TRINITY GRAMMAR SCHOOL
MATHEMATICS DEPARTMENT

1997 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

MATHEMATICS

2 UNIT

Time Allowed -Three hours
(Plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES:

Answer each question in a *separate* Writing Booklet.
All questions are of equal value.
All necessary working should be shown in every question.
Standard integrals are supplied.
Approved calculators may be used.
Hand up the questions in ten separate bundles, each clearly marked with your NAME, CLASS, QUESTION NUMBER on each bundle.
The question paper must be handed to the supervisor at the end of the examination.

Question1

i) Solve $x^2 = 6x$

ii) Evaluate $\pi \times (3.1)^2 \times \frac{1}{3.5}$ correct to 3 significant figures.

iii) Evaluate $\sqrt{x^2 + 4y^{-2}}$ when $x = -3,\ y = 0.5$

iv) Write the exact value of $\sin 225°$

v) Factorise and simplify $\frac{4a^2 - 4}{a+1}$

vi) Rationalise denominator and simplify $\frac{\sqrt{2}}{\sqrt{2}-1}$

vii) Find the value of 'a' such that $\sqrt{60} + \sqrt{5} = 2\sqrt{a}$

Question2

START A NEW PAGE

i) Solve $|4 - 2x| = 6$

ii) Solve $9^x = \frac{1}{27}$

iii) A rectangle has sides 12cm and 11cm. If the sides are each increased by 3cm, find the percentage increase in the area correct to 1 decimal place.

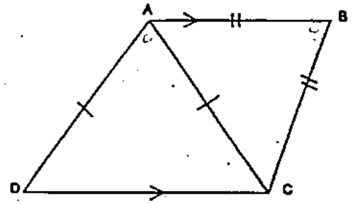
iv) Simplify $(a+2b)^2 - (a-2b)^2$

v) If $S = ut + 0.5at^2$, make the subject 'a'.

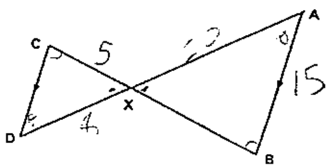
vi) Factorise $3a^2 - 48$

Question 3 START A NEW PAGE

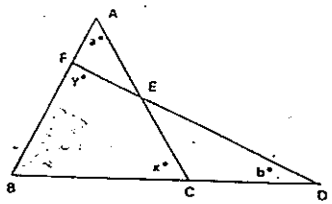
i) In the diagram below AB is parallel to DC, AD = AC and AB = BC. Copy the diagram and show that $\angle DAC = \angle ABC$



ii) In the diagram below, AB is parallel to CD, AX = 20cm, XD = 8cm, CX = 5cm, AB = 15cm. Find CD and BX.



iii) In the figure below show that $a+x=b+y$. Also show that if $a=b$ then $x=y$.



Question4 START A NEW PAGE

i) In triangle ABC $b=5$, $c=4$, $\cos A = \frac{1}{8}$.

a) Calculate a

b) Find angle A (to nearest degree)

c) Find area of the triangle ABC correct to 1 decimal place.

ii) A ship is 180km from a lighthouse on a bearing of 153degrees. The ship then sails west until it is directly south of the lighthouse.What is its distance from the lighthouse?

iii) In triangle ABC , $\angle A = 47^\circ$, $\angle B = 73^\circ$ and $b = 14.6$ cm. Calculate the length of side a.

Question 5 START A NEW PAGE

i)The points A and B have coordinates (-3 , 6) and (5 , 2) respectively. Plot these points on a number plane.

a) Find the equation of the line joining the points A and B

b) The line k is drawn through B with a gradient of 2. Show the equation of k is : $2x - y - 8 = 0$

c) Show the line k passes through P (2 , -4)

d) Prove that triangle ABP is right angled.

e) Find the area of triangle ABP.

ii) Sketch the graph of $y = \sqrt{4 - x^2}$ State its domain and range.

Question 6 START A NEW PAGE

i) Differentiate with respect to x:

a) $y = 4x^3 - \frac{1}{x^2}$

b) $y = xe^x$

c) $y = \frac{x+2}{x-3}$

ii) Evaluate the following

a) $\int_1^2 \left(x + \frac{1}{x^2}\right) dx$

[redacted]

b) $\int_0^2 xe^{x^2}\, dx$

c) $\int_2^3 (3x-4)^5\, dx$

Question 7 START A NEW PAGE

i) Given that α and β are the roots of the quadratic equation $3x^2 + 5x - 1 = 0$, find:

a) $\alpha + \beta$

b) $\alpha\beta$

c) $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$

ii) Find the values of k for which $5x^2 + kx + 1 = 0$ will have equal roots.

iii) For the parabola $(x+3)^2 = -20(y-5)$

Find a) Coordinates of the focus

b) Focal length

c) Equation of the directrix

d) Length of the latus rectum

e) Sketch the curve.

Question 8 START A NEW PAGE

i) Robert is employed at an initial salary of $18400 per annum. Each year the salary is increased by $1200.

a) Find the salary for the tenth year.

b) Find his total salary for the first ten years.

ii) A ball is dropped from a height of 60 cm and continues to bounce to five sixths of the height of the preceding bounce until it comes to rest. Find the total distance travelled by the ball.

iii) Sketch the curve $y = x^2$

That part of the curve from $x=0$ to $x=3$ is rotated about the x axis. Find the exact volume generated in terms of π

Question 9 START A NEW PAGE

i) Copy and complete the following table and plot the points on the graph of $y = \frac{1}{x^2+1}$

x	0	1	2
y			

Use Simpson's Rule with these 3 function values to evaluate to 1 decimal place $\int_0^2 \frac{1}{x^2+1} dx$

ii) a) Evaluate $\int_0^2 (x^2 - 1) dx$

b) Find the area enclosed by the curve $y = x^2 - 1$, the x axis and the ordinates $x=0$ and $x=2$.

iii) By comparing answers to a) and b) above explain why $\int_a^b f(x)dx$ is not always equal to the area between the curve $y = f(x)$, the x axis and the ordinates $x=a$ and $x=b$.

Question 10 START A NEW PAGE

i) a) For the curve $y = 4 + 3x - x^3$, find the coordinates of the stationary points, and distinguish their nature.
b) Find any points of inflexion.
c) Clearly sketch the curve.

ii) The gradient function of a curve is given by $\frac{dy}{dx} = x^{-2}$. If the curve passes through the point $(\frac{1}{2}, 4)$ find its equation.

1997 2 UNIT TRIAL SOLUTIONS

Q1 i) $x^2 = 6x$

$x^2 - 6x = 0$

$x(x-6) = 0$

$x = 0$ or 6. 2

ii) 8.63 3 Sig Figs 2

iii) 5 1

iv) $-\frac{1}{\sqrt{2}}$ 1

v) $\frac{4(a-1)(a+1)}{a+1} = 4(a-1)$ 2

vi) $\frac{\sqrt{2}}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = 2+\sqrt{2}$ 2

vii) $\sqrt{60} \div \sqrt{5} = \sqrt{12}$ 2

$= 2\sqrt{3}$ $a = 3$

2 i) $4 - 2x = 6$ or $4 - 2x = -6$

$x = -1$ $x = 5$ 2

ii) $9^x = \frac{1}{27}$

$3^{2x} = 3^{-3}$

$2x = -3$

$x = -1\frac{1}{2}$ 2

iii) ORIGINAL AREA = 132 cm²

NEW AREA = 210 cm²

% INCREASE $= \frac{78}{132} \times 100$

$= 59.1\%$ 2

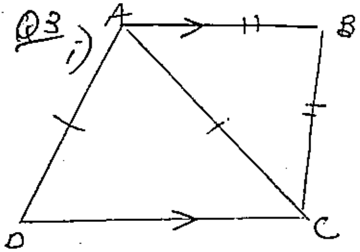
) $8ab$ 2

) $S = ut + \frac{1}{2}at^2$

$a = \frac{2S - 2ut}{t^2}$ 2

vi) $3(a-4)(a+4)$ 2

Q3 i)



1

Let $D\hat{A}C = x°$

Then $A\hat{C}D = (90 - \frac{x}{2})°$ (Isosceles)

$B\hat{A}C = (90 - \frac{x}{2})°$ (Alternate, AB

Then $A\hat{B}C = x°$ (Isos, Angle Sum

ie $D\hat{A}C = A\hat{B}C$

ii) Triangles are Similar, AB//

Then $\frac{AB}{CD} = \frac{AX}{XD} = \frac{BX}{XC}$

$\frac{15}{CD} = \frac{20}{8}$

$CD = 6$. 1

$\frac{20}{8} = \frac{BX}{5}$

$BX = 12.5$ 1

iii) In Δ's ABC and BFD,

$\angle B$ is Common 1

$\therefore a + x = b + y$ (Angle Sum 1

If $a = b$, $a + x = a + y$ 1

$x = y$ 1

4 i) a) $a^2 = b^2 + c^2 - 2bc \cos A$ 1

$= 25 + 16 - 2 \times 5 \times 4 \times \frac{1}{8}$ 1

$= 41 - 5$

$a^2 = 36$

$a = 6$ 1

b) $\cos A = \frac{1}{8}$

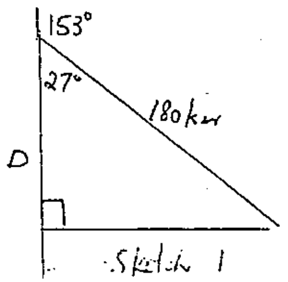
$A = 83°$ Nearest degree 1

c) Area $= \frac{1}{2} bc \sin A$ 1

$= \frac{1}{2} \times 5 \times 4 \times \sin 83°$

Area $= 9.9$ cm² 1 Dec Pl 1

)



Sketch 1

$\cos 27° = \frac{D}{180}$ 1

$D = 160.4$ km

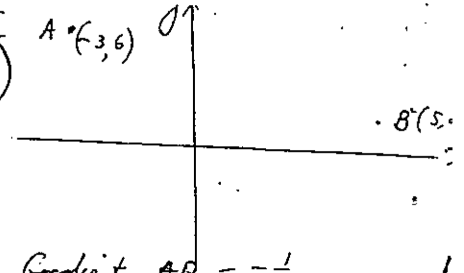
1

) $\frac{a}{\sin 47°} = \frac{14.6}{\sin 73°}$ 1

$a = \frac{14.6 \times \sin 47°}{\sin 73°}$ 1

$a = 11.17$ 2 Dec Pl. 1

Q5 i)



a) Gradient AB $= -\frac{1}{2}$ 1

Equation AB: $y - 2 = -\frac{1}{2}(x - 5)$

$2y - 4 = -x + 5$

$x + 2y - 9 = 0$ 1

b) (5, 2) $y - 2 = 2(x - 5)$

$y - 2 = 2x - 10$

Line k: $2x - y - 8 = 0$

c) $2x - y - 8 = 0$

Sub $(2, -4)$: $4 + 4 - 8 = 0$ True

ie k passes through (2,

d) Side BP has gradient 2

Side AB has gradient $-\frac{1}{2}$

$2 \times -\frac{1}{2} = -1$

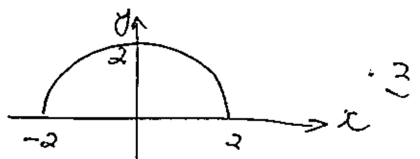
ie Δ ABP is Rt $\angle$ d

e) Area $= \frac{1}{2} \times AB \times BP$

$= \frac{1}{2} \times \sqrt{80} \times \sqrt{45}$

$= 30$ UNITS²

ii)



DOMAIN: $-2 \le x \le$

RANGE $0 \le y \le$

Q6 i) a) $y = 4x^3 - \frac{1}{x^2}$

$\frac{dy}{dx} = 12x^2 + \frac{2}{x^3}$ 2

b) $y = xe^x$

$\frac{dy}{dx} = xe^x + e^x$ 2

c) $y = \frac{x+2}{x-3}$

$\frac{dy}{dx} = \frac{(x-3) - (x+2)}{(x-3)^2}$

$= \frac{-5}{(x-3)^2}$ 2

ii) $\int_1^2 \left(x + \frac{1}{x^2}\right) dx = \left[\frac{x^2}{2} - \frac{1}{x}\right]_1^2$

$= \left(2 - \frac{1}{2}\right) - \left(\frac{1}{2} - 1\right)$

$= 2$ 2

iii) $\int_0^2 x e^{x^2} = \left[\frac{1}{2} e^{x^2}\right]_0^2$

$= \frac{1}{2}(e^4 - 1)$ 2

iv) $\int_2^3 (3x-4)^5 = \left[\frac{(3x-4)^6}{18}\right]_2^3$

$= \frac{1}{18}(5^6 - 2^6)$

$= 864 \cdot 5$ 2

Q7 i) $3x^2 + 5x - 1 = 0$

a) $\alpha + \beta = -\frac{5}{3}$ 1

b) $\alpha\beta = -\frac{1}{3}$ 1

c) $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2+\beta^2}{(\alpha\beta)^2}$

$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$

$= \frac{\frac{31}{9}}{\frac{1}{9}}$

$= 31$ 1

ii) For equal roots

$b^2 - 4ac = 0$ 1

$k^2 - 4 \times 5 \times 1 = 0$

$k^2 = 20$

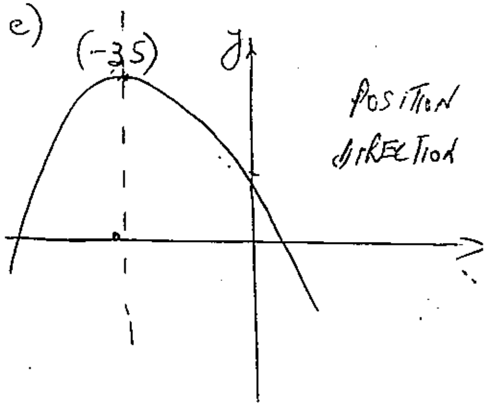
$k = \pm\sqrt{20}$ 1

iii) a) Focus $(-3, 0)$

b) Focal length 5 UNIT

c) DIRECTRIX: $y = 10$

d) LATUS RECTUM $= 20$ UN

e) 

POSITION

DIRECTION

8 i) Arithmetic Progression

$a = 18400$, $d = 1200$, $n = 10$

a) $T_{10} = a + 9d$ 1

$= 18400 + 9 \times 1200$

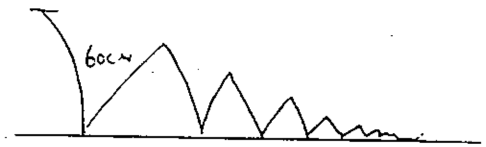
$= \$29200$ 1

b) $S_{10} = \frac{n}{2}(a + l)$ 1

$= 5(18400 + 29200)$

$= \$238000$ 1

ii) GEOMETRIC Progression 1

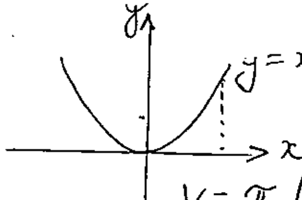


Limiting Sum $\times 2$ $- 60$ 1 (OR SIMILAR)

Total $= 2 \times \frac{a}{1-r} - 60$

$= 2 \times \frac{60}{\left(1 - \frac{5}{6}\right)} - 60$ 1

Total $= 660$ cm 1

iii) 

Sketch 1

$V = \pi \int y^2 dx$ 1

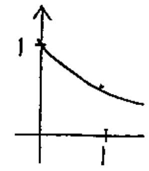
$= \pi \int_0^3 x^4 dx$ 1

$= \pi \left[\frac{x^5}{5}\right]_0^3 = \frac{243\pi}{5}$

Q9 i) $y = \frac{1}{x^2+1}$

x	0	1	2
y	1	0.5	0.2

1



$h = 1$.

$\int_0^2 \frac{1}{x^2+1} dx = \frac{1}{3}\left[1 + 0.2 + 4\times\right.$

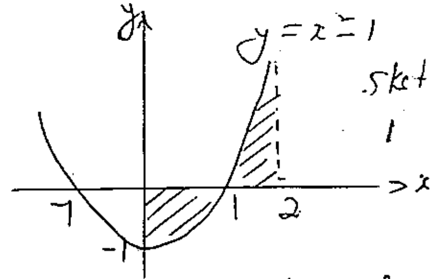
$= 1 \cdot 1$ 1 Dec 1

ii) a) $\int_0^2 (x^2 - 1) dx$

$= \left[\frac{x^3}{3} - x\right]_0^2$ 1

$= \frac{8}{3} - 2$

$= \frac{2}{3}$ 1

b) 

Sketch 1

Area $= \left|\int_0^1 (x^2-1) dx\right| + \int_1^2 (x^2$

$= \left|\left[\frac{x^3}{3} - x\right]_0^1\right| + \left[\frac{x^3}{3} - x\right]_1^2$

$= \left|\frac{1}{3} - 1\right| + \left(\frac{8}{3} - 2\right) - \left(\frac{1}{3}\right.$

$= \frac{2}{3} + \frac{4}{3}$

Area $= 2$ UNITS^2 1

Q + ...) OVER

iii) $\int_a^b f(x)\,dx$ is not equal to the area between $y = f(x)$ and ordinates $x = a$, $x = b$ if the function crosses the x Axis between the limits of the integral. 2

Question 10

i) a) $y = 4 + 3x - x^3$

$\frac{dy}{dx} = 3 - 3x^2 = 0$ for stationary points 1

$3x^2 = 3$

$x^2 = 1$ 1

$x = \pm 1$ $(1, 6)$ 1

$(-1, 2)$ 1

$\frac{d^2y}{dx^2} = -6x$ or test from $\frac{dy}{dx}$ 1

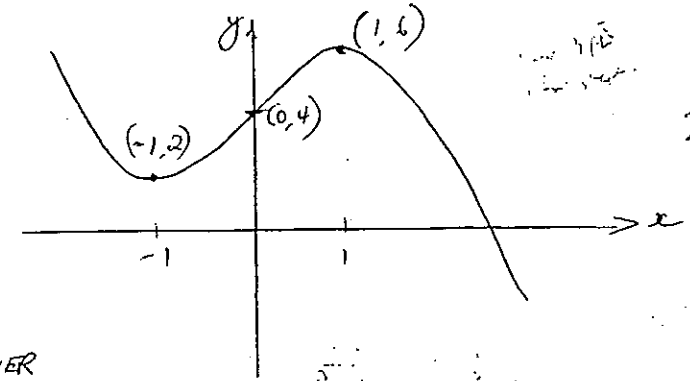
When $x = 1$, $f''(x) < 0$ $\therefore$ MAXIMUM at $(1, 6)$

When $x = -1$, $f''(x) > 0$ $\therefore$ MINIMUM at $(-1, 2)$ 1

b) When $f''(x) = 0$, $-6x = 0$ for inflexion 1

$x = 0$

Must have: Test $x < 0$, $f''(x) > 0$; $x > 0$, $f''(x) < 0$ } ie. Inflexion at (0 1

c) 

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Part ii) OVER

(10) (i) Let X be the intersection of the diagonals.

$\angle XAD = \angle XAC = \theta$ [property of rhombi]

$AX = 2\cos\theta \Rightarrow AC = 4\cos\theta$

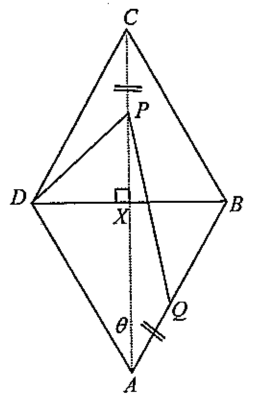
$\therefore AP = 4\cos\theta - x$

The shaded area is the sum of triangles ADP and APQ.

$$S = \frac{1}{2}\times 2\times\left(4\cos\theta - x\right)\sin\theta + \frac{1}{2}\times\left(4\cos\theta - x\right)\times x\sin\theta$$

$$= \frac{\sin\theta}{2}\left(4\cos\theta - x\right)\left(x+2\right)$$

[**NB** S is a concave down parabola in x]



(ii) $$S = \frac{\sin\theta}{2}\left[8\cos\theta + \left(4\cos\theta - 2\right)x - x^2\right]$$

$$\frac{dS}{dx} = \frac{\sin\theta}{2}\left[\left(4\cos\theta - 2\right) - 2x\right] = \sin\theta\left(2\cos\theta - 1 - x\right)$$

$$\therefore \frac{dS}{dx} = 0 \Rightarrow x = 2\cos\theta - 1 \qquad [\because \sin\theta \neq 0]$$

(iii) $$\frac{dS}{dx} = \sin\theta\left(2\cos\theta - 1 - x\right)$$

$$\therefore \frac{d^2S}{dx^2} = -\sin\theta \qquad \left[< 0 \text{ for } 0 < \theta < \frac{\pi}{2}\right]$$

(iv) $\theta = \dfrac{\pi}{6}$

$$\frac{dS}{dx} = 0 \Rightarrow x = 2\cos\left(\frac{\pi}{6}\right) - 1 = \sqrt{3} - 1$$

$\dfrac{d^2S}{dx^2} < 0 \Rightarrow S$ is a maximum

$$AC = 4\cos\left(\frac{\pi}{6}\right) = 2\sqrt{3}$$

$$\therefore \frac{PC}{AC} = \frac{\sqrt{3}-1}{2\sqrt{3}}$$

(v) $\theta = \dfrac{\pi}{4}$

$$\frac{dS}{dx} = 0 \Rightarrow x = 2\cos\left(\frac{\pi}{4}\right) - 1 = \sqrt{2} - 1$$

$\dfrac{d^2S}{dx^2} < 0 \Rightarrow S$ is a maximum

$$AC = 4\cos\left(\frac{\pi}{4}\right) = 2\sqrt{2}$$

$$\therefore \frac{PC}{AC} = \frac{\sqrt{2}-1}{2\sqrt{2}}$$

So the statement is **FALSE.**

(vi) If $\theta = \dfrac{\pi}{3}$ then

$$\frac{dS}{dx} = 0 \Rightarrow x = 2\cos\left(\frac{\pi}{3}\right) - 1 = 0$$

$\dfrac{d^2S}{dx^2} < 0 \Rightarrow S$ is a maximum

So if $\theta = \dfrac{\pi}{3}$ then S **STARTS** at its maximum value and then decreases to 0.

So the statement is **FALSE.**