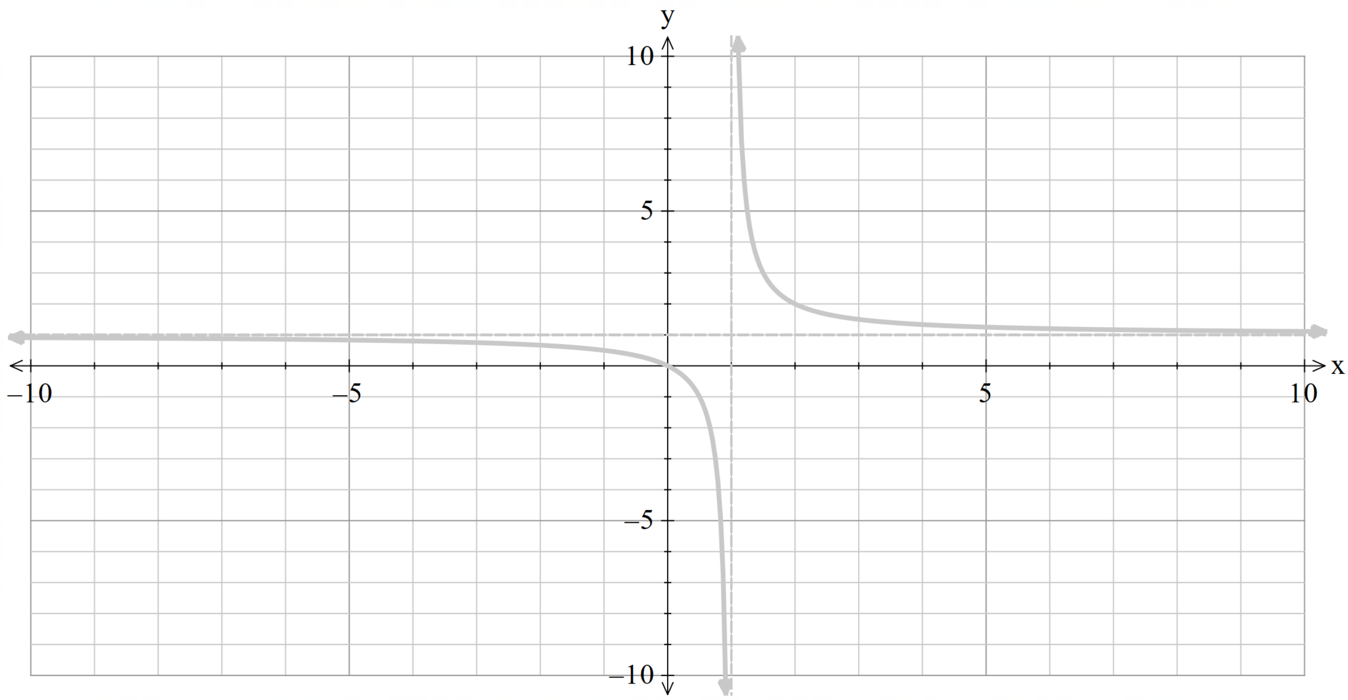
Question 24 (2 marks)

The graph of $y = f(x)$ is shown.



On the graph below, draw the curve $y = f(x - 2) + 2$, showing all important features. **2**



Question 25 (4 marks)

(a) Show that $\log_9 x$ can be expressed as $\dfrac{1}{2}\log_3 x$. **2**

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(b) Using this result, or otherwise, show that the sum to infinity of the series **2**

$\log_3 x + \log_9 x + \log_{81} x + ...$ is equal to $\log_3 x^2$

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Question 26 (3 marks)

The displacement of a particle at time (t) seconds is given by:

$$x = e^{t^2} - 2t^2$$

(a) Given that the velocity is the rate of change of the displacement, find an expression for the velocity. **1**

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(b) Find the two times at which the particle comes to rest. **2**

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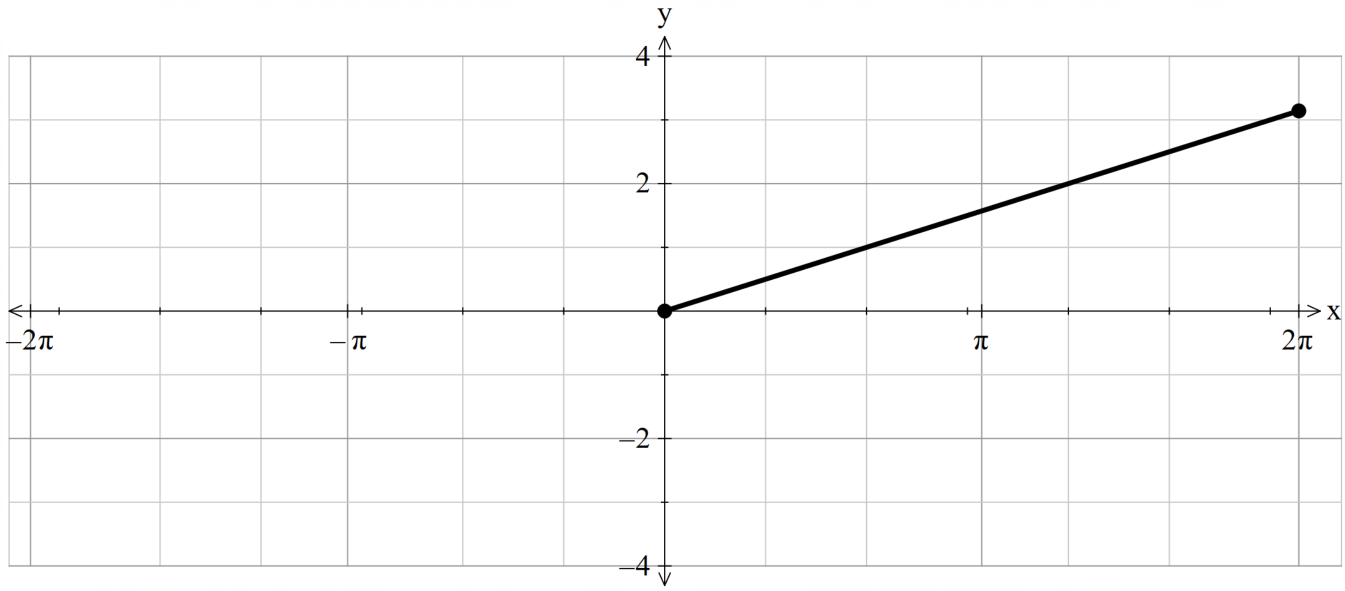
Question 27 (3 marks)

(a) Complete the graph of $y = \left|\frac{x}{2}\right|$ over the domain $[-2\pi, 2\pi]$ **1**



(b) By graphing $y = 3\sin x$ on the same number plane, determine how many solutions there are to the equation $3\sin x = \left|\frac{x}{2}\right|$ in the domain $[-2\pi, 2\pi]$ **2**

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Question 28 (2 marks)

A factory produces boxes of matches. The mean number of matches in each box is 30 and the standard deviation is 3. The number of matches in each box is normally distributed.

(a) What is the probability that a box chosen at random will have between 21 and 33 matches in it? **1**

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(b) If 4000 boxes of matches are produced, how many are expected to have more than 39 matches in them? **1**

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Question 29 (6 marks)

A company manufactures light bulbs. The expected life of the light bulb, t days, is given by the probability density function.

$$f(t)=\begin{cases}\dfrac{1}{4000}e^{-\frac{t}{4000}} & \text{t} \geq 0 \\ 0 & \text{otherwise}\end{cases}$$

(a) Find the Cumulative Density Function $F(t)$. **1**

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(b) A light bulb is considered faulty if its life span is less than 100 days. **2**

Determine the percentage of light bulbs that are considered faulty. Give your answer as a percentage to 1 decimal place.

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(c) Determine the z-score that 100 days would represent on a normal distribution curve **1**

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(d) Find the median number of days a light bulb lasts if it is produced by this company. **2**

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Question 30 (5 marks)

The population of a town over a 7 year period and the number of boats stolen each year is recorded below.

Population (P)	1000	1800	2400	3000	3600	4200	5000
Number of boats stolen (N)	80	78	92	104	104	120	116

(a) Find the correlation coefficient *r*, correct to 2 decimal places, and comment on the strength of the relationship. **2**

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(b) Find the equation of the least squares regression line for this data. **1**

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(c) Using the least squares regression line, estimate the number of boats stolen if the population is 9000. **1**

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(d) Comment on the reliability of this prediction from your answer in part c). **1**

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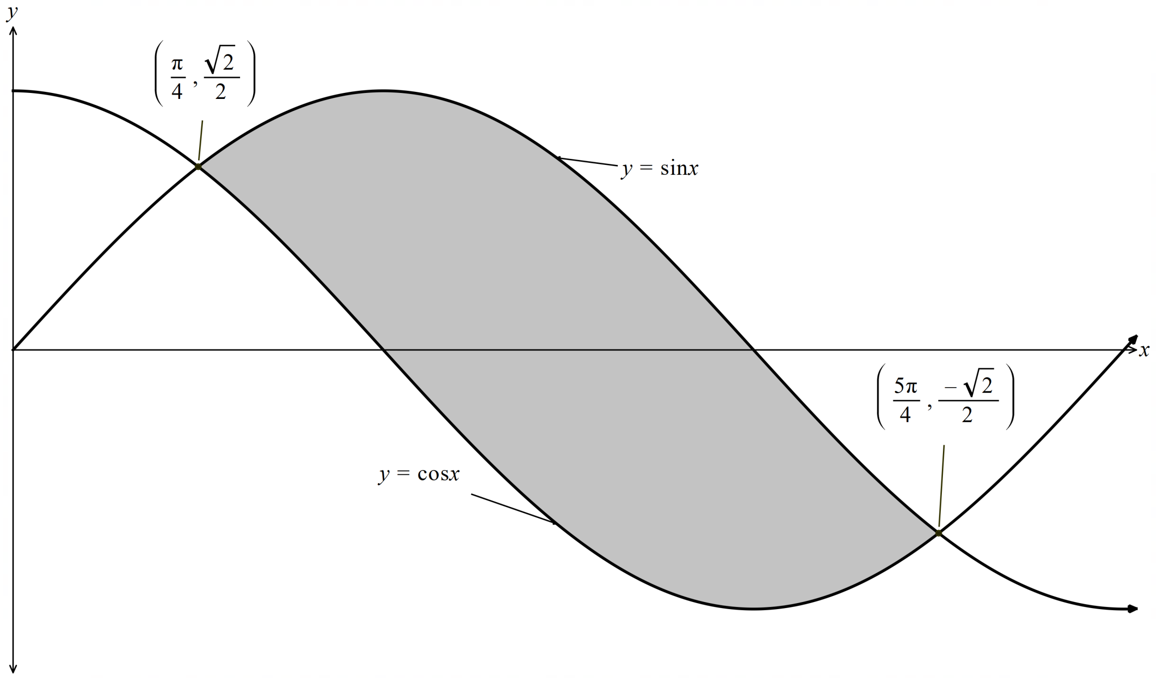
Question 31 (3 marks)

The graph below shows an area enclosed between the curves $y = \sin x$ and $y = \cos x$. **3**



Find an exact value for the shaded area.

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Question 32 (5 marks)

The price $P(t)$ in cents per litre of unleaded petrol during an average year in Broome WA, can be modelled by the function $P(t) = 180 + 44\sin\left(\frac{2\pi t}{183}\right)$ where t is the number of days after 22 March 2023, for $0 \le t \le 366$.

(a) What is the maximum price of petrol during the year? **1**

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(b) Sketch the function $P(t)$ for $0 \le t \le 366$ **2**

(c) What are the values of t for when petrol will cost 202 cents per litre? **2**

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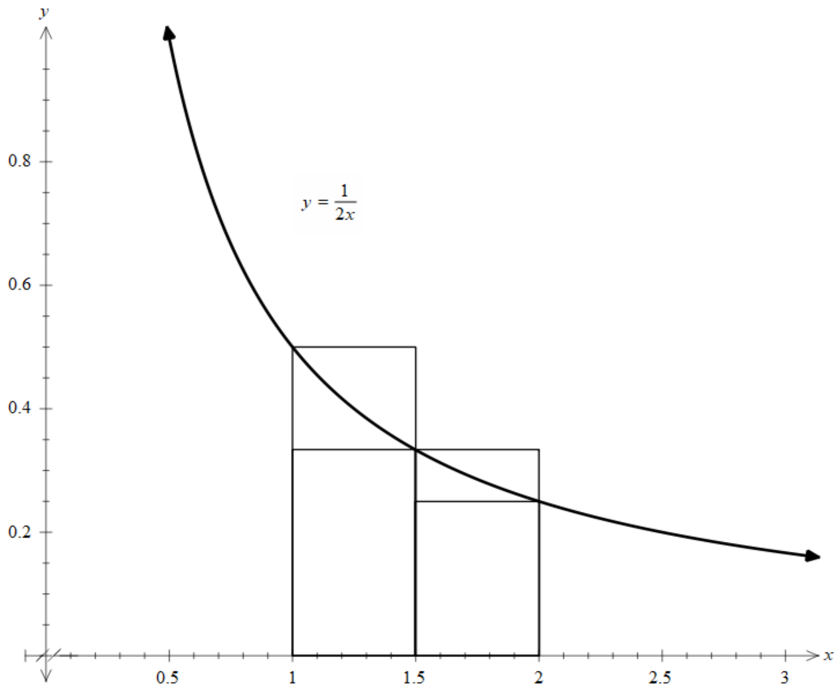
Question 33 (3 marks)

The graph of $f(x)=\dfrac{1}{2x}$ is shown below.



By considering the areas of the rectangles, show that $\dfrac{7}{24}<\ln\sqrt{2}<\dfrac{5}{12}$ **3**

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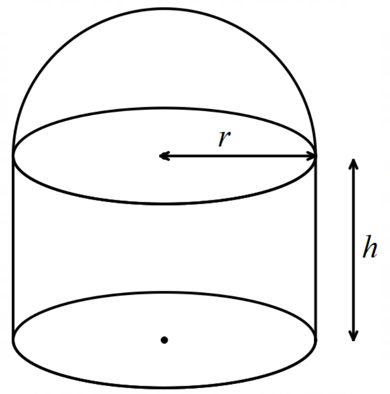
Question 34 (5 marks)



A metal container of volume $0.36\pi\ \mathrm{m}^3$ is to be made in the shape of a hollow cylinder of radius r meters and height h meters, **closed at the bottom** and capped at the top with a hemispherical shell of the same radius. The area of metal required is A m^2. Given that a sphere of radius r has volume $V = \frac{4}{3}\pi r^3$ and surface area $A = 4\pi r^2$

(a) Show that $A = \frac{18\pi}{25r} + \frac{5\pi}{3}r^2$ **2**

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(b) Hence find the dimensions of such a container that uses the least amount of metal. **3**

END OF PAPER

Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

Roseville College

2023 Trial Higher School Certificate Examination
Mathematics Advanced

Name ________________________________ Teacher ________________________

Section I – Multiple Choice Answer Sheet

Allow about 25 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

correct

A ⊗ B ⊗ C ○ D ○

1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○
6. A ○ B ○ C ○ D ○
7. A ○ B ○ C ○ D ○
8. A ○ B ○ C ○ D ○
9. A ○ B ○ C ○ D ○
10. A ○ B ○ C ○ D ○

Fiona · Q29 – Q34.
27 marks

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2023

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Approved calculators may be used
- A reference sheet is provided at the back of this paper
- In Questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks : 100

Section I – 10 marks (pages 2 – 6)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 7 – 28)

- Attempt Questions 11 – 32
- Allow about 2 hours and 45 minutes for this section

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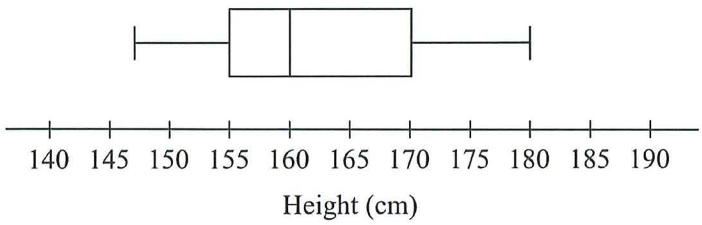
Section I - 10 marks

Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10

1. The heights of students in a class are represented in the boxplot below. A new student, who will become the tallest member, is joining the class.



What is the maximum height the new student can be so they are not classified as an outlier?

A. 187.5 cm

B. 192.5 cm

C. 197.5 cm

D. 202.5 cm

IQR = 170 − 155 = 15.
Upperfence = $Q_3 + 1.5 \times IQR$.
= $170 + 1.5 \times 15$
= 192.5

2. The table below shows the values of the functions $f(x)$ and $g(x)$ for various values of x.

x	1	2	3	4	5
$f(x)$	3	4	5	1	2
$g(x)$	5	3	2	1	4

What is the value of $f(g(3))$?

$g(3) = 2$
$f(g(3)) = f(2)$
$= 4.$

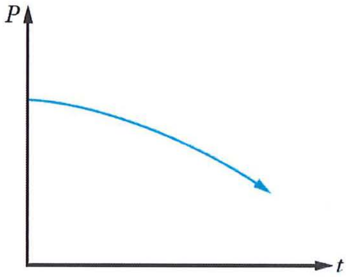
A. 2

B. 3

C. 4

D. 5

3. The population (P) of Marvel, a super villain's town is shown over time (t) by the graph below:



Which statement best describes the population of Marvel.

A. The population is increasing at an increasing rate.

B. The population is increasing at a decreasing rate.

C. The population is decreasing at an increasing rate.

D. The population is decreasing at a decreasing rate.

4. The curve $y = \ln x$ is translated to the left by π units and then dilated horizontally by a scale factor of 3.

What is the equation which describes this new curve?

A. $y = \ln\left(\frac{x}{3} + \pi\right)$

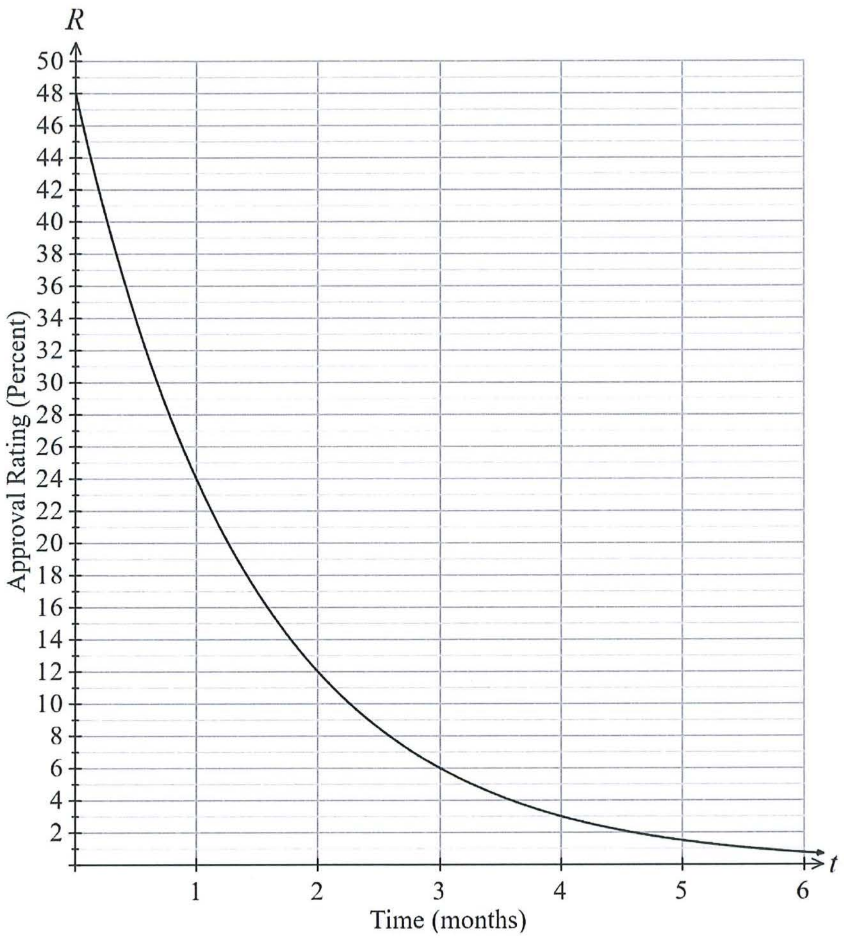
B. $y = \ln\left(\frac{x}{3} - \pi\right)$

C. $y = 3\ln(x + \pi)$

D. $y = \ln\left(\frac{x + \pi}{3}\right)$

5. The approval rating of a politician falls dramatically after an election. The rate of decrease can be modelled by the equation: $R = a \times b^{-t}$, where R is the percentage approval rating, t is the time in months since the election and a and b are positive constants.

The graph below shows the relationship.



Which equation could be used to model the graph shown?

A. $R = 48 \times 2^{-t}$ ✓

B. $R = 24 \times 2^{-t}$

C. $R = 2 \times 48^{-t}$

D. $R = 48 \times 4^{-t}$ ✓

0	1
48	24

6. Find the range of $y = \frac{1}{2} - \frac{3}{2}\sin\left(2x + \frac{\pi}{3}\right)$.

A. $\left[-\frac{3}{2}, \frac{3}{2}\right]$

B. $[-1, 2]$

C. $[-1, 1]$

D. $\left[-\frac{1}{2}, \frac{1}{2}\right]$

$\frac{1}{2} - \frac{3}{2} = -1$

$\frac{1}{2} + \frac{3}{2} = 2$

7. Differentiate $x \sin\left(\frac{1}{x}\right)$

A. $\sin\left(\frac{1}{x}\right) - \frac{1}{x}\cos\left(\frac{1}{x}\right)$

B. $\cos\left(\frac{1}{x}\right) - \frac{1}{x}\sin\left(\frac{1}{x}\right)$

C. $\sin\left(\frac{1}{x}\right) + \frac{1}{x}\cos\left(\frac{1}{x}\right)$

D. $x \cos\left(\frac{1}{x}\right)$

$u = x \quad v = \sin\frac{1}{x}$.

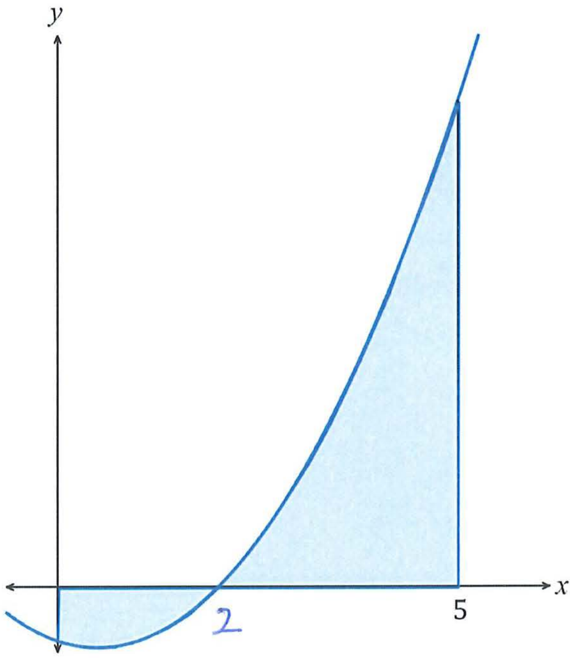
$u' = 1 \quad v' = \left(\cos\frac{1}{x}\right) \times \frac{-1}{x^2}$.

$uv' + vu'$

$= \sin\frac{1}{x} + x\left(\frac{-1}{x^2}\right)\left(\cos\frac{1}{x}\right)$

$= \sin\frac{1}{x} - \frac{1}{x}\cos\left(\frac{1}{x}\right)$

8.



Which expression will give the area of the shaded region bounded by the curve $y = x^2 - x - 2$ the x-axis and the lines $x = 0$ and $x = 5$?

A. $A = \left| \int_0^1 x^2 - x - 2 \, dx \right| + \int_1^5 x^2 - x - 2 \, dx$

B. $A = \int_0^1 x^2 - x - 2 \, dx + \left| \int_1^5 x^2 - x - 2 \, dx \right|$

C. $A = \left| \int_0^2 x^2 - x - 2 \, dx \right| + \int_2^5 x^2 - x - 2 \, dx$

D. $A = \int_0^2 x^2 - x - 2 \, dx + \left| \int_2^5 x^2 - x - 2 \, dx \right|$

$y = x^2 - x - 2.$

$y = (x-2)(x+1)$

$\Rightarrow x = 2 \quad x = -1.$

9. The probability density function for the continuous random variable X is:

$$f(x)=\begin{cases}2xe^{-x^2} & x\geq 0\\ 0 & \text{otherwise}\end{cases}$$

What is the mode of the continuous random variable X?

A. $\ln 2$

B. $\ln\sqrt{2}$

C. $\sqrt{2}$

D. $\dfrac{1}{\sqrt{2}}$

Mode when $f'(x)=0$.

$2xe^{-x^2}$.

$u=2x \quad v=e^{-x^2}$.

$u'=2 \quad v'=-2xe^{-x^2}$.

$-4x^2e^{-x^2}+2e^{-x^2}$.

$2e^{-x^2}(-2x^2+1)=0$

$2e^{-x^2}\neq 0$. $-2x^2+1=0$.

$1=2x^2$.

$\frac{1}{2}=x^2$. $x=\frac{1}{\sqrt{2}}$.

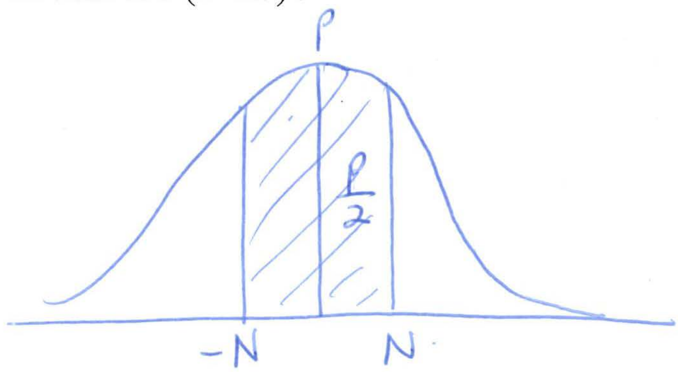
10. The z scores on a test are normally distributed such that $P(-N<x<N)=p$ for some constants $N>0$ and $0<p<1$. What is the value of $P(z<N)$?

A. $\dfrac{1-p}{2}$

B. $1-\dfrac{p}{2}$

C. $\dfrac{1+p}{2}$

D. $\dfrac{1+2p}{3}$



$P(z<N)=\frac{1}{2}+\frac{p}{2}$

$=\frac{1+p}{2}$.

END OF SECTION 1

Roseville College

2023 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Class and Teacher

Student Number

Student Name

Mathematics Advanced

Section II Answer Booklet

90 marks
Attempt Questions 11- 32
Allow about 2 hours and 45 minutes for this section

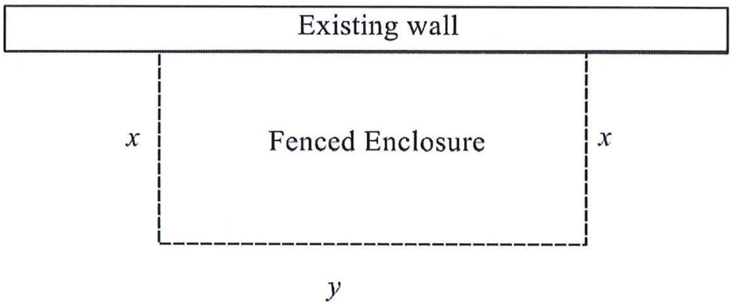
Instructions

- Write your Name and Class details in the spaces provided at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of the booklet.
  If you use this space, clearly indicate which question you are answering.

Please turn over

Question 11 (4 marks)

Chloe plans to build a rectangular enclosure for her new chickens against an existing wall, using 16 metres of wire fencing as shown below. She calls the dimensions of the enclosure x and y.



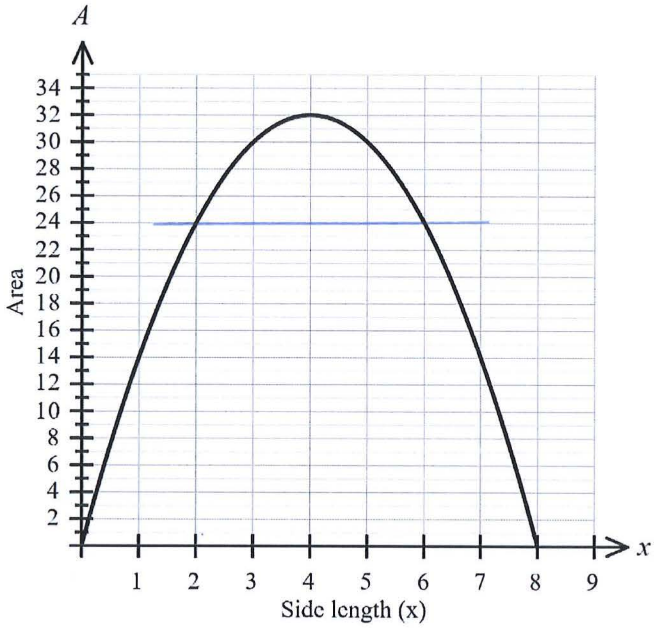
(a) She reasons that $2x + y = 16$. Write y in terms of x and hence, show that the area of the enclosure can be written as $A = 16x - 2x^2$. **1**

$y = 16 - 2x$.

$A = x(16 - 2x)$

$= 16x - 2x^2$.

(b) The graph of $A = 16x - 2x^2$ is shown below. **2**



Use the graph to determine the value(s) of x which would give the enclosure an area of 24 m^2 and give the dimensions of the enclosure in each case.

$x = 2$ or $x = 6$ 6m × 4m

$y = 16 - 4$ $y = 4$. 2m × 12m.

$= 12$

Question 11 continues on page 10.

Question 11 (continued)

(c) Determine the dimensions (both x and y) that will achieve the maximum possible area for the enclosure. **1**

Max area when $x = 4$ and $y = 16 - 2(4)$
$= 8$.

Question 12 (4 marks)

Differentiate the following with respect to x:

(a) $\dfrac{\log_e x}{x^2}$ **2**

$u = \log_e x \quad v = x^2$

$u' = \frac{1}{x} \quad v' = 2x$.

[1] correct set up of Quotient rule

$y' = \dfrac{vu' - uv'}{v^2} = \dfrac{x^2 \cdot \frac{1}{x} - \log_e x \; 2x}{x^4}$ [1] correct application

$= \dfrac{x - 2x\log_e x}{x^4} = \dfrac{\cancel{x}(1 - 2\log_e x)}{x^4}$

$= \dfrac{1 - 2\log_e x}{x^3}$

(b) $\sqrt{\sin x}$ **2**

$y = \sqrt{\sin x} = \sin^{\frac{1}{2}} x$.

$y' = s \quad = \frac{1}{2} \sin^{-\frac{1}{2}} x \cdot \cos x$. [2] correct differentiation

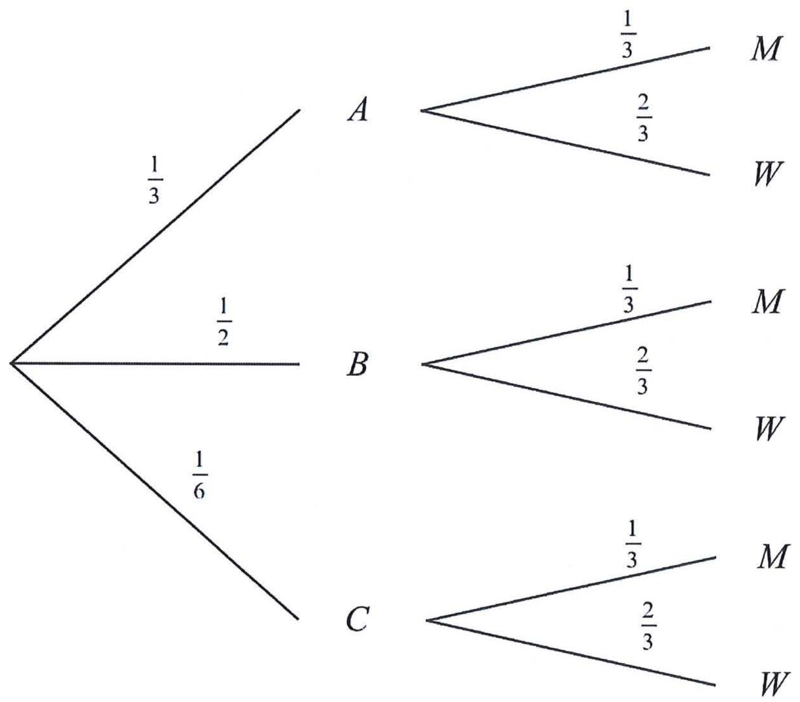
$= \dfrac{1}{2} \dfrac{\cos x}{\sqrt{\sin x}}$.

Question 13 (3 marks)

There are three candidates for team leader on a camp, Andie, Bec and Cat.

There are two candidates for team mascot at the camp, Miao and Woof.

Based on a survey of camp attendees, the probability tree shows the likelihood of the candidates being chosen for each position.



(a) What is the probability that Andie will be leader with Miao as mascot? **1**

$P(A\cancel{A}M) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$.

(b) What is the probability that Andie will be the leader or Miao will be the mascot, but not both? **2**

$P(AW + BM + CM)$

$= \frac{1}{3} \times \frac{2}{3} + \frac{1}{2} \times \frac{1}{3} + \frac{1}{6} \times \frac{1}{3}$. [1] for one correct combination

$= \frac{2}{9} + \frac{1}{6} + \frac{1}{18}$.

$= \frac{4}{9}$

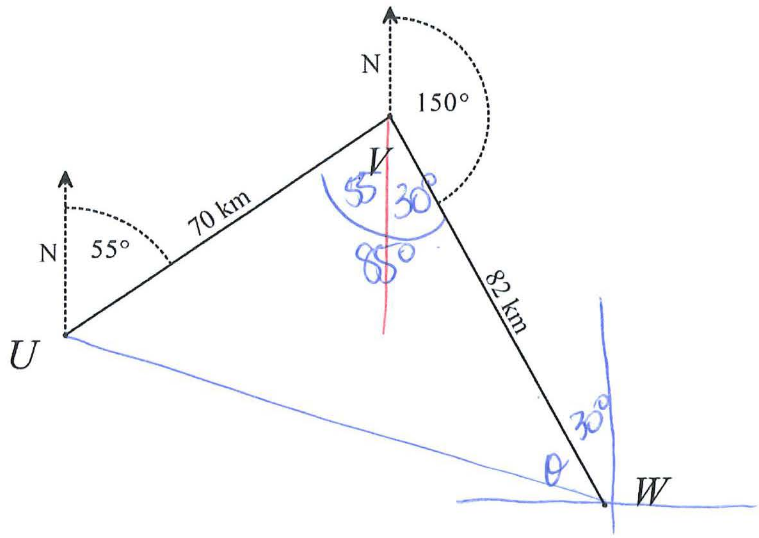
[1] final answer

Question 14 (4 marks)

A helicopter leaves Underwood and flies 70 km on a bearing of 055° to Vanna Beach.

It then flies 82 km on a bearing of 150° to Weston.

The diagram below illustrates the journey.



(a) Calculate, to the nearest km, the distance that the helicopter will fly from Weston back to Underwood directly. **2**

$UW^2 = 70^2 + 82^2 - 2(70)(82)\cos 85$

$\simeq 10623.45$

$UW = 103.07 \text{ km}$

$= 103 \text{ km}$

(b) Determine the bearing (from Weston) on which it should fly to return to Underwood. Give your answer to the nearest degree. **2**

$\frac{\sin\theta}{70} = \frac{\sin 85}{103}$

$\sin\theta = \frac{70 \sin 85}{103}$

$= 42.61.$ [1] correct angle using sine rule.

Bearing $= 360 - 42.61 - 30$

$= 281.39.$

$= 287°.$ [1] correct

Question 15 (2 marks)

Find the domain for the circle $x^2 - 4x + y^2 + 6y - 3 = 0$ **2**

$(x-2)^2 + (y+3)^2 = 16$. [1] for correct equation

centre $= (2, -3)$ radius $= 4$.

Domain $[-2, 6]$ [1] for correct domain.

Question 16 (4 marks)

Find the integrals.

(a) $\int 2^x dx$ **1**

$\frac{2^x}{\ln 2} + C$ [1] for use of formula

(b) $\int \frac{3x}{x^2+2} dx$ **1**

$= \frac{3}{2} \ln|x^2+2| + C$ $f(x) = x^2+2$

$f'(x) = 2x$.

(c) $\int_0^{\frac{\pi}{2}} \sin\frac{x}{2} \, dx$ **2**

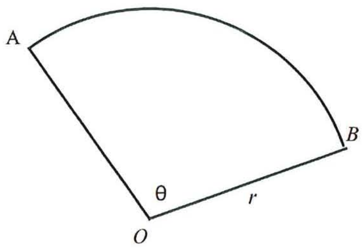
$= \left[-2\cos\frac{x}{2}\right]_0^{\frac{\pi}{2}}$ [1] correct integration

$= -2\cos\frac{\pi}{4} + 2\cos 0$

$= -\frac{2 \times 1}{\sqrt{2}} + 2$

$= 2 - \frac{2}{\sqrt{2}} \frac{\sqrt{2}}{\sqrt{2}} = 2 - \sqrt{2}$. [1] correct substitution + evaluation

Question 17 (3 marks)



In the diagram AOB, is a sector of a circle of radius r cm that contains an angle θ radians at the centre O of the circle. The area of the sector is $9\ cm^2$ and its perimeter is 5 times its arc length. By solving simultaneous equations, find the values of r and θ. **3**

Area $= \frac{1}{2}r^2\theta$.

$9 = \frac{1}{2}r^2\theta$. [1]

or

$18 = r^2\theta$ —(1)

$\theta = \frac{18}{r^2}$

[1] one correct equation

$5r\theta = 2r + r\theta$. [1]

$4r\theta = 2r$

$4r\frac{18}{r^2} = 2r$

$72 = 2r^2$

$r^2 = 36$.

$r = 6$. [1]

since $r > 0$

Sub $r = 6$ into (1)

$\theta = \frac{18}{36}$

$= \frac{1}{2}$ [1]

Question 18 (3 marks)

(a) Show $\dfrac{3\sec^2 3x}{\tan 3x} = 3\sec 3x \ \text{cosec}\ 3x$ **2**

$LHS = \dfrac{\frac{3}{\cos^2 3x}}{\frac{\sin 3x}{\cos 3x}}$

$= \dfrac{3}{\cos^2 3x} \cdot \dfrac{\cos 3x}{\sin 3x}$

$= \dfrac{3}{\cos 3x \sin 3x}$

$= 3\,\text{cosec}\,3x \sec 3x$

As Required

[1] correct $\sec^2 3x = \dfrac{1}{\cos^2 3x}$

or

[1] correct $\tan 3x = \dfrac{\sin 3x}{\cos 3x}$

[1] correct final answer

(b) Hence or otherwise find $\int 3\sec 3x \ \text{cosec}\ 3x \, dx$ **1**

$= \int \dfrac{3\sec^2 3x}{\tan 3x}\, dx$

$= \ln|\tan 3x| + c$ ✓

Question 19 (4 marks)

The random variable X has the probability distribution given in the following table

X	1	2	3	4	5	6
$P(X = x)$	0.15	0.2	0.35	0.05	0.2	0.05

(a) Find E(X) **1**

$= 0.15 + 2\times 0.2 + 3\times 0.35 + 4\times 0.05 + 5\times 0.2 + 6\times 0.05$

$= 3.1$

(b) Calculate the variance of X **1**

$var(X) = 0.15 + 2^2\times 0.2 + 3^2\times 0.35 + 4^2\times 0.05 + 5^2\times 0.2 + 6^2 + 0.05 - 3.1^2$

$= 11.7 - 3.1^2$

$= 2.09$

(c) Find $P(X > 2 \mid X \leq 5)$ **2**

$= \dfrac{P(X > 2 \cap X \leq 5)}{P(X \leq 5)} = \dfrac{P(2 < x \leq 5)}{P(x \leq 5)}$

$= \dfrac{\cancel{0.2} + 0.35 + 0.05 + 0.2}{0.95}$ [1]

$= \dfrac{\cancel{0.8}\ 0.6}{0.95}$

$= 0.6315\ldots$ [1]

$= 63.16\%$

Question 20 (3 marks)

The rate of change of volume V with respect to time (t) is $\frac{dV}{dt} = (2t-1)^2$. If $V = 5$ when $t = \frac{1}{2}$, find the volume when $t = 3$. **3**

$\frac{dV}{dt} = (2t-1)^2$

$\frac{dV}{dt} = 4t^2 - 2t - 2t + 1 \; dt$

$V = \frac{4t^3}{3} - \frac{4t^2}{2} + t + c$

$= \frac{4t^3}{3} - 2t^2 + t + c$ [1] correct integration

At $t = \frac{1}{2}$ $V = 5$ $5 = \frac{4}{3}\left(\frac{1}{2}\right)^3 - 2\left(\frac{1}{2}\right)^2 + \frac{1}{2} + c$

$5 = \frac{4}{24} - \cancel{\frac{1}{2}} + \cancel{\frac{1}{2}} + c$

$5 - \frac{1}{6} = c$

$c = 4\frac{5}{6}$ [1] correct substitution

If $t = 3$ $V = \frac{4}{3}(3)^3 - 2(3)^2 + 3 + 4\frac{5}{6}$

$= 36 - 18 + 3 + 4\frac{5}{6}$

$= 25\frac{5}{6}$ [1] correct rate of change of volume

Question 21 (4 marks)

For an arithmetic series the third term (T_3) is 32 and the sixth term (T_6) is 17.

(a) Find the common difference and the first term. **2**

for an equation [1]

$T_3 = a+(3-1)d$ $\qquad$ $T_6 = a+(6-1)d$

$32 = a+2d$ $-①$ $\qquad$ $17 = a+5d$ $-②$

$17 = a+5d$ $-②$

$① - ②$

$15 = -3d$

[1] for both $d=-5$ $a=42$

$d = -5$

sub $d=-5$ into ①

$32 = a+(2(-5))$

$42 = a$

(b) Find the sum of the terms from T_{10} to T_{20} inclusive. **2**

$S_{20} = \frac{n}{2}\{2a+(n-1)d\}$

$= \frac{20}{2}\{2(42)+19(-5)\}$

$= 10\{84-95\} = -110.$ [1]

$S_9 = \frac{9}{2}(2(42)+8(-5)).$

$= 198.$

$T_{10}-T_{20}$ inclusive means $S_{20}-S_9$

$-110-198 = -308.$ [1].

correct final answer

$T_1 \to T_{20}.$

$T_1 \to T_9 \quad T_{10} \to T_{20}.$

$S_9 \qquad S_{20}.$

$S_{20} - S_9.$

Question 22 (6 marks)

Consider the curve given by $y = 2x^3 - \frac{1}{2}x^4$

(a) Find the x intercepts **1**

$y=0 \quad 0 = 2x^3 - \frac{1}{2}x^4$

$= x^3(2 - \frac{1}{2}x)$

$\therefore x = 0$ or $2 = \frac{1}{2}x$

$x = 4$ $\quad (4,0)\ (0,0)$

(b) Find the stationary points and determine their nature. **3**

$y' = 6x^2 - 4 \times \frac{1}{2}x^3$

$= 6x^2 - 2x^3$

$y'' = 12x - 6x^2$

Stationary Points when $y' = 0$.

$6x^2 - 2x^3 = 0$

$2x^2(3 - x) = 0$

$x = 0$ or $x = 3$ [1]

At $x = 0$, $y = 0$

At $x = 3$, $y = 2(3)^3 - \frac{1}{2}(3)^4$

$= 54 - 40.5$

$= 13.5$

Check nature.

At $x = 0$, $y'' = 12(0) - 6(0) = 0$.

x	-1	0	1
y''	-18		6

At $x = -1$ $y'' = 12(-1) - 6(-1)^2 = -18 < 0$

At $x = 1$ $y'' = 12(1) - 6(1) = 6 > 0$

$\therefore$ Horizontal Point of Inflection since there is a change of concavity. [1]

At $x = 3$

$y'' = 12(3) - 6(3)^2$

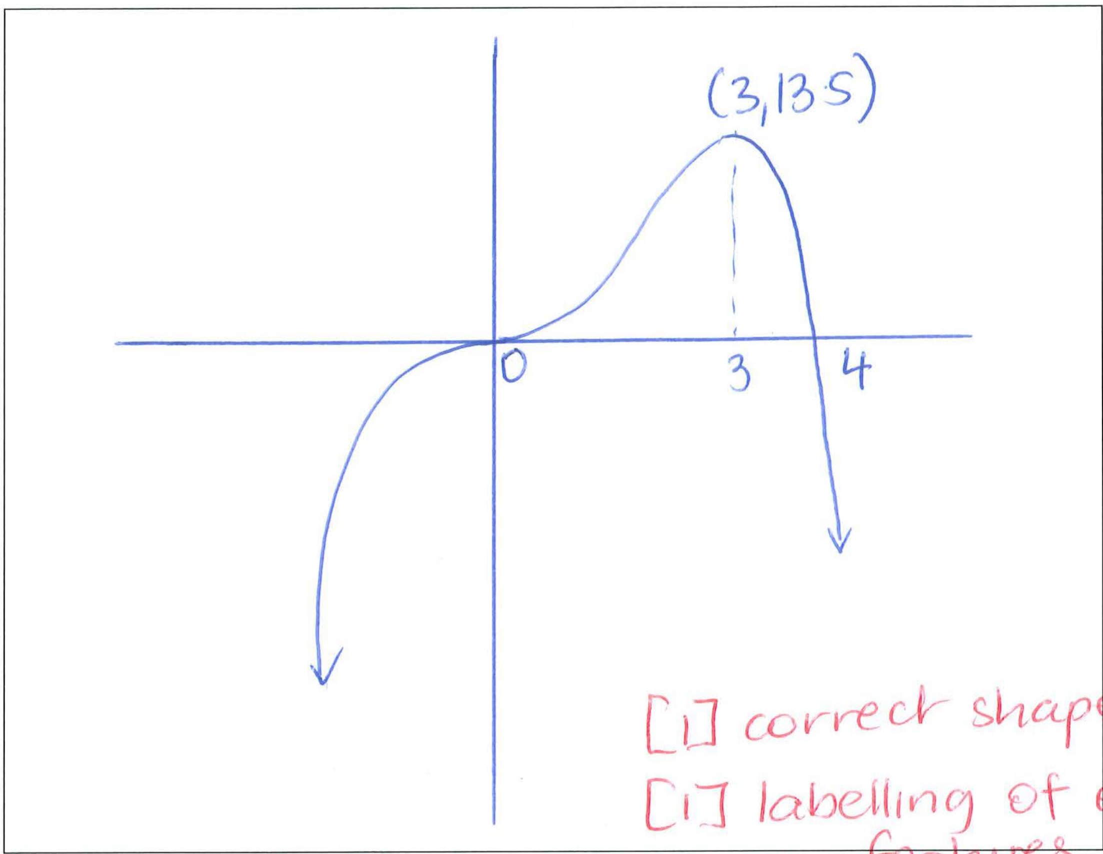
$= 36 - 54 = -18 < 0$

$\therefore$ A max turning point at $(3, 13.5)$ [1]

Question 22 continued on the next page

Question 22 (continued)

(c) Sketch the curve, showing essential features. **2**



[1] correct shape.
[1] labelling of essentially features

Question 23 (5 marks)

At 7 pm on a Wednesday evening after a storm, Jack's water tank was full. The capacity of the tank was 3 000 litres. The tap on the tank, however, was leaking such that the change in volume at any time (t) hours was proportional to the volume (V) of the tank and can be modelled by the equation $V = V_0 e^{-kt}$.

(a) Find V_0, the initial volume of the tank. **1**

$V_0 = 3000$

(b) Given that the volume of the tank after 3 hours is 1 900 litres, find the value of k correct to 4 decimal places. **2**

At $t=3$ $V=1900$ $\quad 1900 = 3000\, e^{-k3}$. [1] correct substitution

Take logs of both sides

$\ln\left(\frac{1900}{3000}\right) = -3k$

$k = \frac{\ln \frac{1900}{3000}}{-3}$

$= 0.1523$ (4dp)

[1] correct value of 'k'

(c) By the time Jack discovered that the tank was leaking, there were only 250 litres of water remaining. At what time and on which day did Jack discover the leak (correct to the nearest minute)? **2**

$250 = 3000\, e^{-0.1523t}$. [1] correct substitution

$\frac{250}{3000} = e^{-0.1523t}$

$\ln\left(\frac{250}{3000}\right) = -0.1523t$

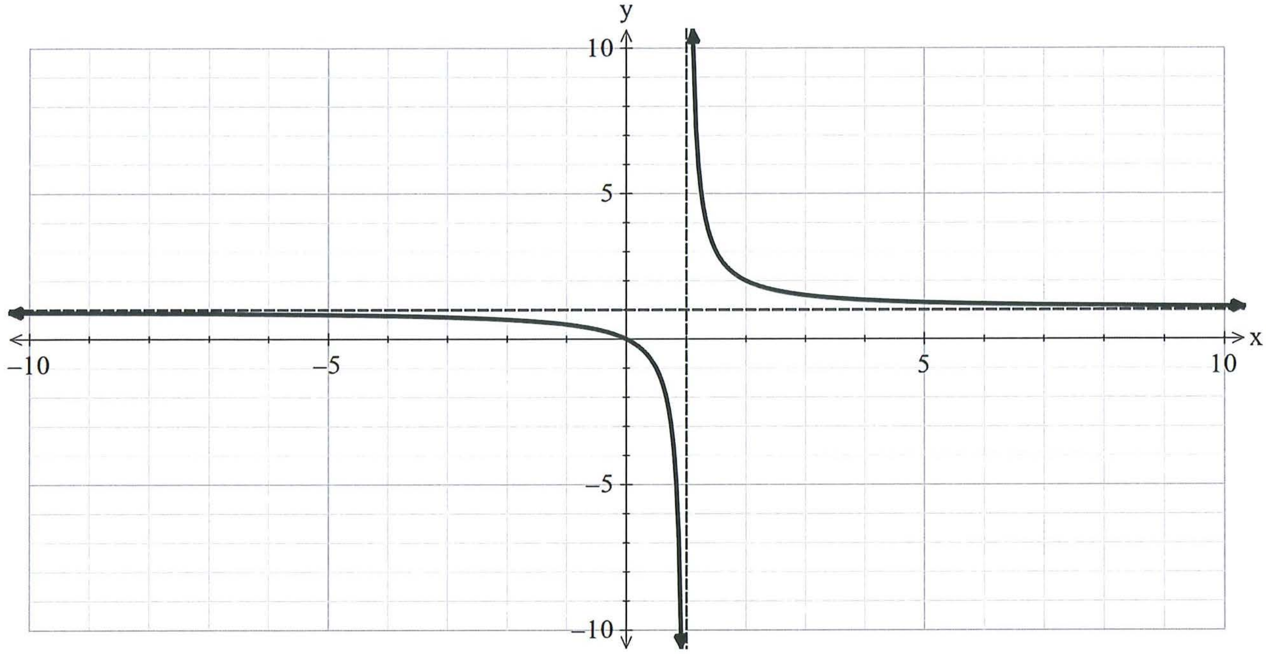
$t = \frac{\ln \frac{250}{3000}}{-0.1523}$

$= 16.31$ mins. [1] correct time and day

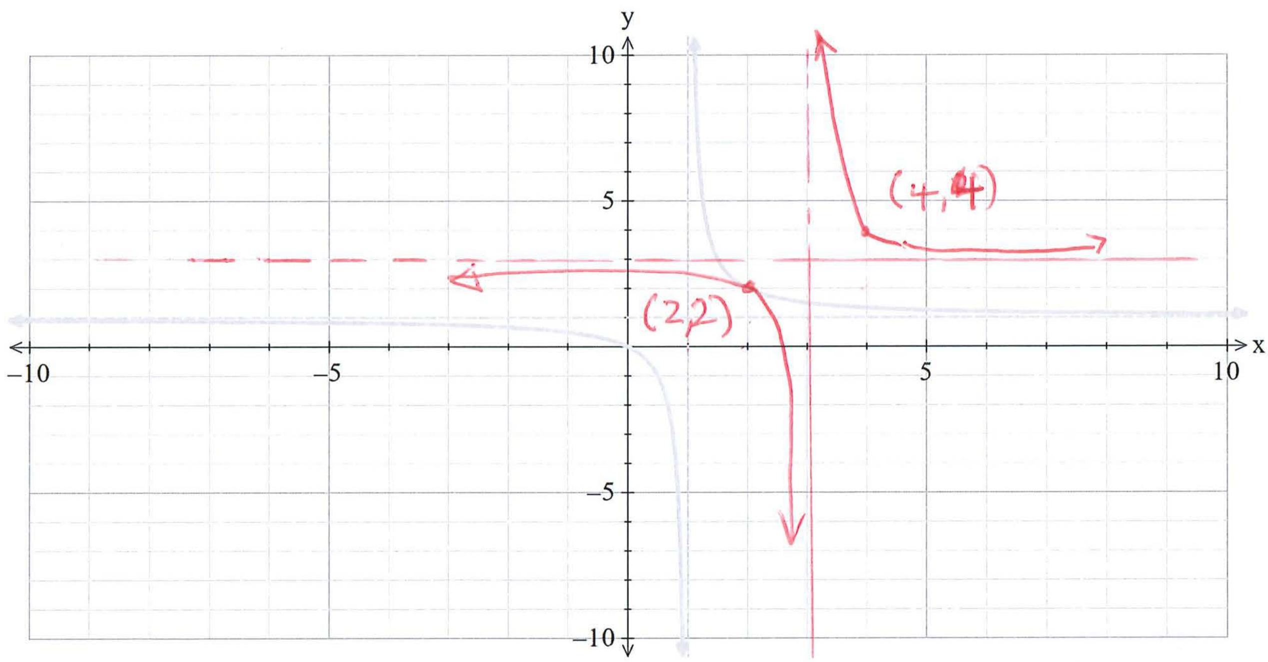
$\therefore$ 11 am 19 mins Thursday.

Question 24 (2 marks) **2**

The graph of $y = f(x)$ is shown.



On the graph below, draw the curve $y = f(x - 2) + 2$, showing all important features.



[1] [correct assymptotes]
[1] [correct shape

Question 25 (4 marks)

(a) Show that $\log_9 x$ can be expressed as $\frac{1}{2}\log_3 x$. **2**

$\log_9 x = \frac{\log_3 x}{\log_3 9} = \frac{\log_3 x}{\log_3 3^2}$

$= \frac{\log_3 x}{2}$

[1] change of base

$= \frac{1}{2}\log_3 x$ As Required

[1] correct final answer

(b) Using this result, or otherwise, show that the sum to infinity of the series **2**

$\log_3 x + \log_9 x + \log_{81} x + \ldots$ is equal to $\log_3 x^2$

$a = \log_3 x$ [1] correct 'a' or 'r'

$r = \frac{\log_9 x}{\log_3 x} = \frac{\frac{1}{2}\log_3 x}{\log_3 x}$ from (a)

$= \frac{1}{2}$ [

$S_\infty = \frac{a}{1-r}$

$= \frac{\log_3 x}{1-\frac{1}{2}}$

$= 2\log_3 x$

$= \log_3 x^2$ correct substitution

Question 26 (3 marks)

The displacement of a particle at time (t) seconds is given by:

$$x = e^{t^2} - 2t^2$$

(a) Given that the velocity is the rate of change of the displacement, find an expression for the velocity. 1

$v = 2te^{t^2} - 4t$. [1] correct differentiation

(b) Find the two times at which the particle comes to rest. 2

At rest when $v = 0$.

$2te^{t^2} - 4t = 0$. [1] $v=0$.

$2t(e^{t^2} - 2) = 0$.

$t = 0$ or $e^{t^2} = 2$.

$\ln e^{t^2} = \ln 2$.

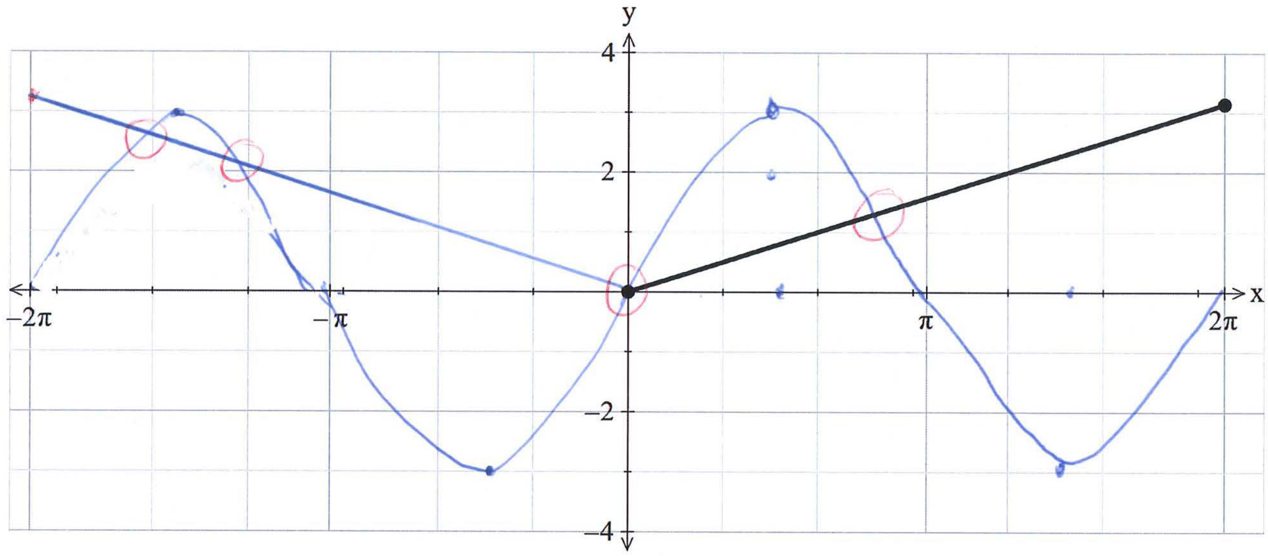
$t^2 = \ln 2$.

$t = \sqrt{\ln 2}$ [1] correct use of logs.

Question 27 (3 marks)

(a) Complete the graph of $y = \left|\frac{x}{2}\right|$ over the domain $[-2\pi, 2\pi]$ **1**

[1] for completing $y = \left|\frac{x}{2}\right|$



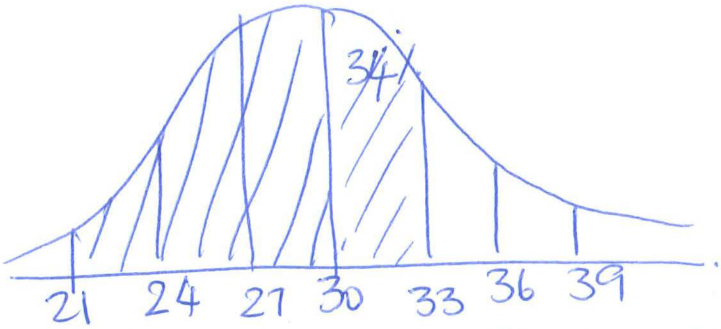
[1] correct shape + period $= 2\pi$ + amplitude $= 3$.

(b) By graphing $y = 3\sin x$ on the same number plane, determine how many solutions there are to the equation $3\sin x = \left|\frac{x}{2}\right|$ in the domain $[-2\pi, 2\pi]$ **2**

4 solutions [1] correct number of solutions

Question 28 (2 marks)

A factory produces boxes of matches. The mean number of matches in each box is 30 and the standard deviation is 3. The number of matches in each box is normally distributed.



(a) What is the probability that a box chosen at random will have between 21 and 33 matches in it? **1**

$\frac{99.7}{2} + 34\%$

$= 83.85\%$ [1]

(b) If 4000 boxes of matches are produced, how many are expected to have more than 39 matches in them? **1**

$0.15\% \times 4000$

$= 6$ [1]

Question 29 (6 marks)

A company manufactures light bulbs. The expected life of the light bulb, t days, is given by the probability density function.

$$f(t) = \begin{cases} \dfrac{1}{4000} e^{-\frac{t}{4000}} & t \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the Cumulative Density Function $F(t)$. **1**

$F(t) = \frac{1}{4000} e^{-\frac{t}{4000}} \times -4000.$

$= \left[-e^{-\frac{t}{4000}} \right]_0^x$

$= -e^{-\frac{x}{4000}} + 1$

$= 1 - e^{-\frac{x}{4000}}$ [1]

(b) A light bulb is considered faulty if its life span is less than 100 days. **2**

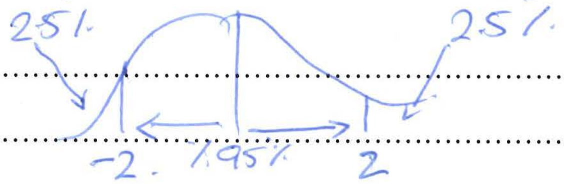
Determine the percentage of light bulbs that are considered faulty. Give your answer as a percentage to 1 decimal place.

$P(X < 100) = 1 - e^{-\frac{100}{4000}}$ [1] correct substitution into CDF

$= 0.0247$

$= 2.5\%$ [1]

(c) Determine the z-score that 100 days would represent on a normal distribution curve **1**



z-score $= -2$ [1]

95% with 2 std deviations.

∴ 5% outside of 95%

(d) Find the median number of days a light bulb lasts if it is produced by this company. **2**

median $1 - e^{-\frac{t}{4000}} = 0.5$ [1]

$-e^{-\frac{t}{4000}} = -0.5$

Multiply both sides by -1

$e^{-\frac{t}{4000}} = 0.5$

Take logs of both sides.

$\frac{-t}{4000} = \ln(0.5)$

$-t = 4000(\ln(0.5))$

$t = -4000(\ln 0.5)$

$= 2772.58$

[1]

Median number of days = 2772.58 days.

Question 30 (5 marks)

The population of a town over a 7 year period and the number of boats stolen each year is recorded below.

Population (P)	1000	1800	2400	3000	3600	4200	5000
Number of boats stolen (N)	80	78	92	104	104	120	116

(a) Find the correlation coefficient r, correct to 2 decimal places, and comment on the strength of the relationship. **2**

$r = 0.94$ (2dp). [1]

strong positive correlation. [1]

(b) Find the equation of the least squares regression line for this data. **1**

$A = 65.63$ (2dp) $y = A + Bx$

$B = 0.011$ (3dp).

$N = 65.63 + 0.011P$ [1]

(c) Using the least squares regression line, estimate the number of boats stolen if the population is 9000. **1**

When $P = 9000$ $N = 65.63 + 0.011 \times 9000$

$= 164.63$.

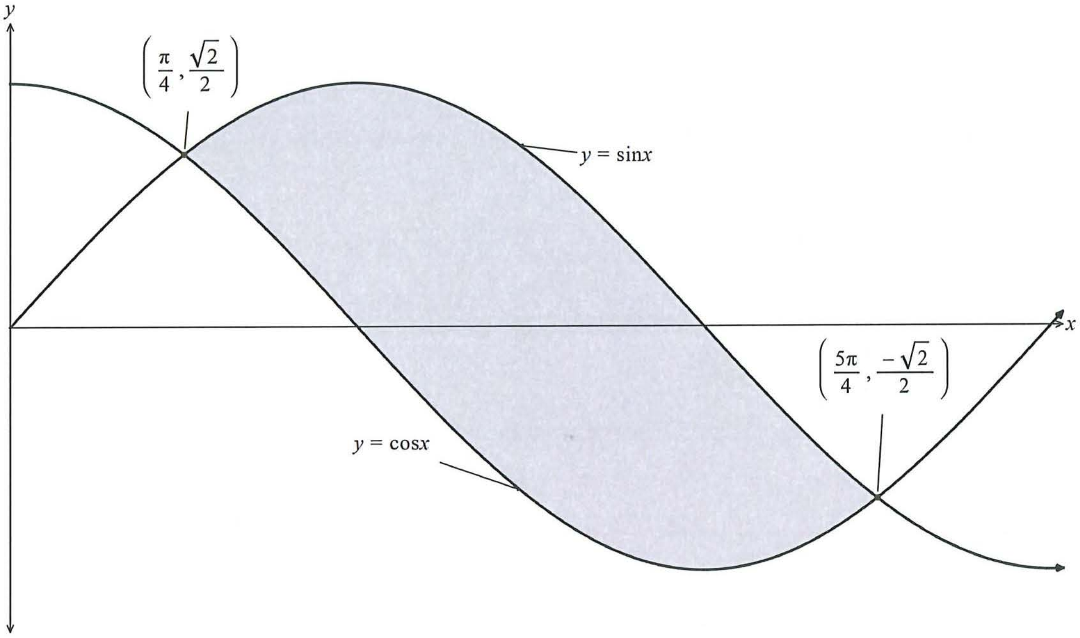
approx. 165 boats stolen.

(d) Comment on the reliability of this prediction from your answer in part c). **1**

Not very reliable as this is extrapolation and outside the range of the data provided

Question 31 (4 marks)

The graph below shows an area enclosed between the curves $y = \sin x$ and $y = \cos x$. **4**



Find an exact value for the shaded area.

$\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \sin x - \cos x \, dx$

$= \left[-\cos x - \sin x\right]_{\frac{\pi}{4}}^{\frac{5\pi}{4}}$ [1]

$= -\cos\frac{5\pi}{4} - \sin\frac{5\pi}{4} - \left[-\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\right]$

$= -\left(-\frac{1}{\sqrt{2}}\right) - \left(-\frac{1}{\sqrt{2}}\right) + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$ [1]

$= \frac{4}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = 2\sqrt{2}$. [1]

Question 32 (5 marks)

The price $P(t)$ in cents per litre of unleaded petrol during an average year in Broome WA, can be modelled by the function $P(t) = 180 + 44\sin\left(\dfrac{2\pi t}{183}\right)$ where t is the number of days after 22 March 2023, for $0 \leq t \leq 366$.

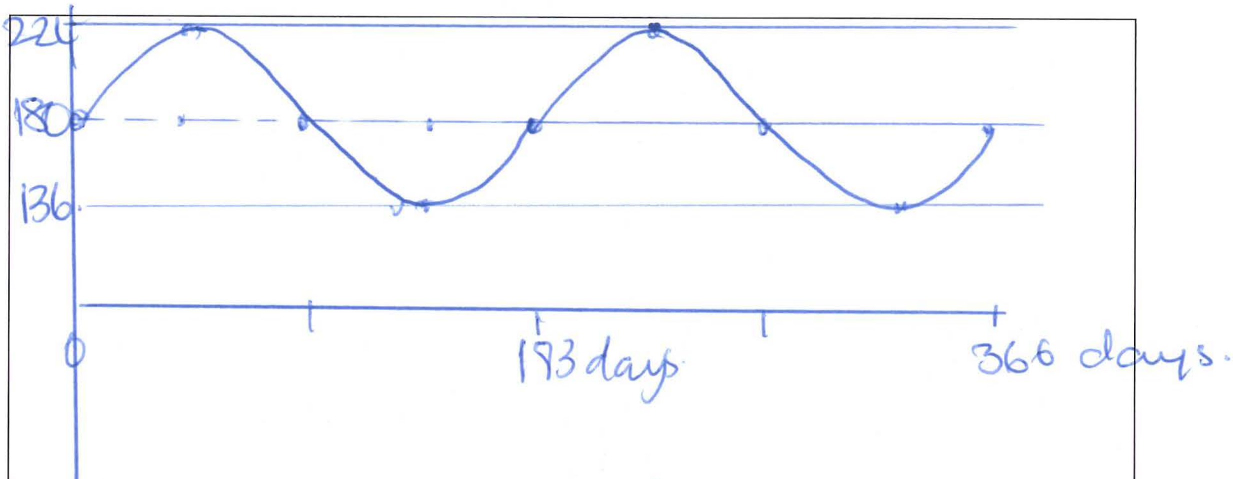
Period $= \dfrac{2\pi}{2\pi} \times 183$.

(a) What is the maximum price of petrol during the year? **1**

$180 + 44 = 224$ cents. [1]

min price $= 180 - 44 = 136$ cents.

(b) Sketch the function $P(t)$ for $0 \leq t \leq 366$ **2**



[1] correct period.
[1] correct shape + amplitude.

(c) What are the values of t for when petrol will cost 202 cents per litre. **2**

$202 = 180 + 44\sin\left(\dfrac{2\pi t}{183}\right)$ [1]

$22 = 44\sin\dfrac{2\pi t}{183}$

$0.5 = \sin\dfrac{2\pi t}{183}$

S | A
T | C

$\dfrac{2\pi t}{183} = \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{\pi}{6} + 2\pi, \dfrac{5\pi}{6} + 2\pi$.

$= \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{13\pi}{6}, \dfrac{17\pi}{6}$

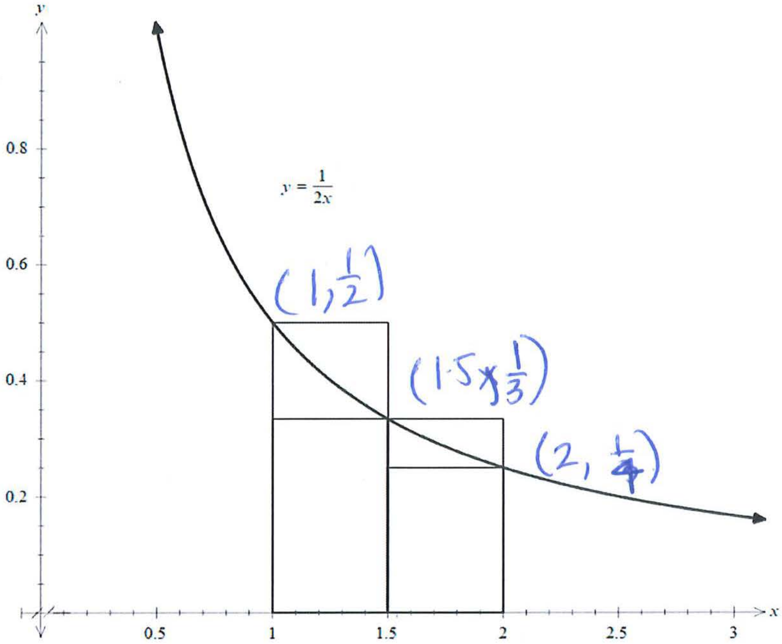
$t = \dfrac{\pi}{6} \times \dfrac{183}{2\pi}, \dfrac{5\pi}{6} \times \dfrac{183}{2\pi}, \dfrac{13\pi}{6} \times \dfrac{183}{2\pi}, \dfrac{17\pi}{6} \times \dfrac{183}{2\pi}$.

$= 15.25, 76.25, 198.25, 259.25$.

[1] For all solns.

Question 33 (3 marks)

The graph of $f(x) = \dfrac{1}{2x}$ is shown below.



By considering the areas of the rectangles, show that $\dfrac{7}{24} < \ln\sqrt{2} < \dfrac{5}{12}$ 3

lower rectangles $= 0.5 \times \frac{1}{3} + 0.5 \times \frac{1}{4}$.

$\frac{1}{6} + \frac{1}{8} = \frac{7}{24}$ [1]

upper Rectangles $= 0.5 \times \frac{1}{2} + 0.5 \times \frac{1}{3}$. or

$= \frac{1}{4} + \frac{1}{6} = \frac{10}{24} = \frac{5}{12}$. [1]

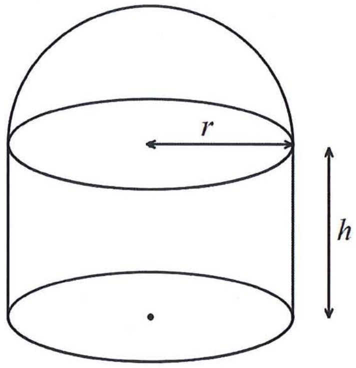
$\int_1^2 \frac{1}{2x} = \frac{1}{2}\left[\ln x\right]_1^2 = \frac{1}{2}\left[\ln 2 - \ln 1\right]$

$= \ln 2^{\frac{1}{2}}$

$= \ln\sqrt{2}$ [1]

$\frac{7}{24} < \ln\sqrt{2} < \frac{5}{12}$. [1]

Question 34 (5 marks)



bold

A metal container of volume $0.36\pi\ \text{m}^3$ is to be made in the shape of a hollow cylinder of radius r meters and height h meters, closed at the bottom and capped at the top with a hemispherical shell of the same radius. The area of metal required is A m^2. Given that a sphere of radius r has volume $V = \frac{4}{3}\pi r^3$ and surface area $A = 4\pi r^2$

(a) Show that $A = \frac{18\pi}{25r} + \frac{5\pi}{3}r^2$ **2**

$V = 0.36\pi = \pi r^2 h + \frac{1}{2} \times \frac{4}{3}\pi r^3$

$\frac{36\cancel{\pi}}{100} = \cancel{\pi} r^2 h + \frac{4}{6}\cancel{\pi} r^3$

$\frac{9}{25} = r^2 h + \frac{2r^3}{3}$

$h = \frac{9}{25r^2} - \frac{2r^3}{3r^2}$

$\boxed{h = \frac{9}{25r^2} - \frac{2r}{3}}$ [1]

$A = \pi r^2 + \frac{1}{2} \times 4\pi r^2 + 2\pi r h$

$A = 3\pi r^2 + 2\pi r h$

$A = 3\pi r^2 + 2\pi r\left(\frac{9}{25r^2} - \frac{2r}{3}\right)$ [1]

$= 3\pi r^2 + \frac{18\pi}{25r} - \frac{4\pi r^2}{3}$

$= \frac{9\pi r^2}{3} - \frac{4\pi r^2}{3} + \frac{18\pi}{25r}$

$A = \frac{5\pi r^2}{3} + \frac{18\pi}{25r}$ As Required.

(b) Hence find the dimensions of such a container that uses the least amount of metal. **3**

$A = \frac{18\pi}{25r} + \frac{5\pi r^2}{3}$

$\frac{dA}{dr} = \frac{-18\pi}{25r^2} + \frac{10\pi r}{3}$

min when $\frac{-18\pi}{25r^2}^{\times r^2} + \frac{10\pi r}{3}^{\times r^2} = 0^{\times r^2}$

$-\frac{18\pi}{25} + \frac{10\pi r^3}{3} = 0$

$\frac{-54\pi + 250\pi r^3}{75} = 0$

$250\cancel{\pi} r^3 = 54\cancel{\pi}$

$r^3 = \frac{54}{250} = \frac{27}{125}$

$r = \frac{3}{5}$ [1]

Check $\frac{d^2A}{dr^2} = \frac{36\pi}{25r^3} + \frac{10\pi}{3}$

At $r = \frac{3}{5}$ $\frac{d^2A}{dr^2} = \frac{36\pi}{25(\frac{3}{5})^3} + \frac{10\pi}{3} = 31.4 > 0$.

$\therefore$ Local min at $r = \frac{3}{5}$. [1]

$h = \frac{9}{25(\frac{3}{5})^2} - \frac{2(\frac{3}{5})}{3}$

$= 1 - \frac{2}{5} = \frac{4}{5}$ [1]

The dimensions are $r = \frac{3}{5}$ $h = \frac{4}{5}$

END OF PAPER