SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2004

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes.
- Working time – 3 hours.
- Write using black or blue pen.
- Board approved calculators may be used.
- All *necessary* working should be shown in every question if full marks are to be awarded.
- Marks may **NOT** be awarded for messy or badly arranged work.
- Hand in your answer booklets in **5** sections.
  Section A (Questions 1 & 2),
  Section B (Questions 3 & 4),
  Section C (Questions 5 & 6),
  Section D (Questions 7 & 8) and
  Section E (Questions 9 & 10).
- Start each **NEW** section in a separate answer booklet.

Total Marks - 120 Marks

- Attempt Sections A - E
- All questions are of equal value.

Examiner: *P. Parker*

This is an assessment task only and does not necessarily reflect the content or format of the Higher School Certificate.

Total marks – 120
Attempt Questions 1 – 10
All questions are of equal value

Answer each section in a SEPARATE writing booklet. Extra writing booklets are available.

SECTION A (Use a SEPARATE writing booklet)

Question 1 (12 marks) **Marks**

(a) Evaluate $\log_e\left(\tan\frac{5\pi}{12}\right)$ leaving your answer correct to 3 significant figures **2**

(b) Differentiate $\sqrt{5x}$ **2**

(c) Solve $2t^2 - t - 15 = 0$ **2**

(d) Find a primitive of $3 - 2x$ **2**

(e) Solve the pair of simultaneous equations **2**

$$y = 2x$$
$$3x + 2y = 14$$

(f) Solve $3 - 4x < 1$ and graph the solution on a number line **2**

Question 2 (12 marks) **Marks**

(a) Differentiate

(i) $\left(1+\cos 2x\right)^3$ **2**

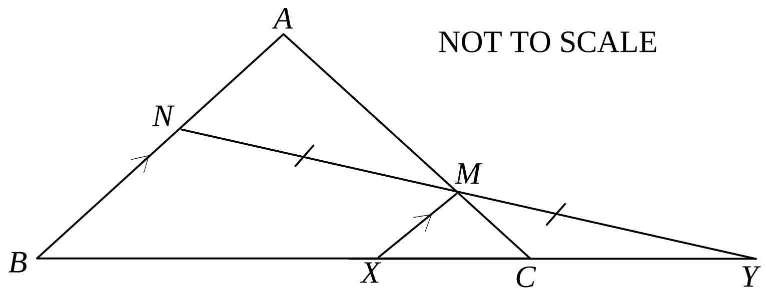
(ii) x^2e^{x+2} **2**

(b) Find:

(i) $\int \frac{\cos x}{\sin x}dx$ **1**

(ii) $\int_{\frac{1}{2}}^{2}\left(1-\frac{1}{x^2}\right)dx$ **2**

(c)



In the diagram above ΔABC is isosceles, with $AB = AC$.
M is the midpoint of the line NY and $XM \parallel AB$.

(i) By using similar triangles, or otherwise, show that $\frac{MX}{NB}=\frac{1}{2}$ **2**

(ii) Hence show that $\frac{MC}{NB}=\frac{1}{2}$ **1**

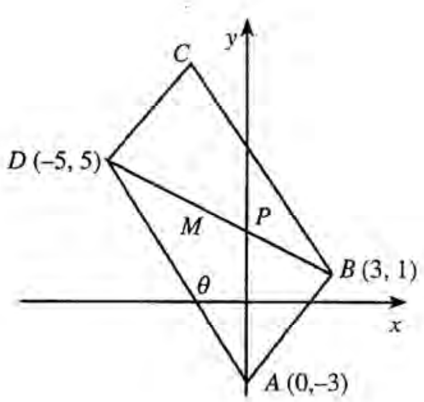
(d) The graph of $y=g(x)$ passes through the point $(2,4)$ and $g'(x)=4-3x^2$. **2**

Find $g(x)$.

SECTION B (Use a SEPARATE writing booklet)

Question 3 (12 marks) **Marks**

In the diagram below *A*, *B* and *D* have coordinates $(0,-3)$, $(3,1)$ and $(-5,5)$ respectively.
The angle θ is the angle the line *AD* makes with the positive direction of the *x* axis.



(i) Find the gradient of the line *AD*. **2**
Hence find θ to the nearest degree.

(ii) Find the coordinates of *M*, the midpoint of *BD*. **1**

(iii) Find the coordinates of *C*, so that *ABCD* is a parallelogram. **1**

(iv) Show that the line *AB* has equation $4x-3y-9=0$. **2**

(v) Find the perpendicular distance between *D* and *AB*. **1**

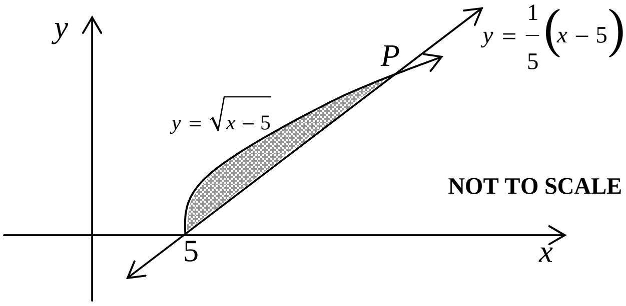
(vi) Find the area of parallelogram *ABCD*. **2**

(vii) The line *BD* has equation $x+2y-5=0$ and meets the y axis at *P*. **3**
Write down the three inequalities that define the region inside ΔABP.

Question 4 (12 marks) **Marks**

(a) Solve $\cos 2x° = -\frac{1}{2}$ for $0° \le x° \le 360°$ **2**

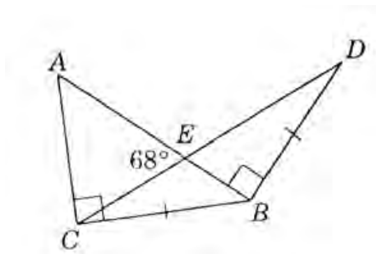
(b)



(i) Find the coordinates of P. **2**

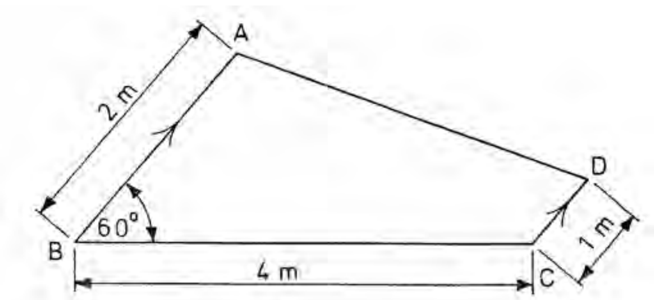
(ii) Find the area of the shaded region bounded by $y = \sqrt{x-5}$ and $y = \frac{1}{5}(x-5)$. **3**

(c) **2**



ABC is a right-angled triangle in which $\angle ACB = 90°$.
ΔCDB is isosceles, in which $CB = DB$.
$\angle AEC = 68°$ and $\angle EBD = 90°$.
Find $\angle DCB$, giving reasons.

(d)



The diagram shows a quadrilateral $ABCD$ with $\angle ABC = 60°$.
AB = 2 m, BC = 4 m and DC = 1 m and $AB \parallel DC$.

(i) Using the cosine rule, calculate AC. **1**

(ii) Hence find AD, correct to 3 significant figures **3**

SECTION C (Use a SEPARATE writing booklet)

Question 5 (12 marks) **Marks**

(a) A curve $\mathscr{C}$ has equation $y = x^3 - 5x^2 + 7x - 14$.

(i) Show $\dfrac{dy}{dx} = (3x-7)(x-1)$ **1**

(ii) Find the coordinates of the stationary points and determine their nature. **3**

(iii) Sketch the graph of $\mathscr{C}$, given that an x intercept occurs in the interval $4 \le x \le 5$. **2**

(iv) Find the values of x for which $\mathscr{C}$ is concave down. **1**

(b) A polygon has 40 sides.

The lengths of the sides, starting with the smallest, form an arithmetic series.

The perimeter of the polygon is 495 cm and the length of the longest side is twice that of the shortest side.

For this series:

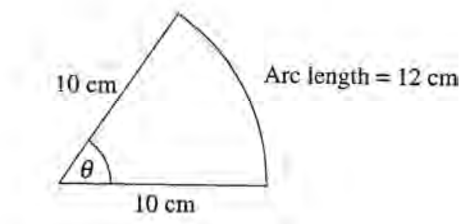
(i) Find the first term. **3**

(ii) The common difference. **2**

Question 6 (12 marks) **Marks**

(a) The diagram below shows a sector of a circle of radius 10 cm. **2**

Find the value of θ to the nearest degree.



(b) Consider the series $\cos^2 x + \cos^4 x + \cos^6 x + \cdots$ for $0 < x < \dfrac{\pi}{2}$

(i) Explain why a limiting sum exists. **1**

(ii) Find the limiting sum, expressing the answer in simplest form. **2**

(c) The rate at which people, N, are admitted to Homebake, a music festival in the Domain, is given by

$$\frac{dN}{dt} = 450t(8 - t)$$

where t is measured in hours.

(i) Find the maximum rate of people being admitted to the festival. **1**

(ii) If initially $N = 0$, find an expression for the amount of people present at time t. **2**

(iii) The festival *lasted* as long as there was a person there. How long did the festival last for? **1**

(d) For the parabola $(y-1)^2 = 16 - 8x$

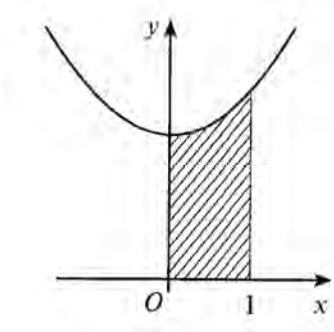
(i) State the coordinates of the vertex and the focus. **2**

(ii) Sketch the graph of the parabola showing the information above **1**

SECTION D (Use a SEPARATE writing booklet)

Question 7 (12 marks) **Marks**

(a) **3**



The diagram above shows the shaded region bounded by the curve $y = x^2 + 3$, the lines $x = 1$, $x = 0$ and the x axis. This region is rotated $360°$ about the y axis.

Find the volume generated.

(b) At time t, the mass M of a material decaying radioactively is given by $M = 5e^{-0.1t}$.

(i) If at time t_1, the mass is M_1 and at time t_2 the mass is $\frac{1}{2}M_1$, show that **2**

$$t_2 - t_1 = 10\ln 2$$

(ii) Calculate the time taken for the initial mass to reduce to a mass of $\dfrac{5}{32}$. **2**

(c)

The spinner above is used in a game. *Once spun*, it is equally likely to stop at any one of the letters **A**, **E**, **I**, **O** or **U**.

(i) If the spinner is spun twice, find the probability that it stops on the same letter twice. **2**

(ii) How many times must the spinner be spun for it to be 99% certain that it will stop on the letter **E** at least once? **3**

Question 8 (12 marks) **Marks**

(a) The velocity v (in km/min) of a train travelling from Olympack Park to Lydcome, non-stop, is given by $v = 20t^2(3-t)$, where t is the time (in minutes) during which the train has been in motion between the two stations.
Find:

(i) The acceleration of the train at the end of the second minute. **1**

(ii) Find an expression for the displacement x km of the train from Olympic Park. **2**

(iii) Hence calculate the distance travelled from Olympic Park to Lidcombe. **1**

(iv) Where and when, between the two stations, was the train travelling the fastest? **2**

(b) (i) Show $\int_0^1 \frac{dx}{1+x} = \ln 2$ **1**

(ii) By using Simpson's rule with five function values, find an approximation to $\ln 2$. **2**

(c) Yddap is given on his 18th birthday a present of $500 from his grandparents. **3**

Yddap immediately deposits this into his Credit Union account. His Credit Union gives him a return of 4% pa, compounded annually.

Each birthday from then on, Yddap decides to deposit $500 into the same account. He does this up until his 39th birthday.

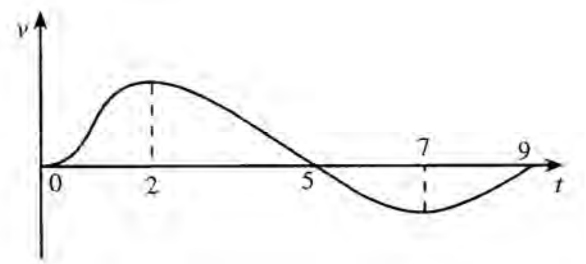
His last deposit of $500 is on his 39th birthday and when Yddap turns 40 he decides to transfer the total of this investment to another account.

How much does Yddap transfer?

SECTION E (Use a SEPARATE writing booklet)

Question 9 (12 marks) **Marks**

(a)



The above graph shows the velocity, $v\ \text{ms}^{-1}$, of a particle moving on a straight line, for $0 \le t \le 9$.

(i) State all the times, or intervals of time, for which the particle

(α) is at rest, **1**

(β) is moving in the positive direction, **1**

(γ) the acceleration is positive, **1**

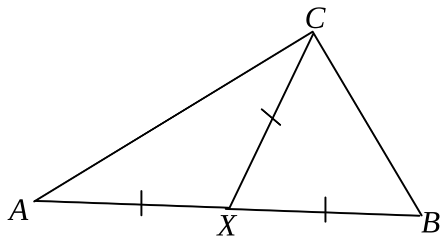
(δ) is slowing down. **1**

(ii) Using the graph, determine whether the particle has returned to its starting point when $t = 9$. Justify your answer. **2**

Question 9 continues on page 11

Question 9 continued **Marks**

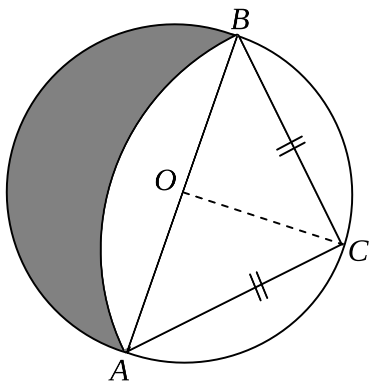
(b) (i) **2**



The diagram above shows triangle *ABC*.
X is a point on *AB* such that $AX = XB = XC$.

Prove $\angle ACB = 90°$

(ii)



AB is a diameter of the circle *ABC* whose centre is *O*.

C is equidistant from *A* and *B*.

The arc *AB* is drawn with *C* as centre.

(α) If the radius of the circle is *r*, using (i) show that $AC = r\sqrt{2}$. **1**

(β) Hence show that the shaded area is equal to the area of the triangle *ABC*. **3**

Question 10 starts on page 12

Question 10 (12 marks) **Marks**

A derrick crane is used to lift and move a heavy block across flat ground.
The crane consists of a fixed vertical mast of height *m*, a boom of fixed length *b* hinged at the base of the mast, and a hoist rope.

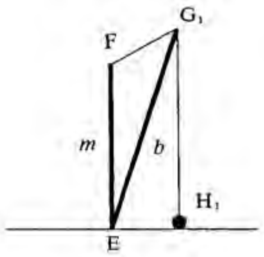


Figure 1

Figure 1 above shows the block is at H_1 on the ground. The hoist rope, anchored at *E*, passes over pulleys at *F* and G_1, then reaches vertically downwards and is attached to the block.

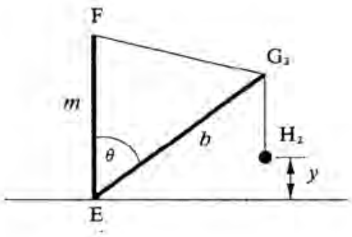


Figure 2

The length of the rope remains constant during the subsequent manoeuvre.
Figure 2 above shows that as the boom is lowered to G_2, the block moves outwards to H_2.

Let $\theta = \angle FEG_2$, $0 < \theta < \frac{\pi}{2}$ and let *y* be the height of the block above the ground.
Assume that $b < 2m$ and that the ground is level. Ignore the size of the pulleys.

(i) If *R* is the length of the rope, show that **2**

$$y = m + b\cos\theta + \sqrt{b^2 + m^2 - 2bm\cos\theta} - R$$

(ii) Show that **3**

$$\frac{dy}{d\theta} = \frac{bm\sin\theta}{\sqrt{b^2 + m^2 - 2bm\cos\theta}} - b\sin\theta\,,$$

Question 10 continues on page 13

Question 10 continued **Marks**

(iii) Show that when $\frac{dy}{d\theta} = 0$ then either $\cos\theta = \frac{b}{2m}$ or $\theta = 0$. **4**

(iv) Assume that y is a maximum when $\cos\theta = \frac{b}{2m}$. **3**

The horizontal distance from the mast to the vertical rope is called the *lifting radius* of the crane.

Find the lifting radius in terms of m and b when y is a maximum.

End of paper

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1}x^{n+1},\ n \neq -1;\ x \neq 0, \text{if } n < 0$$

$$\int \frac{1}{x} dx = \ln x,\ x > 0$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax},\ a \neq 0$$

$$\int \cos ax\, dx = \frac{1}{a}\sin ax,\ \ a \neq 0$$

$$\int \sin ax\, dx = -\frac{1}{a}\cos ax,\ \ a \neq 0$$

$$\int \sec^2 ax\, dx = \frac{1}{a}\tan ax,$$

$$\int \sec ax \tan ax\, dx = \frac{1}{a}\sec ax,\ \ a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ \ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\frac{x}{a},\ \ a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right),\ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x,\ \ x > 0$

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TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

Sample Solutions

Section	Marker
A	Mr Bigelow
B	Mr Hespe
C	Mr Choy
D	Mr Fuller
E	Mr Gainford

QUESTION 1.

(a) $\log(\tan 5/\pi) = \boxed{1 \cdot 32.}$ ✓✓

(b) $y = (5x)^{\frac{1}{2}}$

$y' = \frac{1}{2}(5x)^{-\frac{1}{2}} \times 5$

$= \boxed{\dfrac{5}{2\sqrt{5x}}}$ ✓✓ $\left(\frac{\sqrt{5}}{2} x^{-\frac{1}{2}} \right)$

(c) $2t^2 - t - 15 = 0$

$(2t+5)(t-3) = 0$

$\boxed{t = 3, -\frac{5}{2}}$ ✓✓

(d) $y' = 3 - 2x$

$\boxed{y = 3x - x^2 + c}$ ✓✓

(e) $y = 2x$ —— ①

$3x + 2y = 14$ —— ②

Sub ① into ②

$3x + 4x = 14$

$7x = 14$

$x = 2$

Sub in ①

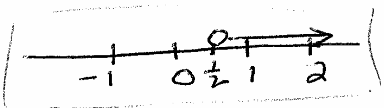
$y = 4$

$\therefore$ Soln. is $\boxed{(2,4)}$ ✓✓

(f) $3 - 4x < 1$

$-4x < -2$

$\boxed{x > \frac{1}{2}}$ ✓

 ✓

QUESTION 2.

(a)(i) $f(x) = (1+\cos 2x)^3$

$\therefore f'(x) = 3(1+\cos 2x)^2 \times -2\sin 2x.$

$= \boxed{-6\sin 2x(1+\cos 2x)^2.}$ ✓✓

(ii) $f(x) = x^2 e^{x+2}$

$f'(x) = x^2 e^{x+2} + 2x e^{x+2}$

$= \boxed{x e^{x+2}(x+2)}$ ✓✓

(b)(i) $\int \frac{\cos x}{\sin x}\, dx = \boxed{\ln(\sin x) + c}$ ✓

(ii) $\int_{\frac{1}{2}}^{2} \left(1 - \frac{1}{x^2}\right) dx = \int_{\frac{1}{2}}^{2} (1 - x^{-2})\, dx$

$= \left[x + x^{-1}\right]_{\frac{1}{2}}^{2}$

$= (2 + \tfrac{1}{2}) - (\tfrac{1}{2} + 2).$

$\boxed{= 0.}$ ✓✓

(c)(i) $N\hat{B}Y = M\hat{X}C$ (corresponding angles).

$\hat{Y}$ is common

$\therefore \triangle NBY \,|||\, \triangle MXY$ (equiangular). ✓✓

$\therefore \boxed{\frac{MX}{NB} = \frac{1}{2}}$ (ratio of corresponding sides equal)

$\left[\frac{NB}{}\ \frac{MY}{NY} = \frac{1}{2}\right.$ (M is the midpt of NY)$\left.\right]$

(ii) $M\hat{C}X = A\hat{B}C$ (base angles of an isosceles triangle)

$M\hat{X}C = A\hat{B}C$ (corresponding angles). ✓

$\therefore M\hat{C}X = M\hat{X}C$

$\therefore \triangle XMC$ is isosceles $\therefore MX = MC$ $\therefore \boxed{\frac{MX}{NB} = \frac{MC}{NB} = \frac{1}{2}.}$

(d) $g'(x) = 4 - 3x^2$

$\therefore g(x) = 4x - x^3 + c.$

now $4 = 4\times 2 - 8 + c.$

$\therefore c = 4$

$\therefore \boxed{g(x) = 4x - x^3 + 4}$ ✓✓

3(i) $m_{AD} = \dfrac{-3-5}{0-^-5}$

$= -8/5$ ✓

$\theta = \tan^{-1}(-8/5)$

$= 180° - 58°$

$= 122°$ ✓

(ii) $M = \left(\dfrac{-5+3}{2}, \dfrac{1+5}{2}\right)$

$= (-1, 3)$ ✓

(iii) $C = (3+(-5-0), 1+(5-^-3))$

$= (-2, 9)$ ✓

(iv) $m_{AB} = \dfrac{1-^-3}{3-0}$

$= 4/3$ ✓

$\therefore$ Eqn of AB is $y = \dfrac{4x}{3} - 3$ ✓

or $4x - 3y - 9 = 0.$

(v) Distance $= \dfrac{|4\times(-5) - 3\times 5 - 9|}{\sqrt{4^2+3^2}}$

$= \dfrac{44}{5}$ ✓ (or 8.8).

3 (vi) Length $AB = \sqrt{9+16}$

$= 5$ ✓

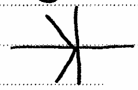
$\therefore$ Area $= {}^{44}/_{5} \times 5$

$= 44$ ✓

(vii) $x + 2y < 5$ ✓ $\cap$ $x > 0$ ✓ $\cap$ $4x - 3y < 9$ ✓

4(a) $2x° = 120°, 240°$ ✓$, 480°, 600°$ ✓

$x° = 60°, 120°, 240°, 300°$ ✓



(b)(i) $\sqrt{x-5} = \dfrac{x-5}{5}$

$25x - 125 = x^2 - 10x + 25$

$x^2 - 35x + 150 = 0$

$(x-5)(x-30) = 0$ ✓

$x = 5, 30$

So P is $(30, 5)$ ✓

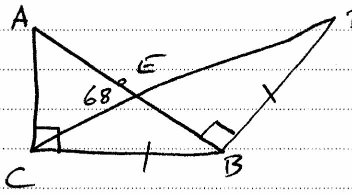
(ii) Area $= \int_5^{30} \left\{ (x-5)^{1/2} - \left(\dfrac{x-5}{5}\right) \right\} dx$ ✓

$= \left[\dfrac{2}{3}(x-5)^{3/2} - \dfrac{x^2}{10} + x \right]_5^{30}$ ✓

$= \frac{2}{3}(125 - 0) - \left(\frac{900}{10} - \frac{25}{10}\right) + 30 - 5$

$= \dfrac{125}{6}$ ✓ or $20\,{}^5/_6$ or $20.8\dot{3}$

(c)

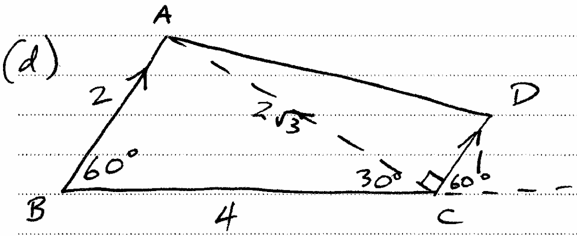


$B\hat{E}D = 68°$ (vert. opp. $\angle$s) ✓

$B\hat{D}E = 22°$ (angle sum of Δ)

$D\hat{C}B = 22°$ ✓ (base angle of isosceles Δ).

(d)



(i) $AC^2 = 4^2 + 2^2 - 2 \times 4 \times 2 \times \cos 60°$ m^2

$= 16 + 4 - 16 \times 1/2$ m^2

$= 12$ m^2

$AC = 2\sqrt{3}$ m. ✓ (3.464101615 etc).

4(d)(ii) $\dfrac{\sin B\hat{C}A}{2} = \dfrac{\sin 60^\circ}{2\sqrt{3}}$

$\sin B\hat{C}A = 1/2$

$B\hat{C}A = 30^\circ$ ✓

$A\hat{C}D = 180^\circ - 60^\circ - 30^\circ$

$= 90^\circ$ ✓

$\therefore AD^2 = 12 + 1$

$\therefore AD = \sqrt{13}$ ✓ (3.605551275 calculator)

$= 3.61$ m (3 sig. fig.)

5

$y = x^3 - 5x^2 + 7x - 14$

(a) (i) $\frac{dy}{dx} = 3x^2 - 10x + 7$ [1]

$= (3x - 7)(x - 1)$

1

(ii) $\frac{dy}{dx} = 0$, $x = \frac{7}{3}$, 1

When $x = 1$, $y = -11$ [1]

$x = \frac{7}{3}$, $y = -12\frac{5}{27}$ (-12.185)

[1]

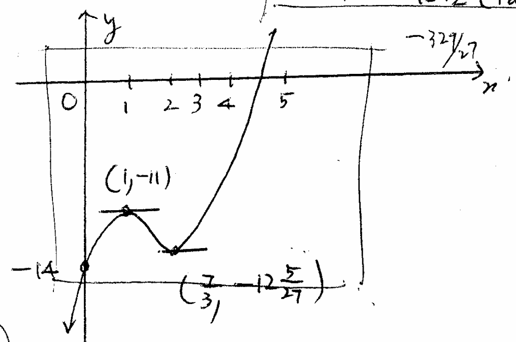
3

$\frac{d^2y}{dx^2} = 6x - 10$

$f''(1) = -4 < 0$ ∴ $(1, -11)$ max [1]

$f''(\frac{7}{3}) = 4 > 0$ ∴ $(\frac{7}{3}, -12\frac{5}{27})$ min
-12.2 (1dp)

(iii)



2

(iv) 1

$f''(x) < 0$, $6x - 10 < 0$, $6x < 10$
$x < 5/3$.

(b) a, $a + d$, $a + 2d$, ... $a + 39d$

$S_{40} = \frac{40}{2}[2a + 39d]$

∴ $495 = 20(2a + 39d)$

$99 = 4(2a + 39d)$ —— (1)

$a + 39d = 2a$ —— (2)

$a = 39d$ ∴ $d = \frac{a}{39}$ —— (3)

Subst (3) into (1)

$99 = 4(2a + 39 \times \frac{a}{39})$

∴ $99 = 4(3a) \Longrightarrow a = \frac{33}{4}$ (8.25)

3

$\Longrightarrow d = \frac{11}{52}$ cm (0.212 cm)

2

$\frac{495}{2340}$

Q (6). (a) $l = 10\theta$ $\theta = 1.2^c$

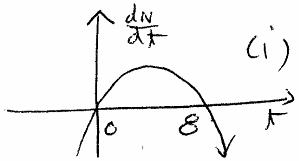
$\therefore 12 = 10\theta$

$1^c = \frac{180}{\pi}$ $\therefore 1.2^c = \boxed{\frac{180 \times 1.2}{\pi} \doteqdot 69^\circ}$ 2

(b) (i) $\boxed{r = \cos^2 x \ (<1).}$ 1

(ii) $S = \frac{\cos^2 x}{1 - \cos^2 x} = \frac{\cos^2 x}{\sin^2 x} = \boxed{\cot^2 x.}$ 2

(c). $\frac{dN}{dt} = 450t(8-t).$



(i) $\frac{dN}{dt}$ (max) $= 450 \times 4 \times 4$ $= 7200$ 1, When $t=4$

$\frac{dN}{dt} = 3600t - 450t^2$

$N = 1800t^2 - 150t^3 + c$ ✓

When $t=0$, $N=0$, $\Rightarrow c=0$

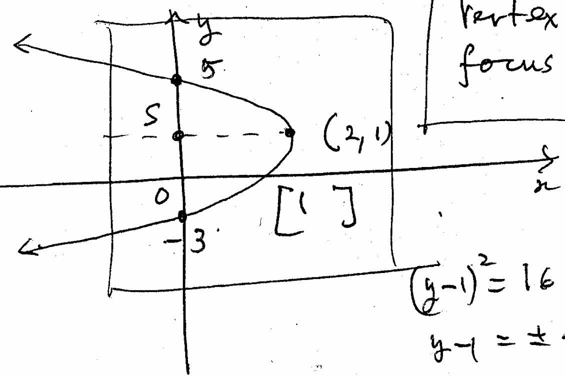
(ii) $\therefore$ $\boxed{N = 150t^2(12-t).}$ 2

(iii) $N = 0$, When $t=0$, or 12. 1

$\therefore$ festival last for 12 hrs.

(d) $(y-1)^2 = -8(x-2)$

$4a = 8$, $a = 2$.



Vertex (2,1). [1]
focus (0,1) [1]

$(y-1)^2 = 16$

$y-1 = \pm 4$

3

SECTION D

Question 7

(a) when $x=0$ $x=1$

$y=3$ $y=4$

$$V = \pi r^2 h - \pi \int_3^4 x^2 dy$$

$$= \pi(1)^2(4) - \pi \int_3^4 (y-3)dy$$

$$= 4\pi - \pi\left[\tfrac{1}{2}y^2 - 3y\right]_3^4$$

$$= 4\pi - \pi\left\{\left[\tfrac{1}{2}(4)^2 - 3(4)\right] - \left[\tfrac{1}{2}(3)^2 - 3(3)\right]\right\}$$

$$= 4\pi - \frac{\pi}{2}$$

$$= \frac{7\pi}{2}\text{units}^3$$

(b)(i) when $t = t_1,\ M = M_1$, ie $M_1 = 5e^{-0\cdot 1t_1}$

when $t = t_2,\ M = \frac{1}{2}M_1$, ie $\frac{1}{2}M_1 = 5e^{-0\cdot 1t_2}$

$M_1 = 10e^{-0\cdot 1t_2}$

$$5e^{-0\cdot 1t_1} = 10e^{-0\cdot 1t_2}$$
$$e^{-0\cdot 1t_1} = 2e^{-0\cdot 1t_2}$$
$$-0\cdot 1t_1 = \ln\left(2e^{-0\cdot 1t_2}\right)$$
$$-0\cdot 1t_1 = \ln(2) + \ln\left(e^{-0\cdot 1t_2}\right)$$
$$-0\cdot 1t_1 = \ln(2) + -0\cdot 1t_2$$
$$t_1 = -10\ln(2) + t_2$$
$$t_2 - t_1 = 10\ln(2)$$

(ii) $\dfrac{5}{32} = 5e^{-0\cdot 1t}$

$$e^{-0\cdot 1t} = \frac{1}{32}$$
$$-0\cdot 1t = \ln\left(\tfrac{1}{32}\right)$$
$$t = -10\ln\left(\tfrac{1}{32}\right)$$

$t = 34\cdot 657$ correct to 3 decimal places

(c)(i) P(same letter twice)= $1 \times \dfrac{1}{5}$

(ii) P(E at least once) = 1 − P(no E)

$$= 1 - \left(\tfrac{4}{5}\right)^n$$

P(E at least once) $\geq \dfrac{99}{100}$

$$1 - \left(\frac{4}{5}\right)^n \geq \frac{99}{100}$$
$$\left(\frac{4}{5}\right)^n \leq \frac{1}{100}$$
$$n\ln\left(\frac{4}{5}\right) \leq \ln\left(\frac{1}{100}\right)$$
$$n \geq \frac{\ln\left(\frac{1}{100}\right)}{\ln\left(\frac{4}{5}\right)}$$
$$n \geq 20\cdot 6377....$$
$$\therefore n = 21$$

Question 8

(a) $v = 20t^2(3-t)$ $v = 60t^2 - 20t^3$

(i) $a = \dfrac{dv}{dt}$

$$a = \frac{d\left(60t^2 - 20t^3\right)}{dt}$$
$$a = 120t - 60t^2$$

when $a = 2$, $a = 120(2) - 60(2)^2$

$a = 0$

(ii) $x = \displaystyle\int \frac{dx}{dt}dt$

$$x = 20t^3 - 5t^4 + C$$

Note: $t = 0, x = 0 \quad \therefore C = 0$

Hence, $x = 20t^3 - 5t^4$

(iii) Let $v = 0$.

$$0 = 20t^2(3-t)$$

$t = 0$ or $t = 3$

when $t = 3$, $\quad x = 20(3)^3 - 5(3)^4$

$x = 135$ km

Therefore the distance travelled from station to station is 135km.

(iv) train is travelling the fastest when $a = 0$

this occurs when $t = 2$ (min) as found in (i).

when $t = 2$, $x = 20(2)^3 - 5(2)^4$

$x = 80$ km from Olympic Park.

(b)(i) $\int_0^1 \frac{dx}{1+x} = [\ln(1+x)]_0^1$

$= \ln(1+(1)) - \ln(1+(0))$

$= \ln(2) - \ln(1)$

$= \ln(2)$

(ii)

$$\int_a^b f(x)dx \approx \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$

5 Function values means Simpsons Rule using 2 applications

$$\int_0^1 \frac{dx}{1+x} = \int_0^{0\cdot5} \frac{dx}{1+x} + \int_{0\cdot5}^1 \frac{dx}{1+x}$$

$$\approx \frac{\frac{1}{2}-0}{6}\left[1 + 4\left(\tfrac{4}{5}\right) + \tfrac{2}{3}\right] + \frac{1-\frac{1}{2}}{6}\left[\tfrac{2}{3} + 4\left(\tfrac{4}{7}\right) + \tfrac{1}{2}\right]$$

$$= \frac{1747}{2520} \text{ or } 0\cdot693$$

(c)

On 18th B'day deposits \$500,

matures to $500(1\cdot04)^{22}$

On 19th B'day deposits \$500,

matures to $500(1\cdot04)^{21}$

On 20th B'day deposits \$500,

matures to $500(1\cdot04)^{20}$

↓

↓

On 39th B'day deposits \$500,

matures to $500(1\cdot04)$

Yddap transferrs

$$500(1\cdot04)^1 + 500(1\cdot04)^2 + \ldots + 500(1\cdot04)^{22}$$

$a = 500(1\cdot04)$

$r = 1\cdot04$

$n = 22$

$$S_n = \frac{a(r^n - 1)}{r-1}$$

$$S_{22} = \frac{500 \times 1\cdot04(1\cdot04^{22} - 1)}{1\cdot04 - 1}$$

$$S_{22} = \$17808\cdot94$$

SECTION E

Question 9

(a) (i) (α) At rest $t=0$

$t=5$

$t=9$ [1]

(β) $0<t<5$ [1]

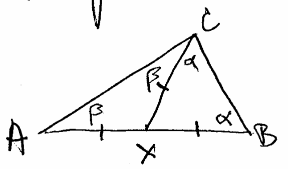
(γ) $0<t<2$, $7<t<9$ [1]

* (δ) $2<t<5$, $7<t<9$ [1]

(ii) No, it has not returned.

The area under the curve is the measure of distance travelled. The negative area ($[5,9]$) is less than the positive area ($[0,5]$). [2]

(b)



Aim To prove $\angle ACB = 90°$

Proof Let $\angle ABC = \alpha$

$\angle BAC = \beta$

Now $\angle BCX = \alpha$ (isos Δ)

and $\angle ACX = \beta$ (")

But $\angle BAC + \angle ACB + \angle CBA = 180°$ (angle sum)

$\therefore\ \beta + (\beta+\alpha) + \alpha = 180°$

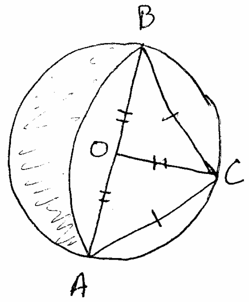
$2(\alpha+\beta) = 180°$

$\therefore\ \alpha+\beta = 90°$

$\therefore\ \angle ACB = 90°$ [2]

QED

(iii)



(α) In ΔABC, $AO=BO=CO$ (radii)

$\therefore\ \angle ACB = 90°$

$\therefore$ Since $AO = r$, in ΔABC, using Pythagoras

$AB^2 = 2AC^2$

$(2r)^2 = 2AC^2$

$4r^2 = 2AC^2$

$AC^2 = 2r^2$

$\therefore\ AC = r\sqrt{2}$ [1]

(β) $\Delta_{ABC} = \frac{1}{2}(r\sqrt{2})^2$

$= r^2$

$A_{shaded} = \frac{1}{2}\pi r^2 - A_{segment}$

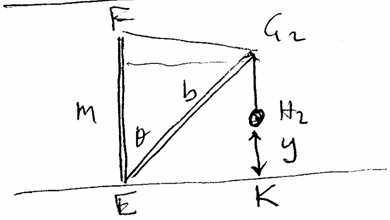
$= \frac{1}{2}\pi r^2 - \frac{1}{2}(r\sqrt{2})^2\left(\frac{\pi}{2} - \sin\frac{\pi}{2}\right)$

$= \frac{1}{2}\pi r^2 - r^2\left(\frac{\pi}{2} - 1\right)$

$= r^2$ [3]

$= \Delta_{ABC}$ QED

Question 10



(i) $R = FG_2 + G_2K - y$

$\therefore y = FG_2 + G_2K - R$

$= \sqrt{b^2+m^2-2bm\cos\theta} + b\cos\theta - R$

as required. [2]

(ii) $\dfrac{dy}{d\theta} = -b\sin\theta + \dfrac{1 \times \cancel{2}bm\sin\theta}{\cancel{2}\sqrt{b^2+m^2-2bm\cos\theta}}$

$= \dfrac{bm\sin\theta}{\sqrt{b^2+m^2-2bm\cos\theta}} - b\sin\theta$ [3]

(iii) When $\dfrac{dy}{d\theta} = 0$

$0 = \dfrac{bm\sin\theta - b\sin\theta\sqrt{b^2+m^2-2bm\cos\theta}}{\sqrt{b^2+m^2-2bm\cos\theta}}$

ie $0 = b\sin\theta\left(m - \sqrt{b^2+m^2-2bm\cos\theta}\right)$

$\therefore$ Either $\sin\theta = 0$ ie $\theta = 0$

or $m - \sqrt{b^2+m^2-2bm\cos\theta} = 0$

ie $m = \sqrt{b^2+m^2-2bm\cos\theta}$

That is $b^2 = 2bm\cos\theta$

ie $\cos\theta = \dfrac{b}{2m}$ [4]

(iv) [Clearly y is min when $\theta = 0$

$\therefore$ y is max when $\cos\theta = \dfrac{b}{2m}$]

Now lifting radius is $b\sin\theta$

$= b\sqrt{1-\cos^2\theta}$

$= b\sqrt{1-\dfrac{b^2}{4m^2}}$

$= \dfrac{b}{2m}\sqrt{4m^2-b^2}$

[3]