Student Number:
Teacher:

St George Girls High School

Mathematics Advanced

2025 HSC Trial Examination

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in **Section I**, use the Multiple-Choice answer sheet provided
- For questions in **Section II:**
    - Answer the questions in the space provided
    - Show relevant mathematical reasoning and/or calculations
    - Marks may not be awarded for incomplete or poorly presented solutions, or where multiple solutions are provided

Total marks: 100

Section I – 10 marks (pages 3 – 8)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 9 – 40)

- Attempt Questions 11 – 35
- Allow about 2 hours and 45 minutes for this section

Q1 – Q10	/10
Q11 – Q15	/ 16
Q16 – Q22a	/ 15
Q22b – Q26	/ 16
Q27 – Q30	/ 15
Q31 – Q33a	/ 13
Q33b – Q35	/ 15
Total	**/100**
	%

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet provided for Questions 1 to 10.

1. An electrician charges a call-out fee of $130 and $120/hour for labour.

   Which equation expresses the amount the electrician charges ($C) as a function of time (t hours)?

   (A) $C = 2 + 130t$

   (B) $C = 130 + 2t$

   (C) $C = 120 + 130t$

   (D) $C = 130 + 120t$

2. The period and range of $y = \cos(\pi x) + 2$ are

   (A) period $=$ 2 and range $= [1,3]$.

   (B) period $=$ 2 and range $= [-3,-1]$.

   (C) period $=$ 1 and range $= [0,2]$.

   (D) period $=$ 1 and range $= [-1,1]$.

3. The domain of $y = \log_e(x^2)$ is

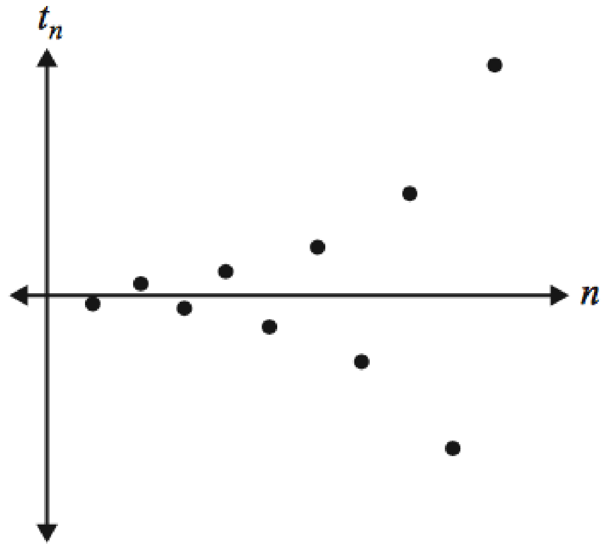
   (A) $(-\infty, 0)$.

   (B) $(0, \infty)$.

   (C) $(-\infty, 0) \cup (0, \infty)$.

   (D) $(-\infty, 0) \cap (0, \infty)$.

4. The graph below displays the value of the first 10 terms of a geometric sequence, t_n.



Which of the following is true of the first term, a, and the common ratio, r?

(A) $a > 0$ and $r > 0$

(B) $a < 0$ and $r < 0$

(C) $a < 0$ and $r > 0$

(D) $a > 0$ and $r < 0$

5. The function $f(x) = 2^x$ is to be dilated vertically by a factor of 8, and **then** translated left by 1 unit.

What is the equation of the transformed function $g(x)$?

(A) $g(x) = 8 \cdot 2^x + 1$

(B) $g(x) = 8 \cdot 2^{x+1}$

(C) $g(x) = 8 \cdot 2^{x-1}$

(D) $g(x) = 2^{8x+1}$

6. What is $\int \frac{4}{(5-2x)^3}\,dx$?

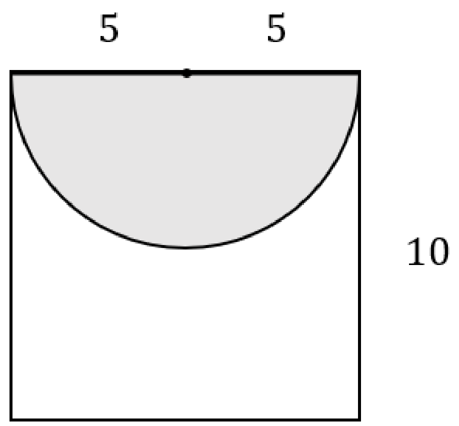
(A) $\frac{1}{(5-2x)^2} + C$

(B) $\frac{1}{2(5-2x)^2} + C$

(C) $\frac{-1}{2(5-2x)^2} + C$

(D) $\frac{-1}{(5-2x)^2} + C$

7. A dart thrown at the square shown below is equally likely to land anywhere inside the square.



What is the probability that the dart lands in the shaded semi-circle?

(A) $\frac{\pi}{20}$

(B) $\frac{\pi}{10}$

(C) $\frac{\pi}{8}$

(D) $\frac{\pi}{4}$

8. Consider the series $1 + 1 + 1 + 1 + \cdots$

What could NOT be said about the series?

(A) This is an arithmetic progression with a common difference of 0.

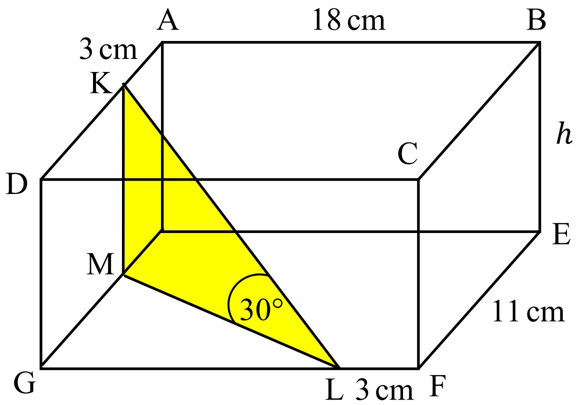
(B) This is a geometric progression with a common ratio of 1.

(C) This series has a limiting sum of $2n$, where n represents the number of terms.

(D) The terms of this series can be described by $T_n = T_{n-1}$, where $T_1 = 1$.

9. The diagram shows the triangle KLM inside a rectangular prism.

The dimensions are as shown.



What is the height, h, of the rectangular prism?

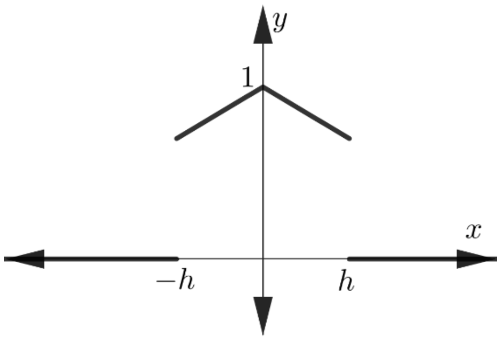
(A) $17\sqrt{3}$ cm

(B) $\dfrac{17\sqrt{3}}{3}$ cm

(C) $15\sqrt{3}$ cm

(D) $\dfrac{17\sqrt{3}}{2}$ cm

10. A function and its graph is displayed below, where h is a positive constant.



$$f(x) = \begin{cases} 0 & \text{for } x < -h \\ 1 - \left|\frac{x}{2}\right| & \text{for } -h \leq x \leq h \\ 0 & \text{for } x > h \end{cases}$$

If $\int_{-h}^{h} f(x)\,dx = 2$, what is the value of h?

(A) 0.5

(B) 1

(C) 1.5

(D) 2

END OF SECTION I – MULTIPLE CHOICE

Mathematics Advanced

Section II Answer Booklet 1

Student Number:	
Teacher:	

Section II
90 marks
Attempt Questions 11 – 35
Allow about 2 hours and 45 minutes for this section

Booklet 1 – Attempt Question 11 – 26 (47 marks)

Instructions

- Write your Teacher's Name and Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on pages 23 to 24 of Booklet 1. If you use this space, clearly indicate which question you are answering.

Q11 – Q15	/ 16
Q16 – Q22a	/ 15
Q22b – Q26	/ 16

Please turn over

Question 11 (**3** marks)

The nth term of an arithmetic sequence is given by $T_n = 5n - 3$.

(a) Write down the first 3 terms of the sequence. **1**

(b) Find the sum of the first 50 terms. **2**

Question 12 (**3** marks)

If $f(x) = -x^2 - 2x + 3$ and $g(x) = 2x + 1$,

(a) Write a simplified expression for $f\left(g(x)\right)$. **2**

(b) What is the axis of symmetry of $y = f\left(g(x)\right)$? **1**

Question 13 (**4** marks)

Differentiate the following with respect to x.

(a) $y = 5x^2 - \dfrac{4}{x}$ **1**

(b) $f(x) = 4x(6 - x)^3$ **2**

(c) $f(x) = \tan(x^2 - 2)$ **1**

Question 14 (**3** marks)

Determine the following with respect to x.

(a) $\displaystyle\int \frac{x^2 - 3x + 4}{\sqrt{x}}\,dx$ **2**

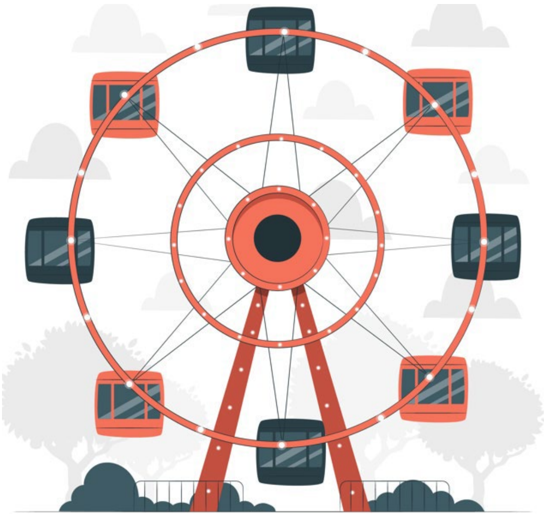
(b) $\displaystyle\int 16\cos(2x - 1)\,dx$ **1**

Question 15 (**3** marks)

Brendan is sitting in a cabin of a Ferris wheel which is revolving. The height, $B(t)$, in metres above the ground of her cabin is given by

$$B(t) = c - k\sin\left(\frac{\pi t}{28}\right),$$

where t is the time in seconds after Brendan's cabin first reaches the bottom of its revolution and c and k are constants.



The top of each carriage reaches a greatest height of 28 metres and a smallest height of 4 metres.

(a) Find the value of c and k. **2**

(b) How many seconds does it take for one complete revolution of the Ferris wheel? **1**

Question 16 (**2** marks)

The probability distribution table for a random variable X is given below.

x	1	2	3	4
$P(X = x)$	k	$2k$	$3k$	$4k$

(a) Determine the value of k. **1**

(b) Determine the standard deviation. **1**

Question 17 (**2** marks)

Determine the last term under 100 in the sequence below. **2**

$$-11, \quad -4, \quad 3, \quad 10, \quad 17, \quad \ldots$$

Question 18 (**2** marks)

Differentiate $\log_e\left(\frac{x+1}{x-2}\right)$ with respect to x. **2**

Question 19 (**2** marks)

Solve $3\tan\left(\frac{x}{2}\right) = -1$ to the nearest minute for $0° \leq x \leq 360°$. **2**

Question 20 (**3** marks)

(a) Show that $1 + \dfrac{10}{x-3} = \dfrac{x+7}{x-3}$. **1**

(b) Hence, evaluate $\displaystyle\int_4^{e+3} \frac{x+7}{x-3}\,dx$. Leave your answer in exact form. **2**

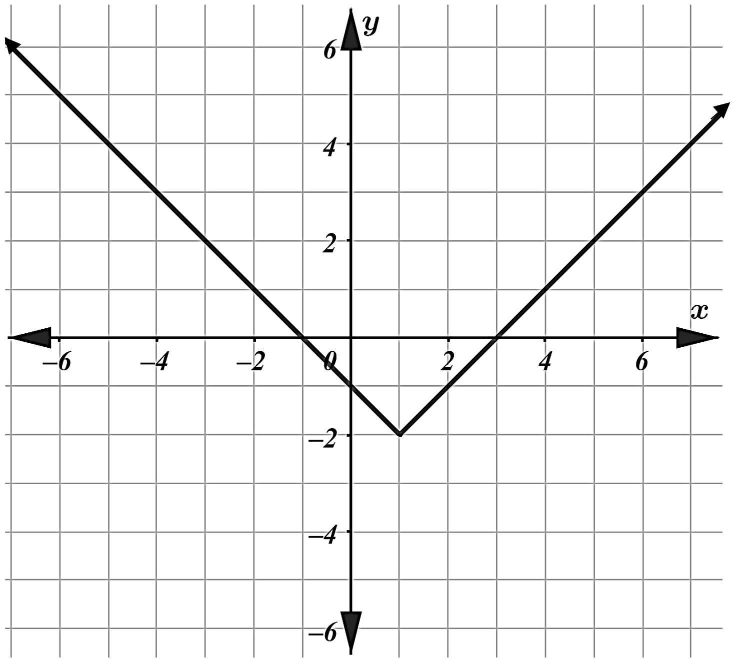
Question 21 (**3** marks)

Determine the sum of the geometric series below. **3**

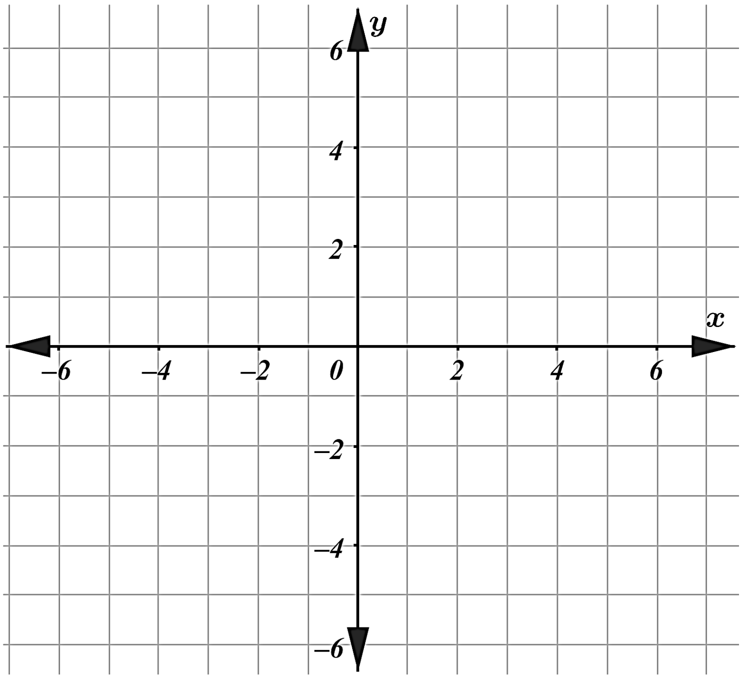
$$2 + 6 + 18 + \cdots + 39\,366$$

Question 22 (**3** marks)

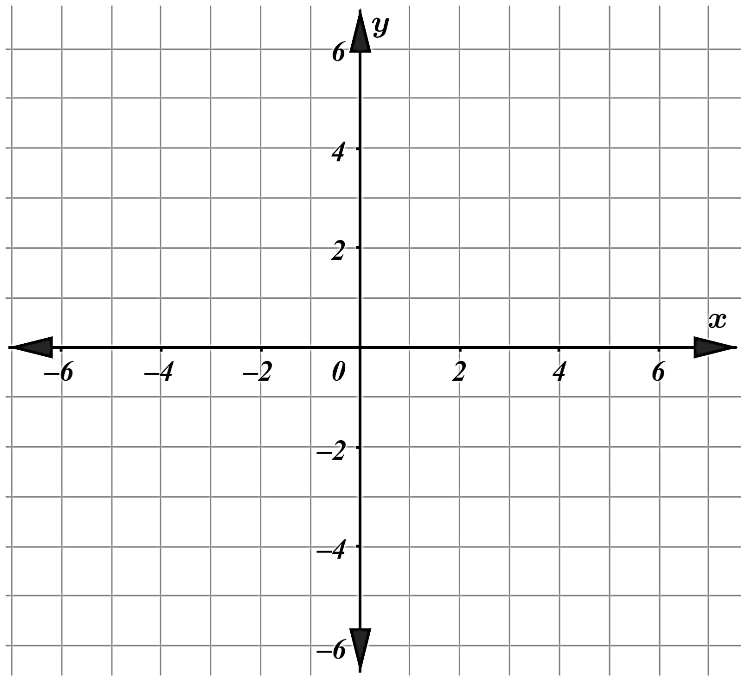
The graph of the function $y = f(x)$ has been shown below.



(a) Sketch the graph of $y = f\left(\frac{x}{2}\right)$ and show all intercepts. **1**



(b) Sketch the graph of $y = -f(x) - 2$ and show all intercepts. **2**



Question 23 (**3** marks)

The following sequence is geometric. **3**

$$6 - x, \qquad x - 8, \qquad x + 4$$

Determine the possible values of the common ratio.

Question 24 (**5** marks)

Anthony plays a game involving two standard dice.
The two dice are rolled. If the sum is

- a prime number, he receives $5
- a square number, he receives $10
- anything else, he loses $5.

(a) By filling in the remainder of the array, determine the probability of losing $5. **3**

		Dice 2					
		1	**2**	**3**	**4**	**5**	**6**
Dice 1	**1**	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6			
	4	5	6				
	5	6					
	6						

(b) Is Anthony expected to gain or lose money in the long run? Justify your answer with relevant working. **2**

Question 25 (**3** marks)

Point $P(3, -5)$ lies on the function $y = f(x)$. **3**

The function is transformed to $y = f(1 - x) + 2$.

What are the coordinates of the image of P?

Question 26 (**3** marks)

(a) Differentiate $x \ln x$. **1**

(b) Hence, find $\int \ln x \, dx$. **2**

END OF BOOKLET 1

Mathematics Advanced

Section II Answer Booklet 2

Student Number:	
Teacher:	

Section II

Booklet 2 – Attempt Question 27 – 35 (43 marks)

Instructions

- Write your Teacher's Name and Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on pages 39 and 40 of Booklet 2. If you use this space, clearly indicate which question you are answering.

Q27 – Q30	/ 15
Q31 – Q33a	/ 13
Q33b – Q35	/ 15

Please turn over

Question 27 (**3** marks)

A toy racetrack consists of multiple track pieces which join together. **3**

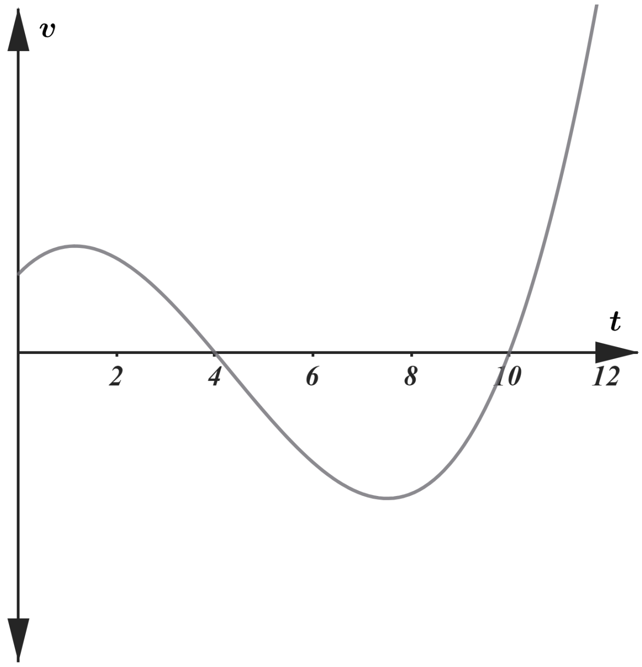
The first and longest piece of the track is 35 cm long, and each piece of track after the longest is 2 centimetres shorter than the previous piece.

The total length of the completed track is 2.6 metres.

Find the length of the shortest piece of track.

Question 28 (**4** marks)

The velocity of a particle moving on a number line is graphed below, where t is in seconds and v is in metres per seconds.
The particle begins its movement from the origin.



(a) State when the particle is at rest. **1**

(b) State when the particle is moving in the negative direction. **1**

(c) State the direction of the acceleration at $t = 6$. Justify your answer. **1**

(d) Explain why the particle will be on the negative side of the origin at $t = 10$, given $\int_0^4 f(t)\,dt < \left| \int_4^{10} f(t)\,dt \right|$. **1**

Question 29 (**4** marks)

Given $x^2 + y^2 = 14xy$,

(a) Show that $\dfrac{(x+y)^2}{16}$ simplifies to xy. **2**

(b) Hence or otherwise, show that $\dfrac{1}{2}(\log x + \log y) = \log\left(\dfrac{x+y}{4}\right)$. **2**

Question 30 (**4** marks)

Determine the area of the region bounded by $y = x^2 - 3x - 3$ and $y = x + 2$. **4**

Question 31 (**4** marks)

Consider the infinite series below.

$$S = \sin^2 x \cos^2 x + 2\sin^2 x \cos^4 x + 4\sin^2 x \cos^6 x + \cdots$$

(a) Given that the limiting sum of S exists, show that it is $\dfrac{\sin^2 x \cos^2 x}{\sin^2 x - \cos^2 x}$. **2**

(b) Hence, show that $\dfrac{1}{\operatorname{cosec}^2 x\,(\tan^2 x - 1)}$ can also be expressed as the limiting sum of S. **2**

Question 32 (**6** marks)

Consider $y = x^2 e^{-x}$.
This will have the following derivatives (do NOT prove this).

$$\frac{dy}{dx} = \frac{x(2-x)}{e^x}$$

$$\frac{d^2y}{dx^2} = \frac{x^2 - 4x + 2}{e^x}$$

(a) Determine any stationary points and their nature. **2**

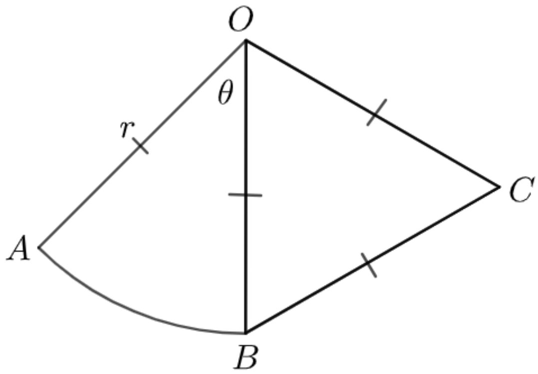
(b) Describe what occurs to the function as $x \to \pm\infty$. **2**

(c) You are given that $(3.4, 0.4)$ and $(0.6, 0.2)$ are points of inflection. **2**

Sketch $y = f(x)$.

Question 33 (**6** marks)

The shape below is made of a sector OAB and equilateral triangle OBC.
The sector OAB has a radius r cm.
The arc AB subtends an angle θ radians at O.



Let A represent the area of the shape.

(a) Given that the perimeter of the shape is 10 cm, show that **3**

$A = 5r + \dfrac{\sqrt{3}-6}{4}r^2$.

(b) Hence, determine the maximum area of the shape to the nearest cm^2. **3**

Question 34 (**7** marks)

A particle moves along a straight line such that its velocity in metres per second, v, is given by the equation below, where t is measured in seconds from $t = 0$.

$$v = 2 + \frac{t}{t^2 + 1}$$

(a) What is the particle's initial speed? **1**

(b) Explain why the particle can never be at rest. **2**

(c) What is the particle’s change in displacement in the first 4 seconds? Leave your answer in exact form. **2**

(d) Determine the particle's exact acceleration at $t = 5$. **2**

Question 35 (**5** marks)

Consider the function $f(x) = e^{-x} - e^{x}$.
It is known that $e^{-x} < e^{x}$ for all $x > 0$ (do NOT prove this).

(a) Show that $f''(x) < 0$ for all positive values of x. **2**

(b) Use the table of function values and the trapezoidal rule to estimate the area of the region bounded by the x-axis, $x = 3$, and the function. **2**
Leave your answer correct to 4 decimal places.

x	0	1	2	3
$f(x) = e^{-x} - e^{x}$	0	-2.3504	-7.2537	-20.0537

(c) Is the answer to part (b) an overestimate or an underestimate? Give a reason for your answer. **1**

END OF EXAMINATION

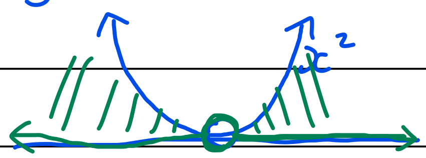
MARKER'S COMMENTS

1. D 2. A 3. C 4. B 5. B
6. A 7. C 8. C 9. B 10. D

1. rate = \$120/h call-out fee = \$130
$\therefore m = 120$ $\therefore$ y-int = 130
$\therefore C = 120t + 130$
$C = 130 + 120t$

2. $T = \frac{2\pi}{\pi} = 2$ Range of $\cos x : [-1, 1]$
$\cos(\pi x) : [-1, 1]$
$\cos(\pi x) + 2 : [1, 3]$

3. $\log_e(x^2) \rightarrow x^2 > 0$



$\therefore x < 0$ and $x > 0$
$(-\infty, 0) \cup (0, \infty)$

4. terms alternate in sign $\rightarrow r < 0$
first term is under n-axis $\rightarrow a < 0$
$\therefore a < 0$ and $r < 0$

MARKER'S COMMENTS

5. $y = 2^x$

dilated vertically by factor 8 $\downarrow$ replace y with $\frac{y}{8}$

$\frac{y}{8} = 2^x$

$y = 8 \cdot 2^x$

translated left by 1 unit $\downarrow$ replace x with $x+1$

$y = 8 \cdot 2^{x+1}$

6. $\int \frac{4}{(5-2x)^3} dx = \int 4(5-2x)^{-3} dx$

$= -2 \int -2(5-2x)^{-3} dx$

$= -2 \cdot \frac{(5-2x)^{-2}}{-2} + C$

$= (5-2x)^{-2} + C$

$= \frac{1}{(5-2x)^2} + C$

7. Area of semi-circle: $\frac{1}{2} \times \pi \times 5^2$

$= \frac{25\pi}{2}$

Area of square : 10^2

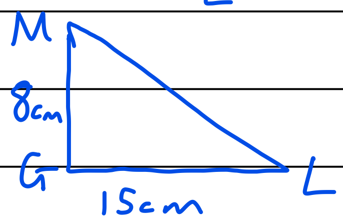
$= 100$

Probability $= \frac{\frac{25\pi}{2}}{100} = \frac{\pi}{8}$

MARKER'S COMMENTS

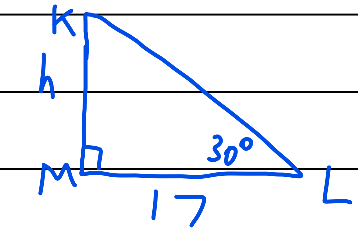
8. Option C: No limiting sum for the series as $r = 1$.

9. $\triangle MGL$



$ML = 17$ cm (8, 15, 17 Pythagorean triad)

$\triangle KML$



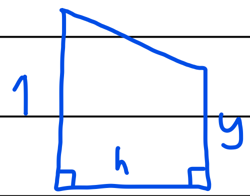
$$\tan 30° = \frac{h}{17}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{17}$$

$$h = \frac{17}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{17\sqrt{3}}{3} \text{ cm}$$

10. $\int_{-h}^{h} f(x)\,dx = 2 \rightarrow \int_{0}^{h} f(x)\,dx = 1$



$$y = 1 - \left|\frac{h}{2}\right|$$

$$= 1 - \frac{h}{2}$$

$$\text{Area} = \frac{h}{2}\left(1 + 1 - \frac{h}{2}\right) = 1$$

$$\frac{h}{2}\left(2 - \frac{h}{2}\right) = 1$$

$$h - \frac{h^2}{4} = 1$$

$$4h - h^2 = 4$$

$$\therefore h^2 - 4h + 4 = 0$$

$$(h-2)^2 = 0$$

$\therefore h = 2$

2025 12MAT HSC AT4

Q11

MARKER'S COMMENTS

a) $T_1 = 5\times1-3$
$= 2$

$T_2 = 5\times2-3$
$= 7$

$T_3 = 5\times3-3$
$= 12$

$\therefore$ First 3 terms are 2, 7, 12.

1 mark for correct answer – no half marks!

b) $a=2,\ d=5,\ n=50$

$S_{50} = \frac{50}{2}[2\times2+(50-1)\times5]$

$= 25\times(4+245)$

$= 6225$

1 mark for correct substitution into $S_n = \frac{n}{2}[2a+(n-1)d]$ or $S_n = \frac{n}{2}(a+l)$ with correct 50^{th} term.

1 mark for final answer (CFPE awarded from part a))

- Well-attempted by all.

2025 12MAT HSC AT4

MARKER'S COMMENTS

Q12

a) $f(g(x)) = f(2x+1)$

$= -(2x+1)^2 - 2(2x+1) + 3$ — 1 mark for correct substitution

$= -(4x^2+4x+1) - 4x - 2 + 3$

$= -4x^2 - 4x - 1 - 4x - 2 + 3$

$= -4x^2 - 8x$ → 1 mark for final simplified expression (either line)

$= -4x(x+2)$

b) $f(g(x)) = -4x^2 - 8x$

$= -4x(x+2)$

i.e. a parabola with x-intercepts at $x=0$ and $x=-2$.

$\therefore$ Axis of symmetry is the line $x=-1$. — 1 mark for correct axis of symmetry

OR Axis of symmetry: $x = \frac{-b}{2a}$

$= \frac{-(-8)}{2(-4)}$

$= \frac{8}{-8}$

$\therefore x = -1$

(CFPE awarded from part a))

- Poorly attempted across the whole cohort (part b))!!
- Many students confused the axis of symmetry with the vertex/turning point – remember, axis implies that it's a line!! others thought the question was asking for even/odd symmetry.
- $\frac{1}{2}$ mark was deducted for just "-1" as the answer (not $x=-1$).
- Be careful with your negative signs!

2025 12MAT HSC AT4

MARKER'S COMMENTS

Q13

a) $y = 5x^2 - \frac{4}{x}$

$= 5x^2 - 4x^{-1}$

$y' = 10x + 4x^{-2}$

$= 10x + \frac{4}{x^2}$

1 mark for correct derivative (either line)

b) $f(x) = 4x(6-x)^3$

$f'(x) = (6-x)^3 \times 4 + 4x \times 3(6-x)^2 \times -1$ — 1 mark for correct use of product rule

$= 4(6-x)^3 - 12x(6-x)^2$

$= 4(6-x)^2(6-x-3x)$

$= 4(6-x)^2(6-4x)$

$= 8(6-x)^2(3-2x)$

1 mark for some correct simplification (any of these lines)

- Many students did not use product rule to differentiate.
- When using chain rule to differentiate $(6-x)^3$, don't forget to multiply by the derivative of $(6-x)$!

c) $f(x) = \tan(x^2-2)$

$f'(x) = 2x\sec^2(x^2-2)$

1 mark for correct derivative

Q14 | **MARKER'S COMMENTS**

a) $\int \frac{x^2 - 3x + 4}{\sqrt{x}} \, dx$

$= \int \frac{x^2}{x^{\frac{1}{2}}} - \frac{3x}{x^{\frac{1}{2}}} + \frac{4}{x^{\frac{1}{2}}} \, dx$

$= \int x^{\frac{3}{2}} - 3x^{\frac{1}{2}} + 4x^{-\frac{1}{2}} \, dx$ — 1 mark for correct simplification using index laws

$= \frac{x^{\frac{5}{2}}}{\frac{5}{2}} - \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{4x^{\frac{1}{2}}}{\frac{1}{2}} + C$

$= \frac{2x^{\frac{5}{2}}}{5} - 2x^{\frac{3}{2}} + 8x^{\frac{1}{2}} + C$ — 1 mark for correct antiderivative (including +c !)

- Poorly attempted across the whole cohort.
- Simplify first (!!!) before integrating.
- $\frac{1}{2}$ mark deducted only once across Booklet 1 for leaving out the constant of integration.

b) $\int 16 \cos(2x - 1) \, dx$

$= 16 \times \frac{1}{2} \int 2 \cos(2x - 1) \, dx$

$= 8 \sin(2x - 1) + C$ 1 mark for correct answer (including +c !)

- $\frac{1}{2}$ mark deducted if answer was $-8\sin(2x-1) + C$. Use the Reference Sheet if you can't remember your integration rules!
- $\frac{1}{2}$ mark deducted only once across Booklet 1 for leaving out the constant of integration.

2025 12MAT HSC AT4

Q15

MARKER'S COMMENTS

a) c = centre $\qquad k$ = amplitude

$c = \frac{28+4}{2} \qquad k = 28 - 16$

$= 16 \qquad = 12$

$\therefore c = 16, \; k = 12$

1 mark $\qquad$ 1 mark

- $\frac{1}{2}$ mark deducted if $k = -12$.
- 1 mark deducted if values of k and c are switched.
- Many students struggled with this question — rearrange the equation first: $B(t) = c - k\sin\left(\frac{\pi t}{28}\right)$

$$= \underbrace{-k}_{\text{amplitude}}\sin\left(\frac{\pi t}{28}\right) \underbrace{+c}_{\text{centre}}$$

b) Period $= \dfrac{2\pi}{\frac{\pi}{28}}$

$= 2\cancel{\pi} \times \dfrac{28}{\cancel{\pi}}$

$= 56$ seconds — 1 mark for correct answer

- "One complete revolution" → period.

Question 16 (2 marks)

(a) $k + 2k + 3k + 4k = 1$

$10k = 1$

$k = 1/10$

OR $k = 0.1$ } ① for correct answer as fraction or decimal

(b) $\sigma = 1$ – ① correct answer from calculator

OR

$E(x) = \mu = \Sigma[x \times p(x)]$

$= 1 \times 0.1 + 2 \times 0.2 + 3 \times 0.3 + 4 \times 0.4$

$= 3$ – ($\frac{1}{2}$) for expected value

OR

$\sigma^2 = \text{var}(x)$

$= \Sigma[x^2 p(x)] - \mu^2$

$= [1^2 \times 0.1 + 2^2 \times 0.2 + 3^2 \times 0.3 + 4^2 \times 0.4] - 3^2$

$= 10 - 9$

$\sigma^2 = 1$ – ($\frac{1}{2}$) for variance

$\therefore \sigma = \sqrt{1}$

$= 1$ – ($\frac{1}{2}$) for standard deviation

OR $\sigma^2 = \text{var}(x)$

$= \Sigma(x-\mu)^2 \times p(x)$

$= (1-3)^2 \times 0.1 + (2-3)^2 \times 0.2 + (3-3)^2 \times 0.3$

$+ (4-3)^2 \times 0.4$

$\sigma^2 = 1$

$\therefore \sigma = \sqrt{1}$

$= 1$

Question 17 (2 marks)

$a = -11$

$d = 7$

$T_n = a + (n-1)d$

$= -11 + 7(n-1)$

$= -11 + 7n - 7$

$= 7n - 18$

$7n - 18 < 100$

$7n < 118$

$n < 16.857142 86$ ① correct n value

$\therefore n = 16$

$\therefore T_{16} = 7(16) - 18$

$= 94$ — ① correct 16^{th} term

OR

$\therefore$ the last term under 100 is the 16^{th} term which is 94.

$1\frac{1}{2}$ marks — If students stated the 16^{th} term is the last term under 100, with no mention of 94.

Show all working and answer the question completely.

Question 18 (2 marks)

Method ① — THE BEST WAY !!!

$$\frac{d}{dx}\log_e\left(\frac{x+1}{x-2}\right)$$

$$= \frac{d}{dx}\left[\log_e(x+1) - \log_e(x-2)\right]$$ – ① mark for correct use of log law

$$= \frac{1}{x+1} - \frac{1}{x-2}$$ – ① mark for correct differentiation of both.

OR THE LONG WAY !!!

Method ② $y = \ln f(x)$

$y' = \frac{f'(x)}{f(x)}$ and quotient rule

$$\frac{d}{dx}\log_e\left(\frac{x+1}{x-2}\right)$$

$$= \underbrace{\frac{1}{\frac{x+1}{x-2}}}_{①} \times \underbrace{\frac{(x-2)\times 1 - (x+1)\times 1}{(x-2)^2}}_{①}$$

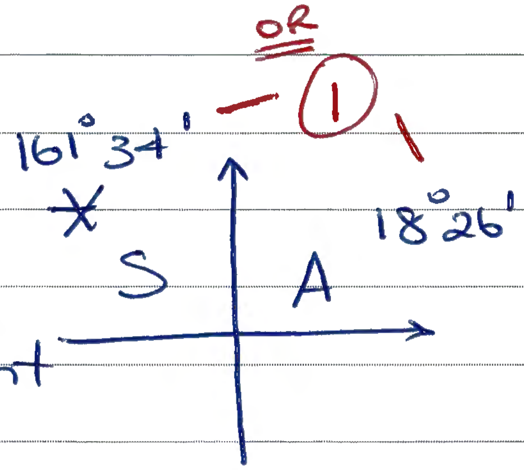
$$= \frac{\cancel{x-2}}{x+1} \times \frac{\cancel{x}-2-\cancel{x}-1}{(x-2)^{\cancel{2}}}$$

OR ① for correct quotient rule $\frac{-3}{(x-2)^2}$

$$= \frac{-3}{(x+1)(x-2)}$$

This type of question has appeared in many past papers and students still don't take markers advice on board to use method ①.

Question 19 (2 marks)



$0° \le x \le 360°$

$\therefore 0° \le \frac{x}{2} \le 180°$

∴ angle is in the 2nd quandrant

$\tan\left(\frac{x}{2}\right) = -\frac{1}{3}$

$\frac{x}{2} = 161° 33' 54.1$

$\frac{x}{2} = 161° 34'$ — ① OR worthwhile progress

$\therefore x = 323° 8'$ — ① correct answer to the nearest minute

$(-\frac{1}{2})$ for incorrect rounding

$(-\frac{1}{2})$ for giving more than one angle in final solution.

OR $\tan^{-1}\left(-\frac{1}{3}\right) = -18° 26'$ — ①

$x = 2\tan^{-1}\left(-\frac{1}{3}\right)$

$= -36° 52'$ — ①

$\therefore -36° 52' + 360°$

$= 323° 8'$ — ①

Question 20 (3 marks)

(a) $LHS = 1 + \dfrac{10}{x-3}$

$= \dfrac{x-3+10}{x-3}$ — ① mark for this line

$= \dfrac{x+7}{x-3}$

$= RHS$

It's a show question so it must be clear how one line is obtained from another.

(b) $\displaystyle\int_4^{e+3} \frac{x+7}{x-3}\,.dx$

$= \displaystyle\int_4^{e+3} 1 + \frac{10}{x-3}\,.dx$

$= \Big[x + 10\ln|x-3|\Big]_4^{e+3}$ — ① correct integration and notation

$= e+3+10\ln|e+3-3| - \big[4+10\ln|4-3|\big]$

$= e+3+10\ln e - \big[4+10\ln 1\big]$

$= e+3+10-(4+0)$

$= e+3+10-4$

$= e+9$ — ① correct answer

$e-1+10$
$= e-9$] ($1\frac{1}{2}$) marks (Be careful!!!)
$-1+10 = +9$

$e+3-4$ — ① or equivalent merit

* no marks if no integration was shown.

Question 21 (3 marks)

$a = 2$
$r = 3$ } ① mark

$T_n = ar^{n-1}$

$2 \cdot 3^{n-1} = 39\,366$

$3^{n-1} = 19\,683$ OR $n-1 = \log_3 19683$

$3^{n-1} = 3^9$ $\quad n-1 = 9$

$\therefore n-1 = 9$ $\quad n = 10$

$n = 10$ — ① mark

$$S_{10} = \frac{2(3^{10}-1)}{3-1}$$

$$= \frac{2(3^{10}-1)}{2}$$

$$= 3^{10} - 1$$

$$= 59048$$ — ① mark

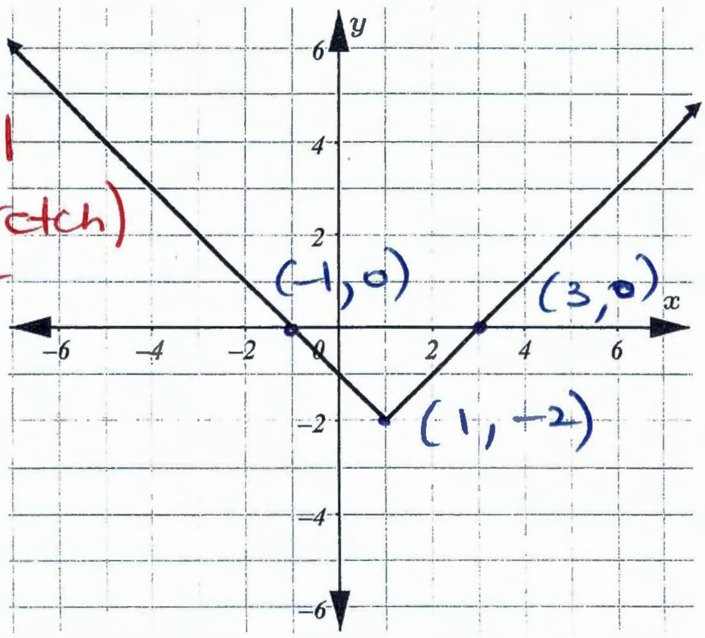
You were told in the question it's a geometric series so <u>don't waste time</u> showing that it is.

Question 22 (3 marks)

$f\left(\frac{x}{2}\right)$
$= f\left(\frac{1}{2} \cdot x\right)$

The graph of the function $y = f(x)$ has been shown below.

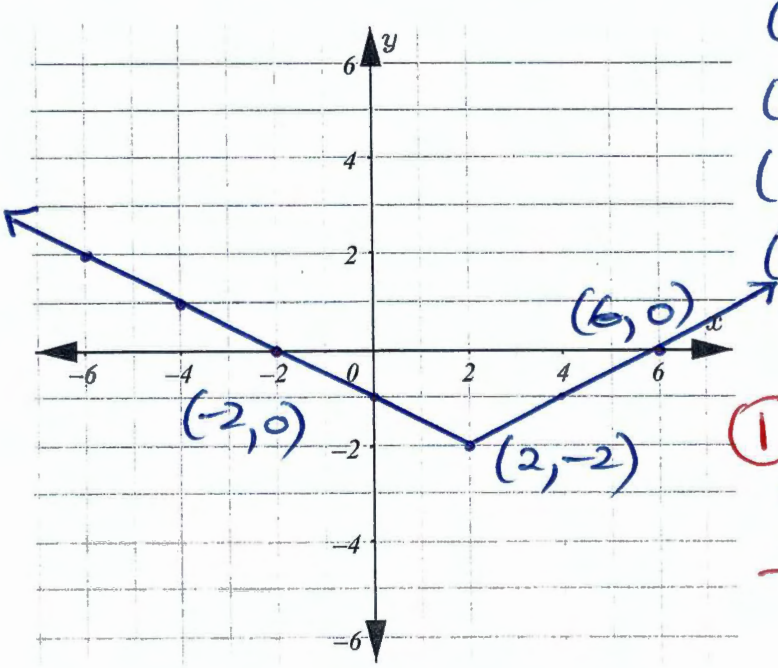
$f\left(\frac{x}{2}\right)$ is a horizontal dilation (stretch) by a factor of $\frac{1}{\frac{1}{2}} = 2$



$(x, y) \to \left(\frac{x}{\frac{1}{2}}, y\right)$
$= (2x, y)$

(a) Sketch the graph of $y = f\left(\frac{x}{2}\right)$ and show all intercepts. 1

$(x, y) \to (2x, y)$
$(-1, 0) \to (-2, 0)$
$(1, -2) \to (2, -2)$
$(3, 0) \to (6, 0)$



1 mark correct graph

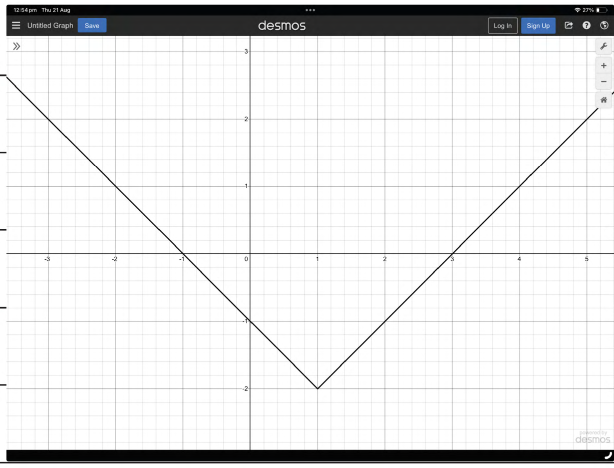
*students need to revise their transformations : $f(bx)$
$\therefore (x, y) \to \left(\frac{x}{b}, y\right)$

Q 22 b)

MARKER'S COMMENTS

Original Function:

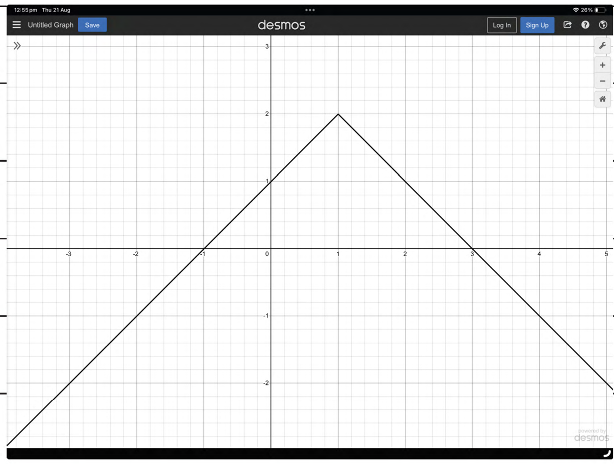
$y = f(x)$



$y = -f(x)$

This is a vertical dilation of factor -1. The original graph is reflected on the x-axis.

1 mark

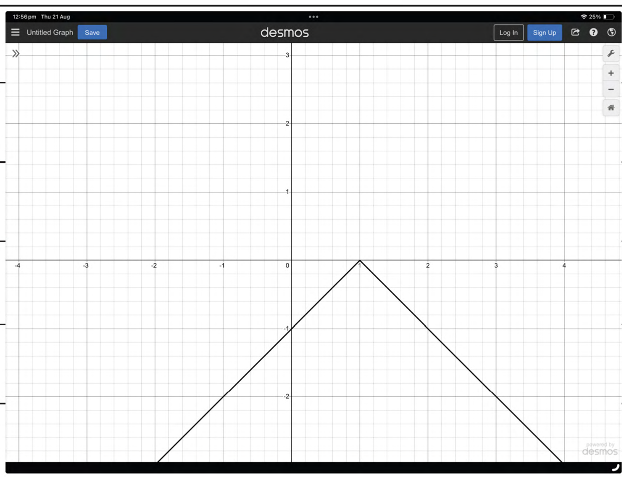


Some students mistakenly reflected on the vertex of the function or on the y-axis.

$y = -f(x) - 2$

This is a vertical translation down 2 units.

1 mark



Some students mistakenly translated left or right.

Lose $\frac{1}{2}$ mark for not showing all intercepts as required.

MARKER'S COMMENTS

Q23

$$\frac{x-8}{6-x} = \frac{x+4}{x-8} \quad - 1$$

$$(x-8)^2 = (6-x)(x+4)$$

$$x^2-16x+64 = 6x+24-x^2-4x$$

$$x^2-16x+64 = -x^2+2x+24$$

$$2x^2-18x+40=0$$

$$2(x^2-9x+20)=0$$

$$2(x-4)(x-5)=0$$

$$\therefore x = 4 \text{ or } 5. \quad - 1$$

$\therefore$ common ratios :

When $x=4$ $r = \frac{x-8}{6-x} = \frac{4-8}{6-4} = -2$

When $x=5$ $r = \frac{x-8}{6-x} = \frac{5-8}{6-5} = -3$

1 - CFPE allowed

Only $\frac{1}{2}$ of this last mark was awarded if r was worked out, but it was concluded that $r = 4$ or 5,

Made with Goodnotes

Q 24

MARKER'S COMMENTS

a)

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Prime Number

Square Number

Anything else.

P(losing)

= P(anything else)

$= \frac{14}{36}$

$= \frac{7}{18}$

1 mark - for filling in table

1 mark - for 14

1 mark - for 36.

Simplify preferred, but no marks allocated to this.

Q 24 **MARKER'S COMMENTS**

b) $P(\text{Prime}) = \frac{15}{36}$

$P(\text{Square}) = \frac{7}{36}$

$P(\text{Anything else}) = \frac{14}{36}$

$\frac{1}{2}$ for working out probabilities (CFPE allowed from part a)) - zero if don't show all 3 probabilities

x	$+\$5$	$+\$10$	$-\$5$
$P(X=x)$	$\frac{15}{36}$	$\frac{7}{36}$	$\frac{14}{36}$

$E(X) = 5 \times \frac{15}{36} + 10 \times \frac{7}{36} + -5 \times \frac{14}{36}$ — $\frac{1}{2}$

$E(X) = \$2.08$ — $\frac{1}{2}$

∴ Anthony is expected to gain money in the long run.

— $\frac{1}{2}$ (this is only awarded if $E(X)$ was found)

- A number of students did not realise that they needed to use expected value to work out what Anthony's mean earnings would be in the long run. They instead used the probabilities only, and did not consider the differing Dollar amounts for win or loss.

Q25

MARKER'S COMMENTS

$y = f(1-x)+2$

Horizontal Dilation (reflect over y-axis)

$y = f(-(x-1))+2$

Horizontal Translation 1 right

Vertical Translation 2 up

Many students tried to interpret the transformations without putting the transformed function in the form $y = kf(a(x+b)+c$ as above, to understand the transformations broken down.

The order in which these transformations should be completed is optional:

Option 1	OR	Option 2
Horizontal Dilation		Vertical Translation
Horizontal Translation		Horizontal Dilation
Vertical Translation		Horizontal Translation

OR Option 3

Horizontal Dilation

Vertical Translation

Horizontal Translation

(Essentially the Horizontal dilation must happen before the Horizontal Translation)

Q25 cont'd

MARKER'S COMMENTS

- Some students did not remember that the Horizontal dilation had to be done before the Horizontal translation.

Completion of the Transformations:
(using option 1)

① Horizontal Dilation – reflect over y-axis

$$P(x, y) = (3, -5)$$

$$\Downarrow$$

$$P'(-x, y) = (-3, -5)$$ – 1 mark

② Horizontal Translation – 1 unit right

$$\Downarrow$$

$$P'(-(x-1), y) = (-2, -5)$$ – 1 mark

③ Vertical Translation – 2 units up

$$\Downarrow$$

$$P'(-(x-1), y+2) = (-2, -3)$$ – $\frac{1}{2}$ mark

All 3 transformations done in the correct order – $\frac{1}{2}$ mark
(zero if reflection order not demonstrated)

Q25 cont'd

MARKER'S COMMENTS

* Many students did not use P' notation for the image.

The quick way of doing the question is as follows:

$y = f(1-x) + 2$

$P(x, y) \longrightarrow P'(1-x, y+2)$

$P(3, -5) \longrightarrow P'(1-3, -5+2)$

$= P'(-2, -3)$

This method does not show understanding but was still awarded. This method would not be helpful if graphing were required.

MARKER'S COMMENTS

Q26

a) $y = x \times \ln x$

using the product rule:

let $u = x$ $\quad \frac{du}{dx} = 1$

let $v = \ln x$ $\quad \frac{dv}{dx} = \frac{1}{x}$

$\frac{dy}{dx} = x \times \frac{1}{x} + 1 \times \ln x$ — 1

$\frac{dy}{dx} = 1 + \ln x$

b) Method 1: 1 mark for some progress such as below.

$\int 1 + \ln x \, dx = x \ln x + c$ — 1

$\int 1 \, dx + \int \ln x \, dx = x \ln x + c$

$x + \int \ln x \, dx = x \ln x + c$

$\int \ln x \, dx = x \ln x - x + c$ — 1

lose $\frac{1}{2}$ for forgetting $+c$ (once only in Booklet 1)

26b continued **MARKER'S COMMENTS**

<u>Method 2:</u>

From part a) $\frac{d}{dx}(x\ln x) = \ln x + 1$

$\therefore \ln x = \frac{d}{dx}(x\ln x) - 1$

$\int \ln x \, dx = \int \frac{d}{dx}(x\ln x) - 1 \, dx$ — 1

$= x\ln x - x + C$ — 1

MARKER'S COMMENTS

Mathematics Advanced Solutions

HSC Trial exam

Question 27

$a = 35 \quad d = -2 \quad S_n = 260\text{cm}$

$$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$\therefore \frac{n}{2}\left(2(35) + (n-1)\cdot(-2)\right) = 260$$ — 1mk

$$n(70 - 2n + 2) = 520$$

$$n(72 - 2n) = 520$$

$$72n - 2n^2 = 520$$

$$2n^2 - 72n + 520 = 0$$

$$n^2 - 36n + 260 = 0$$

$$n = \frac{36 \pm \sqrt{36^2 - 4(1)(260)}}{2}$$

$$= 26 \text{ or } 10.$$

— 1mk
— no ½mk was awarded

$$T_n = a + (n-1)d$$

$$= 35 + (26-1)(-2)$$

$$= 35 + 25(-2)$$

$$= -15\text{ cm}$$

— ½ mk with appropriate justification.

But T_n represents length and length cannot be negative

so $n \neq 26$

$$T_n = a + (n-1)d$$

$$= 35 + (10-1)(-2)$$

$$= 17\text{cm}$$ — ½mk

MARKER'S COMMENTS

∴ length of the shortest piece is 17cm.

Markers Comments

This question was generally done well. However, a few students did not find the T_n values and also did not eliminate the negative T_n with an appropriate justification.

Question 28

a) $t = 4s$ and $t = 10s$ — 1mk

b) $4s < t < 10s$ — 1mk

c) acceleration is negative ($a<0$) — ½mk

as the gradient of tangent at $t = 6s$ is negative. — ½mk

d) $\int_0^{10} f(t)\,dt < 0$. As the particle starts from the origin, it must end on the negative side since the overall 'area' is negative. — 1mk

MARKER'S COMMENTS

Note that area underneath the velocity time graph represents the distance travelled and not displacement. Consequently, the distance travelled to the left is more than the distance travelled to the right. Hence, displacement is in the negative direction. (towards the left)

Marker's Comments

a) Generally, well done.

b) Quite well attempted. A few students included the equal sign which is not correct.

c) Some students had difficulty in providing an appropriate justification.

d) Not attempted well. See the detailed explanation provided above.

MARKER'S COMMENTS

Question 29

a) $\dfrac{(x+y)^2}{16}$

$= \dfrac{x^2 + 2xy + y^2}{16}$ — 1 mk

$= \dfrac{14xy + 2xy}{16}$ $(x^2 + y^2 = 14xy)$

$= \dfrac{16xy}{16}$ — 1 mk

$= xy$

b) $LHS = \frac{1}{2}(\log x + \log y)$

$= \frac{1}{2}(\log xy)$

$= \frac{1}{2}\left(\log \dfrac{(x+y)^2}{16}\right)$ --- from part (a)

$= \frac{1}{2}\left(\log \left(\dfrac{x+y}{4}\right)^2\right)$

$= \frac{1}{2} \times 2 \left(\log \left(\dfrac{x+y}{4}\right)\right)$

$= \log\left(\dfrac{x+y}{4}\right)$

$= RHS$ As required.

$\therefore LHS = RHS$ $\therefore$ proved true.

MARKER'S COMMENTS

1mk – using any 2 relevant logarithmic laws.

1mk – for substantial progress towards the solution

<u>Alternative method</u>

$RHS = \log\left(\frac{x+y}{4}\right)$

$= \log\left(\sqrt{xy}\right)$ from part (a)

$= \log(xy)^{1/2}$

$= \frac{1}{2}\log(xy)$

$= \frac{1}{2}(\log x + \log y)$

$= LHS$

$\therefore RHS = LHS$

$\therefore$ proved true

MARKER'S COMMENTS

Alternative method

$RHS = \log\left(\frac{x+y}{4}\right)$

$= \log\left(\left(\frac{x+y}{4}\right)^2\right)^{1/2}$

$= \frac{1}{2}\log\left(\frac{(x+y)^2}{16}\right)$

$= \frac{1}{2}\log xy$

$= \frac{1}{2}(\log x + \log y)$

Alternative method.

$LHS = \frac{1}{2}\log(xy)$

$= \frac{1}{2}\log\left(\frac{(x+y)^2}{16}\right)$ from part (a)

$= \frac{1}{2}\log(x+y)^2 - \frac{1}{2}\log 16$

$= \log(x+y)^{2\times 1/2} - \log 16^{1/2}$

$= \log(x+y) - \log 4$

$= \log\left(\frac{x+y}{4}\right)$

$= RHS$

$\therefore LHS = RHS$ proved true

MARKER'S COMMENTS

Alternative Method

$$LHS = \frac{1}{2}(\log x + \log y)$$

$$= \frac{1}{2}\log x + \frac{1}{2}\log y$$

$$= \log x^{1/2} + \log y^{1/2}$$

$$= \log (x^{1/2} \times y^{1/2})$$

$$= \log (xy)^{1/2}$$

$$= \frac{1}{2}\log xy$$

$$= \frac{1}{2}\log \left(\frac{x+y}{4}\right)^2$$

$$= \log \left(\frac{x+y}{4}\right)^{2 \times 1/2}$$

$$= \log \left(\frac{x+y}{4}\right)$$

$= RHS$

$\therefore LHS = RHS$

$\therefore$ proved true.

MARKER'S COMMENTS

Alternative Method

Given $\frac{(x+y)^2}{16} = xy$

$\therefore \frac{x+y}{4} = \sqrt{xy}$

Take log on both sides

$\log\left(\frac{x+y}{4}\right) = \log\sqrt{xy}$

$\log\left(\frac{x+y}{4}\right) = \log(xy)^{1/2}$

$\log\left(\frac{x+y}{4}\right) = \frac{1}{2}\log xy$

$\therefore \log\left(\frac{x+y}{4}\right) = \frac{1}{2}(\log x + \log y)$

$\therefore$ LHS = RHS

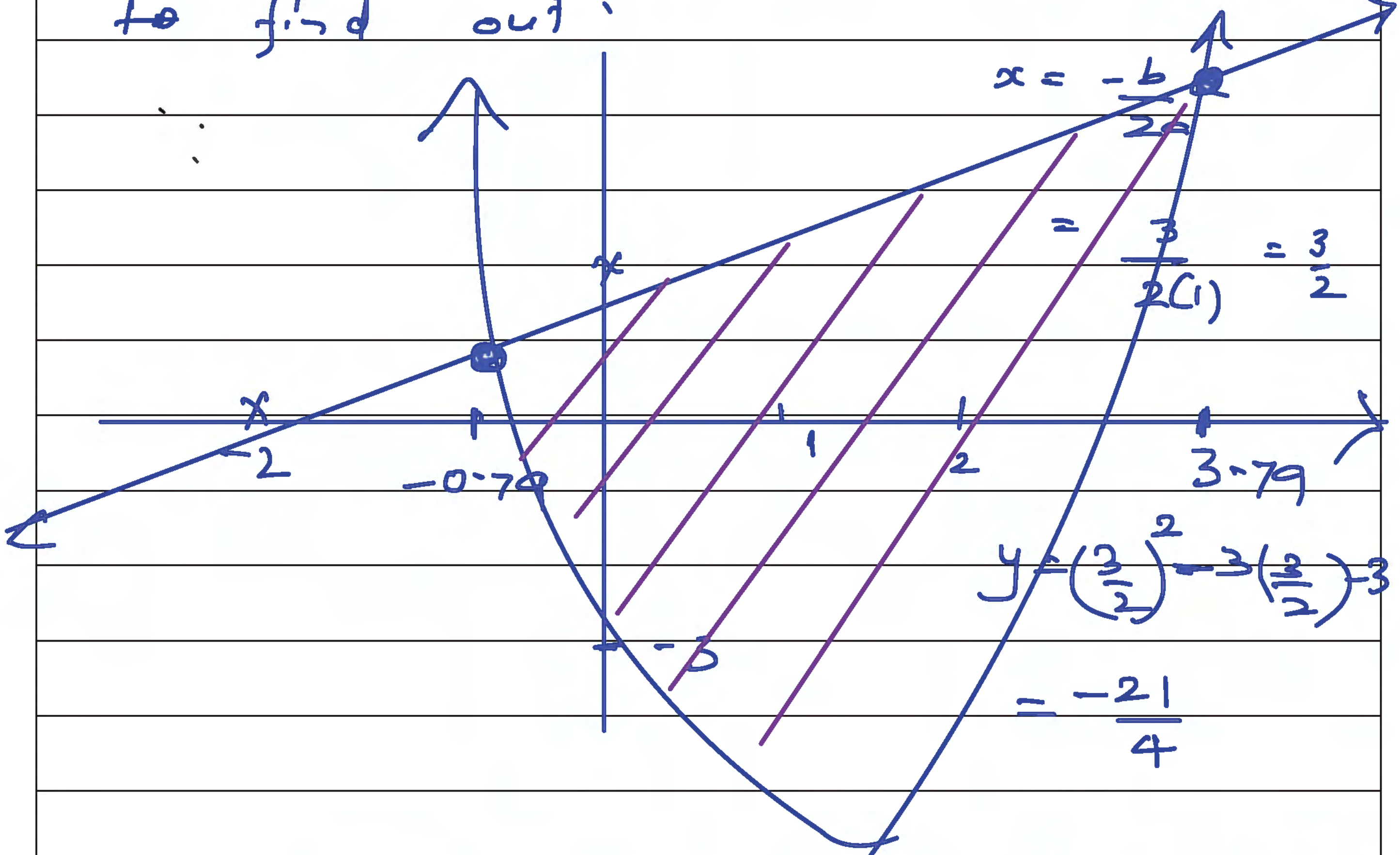
Marker's Comments:

Generally, well done. However, a few students need to re-visit the logarithmic laws. Furthermore, a few students assumed that $\frac{1}{2}(\log x + \log y) = \log\left(\frac{x+y}{4}\right)$ is true and then proved that the statement is true.

MARKER'S COMMENTS

Question 30

Sketch the 2 graphs to give an indication of the area that you have to find out.



$x = \frac{-b}{2a}$

$= \frac{3}{2(1)} = \frac{3}{2}$

$y = \left(\frac{3}{2}\right)^2 - 3\left(\frac{3}{2}\right) - 3$

$= \frac{-21}{4}$

Next, find the point of intersection

$x^2 - 3x - 3 = x + 2$

$x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

$x = 5$ or $x = -1$ — 1 mk

$\int_{-1}^{5}$ (top curve − bottom curve).

Note that this area is going

MARKER'S COMMENTS

to be the same anywhere in the plane as this region just represents translation anywhere in the plane. so, you don't have to actually take the absolute value (if you wish), as long as one graph is above the other anywhere in the domain, even if some area is below the x-axis.

$$\int_{-1}^{5} (x+2) - (x^2-3x-3)\,dx$$

$$= \int_{-1}^{5} (x+2 - x^2 + 3x + 3)\,dx$$

$$= \int_{-1}^{5} (\underbrace{-x^2 + 4x + 5}_{1\,mk})\,dx$$

$$= \left[\frac{-x^3}{3} + 2x^2 + 5x\right]_{-1}^{5} \quad \text{— 1 mk}$$

$$= \left(\frac{-5^3}{3} + 2(5)^2 + 5(5)\right) - \left(\frac{+1}{3} - 2 + 5\right)$$

$= 36$ units^2 — 1 mk

Marker's comments

Poorly attempted. See detailed explanation above.

12MAT MARKER'S COMMENTS 2025 AT4

Q31 a)

$$S = \frac{a}{1-r}$$

$a = \sin^2 x \cos^2 x$

$r = \frac{2\cancel{\sin^2 x}\cos^{\cancel{4}} x}{\cancel{\sin^2 x}\ \cancel{\cos^2 x}}$

$r = 2\cos^2 x$ —— $\frac{1}{2}$

$$\therefore \quad S = \frac{\sin^2 x \cos^2 x}{1 - 2\cos^2 x} \quad \text{—— } 1$$

$$= \frac{\sin^2 x \cos^2 x}{1 - \cos^2 x - \cos^2 x} \quad \text{—— } \tfrac{1}{2}$$

$$\therefore \quad S = \frac{\sin^2 x \cos^2 x}{\sin^2 x - \cos^2 x}$$ as required

2 marks – for correct steps to show S

$1\frac{1}{2}$ marks – for finding the common ratio $r = 2\cos^2 x$ and for using the limiting sum formula correctly and writing $S = \frac{\sin^2 x \cos^2 x}{1 - 2\cos^2 x}$

1 mark – for using the limiting sum formula correctly with an incorrect common ratio.

$\frac{1}{2}$ mark – the common ratio was correctly found.

Markers comment: This question was answered well overall but some students applied the Ext 1 trig identities to finish the proof. I accepted the use of this method however it is not intended to be used in an Advanced paper.

From $\frac{\sin^2 x \cos^2 x}{1 - 2\cos^2 x} = \frac{\sin^2 x \cos^2 x}{-(2\cos^2 x - 1)} = \frac{\sin^2 x \cos^2 x}{-\cos 2x} = \frac{\sin^2 x \cos^2 x}{-(\cos^2 x - \sin^2 x)}$

MARKER'S COMMENTS

b) $$\frac{\sin^2 x\cos^2 x}{\sin^2 x-\cos^2 x} = \frac{\sin^2 x\cos^2 x \div \cos^2 x}{\sin^2 x-\cos^2 x \div \cos^2 x}$$

$$= \frac{\sin^2 x}{\frac{\sin^2 x}{\cos^2 x}-1}$$

1

$$= \frac{\sin^2 x}{\tan^2 x-1}$$

— 1

$$= \frac{1}{\operatorname{cosec}^2 x(\tan^2 x-1)}$$ since $\operatorname{cosec} x = \frac{1}{\sin x}$

Alternative method

$$\frac{1}{\operatorname{cosec}^2 x(\tan^2 x-1)} = \frac{1}{\frac{1}{\sin^2 x}\left(\frac{\sin^2 x}{\cos^2 x}-1\right)}$$

$$= \frac{\sin^2 x}{\frac{\sin^2 x-\cos^2 x}{\cos^2 x}} \times \frac{\cos^2 x}{\cos^2 x}$$

$$= \frac{\sin^2 x\cos^2 x}{\sin^2 x-\cos^2 x}$$

Alternative method

$$\frac{1}{\operatorname{cosec}^2 x(\tan^2 x-1)} = \frac{1}{\frac{1}{\sin^2 x}\left(\frac{\sin^2 x}{\cos^2 x}-1\right)}$$

$\frac{1}{2}$ $\frac{1}{2}$

$$= \frac{1}{\frac{1}{\cos^2 x}-\frac{1}{\sin^2 x}}$$

— $\frac{1}{2}$

$$= \frac{1}{\frac{\sin^2 x-\cos^2 x}{\cos^2 x\sin^2 x}} \times \frac{\cos^2 x\sin^2 x}{\cos^2 x\sin^2 x}$$ — $\frac{1}{2}$

$$= \frac{\sin^2 x\cos^2 x}{\sin^2 x-\cos^2 x}$$

MARKER'S COMMENTS

2 marks — Complete solution

1 mark — Progress towards solution

— ½ mark each for using $cosec^2 x = \frac{1}{\sin^2 x}$ and $\tan^2 x = \frac{\sin^2 x}{\cos^2 x}$

½ mark — incorrect methods used to prove but at least one trig identity used correctly in the proof.

Markers comment:

- This question was not well answered by many students
- Students had difficulty progressing through the proof because their trig identities were incorrect. Many students used $\sec^2 x = \tan^2 x - 1$ which is not correct. The correct trig identity is $1 + \tan^2 x = \sec^2 x$. Please review the trigonometric identities.

MARKER'S COMMENTS

Q32

$$y = x^2 e^{-x}$$

a) $$\frac{dy}{dx} = \frac{x(2-x)}{e^x}$$

For stationary points, $\frac{dy}{dx} = 0$

$$0 = \frac{x(2-x)}{e^x}$$

$x = 2$, $x = 0$

when $x = 2$, $y = 4e^{-2}$
$x = 0$, $y = 0$
} ① mark

At $(0,0)$, $\frac{d^2y}{dx^2} = \frac{2}{1}$
$= 2 > 0$

$\therefore$ Concave up $\therefore$ $(0,0)$ is a local minimum — $\frac{1}{2}$

At $(2, 4e^{-2})$, $\frac{d^2y}{dx^2} = \frac{4-8+2}{e^2}$

$= \frac{-6}{e^2} < 0$

$\therefore$ concave down $\therefore$ is a local maximum — $\frac{1}{2}$

2 marks – for complete solution
1½ marks – for almost the complete solution but not including the stationary points $(0,0)$ and $(2, 4e^{-2})$
1 mark – Finding the stationary points
½ mk – For finding $x=0$, $x=2$

This part was well done overall.

b) As $x \to -\infty$, $y \to \infty$ 1
$x \to \infty$, $y \to 0^+$ 1

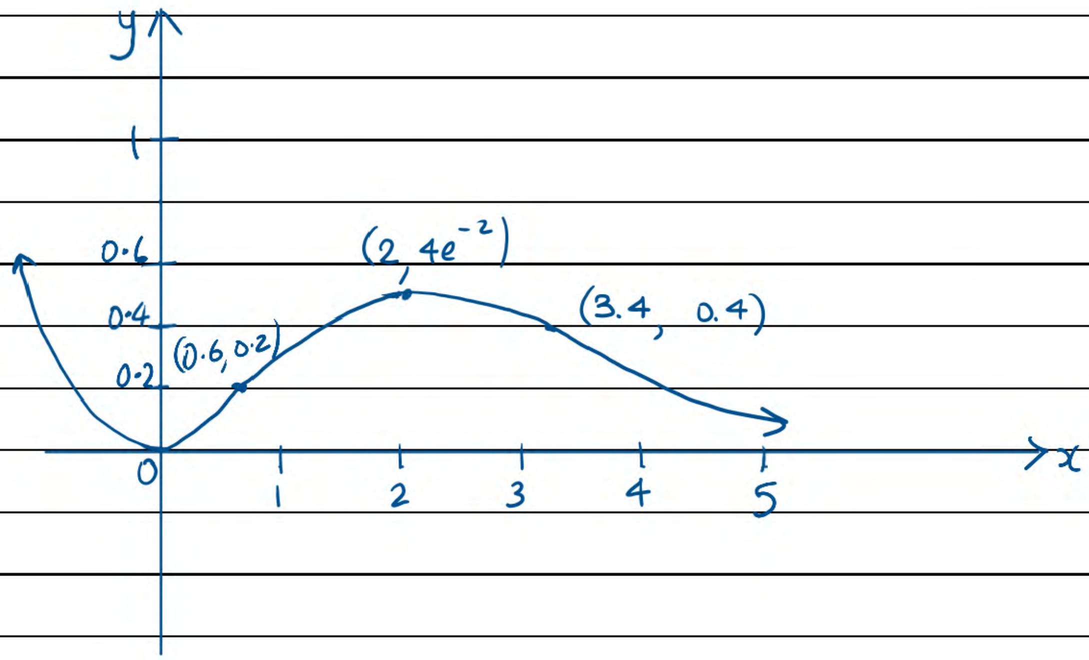
$\boxed{2}$

This part was answered well overall, however some students did not separate large x and wrote $x \to \pm\infty$, $y \to 0$. No marks were awarded if students did this.

MARKER'S COMMENTS

c)

$4e^{-2} \doteq 0.54$



$\frac{1}{2}$ mk – shape

1 mk – turning points

$\frac{1}{2}$ mk – points of inflection

$\boxed{2}$

Overall this graph was well drawn.

MARKER'S COMMENTS

Q33

a)

$$P = 3r + r\theta \quad \text{---} (1) \qquad \text{—— } \tfrac{1}{2}$$

If $P = 10$, sub in (1)

$$3r + r\theta = 10$$

$$r\theta = 10 - 3r$$

$$\theta = \frac{10 - 3r}{r} \quad \text{sub in (2)} \text{ — } \tfrac{1}{2}$$

A_1 = Area of sector

$$= \frac{1}{2} r^2 \theta$$

$$= \frac{1}{2} r^2 \left(\frac{10 - 3r}{r} \right)$$

$$= 5r - \frac{3}{2} r^2 \qquad \tfrac{1}{2}$$

A_2 = Area of triangle

$$= \frac{1}{2} r^2 \sin \theta$$

$$= \frac{1}{2} r^2 \sin(60°)$$

$$= \frac{1}{2} r^2 \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{3}}{4} r^2 \qquad \tfrac{1}{2}$$

$$\therefore A = A_1 + A_2$$

$$= 5r - \frac{3}{2} r^2 + \frac{\sqrt{3}}{4} r^2 \qquad \text{—— } \tfrac{1}{2}$$

$$= 5r + \frac{-6}{4} r^2 + \frac{\sqrt{3}}{4} r^2 \qquad \text{—— } \tfrac{1}{2}$$

$$A = 5r + \frac{\sqrt{3} - 6}{4} r^2$$

3 marks – for correct solution

2 marks – Finding $\theta = \frac{10 - 3r}{r}$ and Area of sector and the Area of triangle correctly

1 mark – for expressing θ as $\frac{10-3r}{r}$

or

– for finding the area of triangle and area of sector

$\frac{1}{2}$ mark – for expressing P as $P = 3r + r\theta$

or – for finding either the Area of a triangle or area of sector correctly.

MARKER'S COMMENTS

Markers comment:

Students need to review the formula for the arc length and area of a sector.

- The arc length formula is $l = r\theta$. Many students just left the arc length as AB, and wrote $P = 3r + AB$.
- The area of a sector formula is

$$A = \frac{1}{2}r^2\theta$$

Many students used $\frac{\theta}{360°} \times \pi r^2$ and did not realise that the angles are in radian measure and should have replaced 360° with 2π.

MARKER'S COMMENTS

(b) $A = 5r + \frac{\sqrt{3}-6}{4}r^2$

① for $\frac{dA}{dr}$

$\frac{dA}{dr} = 5 + \frac{\sqrt{3}-6}{2}r = 0$ at maximum area

$\therefore \quad r = -5 \times \frac{2}{\sqrt{3}-6}$

$= \frac{10}{6-\sqrt{3}}$ ← ① for r

$\frac{d^2A}{dr^2} = \frac{\sqrt{3}-6}{2} \approx -2.134 < 0$ ← 0.5 for justification. This can be gradient table

$\therefore$ maximum occurs at $r = \frac{10}{6-\sqrt{3}}$

$\therefore A = 5\left(\frac{10}{6-\sqrt{3}}\right) + \frac{\sqrt{3}-6}{4}\left(\frac{10}{6-\sqrt{3}}\right)^2$

$\approx 6 \text{ cm}^2$ ← 0.5 for final answer

- $\frac{1}{2}$ deducted for minor errors in simplifying <u>after</u> determining $\frac{dA}{dr}$ correctly or solving for r.
- lack of units not penalised
- Common errors:
  - Not subbing back in A correctly
  - $2 \cdot \frac{\sqrt{3}-6}{4} r = (2\sqrt{3}-12)r$
- Some candidates used $\triangleq$ incorrectly

MARKER'S COMMENTS

34 (a) $V = 2 + 0$

$= 2\ ms^{-1}$

(b) $t > 0$ and $t^2 + 1 > 0$ since square values must be positive

$\therefore \frac{t}{t^2+1} \geq 0$ since the quotient of two positives is positive

$\therefore 2 + \frac{t}{t^2+1} \geq 2$

$\therefore$ V is always a positive value and will never equal zero.

This means it is never at rest

- 0.5 rewarded for any mention of $t^2 \geq 0$ or $t^2 + 1 > 0$
- If $\frac{t}{t^2+1} > 0$ is stated,
  - ↳ 1 mark if $t^2 > 0$ stated previously
  - ↳ 1.5 marks if $t^2 > 0$ AND $t > 0$ stated
  - ↳ 0 marks if only $t > 0$ stated
- 0.5 awarded for observing large values of t

  Inclusive of:

  - ↳ <u>stating</u> $\frac{t}{t^2+1} > 0$, hence $\frac{t}{t^2+1} \neq -2$
  - ↳ $\lim_{t \to \infty} \frac{t}{t^2+1} = 0$

MARKER'S COMMENTS

(b) Using the discriminant:

If the particle is at rest,

$$2 + \frac{t}{t^2+1} = 0$$

$$2t^2+2+t=0$$

$$2t^2+t+2=0 \quad \leftarrow \text{① for reaching this line}$$

$$\Delta = 1^2 - 4(2)(2)$$

$$= -15 < 0 \quad \leftarrow 0.5 \text{ for the discriminant or quadratic formula}$$

$\therefore$ No possible solution

$\therefore$ the particle is never at rest ← 0.5 for interpretation of discriminant

- Observing what occurs when $t \to \infty$ simply looks at what the function approaches for large values of x. It does not consider anything beforehand.

  E.g. $\lim_{t\to\infty} \frac{\sin(t)}{t} + \frac{1}{5} = \frac{1}{5}$

  BUT $\frac{\sin(t)}{t} + \frac{1}{5} = 0$ at $t \approx 4.10462$ and 4.9063

- Some unfortunate substitution or algebraic manipulation errors occurred if using the discriminant method
- candidates need to practice supporting arguments in explain questions more.

MARKER'S COMMENTS

(c) Method 1: 0.5 for integral

$\int_0^4 2 + \frac{t}{t^2+1}\,dt$ 0.5 for boundaries

$= \left[2t + \frac{1}{2}\ln|t^2+1|\right]_0^4$ 0.5 for primitive

$= 8 + \frac{1}{2}\ln 17$ 0.5 for answer

Method 2:

$x = \int 2 + \frac{t}{t^2+1}\,dt$ $\leftarrow 0.5$

$= 2t + \frac{1}{2}\ln|t^2+1| + C$ $\leftarrow 0.5$

At $t=0$, $x = C$ $\leftarrow 0.5$

$t=4$, $x = 8 + \frac{1}{2}\ln 17 + C$ $\leftarrow 0.5$

$\therefore$ Change in displacement is $8 + \frac{1}{2}\ln 17$

- Many candidates assumed $C=0$, or that the initial displacement was 0. This cannot be assumed, but no penalty was given.

- Many candidates did not determine initial displacement

- Some candidates thought this was to do with velocity.

Average velocity $= \frac{\Delta x}{\Delta t}$ } evaluated

the question asked for Δx.

- use dt instead of dx
- d is not displacement

MARKER'S COMMENTS

(d) $a = \frac{dv}{dt}$

$= \frac{t^2+1-2t}{(t^2+1)^2}$ ← ① for quotient rule

$= \frac{1-t^2}{(t^2+1)^2}$

At $t=5$, $a = \frac{1-5^2}{(5^2+1)^2}$

$= \frac{-6}{169}\ ms^{-2}$ ← ① for evaluation

- 0.5 deducted for subsequent minor errors such as incorrect simplification.

- Candidates need to revise the quotient rule.
  - Many forgot to square the denominator.
  - Some did $uv' - vu'$ as the numerator

MARKER'S COMMENTS

35 (a) $f'(x) = -e^{-x} - e^{x}$ ← 0.5

$f''(x) = e^{-x} - e^{x}$ ← 0.5

As $e^{-x} < e^{x}$ for all $x > 0$, ← 0.5

$e^{-x} - e^{x} < 0$ for all $x > 0$

$\therefore f''(x) < 0$ for all $x > 0$ ← 0.5

- Candidates need to work on using supporting statements
- If $f''(x)$ was not determined, no further marks were awarded

(b) $\int_0^3 f(x)\,dx \approx \frac{1}{2}\left(\underbrace{-20.0537 + 2(-2.3504 - 7.2537)}_{0.5}\right)$ 0.5

≈ -19.6310 ← 0.5

$\therefore A \approx 19.6310$ units^2 ← 0.5

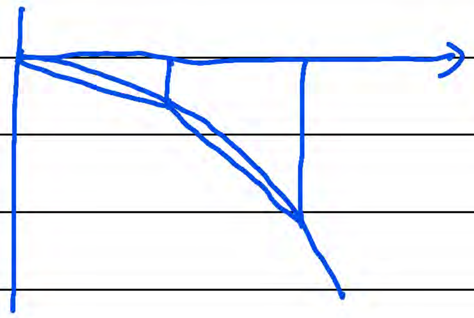
- Many candidates determined '$\frac{1}{2}$' incorrectly. Use '$\frac{h}{2}$' where h is the sub-interval length.

  ↳ $\frac{b-2a}{2n}$ sucks! So many errors came from trying to use this.

- Some candidates forgot that the area is positive

MARKER'S COMMENTS

(c)



It is an overestimate as the chords/line segments are outside the curve.

- Mark rewarded if:
  - ↳ trapeziums/chord/line is outside the curve WITH a supporting diagram
  - ↳ negative concavity AND negative y-values
- 0.5 deducted if the wrong shape was used.

- this question exposed candidates that rote learned 'negative concavity implies underestimate'
  Conceptual understanding is actually important