Student Number:

Sydney Girls High School
2025
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using blue or black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total Marks: 100

Section I – 10 marks (pages 2–7)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 8–41)

- Attempt Questions 11–39
- Allow about 2 hours and 45 minutes for this section

THIS IS A TRIAL PAPER ONLY
It does not necessarily reflect the format or the content of the 2025 HSC Examination Paper in this subject.

Questions	M.C.	Q11-23	Q24-27	Q28-32	Q33-36	Q37-39	Total
Total	/10	/30	/14	/16	/15	/15	/100

Section I

10 marks

Attempt Questions 1-10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for questions 1 – 10

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1 Which one of the following relations is a *many-to-one* relation?

A) $y = 9x + 2$

B) $x^2 + y^2 = 1$

C) $y = \sqrt{4 - x^2}$

D) $x = 4y^2$

2 If A and B are two independent events such that $P(A) = 0.5$ and $P(B) = 0.4$, what is is the value of $P(A \cup B)$?

A) 0.6

B) 0.7

C) 0.8

D) 0.9

3 Given that $f(x) = |x - 3| + 1$ for all real x.

Which one of the following is true about the function $f(x)$?

A) $f(x)$ is differentiable everywhere.

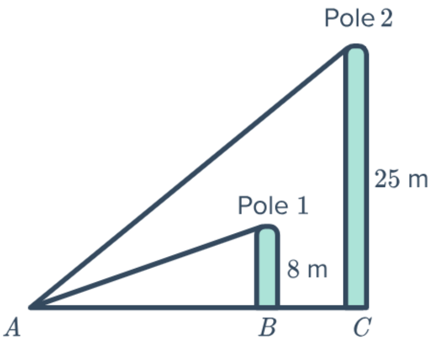
B) $f(x)$ is differentiable at $x = 3$ and $x = 4$.

C) $f(x)$ is differentiable at $x = 3$, but not at $x = 4$.

D) $f(x)$ is differentiable at $x = 4$, but not at $x = 3$.

4 A man stands at point A looking at the top of two poles. Pole 1 has a height 8 metres and an angle of elevation of 37° from point A. Pole 2 has a height 25 metres and an angle of elevation of 61° from point A.

Find the expression for distance BC.



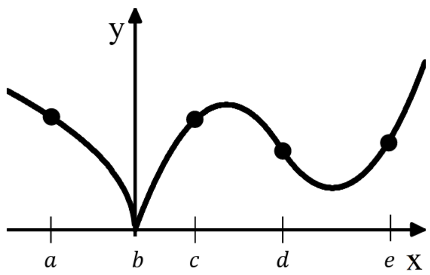
A) $8\tan 37^\circ - 25\tan 61^\circ$

B) $25\tan 61^\circ - 8\tan 37^\circ$

C) $8\cot 37^\circ - 25\cot 61^\circ$

D) $25\cot 61^\circ - 8\cot 37^\circ$

5 The graph of the function $f(x)$ is shown in the diagram below. For which of the following values of x is $f'(x)$ positive and increasing.



A) a

B) c

C) d

D) e

6 Find $\lim_{x \to 0} \frac{4x - \tan 2x}{\sin 5x}$

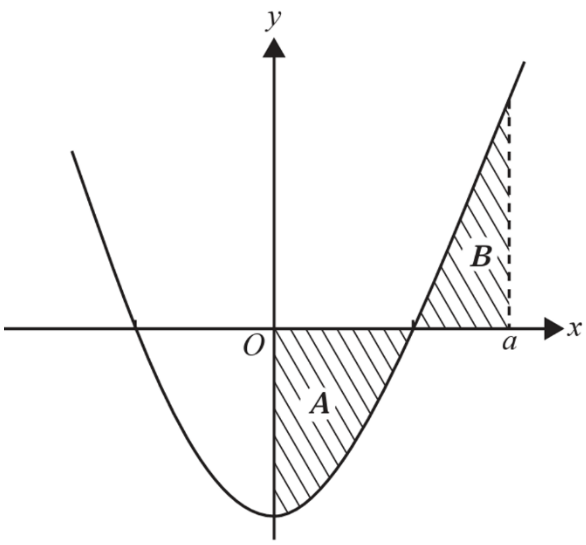
A) $\frac{4}{5}$

B) $\frac{2}{5}$

C) $\frac{6}{5}$

D) 0

7 A part of the graph of $h(x) = 3x^2 - 6$ is shown below. Given that $a > 0$.



The area of the region marked $\boldsymbol{A}$ is the same as the area of the region marked $\boldsymbol{B}$.

The exact value of a is

A) $3\sqrt{3}$

B) 6

C) $\sqrt{6}$

D) 3

8 Let $h(x) = f\left(g(x)\right)$, with $f(-1) = 3$, $f'(-1) = 5$, $g(4) = -1$ and $g'(4) = 3$.

What is the equation of the tangent to the graph of $y = h(x)$ at $x = 4$?

A) $y = 5x + 8$

B) $y = 15x - 57$

C) $y = 5x - 17$

D) $y = 15x - 63$

9 The expression $\dfrac{1}{\log_a(b)} + \dfrac{1}{\log_b(c)} + \dfrac{1}{\log_c(a)}$ is equal to

A) $\log_a(b) + \log_c(b) + \log_a(c)$

B) $\log_b(c) + \log_c(a) + \log_a(b)$

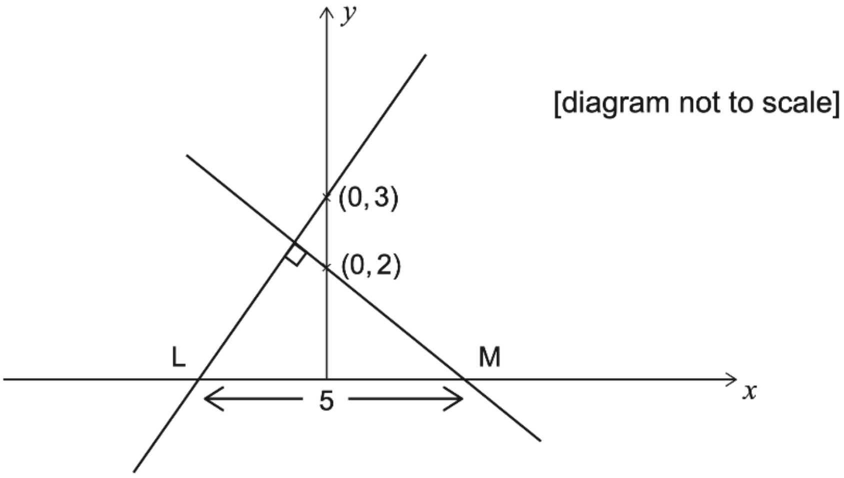
C) $-\log_a(b) - \log_b(c) - \log_c(a)$

D) $\log_b(a) + \log_c(b) + \log_a(c)$

Multiple Chioce Question Continues Overleaf...

10 The straight line with equation $y = ax + 3$, where $a > 1$, is perpendicular to the line with equation $y = bx + 2$. The lines cut the x-axis at the points L and M respectively. The length of LM is 5 units.

What is the value of $a + b$, given that $a > 1$?



A) $-\frac{13}{6}$

B) $\frac{13}{6}$

C) $-\frac{5}{6}$

D) $\frac{5}{6}$

Mathematics Advanced

Section II Answer Booklet 1

Section II

90 marks

Attempt Questions 11 – 40

Allow about 2 hours and 45 minutes for this section

Booklet 1 – Attempt Question 11 – 23 (30 marks)

Booklet 2 – Attempt Question 24 – 39 (60 marks)

Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Please clearly mark the questions written on the papers provided at the end of the paper.

Question 11 (1 mark)

Find $\lim_{x \to 4} \dfrac{x^2 - 7x + 12}{x - 4}$. **1**

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Question 12 (2 marks)

Differentiate with respect to x: $y = \dfrac{e^{2x}}{x+5}$ **2**

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Question 13 (2 marks)

A bag contains red and purple marbles, with at least three of each colour. Three marbles are drawn from the bag, without replacement. The number of purple marbles drawn is given by the random variable X.

The following tables shows a probability distribution for the random variable X.

x	0	1	2	3
$P(X = x)$	a	$\frac{1}{10}$	$\frac{1}{3}$	$\frac{3}{10}$

(i) Find the probability of drawing three red marbles. **1**

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(ii) Find the expected value of X. **1**

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Question 14 (3 marks)

Solve: $|3x-2| \geq 5$ **3**

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Question 15 (2 marks)

Solve for $x : \log(3x+6) = \log 3 + 1$ **2**

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Question 16 (3 marks)

The points $P(1,-2)$ and $Q(7,1)$ lie on the circumference of a circle.
Show that the centre of the circle lies on the line $4x + 2y = 15$. **3**

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Question 17 (3 marks)

Consider the arithmetic progression $-7, 2, 11, 20, \ldots, 713$.

(i) Find the number of terms. **2**

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(ii) Hence, find the sum of terms. **1**

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Question 18 (1 mark)

Evaluate: $\int 2\tan^2 x \, dx$ **1**

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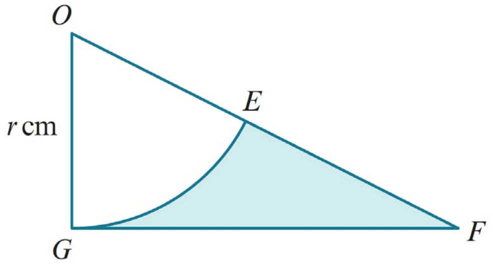
Question 19 (3 marks)

The diagram shows a sector, EOG, of a circle, with centre O and radius r cm. The line GF is a tangent to the circle at G. Point E is the midpoint of OF.



Find the expression for the exact area of the shaded region. **3**

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Question 20 (2 marks)

Find an approximation for $\int_2^6 \frac{1}{\sqrt{3x-5}}\,dx$ correct to 2 decimal places, using 3 applications of the trapezoidal rule. **2**

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Question 21 (2 marks)

Consider functions $f(x) = x + 1$ and $g(x) = \frac{1}{\sqrt{3x-5}}$.

(i) Find the rule for the composite function $g\left(f(x)\right)$. **1**

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(ii) Find the domain of $g\left(f(x)\right)$. **1**

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Question 22 (3 marks)

The time from Amber getting out of bed until her arrival at her office is normally distributed with a mean of 43 minutes and a standard deviation of 7 minutes. Amber's arrival at her office is classified as being late if it occurs after 9:20 am.

(i) If Amber gets out of bed at 8:16 am, find the z-score for the number of minutes she has to get to the office. **1**

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(ii) If Amber gets out bed at 8:16 am, find the probability that she will arrive late using the Empirical Rule. **2**

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Question 23 (3 marks)

Given that $f(x) = -2x^3 + k^3x^2$ where k is a positive constant. Find the complete set of values of x for which the graph of $f(x)$ is concave up, using interval notation. **3**

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EXAMINATION

Mathematics Advanced

Section II Answer Booklet 2

Attempt Questions 24 – 39 (60 marks)

Instructions

- Write your Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Please clearly mark the questions written on the papers provided at the end of the paper.

Question 24 (4 marks)

A ship departs from point A and sails 12 kilometres on a bearing of 156° to reach point B. It then changes direction and sails 16 kilometres on a bearing of 123° to reach point C.

(i) Find the distance AC, correct to the nearest kilometre. **2**

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(ii) Find the true bearing that the ship should follow to return directly to point A from point C. Give your answer correct to the nearest degree. **2**

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Question 25 (3 marks)

Evaluate $\int_1^{\sqrt{3}} \left(\frac{x^3}{6} + \frac{1}{3x^2} \right) dx$, giving your answer in the form $a + b\sqrt{3}$,

where a and b are constants to be determined. **3**

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Question 26 (3 marks)

Solve the equation $3\sin^2 x + 1 = 4\cos x$ for $x \in [-\pi, \pi]$.

Give your answer correct to three decimal places. **3**

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Question 27 (4 marks)

The table below shows the future values of an annuity of $1 for different rates of interest and different numbers of compounding periods. The contribution is made at the end of each time period.

Future Value Interest Factors

Time Period	*Interest Rate*				
	1%	2%	3%	4%	5%
1	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500
3	3.0301	3.0604	3.0909	3.1216	3.1525
4	4.0604	4.1216	4.1836	4.2465	4.3101
5	5.1010	5.2040	5.3091	5.4163	5.5256
6	6.1520	6.3081	6.4684	6.6330	6.8019
7	7.2135	7.4343	7.6625	7.8983	8.1420
8	8.2857	8.5830	8.8923	9.2142	9.5491

(i) Kelly opens an annuity account and makes contributions of $3500 at the end of each quarter over a period of 18 months at an interest rate of 8% per annum, compounding quarterly. Find the total amount in Kelly's account after 18 months. **1**

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(ii) Julie deposits $10 000 into a saving account at the end of each year for 9 years.

For the first 4 years, the interest rate is 3% per annum, compounding annually. From the 5th year, the interest rate increases to 5% per annum, compounding annually.

Find the amount of money in Julie's account at the end of nine years. **3**

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Question 28 (3 marks) (The table on page 25 can be used for this question)

The mass of a certain species of fish caught at sea is normally distributed with mean 5.73 kg and variance 2.56 kg^2. Find the probability that a randomly selected fish caught at sea has a mass that is:

(i) less than 5.9 kg **1**

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(ii) between 5.5 and 8.5 kg **2**

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Table: The standard normal distribution

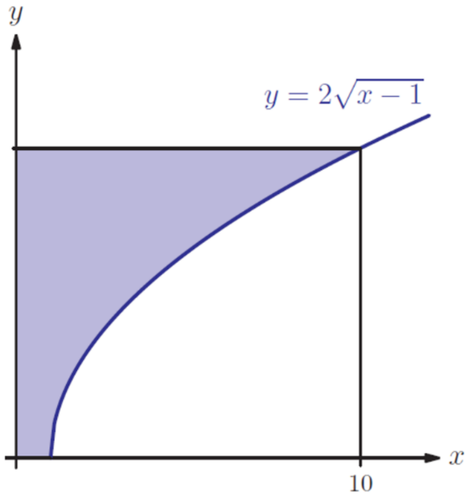
The table below provides some values of the probabilities for the standard distribution.

$$\text{i.e. } \Phi(z) = P(Z \le z) = \int_{-\infty}^{z} \phi(t)dt$$

z	+0.00	+0.01	+0.02	+0.03	+0.04	+0.05	+0.06	+0.07	+0.08	+0.09
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56360	0.56749	0.57142	0.57535
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91308	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900
3.1	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929
3.2	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950
3.3	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965
3.4	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976
3.5	0.99977	0.99978	0.99978	0.99979	0.99980	0.99981	0.99981	0.99982	0.99983	0.99983

Question 29 (3 marks)

The curve $y = 2\sqrt{(x-1)}$ is shown below. Find the shaded area. **3**



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Question 30 (2 marks)

Consider the geometric series $7 + 14e^{-2} + 28e^{-4} + 56e^{-6} + \cdots$

Justify whether this series has a limiting sum or not. **2**

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Question 31 (4 marks)

The continuous random variable X has probability density function $f(x)$, where

$$f(x) = \begin{cases} k(4x^2 - x^3), & 0 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$

(i) Show that $k = \dfrac{3}{20}$. **2**

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(ii) Find $P\left(\dfrac{1}{2} < X < 1\right)$. Give your answer correct to 3 significant figures. **2**

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Question 32 (4 marks)

Eight students in the Standard Mathematics class sat end-of-term tests in *Functions* and *Calculus*. The table below shows their marks out of 100 in each test.

Students	A	B	C	D	E	F	G	H
Functions (x)	63	59	69	83	77	75	85	
Calculus (y)	58	62	70	72	76	76	78	84

Unfortunately, the *Functions* mark for student H has been lost. However, it is known that the correlation coefficient between *Functions* marks and *Calculus* marks is $r = 0.9$ and the equation of the least squares regression line of *Calculus* marks on *Functions* marks is $y = 0.72x + 18$.

(i) Use the mean $\bar{y} = 72$ of the *Calculus* marks and the least squares regression line to find the mean $\bar{x}$ of the *Functions* marks, and hence the missing *Functions* mark for student H.

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(ii) Another student in the same class sat the *Functions* test but misses the *Calculus* test. Explain whether or not you would be justified statistically in using their *Functions* mark and the given least squares regression line to estimate their *Calculus* mark.

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Question 33 (4 marks)

Given that $f(x) = 5x\cos(5x)$,

(i) Find $f'(x)$. **1**

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(ii) Hence find the antiderivative of $x\sin(5x)$. **3**

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Question 34 (3 marks)

Each morning Emily drinks coffee or tea. If she drinks coffee on one morning, the probability she drinks tea the next morning is 0.6, and if she drinks tea on one morning, the probability she drinks coffee the next morning is 0.3.

Suppose she drinks tea on one Sunday morning.

(i) Find the probability that she drinks coffee on each of the next three mornings after Sunday. **1**

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(ii) Find the probability that she drinks coffee on at least two of the next three mornings after Sunday. **2**

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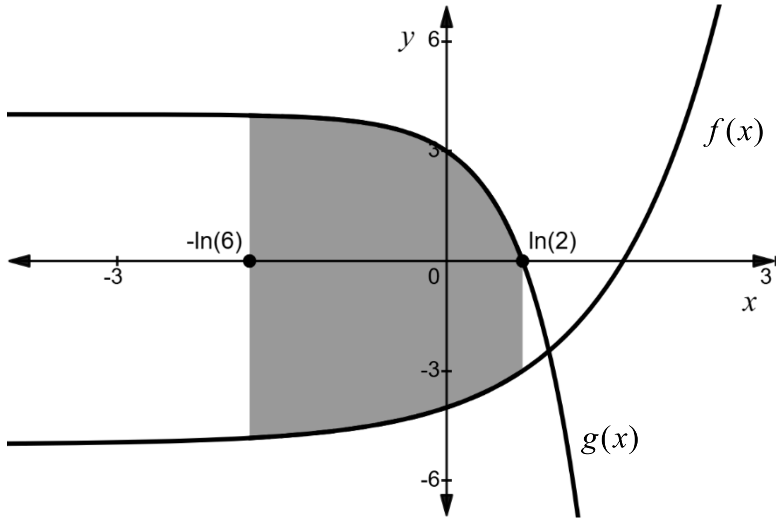
Question 35 (3 marks)

The diagram shows graphs of the functions $f(x) = e^x - 5$ and $g(x) = -e^{2x} + 4$.



Find the exact area enclosed by curves, the line $x = -\ln(6)$ and the line $x = \ln(2)$. **3**

Question 36 (5 marks)

A particle moves along a line with velocity

$$v(t) = 6\cos\left(2t + \frac{\pi}{3}\right),$$

where time t is measured in seconds.

Given that the displacement at $t = 0$ is 2 metres. Find:

(i) the displacement function $s(t)$. **2**

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(ii) the acceleration function $a(t)$. **1**

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Question 36 continues overleaf …

Question 36 Continued (5 marks)

(iii) the time at which the particle changes direction within the interval $0 < t < 1$. **2**

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Question 37 (5 marks)

For the function $y = (4x-1)e^{-\frac{x}{2}}$:

(i) Find any stationary points and determine their nature. **3**

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(ii) Using part i), sketch the graph of $y = f(x)$, showing all key features. **2**

Question 38 (5 marks)

A lake contains an initial population of 500 000 fish at the beginning of 2025. Each year, the fish population increases by 7% due to natural reproduction. After this increase, a fishing company catches fish from the lake. In 2025, 2000 fish are caught. In each subsequent year, the number of fish caught increases by 9% compared to the previous year.

Let F_n be the number of fish remaining in the lake after the nth year's catch.

(i) Show that $F_3 = 500000 \times 1.07^3 - 2000 \times (1.07^2 + 1.07 \times 1.09 + 1.09^2)$ **2**

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(ii) If the fishing was allowed to continue at this rate, find the year which there will be no fish in this lake. **3**

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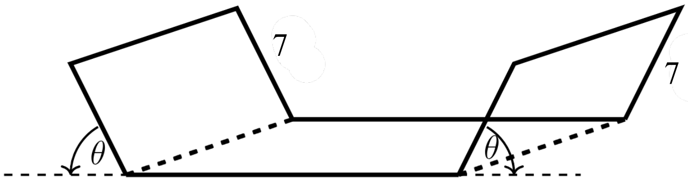
Question 39 (5 marks)

A water trough is to be made from a 21m long strip of tin by bending up by the angle θ with a 7m strip at each side, as shown in the diagram. Find the size of angle θ that would maximize the cross-sectional area, and thus the volume, of the trough.



(i) Show that the cross-sectional area is given by $A = 49(\sin\theta + \sin\theta\cos\theta)$. **2**

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Question 39 Continues Overleaf…

Question 39 (5 marks)

(ii) Hence, find the size of angle θ that would maximise the cross-sectional area of the trough. **3**

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The End

Sydney Girls High School

Mathematics Faculty

Multiple Choice Answer Sheet
Trial HSC Mathematics Advanced

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample **$2 + 4 = ?$** **(A) 2 (B) 6 (C) 8 (D) 9**

A ○ **B** ● **C** ○ **D** ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● **B** ~~●~~ **C** ○ **D** ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:

correct

A ~~●~~ **B** ~~●~~ **C** ○ **D** ○

Completely fill the response oval representing the most correct answer.

1. A ○ B ○ C ● D ○
2. A ○ B ● C ○ D ○
3. A ○ B ○ C ○ D ●
4. A ○ B ○ C ○ D ●
5. A ○ B ○ C ○ D ●
6. A ○ B ● C ○ D ○
7. A ○ B ○ C ● D ○
8. A ○ B ● C ○ D ○
9. A ○ B ○ C ○ D ●
10. A ○ B ○ C ○ D ●

Question 1 – C

Worked solutions:

A is one-to-one; B is many-to-many; C is many-to-one; D is one-to-many.

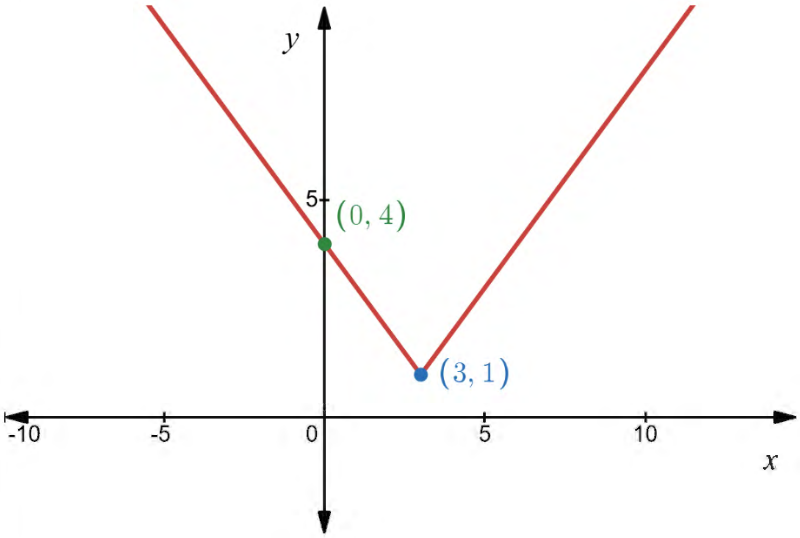
Question 2 – B

Worked solutions:

$$\begin{aligned} P(A\cup B) &= P(A)+P(B)-P(A\cap B) \\ &= 0.5+0.4-P(A)\times P(B) \\ &= 0.9-0.5\times 0.4 \\ &= 0.9-0.2 \\ &= 0.7 \end{aligned}$$

Question 3 - D

Worked solutions:



Question 4 - D

Worked solutions:

$$\tan 61^\circ = \frac{25}{AC} \qquad \tan 37^\circ = \frac{8}{AB}$$

$$AC = \frac{25}{\tan 61^\circ} \qquad AB = \frac{8}{\tan 37^\circ}$$

$$\begin{aligned} BC &= AC - AB \\ &= \frac{25}{\tan 61^\circ} - \frac{8}{\tan 37^\circ} \\ &= 25\cot 61^\circ - 8\cot 37^\circ \end{aligned}$$

Question 5 - D

Worked solutions:

The gradient of the tangent at $x = e$ is positive and the graph of $f(x)$ is concave up.

Question 6 – B

Worked solutions:

$$\begin{aligned} \lim_{x\to 0}\frac{4x - \tan 2x}{\sin 5x} &= \lim_{x\to 0}\left(\frac{4x}{\sin 5x} - \frac{\tan 2x}{\sin 5x}\right) \\ &= \lim_{x\to 0}\frac{4x}{\sin 5x} - \lim_{x\to 0}\frac{\tan 2x}{\sin 5x} \\ &= \lim_{x\to 0}\left(\frac{5x}{\sin 5x}\times\frac{4}{5}\right) - \lim_{x\to 0}\left(\frac{5x}{\sin 5x}\times\frac{1}{5x}\times\frac{\tan 2x}{2x}\times 2x\right) \\ &= \frac{4}{5} - \frac{2}{5} \\ &= \frac{2}{5} \end{aligned}$$

Question 7 – C

Worked solutions:

$$3x^2-6=0$$
$$x^2=2$$
$$x=\pm\sqrt{2}$$
$$\left|\int_0^{\sqrt{2}} 3x^2-6\,dx\right| = \int_{\sqrt{2}}^{a} 3x^2-6\,dx$$
$$\left[x^3-6x\right]_{\sqrt{2}}^{0} = \left[x^3-6x\right]_{\sqrt{2}}^{a}$$
$$-2\sqrt{2}+6\sqrt{2} = a^3-6a-\left(2\sqrt{2}-6\sqrt{2}\right)$$
$$a^3-6a=0$$
$$a(a^2-6)=0$$
$$a=0 \text{ or } a=\pm\sqrt{6}$$

Question 8 – B

Worked solutions:

When $x=4$,

$$h(4)=f\left(g(4)\right)$$
$$=f(-1)$$
$$=3$$
$$\therefore \left(4,3\right)$$
$$h'(x)=f'\left(g(x)\right)\times g'(x)$$
$$h'(4)=f'\left(g(4)\right)\times g'(4)$$
$$=f'\left(-1\right)\times g'(4)$$
$$=5\times 3$$
$$=15$$

Sub $\left(4,3\right)$ into $y-y_1=m(x-x_1)$:

$$y-3=15(x-4)$$
$$y=15x-60+3$$
$$y=15x-57$$

Question 9 – D

Worked solutions:

$$\frac{1}{\log_a(b)}+\frac{1}{\log_b(c)}+\frac{1}{\log_c(a)}=\frac{1}{\frac{\log_b(b)}{\log_b(a)}}+\frac{1}{\frac{\log_c(c)}{\log_c(b)}}+\frac{1}{\frac{\log_a(a)}{\log_a(c)}}$$

$$=\frac{1}{\frac{1}{\log_b(a)}}+\frac{1}{\frac{1}{\log_c(b)}}+\frac{1}{\frac{1}{\log_a(c)}}$$

$$=\log_b(a)+\log_c(b)+\log_a(c)$$

Question 10 – D

Worked solutions:

x – coordinate of L:

$ax+3=0$

$x=-\frac{3}{a}$

x – coordinate of M:

$bx+2=0$

$x=-\frac{2}{b}$

$$\therefore LM=-\frac{2}{b}-\left(-\frac{3}{a}\right)$$

$$=-\frac{2}{b}+\frac{3}{a}$$

$$\begin{cases}-\frac{2}{b}+\frac{3}{a}=5\\ ab=-1\end{cases}\Rightarrow\begin{cases}3b-2a=5ab\\ ab=-1\end{cases}\Rightarrow\begin{cases}3b-2a=-5\\ ab=-1\end{cases}\Rightarrow\begin{cases}3b-2a=-5 & \text{...(1)}\\ a=-\frac{1}{b} & \text{...(2)}\end{cases}$$

Sub (2) into (1):

$$3b-2\left(-\frac{1}{b}\right)=-5$$

$$3b+\frac{2}{b}=-5$$

$$3b^2+5b+2=0$$

$$(b+1)(3b+2)=0$$

$$b=-1 \text{ or } b=-\frac{2}{3}$$

$$\therefore\begin{cases}a=1\\ b=-1\end{cases}\text{ or }\begin{cases}a=\frac{3}{2}\\ b=-\frac{2}{3}\end{cases}$$

Since $a>1$, $a+b=\frac{3}{2}-\frac{2}{3}=\frac{5}{6}$.

Question 11 (1 mark)

Find $\lim_{x\to 4}\frac{x^2-7x+12}{x-4}$. 1

$\lim_{x\to 4}\frac{(x-3)\cancel{(x-4)}}{\cancel{(x-4)}} = \lim_{x\to 4} x-3$

(Nearly all students got this correct)

$= 4-3$

$= 1$

Question 12 (2 marks)

Differentiate with respect to x: $y=\frac{e^{2x}}{x+5}$ $\begin{matrix} u \\ v \end{matrix}$ 2

let $u = e^{2x}$ $v = x+5$

$u' = 2e^{2x}$ $v' = 1$

$\therefore \frac{dy}{dx} = \frac{vu' - uv'}{v^2}$

$= \frac{2e^{2x}(x+5) - e^{2x}}{(x+5)^2}$ 1

$= \frac{e^{2x}(2x+10-1)}{(x+5)^2}$

$= \frac{e^{2x}(2x+9)}{(x+5)^2}$ 1

- 1 mark for applying quotient rule correctly
- 1 mark for simplifying

(Nearly all students received full marks for this question)

Question 13 (2 marks)

A bag contains red and purple marbles, with at least three of each colour. Three marbles are drawn from the bag, without replacement. The number of purple marbles drawn is given by the random variable X.

The following tables shows a probability distribution for the random variable X.

x	0	1	2	3
$P(X = x)$	a	$\frac{1}{10}$	$\frac{1}{3}$	$\frac{3}{10}$

(i) Find the probability of drawing three red marbles. **1**

$P(3\text{red}) = P(0\text{ purple}) = P(X=0) = a$

(Nearly all students got this correct)

$= 1 - \left(\frac{1}{10} + \frac{1}{3} + \frac{3}{10}\right)$

$= 1 - \frac{11}{15}$

$= \frac{4}{15}$ 1

(ii) Find the expected value of X. **1**

$E(x) = \sum x . P(X=x)$

$= (0 \times a) + (1 \times \frac{1}{10}) + (2 \times \frac{1}{3}) + (3 \times \frac{3}{10})$

(Nearly all students got this correct)

$= 0 + \frac{1}{10} + \frac{2}{3} + \frac{9}{10}$

$= 5/3 = 2\,{}^{2}/_{3}$ 1

Question 14 (3 marks)

Solve: $|3x-2| \geq 5$ **3**

$3x - 2 \geq 5$ or $3x - 2 \leq -5$

$3x \geq 7$ $\quad$ $3x \leq -3$

$x \geq 7/3$ 1 $\quad$ $x \leq -1$ 2

(Nearly all students were able to solve $3x-2 \geq 5$ and get $x \geq 7/3$)

(A lot of students weren't able to solve $3x-2 \leq -5$ and get $x \leq -1$)

-1 mark was deducted for every error made.

Question 15 (2 marks)

Solve for $x : \log(3x+6) = \log 3 + 1$ **2**

$\log_e(3x+6) - \log_e 3 = 1$

$\log_e \frac{3x+6}{3} = 1$

$\log_e \frac{\cancel{3}(x+2)}{\cancel{3}} = 1$

$\log_e x+2 = 1$ 1

$x+2 = e^1$

$x = e-2$

$x \doteq 0.72$ (2d.p) 1

(This is the correct solution)

$\log_{10}(3x+6) - \log_{10} 3 = 1$

$\log_{10}(x+2) = 1$ 1

$x+2 = 10^1$

$x = 8$ 1

(Many students used base 10)

(Both solutions were marked correct.)

Question 16 (3 marks)

The points $P(1,-2)$ and $Q(7,1)$ lie on the circumference of a circle.

Show that the centre of the circle lies on the line $4x+2y=15$. **3**

Circle: Centre (a,b) : $(x-a)^2 + (y-b)^2 = r^2$

x, y

$P(1,-2)$: $(1-a)^2 + (-2-b)^2 = r^2$ ... (1)

$Q(7,1)$: $(7-a)^2 + (1-b)^2 = r^2$ ... (2)

– 1 mark awarded for forming circle equations

(1) : $1 - 2a + a^2 + 4 + 4b + b^2 = r^2$

$a^2 - 2a + b^2 + 4b + 5 = r^2$

(2) : $49 - 14a + a^2 + 1 - 2b + b^2 = r^2$

$a^2 - 14a + b^2 - 2b + 50 = r^2$

– 1 mark awarded for expanding and simplifying.

(1) − (2) $12a + 6b - 45 = 0$ $(\div 3)$

$4a + 2b = 15$ where (a,b) is the centre

which satisfies $4x + 2y = 15$.

– 1 mark for showing equation $4a+2b=15$

(Question was completed poorly most students weren't able to form circ equations, and hence solve question)

A lot of students used the following method:

1. Centre of circle – midpoint: $\left(\frac{1+7}{2}, \frac{-2+1}{2}\right) = (4, -\frac{1}{2})$

2. Given $M(4,-\frac{1}{2})$ substitute into $4x+2y=15$

LHS: $4x+2y = 4(4) + 2(-\frac{1}{2})$

$= 16 - 1$

$= 15$

$=$ R.H.S. $\therefore$ Centre lies on line

This method doesn't prove midpoint is centre and 2 marks were awarded.

Question 17 (3 marks)

$T_1 \quad T_2 \quad T_3$

Consider the arithmetic progression $-7, 2, 11, 20, \ldots, 713$.

(i) Find the number of terms. **2**

$T_1 = -7 = a$

$d = T_2 - T_1 = T_3 - T_2$

$= 2 + 7 = 11 - 2$

$d = 9$

i) $\therefore T_n = a + (n-1)d$

$= -7 + (n-1)9$

$= -7 + 9n - 9$

$T_n = 9n - 16$ 1

ii) $713 = 9n - 16$

$713 + 16 = 9n$

$729 = 9n$

$\therefore n = \frac{729}{9}$ 1

$n = 81$.

(Question was completed extremely well with most students receiving full marks)

– 1 mark awarded for general term T_n

– 1 mark for number of terms.

(ii) Hence, find the sum of terms. **1**

$S_n = \frac{n}{2}(a + \ell)$

$S_{81} = \frac{81}{2}(-7 + 713)$

$= \frac{81}{2} \times 706$

$= 28\,593$ 1

(Nearly all students got this correct).

Question 18 (1 mark)

Evaluate: $\int 2\tan^2 x \, dx$ **1**

$2\int \tan^2 x \, dx = 2\int \sec^2 x - 1 \, dx$

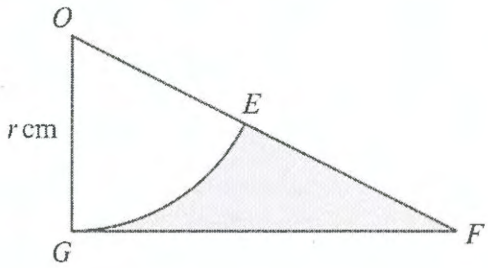
$= 2[\tan x - x] + C$

$= 2\tan x - 2x + C$ 1

(A lot of students weren't able to make the substitution of $\tan^2 x = \sec^2 - 1$. Hence, were unable to answer question correctly.)

Question 19 (3 marks)

The diagram shows a sector, EOG, of a circle, with centre O and radius r cm. The line GF is a tangent to the circle at G. Point E is the midpoint of OF.



Find the expression for the exact area of the shaded region. **3**

$OG = OE$ (radii of circle)

$OE = EF$ (midpoint E, given)

$\therefore OG = OE = EF = r$

$GF = \sqrt{OF^2 - OG^2}$ $\quad\quad \cos \angle GOE = \frac{r}{2r} = \frac{1}{2}$

$= \sqrt{(2r)^2 - r^2}$

$= \sqrt{4r^2 - r^2}$ $\quad\quad \therefore \angle GOE = 60°$

$= \sqrt{3}\, r$ 1

Area of $\triangle OGF = \frac{1}{2} \times \sqrt{3}\, r \times r$

$= \frac{\sqrt{3}\, r^2}{2}$

Area of Sector $OGE = \frac{60}{360} \pi r^2$ $\quad\quad$ or $\frac{1}{2} r^2 \frac{\pi}{3}$

$= \frac{\pi r^2}{6}$ 1 $\quad\quad \left(\frac{\pi r^2}{6}\right)$

Area of shaded region $= \triangle OGF - \text{Sector } OGE$

$= \frac{\sqrt{3}\, r^2}{2} - \frac{\pi r^2}{6}$

$= \frac{(3\sqrt{3} - \pi) r^2}{6}$ 1

2/3 for not finding angle

Question 20 (2 marks)

Find an approximation for $\int_2^6 \frac{1}{\sqrt{3x-5}}\,dx$ correct to 2 decimal places, using 3 applications of the trapezoidal rule. **2**

$$\text{Area} \approx \frac{6-2}{2(3)}\left\{f(2)+f(6)+2\left[f\left(\frac{10}{3}\right)+f\left(\frac{14}{3}\right)\right]\right\}$$

$$\approx \frac{2}{3}\left\{1+\frac{1}{\sqrt{13}}+2\left[\frac{1}{\sqrt{5}}+\frac{1}{3}\right]\right\} \quad (1)$$

$$\approx 1.89 \quad (1)$$

* Poorly done by whole cohort

→ zero for incorrect substitution into first line

Question 21 (2 marks)

Consider functions $f(x)=x+1$ and $g(x)=\frac{1}{\sqrt{3x-5}}$.

(i) Find the rule for the composite function $g(f(x))$. **1**

$$g(x+1)=\frac{1}{\sqrt{3(x+1)-5}}$$

$$=\frac{1}{\sqrt{3x+3-5}}$$

$$=\frac{1}{\sqrt{3x-2}} \quad (1)$$

(ii) Find the domain of $g(f(x))$. **1**

$3x-2>0$ $\left(\frac{2}{3},\infty\right)$

$3x>2$

$x>\frac{2}{3}$ (1)

zero marks for $x \geq \frac{2}{3}$

Question 22 (3 marks)

The time from Amber getting out of bed until her arrival at her office is normally distributed with a mean of 43 minutes and a standard deviation of 7 minutes. Amber's arrival at her office is classified as being late if it occurs after 9:20 am.

(i) If Amber gets out of bed at 8:16 am, find the z-score for the number of minutes she has to get to the office. **1**

9:20 − 8:16 = 64 minutes

$$z = \frac{64-43}{7}$$

$$= 3 \quad (1)$$

(ii) If Amber gets out bed at 8:16 am, find the probability that she will arrive late using the Empirical Rule. **2**

$$P(\text{arriving late}) = P(z > 3)$$

$$= 1 - P(z < 3)$$

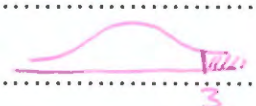
$$= \frac{1 - 0.997}{2}$$

$$= 0.15\% \quad (1)$$

2 marks for showing working (any of these)

$$1 - P(z<3)$$

$$\frac{1-0.997}{2}$$



Question 23 (3 marks)

Given that $f(x) = -2x^3 + k^3x^2$ where k is a positive constant. Find the complete set of values of x for which the graph of $f(x)$ is concave up, using interval notation. **3**

concave up if $f''(x) > 0$

$f(x) = -2x^3 + k^3x^2$

$f'(x) = -6x^2 + k^3 . 2x$

$f''(x) = -12x + 2k^3$ 1

$\therefore -12x + 2k^3 > 0$ ← needs to include this line

$-12x > -2k^3$

$\therefore 6x < k^3$

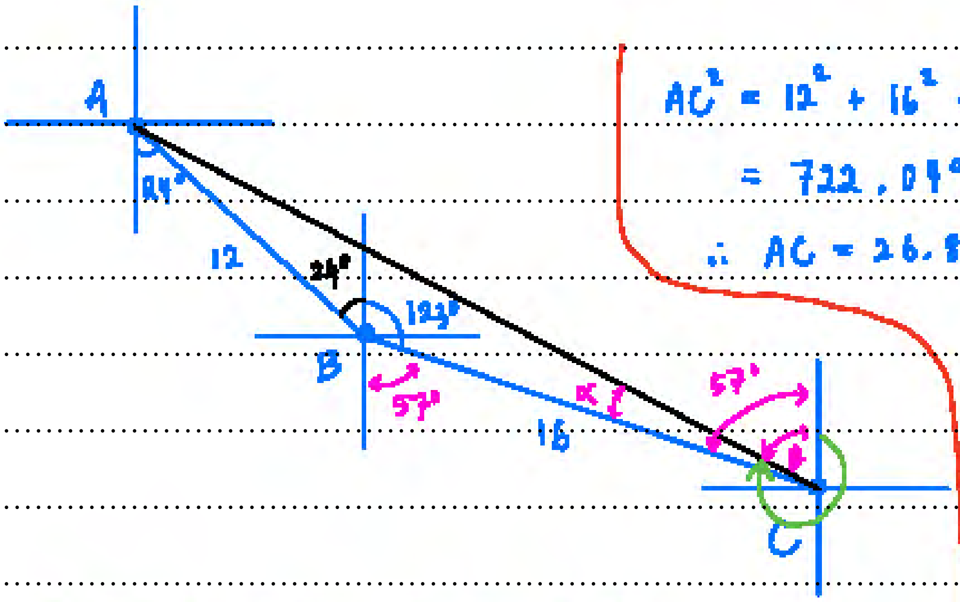
$\therefore x < \frac{k^3}{6}$ 1

$\therefore (-\infty, \frac{k^3}{6})$ 1

Question 24 (4 marks)

A ship departs from point A and sails 12 kilometres on a bearing of 156° to reach point B. It then changes direction and sails 16 kilometres on a bearing of 123° to reach point C.

(i) Find the distance AC, correct to the nearest kilometre. **2**



$AC^2 = 12^2 + 16^2 - 2 \times 12 \times 16 \cos 147°$ ✓

$= 722.049\ldots$

$\therefore AC = 26.8\ldots = 27$ km ✓

Comment Generally well done

(ii) Find the true bearing that the ship should follow to return directly to point A from point C. Give your answer correct to the nearest degree. **2**

$$\frac{\sin\alpha}{12} = \frac{\sin 147°}{27}$$

$\alpha = 14.008\ldots$

$\theta = 57° - \alpha = 42.9917\ldots$ ✓

$\therefore$ required bearing $= 360° - \theta$

$= 317°T$ ✓

Comments

Mixed responses.

Students with a clear diagram were generally more successful.

Question 25 (3 marks)

Evaluate $\int_1^{\sqrt{3}} \left(\frac{x^3}{6} + \frac{1}{3x^2} \right) dx$, giving your answer in the form $a + b\sqrt{3}$,

where a and b are constants to be determined. **3**

$$= \left[\frac{x^4}{24} - \frac{1}{3x} \right]_1^{\sqrt{3}}$$ ✓

$$= \frac{(\sqrt{3})^4}{24} - \frac{1}{3\sqrt{3}} - \left(\frac{1}{24} - \frac{1}{3} \right)$$

$$= \frac{9}{24} - \frac{\sqrt{3}}{9} + \frac{7}{24}$$

✓ correct progress

$$= \frac{2}{3} - \frac{\sqrt{3}}{9}$$

$$= \frac{2}{3} - \frac{1}{9}\sqrt{3}$$ ✓

Comment

- There were some errors made when integrating, substituting and/or simplifying.
- Students are reminded to double check over their work (or redo on spare paper)

Question 26 (3 marks)

Solve the equation $3\sin^2 x + 1 = 4\cos x$ for $x \in [-\pi, \pi]$.

Give your answer correct to three decimal places. **3**

$3(1-\cos^2 x) + 1 = 4\cos x$ ✓

$0 = 3\cos^2 x + 4\cos x - 3$

$0 = (3\cos x - 2)(\cos x + 2)$

$\therefore \cos x = \frac{2}{3}, -2$ ✓

$\cos x = \frac{2}{3} \quad (\cos x \neq -2)$

$\therefore x = \pm 0.841$ ✓

Comment

Common issues revolved around:

- not knowing when/why to convert $\sin^2 x$ into $1 - \cos^2 x$
- solving $\cos x = \frac{2}{3}$ without paying attention to the domain.

Question 27 (4 marks)

The table below shows the future values of an annuity of $1 for different rates of interest and different numbers of compounding periods. The contribution is made at the end of each time period.

Future Value Interest Factors

Time Period	*Interest Rate* 1%	2%	3%	4%	5%
1	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500
3	3.0301	3.0604	3.0909	3.1216	3.1525
4	4.0604	4.1216	4.1836	4.2465	4.3101
5	5.1010	5.2040	5.3091	5.4163	5.5256
6	6.1520	6.3081	6.4684	6.6330	6.8019
7	7.2135	7.4343	7.6625	7.8983	8.1420
8	8.2857	8.5830	8.8923	9.2142	9.5491

(i) Kelly opens an annuity account and makes contributions of $3500 at the end of each quarter over a period of 18 months at an interest rate of 8% per annum, compounding quarterly. Find the total amount in Kelly's account after 18 months. **1**

8% p.a → 2% per quarter

18 months → 6 quarters

3500 × 6.3081 = $22 078.35 ✓

(ii) Julie deposits $10 000 into a saving account at the end of each year for 9 years.

For the first 4 years, the interest rate is 3% per annum, compounding annually. From the 5th year, the interest rate increases to 5% per annum, compounding annually.

Find the amount of money in Julie's account at the end of nine years. **3**

10 000 × 4.1836 = $41 836 ✓

$41\,836(1.05)^5 + 10\,000 \times 5.5256$ ✓

[for either $41836(1.05)^5$ OR $10\,000 \times 5.5256$]

= $108 650.52 ✓

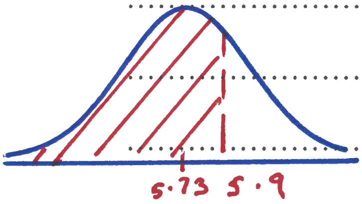
Comment

* Not particularly well done.
* Common errors were:
  - 41836 × 5.5256
  - 41 836 + 10 000 × 5.5256
* Although not wrong, some students solved this by developing the sum of a geometric progression.

Question 28 (3 marks) (The table on page 25 can be used for this question)

The mass of a certain species of fish caught at sea is normally distributed with mean 5.73 kg and variance 2.56 kg^2. Find the probability that a randomly selected fish caught at sea has a mass that is:

(i) less than 5.9 kg **1**



Variance $= 2.56$ * Some students missed this.

$\therefore \sigma = \sqrt{2.56} = 1.6$

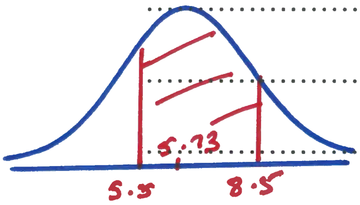
$P(X < 5.9) = P\left(Z < \frac{5.9 - 5.73}{1.6}\right)$

$= P(Z < 0.10625)$

$\doteqdot P(Z < 0.11)$ * some students rounded incorrectly.

$\doteqdot 0.54380$ ✓

(ii) between 5.5 and 8.5 kg **2**



$x = 8.5 \quad z = \frac{8.5 - 5.73}{1.6} \doteqdot 1.73$ No marks awarded for correct z-scores.

$x = 5.5 \quad z = \frac{5.5 - 5.73}{1.6} \doteqdot -0.14$

$P(5.5 < X < 8.5) = P(X < 8.5) - P(X < 5.5)$ ✓ 1 mark for recognising this.

$= P(Z < 1.73) - (1 - P(Z < 0.14))$

$= 0.95818 - (1 - 0.55567)$

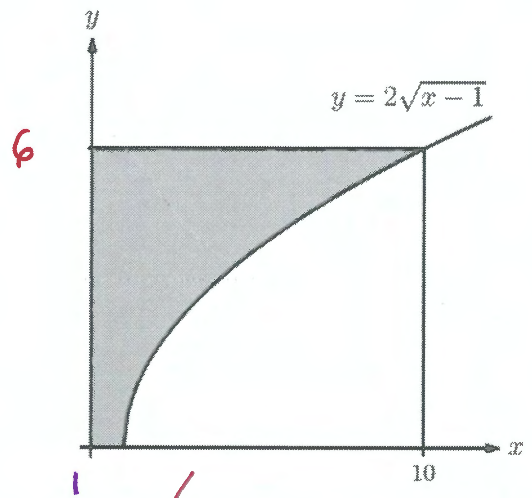
$= 0.51385$ ✓ final mark for correct answer.

* students who left $\sigma = 2.56$ were awarded carry on error if all other working was correct.

Note: A quick sketch of the normal curve is helpful for answering these questions.

Question 29 (3 marks) This question was very well done.

The curve $y = 2\sqrt{(x-1)}$ is shown below. Find the shaded area. **3**



Find x-intercept

Let $2\sqrt{x-1} = 0$

$\sqrt{x-1} = 0$

$x - 1 = 0$

$x = 1$

$x = 10$

$y = 2\sqrt{9} = 6$

Area rectangle $= 10 \times 6$
$= 60$

$$A = 60 - 2\int_1^{10} \sqrt{x-1}\, dx$$

$$= 60 - 2\int_1^{10} (x-1)^{1/2}\, dx$$

$$= 60 - 2\left[\frac{2}{3}(x-1)^{3/2}\right]_1^{10}$$

$$= 60 - \frac{4}{3}(27 - 0)$$

$$= 24u^2$$

Alternative solution: –

$$\frac{y}{2} = \sqrt{x-1}$$

$$x - 1 = \frac{y^2}{4}$$

$$x = \frac{y^2}{4} + 1$$

$$A = \int_0^6 \left(\frac{y^2}{4} + 1\right) dy$$

$$= \left[\frac{y^3}{12} + y\right]_0^6$$

$$= 24u^2$$

Question 30 (2 marks)

Consider the geometric series $7 + 14e^{-2} + 28e^{-4} + 56e^{-6} + \cdots$

Justify whether this series has a limiting sum or not. **2**

G.P $a = 7$ $r = \frac{2}{e^2}$ if a L.S $-1 < r < 1$

Since $r \doteqdot 0.27$ which satisfies above condition the geometric series has a limiting sum.

or mention the fact that $e^2 > 2$
$\therefore r < 1$.

Justification was necessary for full marks.

Question 31 (4 marks) This question was very well done.

The continuous random variable X has probability density function $f(x)$, where

$$f(x) = \begin{cases} k(4x^2 - x^3), & 0 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$

(i) Show that $k = \frac{3}{20}$. **2**

$k\int_0^2 (4x^2 - x^3)\,dx = 1$ property of PDF

$k\left[\frac{4}{3}x^3 - \frac{x^4}{4}\right]_0^2 = 1$

$k\left(\frac{32}{3} - 4\right) = 1$

$k\left(\frac{20}{3}\right) = 1$

$\therefore k = \frac{3}{20}$, as required.

(ii) Find $P\left(\frac{1}{2} < X < 1\right)$. Give your answer correct to 3 significant figures. **2**

must do this for full marks.

$= \frac{3}{20}\int_{1/2}^{1} (4x^2 - x^3)\,dx$

$= \frac{3}{20}\left[\frac{4}{3}x^3 - \frac{x^4}{4}\right]_{1/2}^{1}$

$= \frac{3}{20}\left[\frac{4}{3} - \frac{1}{4} - \left(\frac{1}{6} - \frac{1}{64}\right)\right]$ ✓

$= \frac{179}{1280}$

$\doteqdot 0.140$ (3 sig figs)

Question 32 (4 marks)

Eight students in the Standard Mathematics class sat end-of-term tests in *Functions* and *Calculus*. The table below shows their marks out of 100 in each test.

Students	A	B	C	D	E	F	G	H
Functions (x)	63	59	69	83	77	75	85	
Calculus (y)	58	62	70	72	76	76	78	84

Unfortunately, the *Functions* mark for student H has been lost. However, it is known that the correlation coefficient between *Functions* marks and *Calculus* marks is $r = 0.9$ and the equation of the least squares regression line of *Calculus* marks on *Functions* marks is $y = 0.72x + 18$.

(i) Use the mean $\bar{y} = 72$ of the *Calculus* marks and the least squares regression line to find the mean $\bar{x}$ of the *Functions* marks, and hence the missing *Functions* mark for student H. **2**

Let $y = 72$

$72 = 0.72x + 18$

$0.72x = 54$

$\bar{x} = 75$ ✓

$\dfrac{63+59+69+83+77+75+85+x}{8} = 75$

$x = 89$ ✓

(ii) Another student in the same class sat the *Functions* test but misses the *Calculus* test. Explain whether or not you would be justified statistically in using their *Functions* mark and the given least squares regression line to estimate their *Calculus* mark. **2**

Yes.

$r = 0.9$ indicates a strong positive correlation between the tests ∴ you would be justified statistically. ✓

Could also discuss the limitations of the model ✓ ie extrapolation would not be justified.

Question 33 (i)

Criteria	Marks
• Provide correct answer.	1

Sample answer:

$$\begin{aligned} f'(x) &= 5\cos(5x) - 5x \times 5\sin(5x) \\ &= 5\cos(5x) - 25\sin(5x) \end{aligned}$$

Question 33 (ii)

Comments
• Answered generally well. • Some students were confused with the coefficient of $x\sin(5x)$. • A few students forgot the constant term.

Criteria	Marks
• Correctly evaluate the indefinite integral.	3
• Make progress with correct integrand.	2
• Correctly express $x\sin(5x)$ using the derivative from part (i)	1

Sample answer:

$$\begin{aligned} f'(x) &= 5\cos(5x) - 25x\sin(5x) \\ 25x\sin(5x) &= 5\cos(5x) - f'(x) \\ x\sin(5x) &= \frac{1}{25}\left(5\cos(5x) - f'(x)\right) \\ \int x\sin(5x)\,dx &= \int \frac{1}{25}\left(5\cos(5x) - f'(x)\right)dx \\ &= \frac{1}{25}\int\left(5\cos(5x) - f'(x)\right)dx \\ &= \frac{1}{25}\left(\sin(5x) - f(x)\right) + C \\ &= \frac{\sin(5x) - 5x\cos(x)}{25} + C \end{aligned}$$

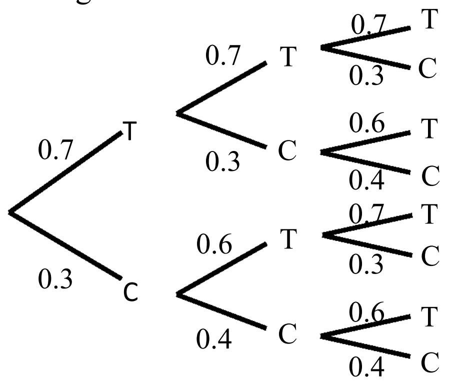
Question 34 (i)

Comments
• A few students had a tree diagram with incorrect given condition. • Some students solved the question without considering the conditional probability.

Criteria	Marks
• Provides correct answer.	1

Sample answer:

Given she drinks tea on one Sunday Morning, the probability she drinks coffee the next morning is 0.3.



$$P(3C \mid T) = 0.3 \times 0.4 \times 0.4$$
$$= 0.048$$

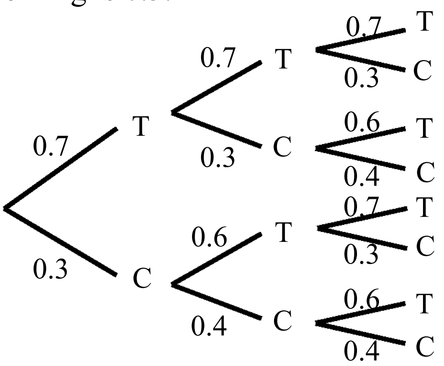
Question 34 (ii)

Comments
• A few students had a tree diagram with incorrect given condition. • Some students solved the question without considering the conditional probability.

Criteria	Marks
• Provides the correct answer.	2
• Correctly finds $P(2C\,|\,T)$, $P(3C\,|\,T)$, $P(1C\,|\,T)$ or $P(\text{no coffee}\,|\,T)$	1

Sample answer:

Given she drinks tea on one Sunday Morning, the probability she drinks coffee the next morning is 0.3.



$$P(\text{at least 2 coffee given drinking tea}) = P(2C\,|\,T) + P(3C\,|\,T)$$

$$= \underbrace{0.3\times0.4\times0.6 + 0.3\times0.6\times0.3 + 0.7\times0.3\times0.4}_{P(CCT|T)+P(CTC|T)+P(TCC|T)} + 0.048$$

$$= 0.258$$

Question 35

Comments
• The easiest way is to use area between two curves formula. • Many students used the sum of two areas between curves and $x-$ axis. When finding the area under the $x-$ axis, many students didn't evaluate the correct integration with absolute values. • Many students made careless errors and failed to simplify the final answer.

Criteria	Marks
• Provides the correct answer, showing all necessary working.	3
• Correctly evaluates to get the constant term or the $\ln$ term.	2
• Provides correct expression for area and correctly integrate it before substituting the boundaries.	1

Sample answer:

$$\begin{aligned}
\text{Area} &= \int_{-\ln 6}^{\ln 2} -e^{2x}+4-e^{x}+5\,dx \\
&= \int_{-\ln 6}^{\ln 2} -e^{2x}-e^{x}+9\,dx \\
&= \left[\frac{-e^{2x}}{2}-e^{x}+9x\right]_{-\ln 6}^{\ln 2} \\
&= \frac{-e^{2\ln 2}}{2}-e^{\ln 2}+9\ln 2-\left(\frac{-e^{-2\ln 6}}{2}-e^{-\ln 6}-9\ln 6\right) \\
&= \frac{-4}{2}-2+9\ln 2-\left(\frac{-6^{-2}}{2}-\frac{1}{6}-9\ln 6\right) \\
&= 9\ln 2+9\ln 6-\frac{275}{72} \\
&= 9\ln 12-\frac{275}{72}
\end{aligned}$$

Question 36 (i)

Comments
• Answered well.

Criteria	Marks
• Provides the correct answer using the initial condition.	2
• Correctly finds $3\sin\left(2t+\frac{\pi}{3}\right)+C$.	1

Sample answer:

$$s(t)=\int 6\cos\left(2t+\frac{\pi}{3}\right)dx$$

$$=3\sin\left(2t+\frac{\pi}{3}\right)+C$$

When $t=0,\ s(0)=2$

$$2=3\sin\left(\frac{\pi}{3}\right)+C$$

$$C=2-3\sin\left(\frac{\pi}{3}\right)$$

$$C=2-\frac{3\sqrt{3}}{2}$$

$$\therefore s(t)=3\sin\left(2t+\frac{\pi}{3}\right)+2-\frac{3\sqrt{3}}{2}$$

Question 36 (ii)

Comments	Criteria	Marks
• Answered well.	• Provides correct answer.	1

Sample answer:

$$a(t)=\frac{dv}{dt}$$

$$=-12\sin\left(2t+\frac{\pi}{3}\right)$$

Question 36 (iii)

Comments
• Most students forgot to test using first derivative table or second derivative.

Criteria	**Marks**
• Tests the nature using first derivative table or second derivative.	2
• Correctly finds $t = \frac{\pi}{12}$.	1

Let $v(t) = 0$:

$$6\cos\left(2t + \frac{\pi}{3}\right) = 0$$

$$\cos\left(2t + \frac{\pi}{3}\right) = 0$$

$$2t + \frac{\pi}{3} = \frac{\pi}{2}$$

$$2t = \frac{\pi}{6}$$

$$t = \frac{\pi}{12} \text{ seconds}$$

Testing the nature:

$$a\left(\frac{\pi}{12}\right) = -12\sin\left(2 \times \frac{\pi}{12} + \frac{\pi}{3}\right)$$

$$= -12$$

$$< 0 \qquad \therefore \text{ It's a turning point,}$$

Therefore, the particle changes directions at $t = \frac{\pi}{12}$ seconds.

Question 37 (5 marks)

For the function $y=(4x-1)e^{-\frac{x}{2}}$:

(i) Find any stationary points and determine their nature. **3**

$y' = 4e^{-\frac{x}{2}} - \frac{1}{2}e^{-\frac{x}{2}}(4x-1)$

$= e^{-\frac{x}{2}}\left(4 - \frac{1}{2}(4x-1)\right)$

$= e^{-\frac{x}{2}}\left(4 - 2x + \frac{1}{2}\right)$

$= e^{-\frac{x}{2}}\left(\frac{9}{2} - 2x\right)$

*If not written $e^{-\frac{x}{2}} \neq 0$ 1 mark was deducted

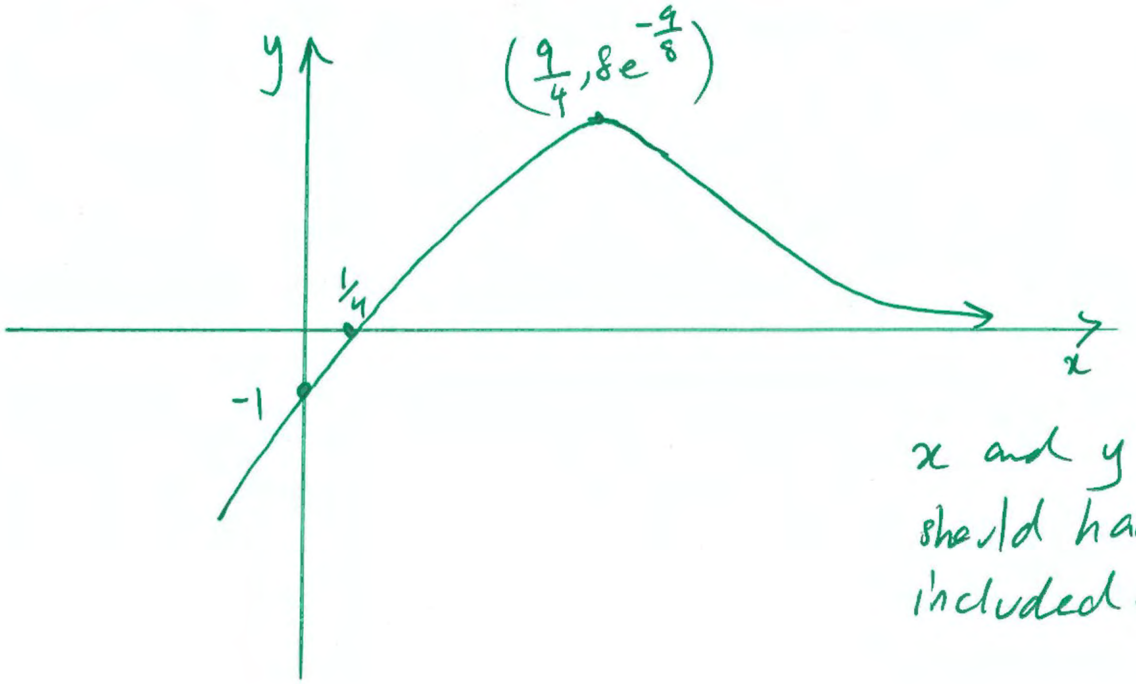
$y'=0 \therefore \frac{9}{2} - 2x = 0 \longrightarrow x = \frac{9}{4} \rightarrow y = 8e^{-\frac{9}{8}}$ $\left(\frac{9}{4}, 8e^{-\frac{9}{8}}\right)$

$e^{-\frac{x}{2}} \neq 0$

x	2	$\frac{9}{4}$	3
y'	$\frac{1}{2e}$	0	$-\frac{3}{2}e^{-\frac{3}{2}}$

max t.p

(ii) Using part i), sketch the graph of $y=f(x)$, showing all key features. **2**



x and y intercepts should have been included.

Question 38 (5 marks)

A lake contains an initial population of 500 000 fish at the beginning of 2025. Each year, the fish population increases by 7% due to natural reproduction. After this increase, a fishing company catches fish from the lake. In 2025, 2000 fish are caught. In each subsequent year, the number of fish caught increases by 9% compared to the previous year.

Let F_n be the number of fish remaining in the lake after the nth year's catch.

(i) Show that $F_3 = 500000 \times 1.07^3 - 2000 \times (1.07^2 + 1.07 \times 1.09 + 1.09^2)$ **2**

$F_1 = 500000(1.07) - 2000$

$F_2 = (500000(1.07) - 2000)1.07 - 2000(1.09)$

$= 500000(1.07)^2 - 2000(1.07) - 2000(1.09)$

$F_3 = (500000(1.07)^2 - 2000(1.07) - 2000(1.09))1.07 - 2000(1.09)^2$

$= 500000(1.07)^3 - 2000(1.07)^2 - 2000(1.09)(1.07) - 2000(1.09)^2$

$= 500000(1.07)^3 - 2000(1.07^2 + (1.09)(1.07) + 1.09^2)$

*For 2 marks the students had to show all these steps. Just putting F_1 or F_2 was not accepted.

(ii) If the fishing was allowed to continue at this rate, find the year which there will be no fish in this lake.

$F_n = 500000(1.07)^n - 2000(1.07^{n-1} + 1.07^{n-2}(1.09) + 1.07^{n-3} 1.09^2 + \dots 1.09^{n-1})$

$F_n = 0$

$500000(1.07)^n = 2000\left(\dfrac{1.07^{n-1}\left(\left(\frac{1.09}{1.07}\right)^n - 1\right)}{\frac{1.09}{1.07} - 1}\right)$

*you had to write F_n for 1 mark.

$500\cancel{000}(1.07)^n = 2\cancel{000}\left(\dfrac{\frac{1.07^n}{\cancel{1.07}}\left(\left(\frac{1.09}{1.07}\right)^n - 1\right)}{\frac{1.09 - 1.07}{\cancel{1.07}}}\right)$

*Full marks for correct answer

$5\cancel{00}(1.07)^n = 1\cancel{00}\left(1.07^n\left(\left(\frac{1.09}{1.07}\right)^n - 1\right)\right)$

$5(1.07)^n = \cancel{1.07^n}\left(\frac{1.09^n}{\cancel{1.07^n}}\right) - 1.07^n$

$6(1.07)^n = 1.09^n$

$6 = \left(\frac{1.09}{1.07}\right)^n$

$\ln 6 = n \ln\left(\frac{1.09}{1.07}\right)$

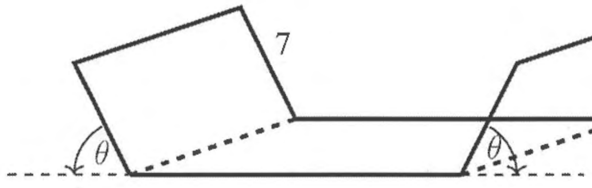
$\therefore n = \dfrac{\ln 6}{\ln\left(\frac{1.09}{1.07}\right)}$

$= 96.75$

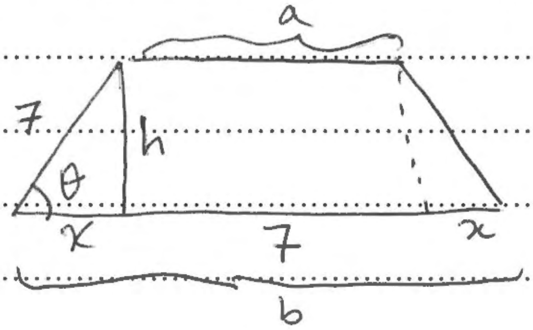
$\therefore$ year 2121

Question 39 (5 marks)

A water trough is to be made from a 21m long strip of tin by bending up by the angle θ with a 7m strip at each side, as shown in the diagram. Find the size of angle θ that would maximize the cross-sectional area, and thus the volume, of the trough.



(i) Show that the cross-sectional area is given by $A = 49(\sin\theta + \sin\theta\cos\theta)$. **2**



$\sin\theta = \frac{h}{7}$

$h = 7\sin\theta$

$\cos\theta = \frac{x}{7}$

$x = 7\cos\theta$

$A = \frac{1}{2}h(a+b)$

$= \frac{1}{2}(7\sin\theta)(7+7+2x)$

$= \frac{1}{2}(7\sin\theta)(14+2(7\cos\theta))$

$= \frac{1}{2}(7\sin\theta)(14+14\cos\theta)$

$= \frac{1}{2}(7\sin\theta)14(1+\cos\theta)$

$= 49\sin\theta(1+\cos\theta)$

$= 49(\sin\theta + \sin\theta\cos\theta)$

* To receive 2 marks you had to show all the steps properly

Question 39 Continues Overleaf…

Question 39 (5 marks)

(ii) Hence, find the size of angle θ that would maximise the cross-sectional area of the trough. **3**

$A = 49(\sin\theta + \sin\theta\cos\theta)$

$A' = 49(\cos\theta + \cos\theta\cos\theta - \sin\theta\sin\theta)$

$= 49(\cos\theta + \cos^2\theta - \sin^2\theta)$

$A'' = 49(-\sin\theta - 2\cos\theta\sin\theta - 2\sin\theta\cos\theta)$

$= 49(-\sin\theta - 4\sin\theta\cos\theta)$

For max or min

$A' = 0$

$\cos\theta + \cos^2\theta - \sin^2\theta = 0$

$\cos\theta + \cos^2\theta - (1 - \cos^2\theta) = 0$

$\cos\theta + \cos^2\theta - 1 + \cos^2\theta = 0$

$2\cos^2\theta + \cos\theta - 1 = 0$

$2\cos^2\theta + 2\cos\theta - \cos\theta - 1 = 0$

$2\cos\theta(\cos\theta + 1) - (\cos\theta + 1) = 0$

$(2\cos\theta - 1)(\cos\theta + 1) = 0$

$\cos\theta = \frac{1}{2}$ or $\cos\theta = -1$ $\quad \theta \neq \pi$ as θ must be acute

$\theta = \frac{\pi}{3}$

*Many students didn't use radian measure, you need to use radian in calculus.

*If you didn't test by A'' or table you lost 1 mark.

Test $A'' = 49\left(-\sin\frac{\pi}{3} - 4\sin\frac{\pi}{3}\cos\frac{\pi}{3}\right)$

$= 49\left(-\frac{\sqrt{3}}{2} - 4\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)\right) = 49\left(\frac{-\sqrt{3}}{2} - \frac{2\sqrt{3}}{2}\right) = \frac{-147\sqrt{3}}{2} < 0$

$\therefore$ max

The End