Trinity Grammar School

Mathematics Department

2015

TRIAL EXAMINATION

HSC ASSESSMENT TASK 4

Year 12

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Only approved calculators for this course are allowed in this task
- A table of Standard Integrals is supplied
- Show all necessary working
- Write your Board of Studies Student Number and your Class teacher on the question paper and on any answer sheets or writing booklets used to write your responses to the questions submitted
- If you do not attempt a question you must submit an answer sheet or writing booklet for that question clearly indicating **N/A** and your Student Number or Name.

- **HSC Assessment Weighting:** 40%

Board of Studies Student Number
(Year 12 only)

Class Teacher:

……

Name:
(Year 11 only)

……

Do NOT write solutions on this question paper. Any working on this question paper will NOT be marked.

Date of examination:

Thursday, 13 August 2015

Total marks – 100

Section I

10 marks

- Attempt Questions 1-10
- Allow about **15** minutes for this section

Section II

90 marks

- Attempt Questions 11-16
- Allow about **2** hour **45** minutes for this section

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Section I 10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section
- Shade the correct response on the multiple-choice answer sheet for Questions 1 – 10
- Each question is worth 1 mark

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1. What is the value of $\lim_{x\to 3}\dfrac{x^2+2x-15}{x-3}$?

(A) 0

(B) 3

(C) 5

(D) 8

2. The solution to $\sum_{k=7}^{10}(3k-1)$ is

(A) 29

(B) 49

(C) 89

(D) 98

3. What is the value of $\int_0^1 (e^{3x}+1)dx$?

(A) $\dfrac{1}{3}e^3$

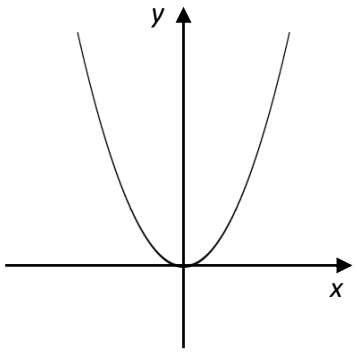
(B) e^3

(C) $\dfrac{1}{3}(e^3+1)$

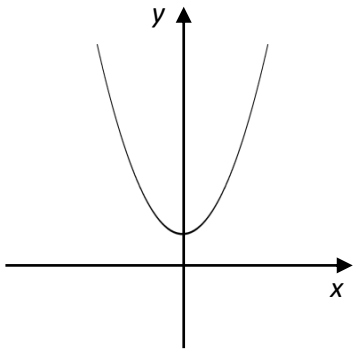
(D) $\dfrac{1}{3}(e^3+2)$

4. Which graph represents a quadratic equation which is positive definite?

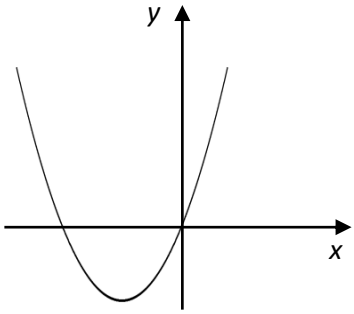
(A)



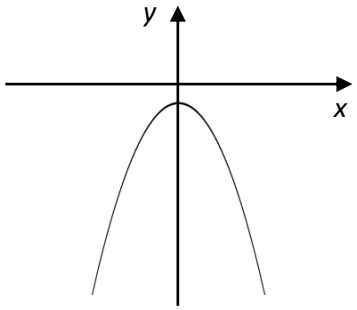
(B)



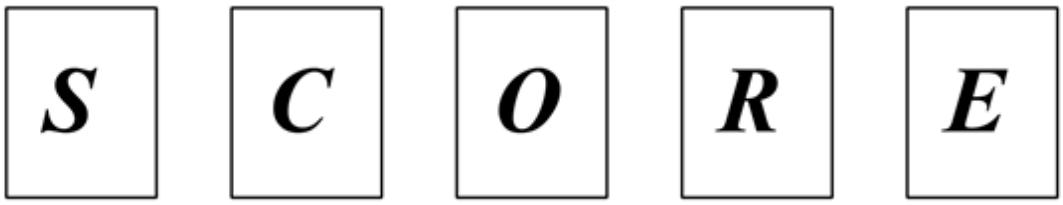
(C)



(D)



5. The letters of the word SCORE are written on separate cards.



When 2 cards are drawn at random the total number of possible outcomes is 20.

Which statement is true?

(A) The first card is drawn with replacement

(B) The first card is drawn without replacement

(C) The second card is drawn with replacement

(D) The second card is drawn without replacement

6. The domain of $y = \log_e(x+2)$ is

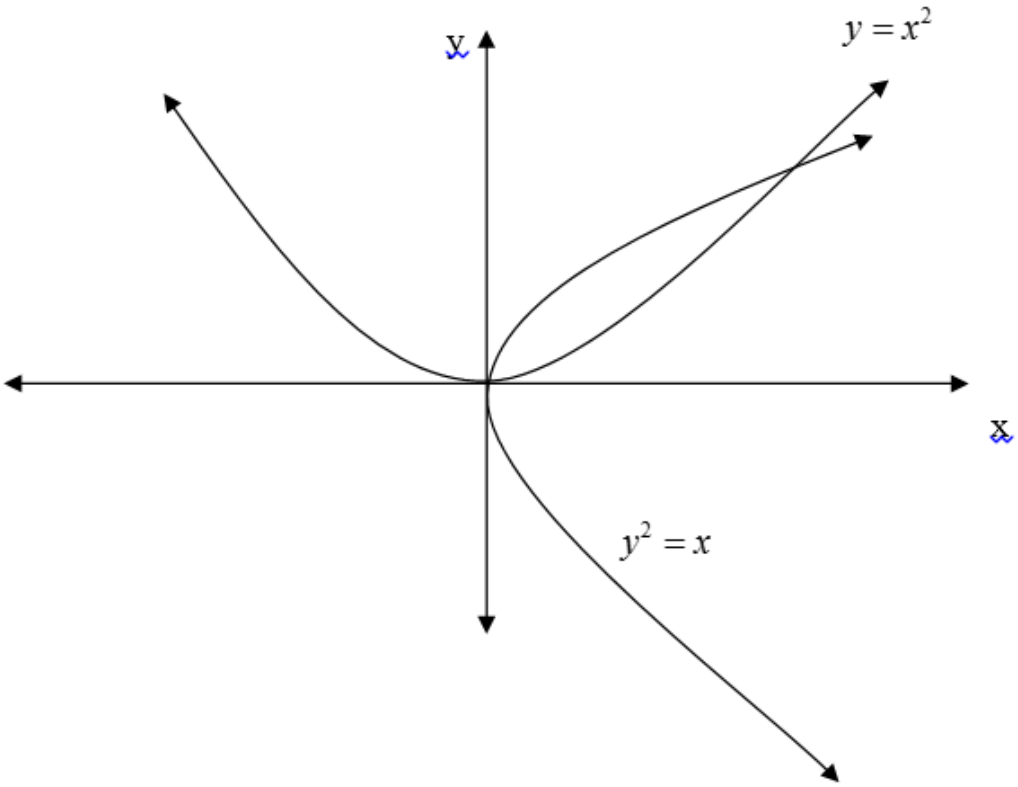
(A) $x > 0$

(B) $x > 1$

(C) $x > -3$

(D) $x > -2$

7.



Not to scale

The area between the curves can be calculated using

(A) $\int_0^1 \sqrt{x} - x^2 dx$

(B) $\int_0^1 y - y^2 dy$

(C) $\int_0^1 x^2 - x dx$

(D) $\int_{-1}^1 x^2 - x dx$

8. When simplified fully $\cos^2\left(\frac{\pi}{2}-\theta\right)\cot\theta$ is:

(A) $\cos^2\theta\cot\theta$

(B) $\sin^2\theta\cot\theta$

(C) $\sin\theta\cos\theta$

(D) $\frac{\sin^3\theta}{\cos\theta}$

9. The acceleration of a particle moving in a straight line is given by the formula $a=12t+6$. Initially the particle is at $x=5$ metres and the initial velocity of the particle is –36 m/s. When is the particle at rest?

(A) $t=0$
(B) $t=1$
(C) $t=2$
(D) $t=3$

10. What are the coordinates of the focus of the parabola $x^2=6y+2x+11$?

(A) $\left(-\frac{3}{2},1\right)$

(B) $\left(-\frac{1}{2},1\right)$

(C) $\left(1,-\frac{3}{2}\right)$

(D) $\left(1,-\frac{1}{2}\right)$

End of Section I
Section II commences on the next page

Section II 90 marks

- Attempt Questions 11 – 16
- Allow about 2 hour and 45 minutes for this section
- Answer each question in a SEPARATE writing booklet or answer sheet. Extra writing booklets are available.
- In Questions 11 – 16, your responses should include all relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet **Marks**

(a) Simplify $\frac{x^2-4}{2x+4}$. **2**

(b) Differentiate $(5x+2)^4$. **2**

(c) Solve

(i) $|2x+3|>1$. **2**

(ii) $5x^2-9x-2=0$. **2**

(d) Find $\int \frac{x}{x^2+3}dx$. **2**

(e) Evaluate $\int_0^{\frac{\pi}{2}} \sec^2 \frac{x}{3}dx$. **2**

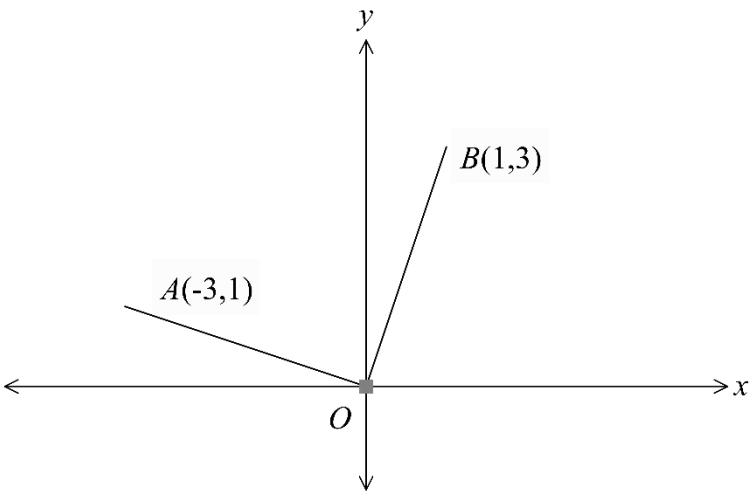
(f) Solve $f'(x)=0$, where $f(x)=3xe^{3x}$. **3**

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet **Marks**

(a) Points $A(-3,1)$ and $B(1,3)$ are on a number plane.

Copy the diagram into your writing booklet.



(i) Find the gradient of line *OA*. **1**

(ii) Show that *OA* is perpendicular to *OB*. **1**

(iii) *OACB* is a quadrilateral in which *BC* is parallel to *OA*. **2**
Show that the equation of *BC* is $x+3y-10=0$.

(iv) The point *C* lies on the line $x=-2$. **1**
What are the coordinates of point *C*?

(v) Show that the length of the line *BC* is $\sqrt{10}$. **1**

(vi) Find the area of *OACB*. **1**

(b) The table shows the values of a function $f(x)$ for five values of x. **2**

x	1	1.5	2	2.5	3
$f(x)$	4	1.5	–2	2.5	8

Use Simpson's rule with these five function values to estimate $\int_1^3 f(x)dx$.

Question 12 continues

Question 12 (Continued)

(c) Two dice are thrown. The probability the sum of the two dice is:

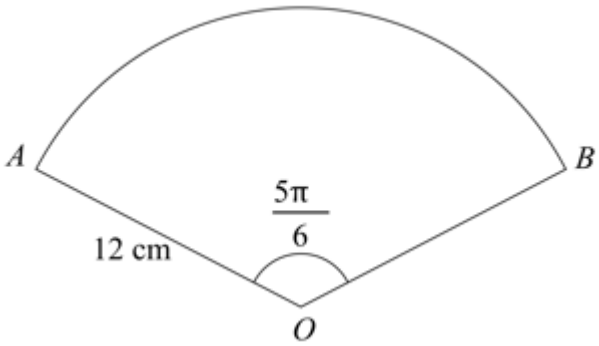
(i) seven or eleven **1**

(ii) multiple of three **1**

(d) In the diagram, AB is an arc of a circle with centre O. **1**

The radius $OA = 12$ cm and the angle AOB is $\frac{5\pi}{6}$ radians.

Find the length of the arc AB.



(e) Consider the functions $y = x^2$ and $y = x^2 - 3x + 2$.

(i) Sketch the two functions on the same axes. **2**

(ii) Hence or otherwise find the values of x such that $x^2 > (x-1)(x-2)$. **1**

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet **Marks**

(a) Jack drops a super bouncy ball from the top of a 56 m building on to a concrete surface below. Its first rebound is 42 m, and each subsequent rebound is three quarters the height of the previous one.

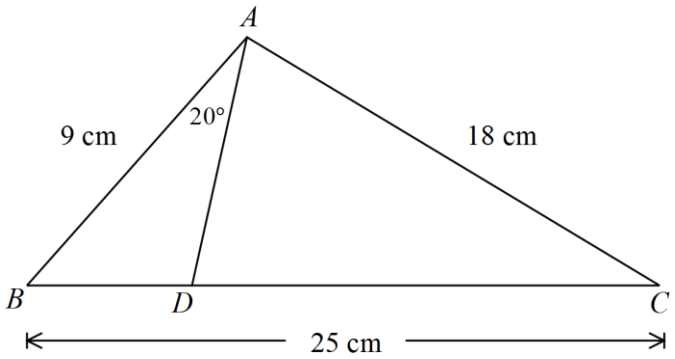
(i) How high will it rise on the fifth rebound? **2**

(ii) How far will it travel in total? **1**

(b) For the domain $0 \leq x \leq 6$, a function $y = f(x)$ satisfies $f'(x) < 0$ and $f''(x) < 0$. **2**

Sketch a possible graph of $y = f(x)$ in this domain.

(c) In ΔABC, $AB = 9$ cm, $AC = 18$ cm and $BC = 25$ cm.
Point D lies on BC and $\angle BAD = 20°$, as shown in the diagram.



NOT TO SCALE

(i) Show that $\angle ABC$ is approximately $32°$. **1**

(ii) Find the length of BD, correct to 1 decimal place. **2**

(iii) Hence, or otherwise, find the area of ΔADC, correct to the nearest cm^2. **2**

(d) (i) Differentiate $\dfrac{x^2 - 2}{x^2 + 2}$. **2**

(ii) Hence evaluate $\displaystyle\int_2^4 \frac{x}{\left(x^2 + 2\right)^2}\, dx$ **3**

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet **Marks**

(a) A function $f(x)$ is defined by $f(x) = x^2(3-x)$.

(i) Find the stationary points for the curve $y = f(x)$ and determine their nature. **2**

(ii) Sketch the graph of $y = f(x)$ showing the stationary points and x-intercepts. **2**

(iii) Find the equation of the tangent to the curve at the point $P(1,2)$. **1**

(b) The quadratic equation $2m^2 - 3m + 6 = 0$ has roots α and β.

Find the value of:

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

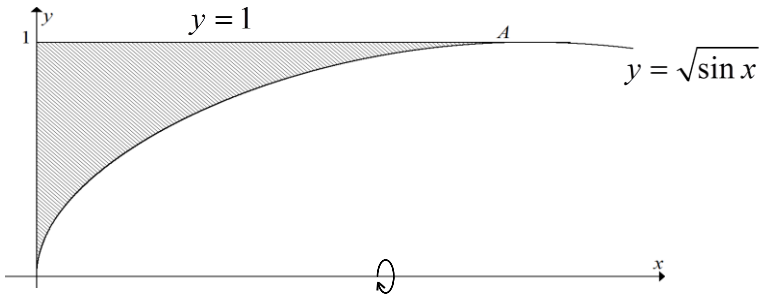
(iii) $\alpha^2 + \beta^2$ **1**

(c) The displacement of a object at time (t) seconds is given by: **3**

$$x = 3e^{-2t} + 10e^{-t} + 4t$$

Find the time the object comes to rest.

(d) The region bounded by the curve $y = \sqrt{\sin x}$, the y-axis and the line $y = 1$ is rotated around the x-axis to form a solid.



(i) $y = \sqrt{\sin x}$ and $y = 1$ meet at the point A.
Find the coordinates of A. **1**

(ii) Find the volume of the solid. **3**

End of Question 14

Question 15 (15 marks) Use a SEPARATE writing booklet **Marks**

(a) Solve for x: $2^x = 5$ **2**

(b) The radiation in a rock after a nuclear accident was 8,000 becquerel (bq). One year later, the radiation in the rock was 7,000 bq. It is known that the radiation in the rock is given by the formula:

$$R = R_0 e^{-kt}.$$

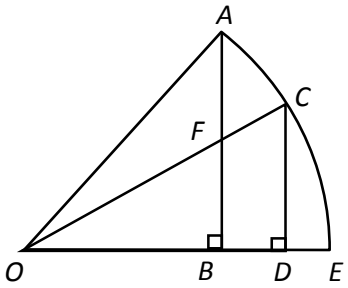
where R_0 and k are constants and t is the time measured in years.

(i) Evaluate the constants R_0 and k. **2**

(ii) What is the radiation of the rock after 10 years? **1**
Answer correct to the nearest whole number.

(iii) The region will become safe when the radiation of the rock reaches 50 bq. After how many years will the region become safe? **2**

(c) In the diagram, ACE is an arc of a circle with centre O and radius $2\sqrt{13}$ cm.
$\angle OBA = \angle ODC = 90°$, $OB = CD = 4$ cm.



NOT TO SCALE

(i) Prove that ΔOAB is congruent to ΔOCD. **3**

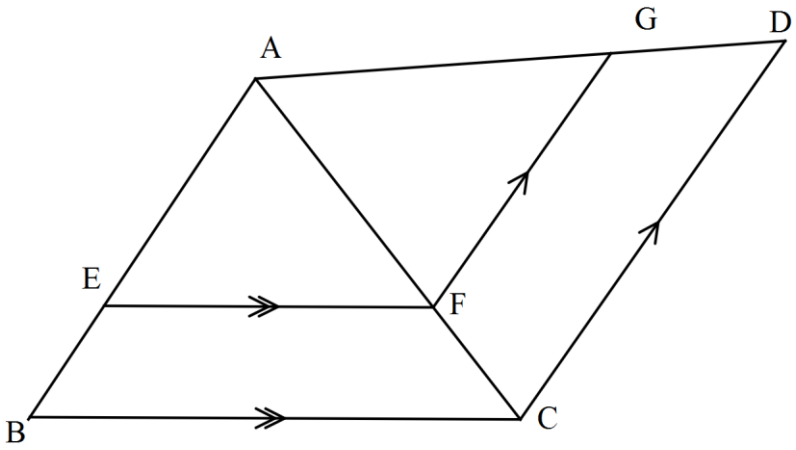
(ii) Find the length of BD. **1**

(iii) ΔFOB is similar to ΔCOD. (Do NOT show this.)
Find the area of $BDCF$. **2**

Question 15 continues

Question 15 (continues) **Marks**

(d) In the figure below, EF || BC and CD || FG.



Prove that $\frac{AE}{AB} = \frac{AG}{AD}$ **2**

End of Question 15

Question 16 (15 marks) Use a SEPARATE writing booklet **Marks**

(a) Georgina borrows \$650 000 to purchase her first home. She takes out a loan over 30 years, to be repaid in equal monthly instalments. The interest rate is 5.4% per annum reducible, calculated monthly.

(i) Show that the amount, $\$A_n$, owing after the nth repayment is given by the formula: **2**

$$A_n = 650000(1.0045)^n - M(1 + 1.0045 + 1.0045^2 + ... + 1.0045^{n-1})$$

(ii) Find the monthly repayment required to repay the loan in 30 years. **2**

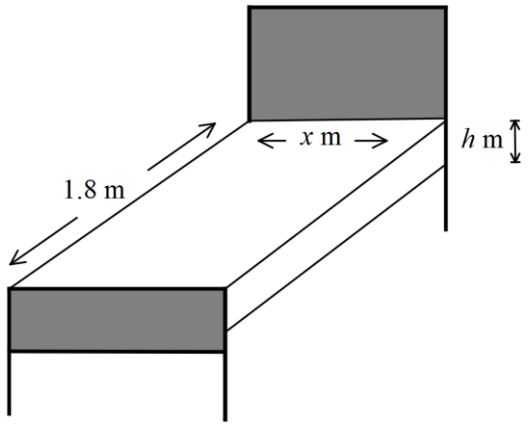
(iii) Georgina wants to pay off the loan in less than 30 years. If she can afford to pay \$5 000 per month, how many months will it take her to pay off the home loan? **2**

(iv) How much will Georgina save in interest if she pays \$5 000 per month? **1**

Question 16 continues

Question 16 (continued) **Marks**

(b) A mattress is sitting in a bedframe which has a headboard and a footboard. The length of the mattress is 1.8 m, its width is x m and its height is h m, as shown in the diagram. A blanket is the same length as the mattress and its width is enough to cover exactly the width of the mattress and its two sides.



NOT TO SCALE

(i) If the volume of the mattress is $0.504\,\text{m}^3$, show that the area A of the blanket is given by **2**

$$A = 1.8\left(x + \frac{0.56}{x}\right).$$

(ii) Hence, or otherwise, find the area of the smallest blanket needed to cover the mattress, correct to 2 decimal places. **3**

(c) Find the value of n such that: **3**

$$\frac{10^{3n} \times 25^{n+2}}{8^n} = 1$$

End of Question 16

End of Section II

End of Examination

SUGGESTED MARKING GUIDELINES for TGS 2015 HSC Mathematics Trial

Section I

1. D 2. D 3. D 4. B 5. B 6. D 7. A 8. C 9. C 10. D

Section II

Question 11

(a) 2 marks

Outcomes Assessed: P3

Targeted Performance Bands: 2-3

Criteria	Marks
• Gives correct expression	2
• Factorises difference of two squares	1

Sample Answer

$$\frac{(x-2)(x+2)}{2(x+2)}$$
$$=\frac{x-2}{2}$$

11(b) 2 marks

Outcomes Assessed: P7

Targeted Performance Bands: 3-4

Criteria	Marks
• Differentiates correctly	2
• Progress towards the derivative	1

Sample Answer

$$4(5x+2)^3 \times 5$$
$$=20(5x+2)^3$$

11(c)

(i) 2 marks

Outcomes Assessed: P4

Targeted Performance Bands: 2-3

Criteria	Marks
• Gives correct solution	2
• Gives one correct solution	1

Sample Answer

$2x+3>1$ or $2x+3<-1$

$2x>-2$ $2x<-4$

$x>-1$ $x<-2$

(ii) 2 marks

Outcomes Assessed: P4

Targeted Performance Bands: 2-3

Criteria	Marks
• Gives correct solutions	2
• Factorises expression correctly	1

Sample Answer

$(5x+1)(x-2)=0$

$x=-\frac{1}{5}$ or $x=2$

11(d) 2 marks

Outcomes Assessed: H5

Targeted Performance Bands: 3-4

Criteria	Marks
• Gives the correct solution with or without the constant of integration *C*	2
• Progress towards the correct primitive	1

Sample Answer

$$\frac{1}{2}\int \frac{2x}{x^2+3}dx$$

$$= \frac{1}{2}\ln\left(x^2+3\right)+C$$

11(e) 2 marks

Outcomes Assessed: H5

Targeted Performance Bands: 3-4

Criteria	Marks
• Gives the correct solution	2
• Integrates correctly	1

Sample Answer

$$\left[3\tan\frac{x}{3}\right]_0^{\frac{\pi}{2}}$$

$$= 3\tan\frac{\pi}{6} - 3\tan 0$$

$$= \frac{3}{\sqrt{3}} = \sqrt{3} \quad or \quad \approx 1.732.....$$

11(f) 3 marks

Outcomes Assessed: H3, H4

Targeted Performance Bands: 3-4

Criteria	Marks
• Gives the correct solution	3
• Differentiates correctly	2
• Progress towards the correct derivative	1

Sample Answer

$f'(x) = 3e^{3x} + 9xe^{3x}$

$3e^{3x}(1+3x) = 0 \qquad (e^{3x} \neq 0)$

$\therefore x = -\frac{1}{3}$

Question 12

12(a) (i)	Gradient of *OA*: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-1}{0--3} = -\frac{1}{3}$	1 Mark: Correct answer.
12(a) (ii)	Gradient of *OB*: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-3}{0-1} = 3$ Perpendicular lines occur when $m_1 m_2 = -1$ $m_1 m_2 = -1$ $-\frac{1}{3} \times 3 = -1$ True	1 Mark: Correct answer.
12(a) (iii)	If *BC* is parallel to *OA* then it has the same gradient or $m = -\frac{1}{3}$ $y - y_1 = m(x - x_1)$ $y - 3 = -\frac{1}{3}(x-1)$ $3y - 9 = -x + 1$ or $x + 3y - 10 = 0$	2 Marks: Correct answer. 1 Mark: Uses the gradient intercept form with at least 1 correct value.

12(a) (iv)	The point *C* lies on the line $x = -2$ Substitute –2 for *x* into $x + 3y - 10 = 0$	1 Mark: Correct answer.

<table>
<tr><td></td><td>$-2+3y-10=0$
$3y=12$ or $y=4$
Coordinates of C are $(-2,4)$.</td><td></td></tr>
<tr><td>12(a)
(v)</td><td>$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
$BC=\sqrt{(-2-1)^2+(4-3)^2}=\sqrt{10}$</td><td>1 Mark: Correct answer.</td></tr>
<tr><td>12(a)
(vi)</td><td>Quadrilateral OACB is a square (Rectangle with all sides equal)
$A=s^2=\left(\sqrt{10}\right)^2=10$ square units</td><td>1 Mark: Correct answer</td></tr>
<tr><td>12(b)</td><td>$\int_1^3 f(x)dx\approx\frac{h}{3}\left[y_0+y_4+4(y_1+y_3)+2y_2\right]$
$\approx\frac{0.5}{3}\left[4+8+4(1.5+2.5)+2\times-2\right]\approx 4$</td><td>2 Marks: Correct answer.
1 Mark: Uses Simpson's rule.</td></tr>
<tr><td>12(c)</td><td>i) P(7 or 11) = $\frac{8}{36}$
$=\frac{2}{9}$
ii) P(*3) = $\frac{12}{36}$
$=\frac{1}{3}$
<table>
<tr><th></th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th></tr>
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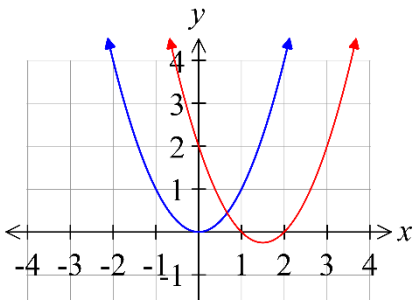
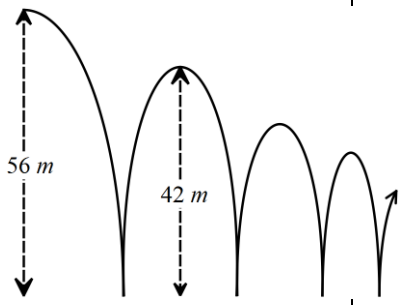
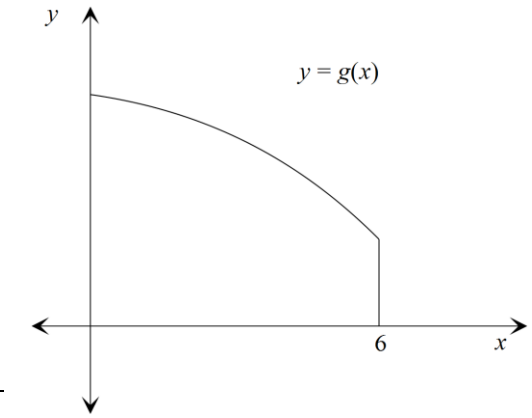
<tr><th>6</th><td>7</td><td>8</td><td>9</td><td>10</td><td>11</td><td>12</td></tr>
</table></td><td>1 mark correct
1 mark correct</td></tr>
<tr><td>12(d)</td><td>$l=r\theta$
$=12\times\frac{5\pi}{6}=10\pi$ radians</td><td>1 Mark: Correct answer.</td></tr>
<tr><td>12(e)
(i)</td><td>$y=x^2-3x+2=(x-1)(x-2)$
 
</td><td>2 Marks: Correct answer.
1 Mark: One graph drawn correctly.</td></tr>
</table>

12(e) (ii)	Now $y = x^2 > (x-1)(x-2)$ Point of intersection from the graph is $x = \frac{2}{3}$ Alternatively $x^2 = (x-1)(x-2)$ $x^2 = x^2 - 3x + 2$ $3x = 2$ or $x = \frac{2}{3}$ Therefore $x^2 > (x-1)(x-2)$ when $x > \frac{2}{3}$	1 Mark: Correct answer.

Question 13

(a)	(i) Taking 42 as the first term as it is the first completed rebound.  $T_n = ar^{n-1}$ For the rise on the 5th bounce $a = 42 \quad r = \frac{3}{4} \quad n = 5$ $T_5 = 42\left(\frac{3}{4}\right)^{5-1}$ $T_5 = 42\left(\frac{3}{4}\right)^{4}$ $\approx 13.29m$ Note if take 56 as first term, need to find the 6th term.	2	1 for determining the series 1 for finding correct term (only 1 mark if found T_5 after using a = 56.)

(a)	(ii) $\|r\| < 1 \qquad r = \frac{3}{4}$ Consider one bounce up and down as a term, so $t_1 = 84$ $\therefore s_\infty = \frac{a}{1-r}$ $= \frac{84}{1-\frac{3}{4}}$ $= \frac{84}{\frac{1}{4}}$ $= 336m$ Total distance travelled will be $336 + 56 = 392m$ Alternately take 42 as first term and double result from $S\infty$	1	1 for correct answer

(b)	$f'(x) < 0 \therefore$ negative gradient (decreasing) $f''(x) < 0 \therefore$ concave down  	2	Graph needs to have negative gradient (1 mark) and concave down (1 mark).

(c)

(i) 1 mark

Outcomes Assessed: P4

Targeted Performance Bands: 3-4

Criteria	Marks
• Gives the correct answer	1

Sample Answer

$\text{let } \angle ABC = \theta$

$$\therefore \cos\theta = \frac{9^2 + 25^2 - 18^2}{2 \times 9 \times 25}$$

$$\theta \approx 32° \,(\text{nearest degree})$$

(ii) 2 marks

Outcomes Assessed: P4

Targeted Performance Bands: 3-4

Criteria	Marks
• Gives the correct solution	2
• Correct substitution into the sine rule	1

Sample Answer

$$\frac{BD}{\sin 20°} = \frac{9}{\sin 128°}$$

$$BD \sin 128° = 9 \sin 20°$$

$$BD = \frac{9 \sin 20°}{\sin 128°} \approx 3.9\,\text{cm}\,(1 \text{ decimal place})$$

(iii) 2 marks

Outcomes Assessed: P4

Targeted Performance Bands: 3

Criteria	Marks
• Gives the correct solution	2
• Progress towards the solution	1

Sample Answer

Method 1

$$A=\left(\frac{1}{2}\times 9\times 25\times \sin 32°\right)-\left(\frac{1}{2}\times 9\times 3.9\times \sin 32°\right)$$

$$\approx 50\,\text{cm}^2 \text{ (nearest cm}^2\text{)}$$

Method 2

$$\frac{AD}{\sin 32°}=\frac{3.9}{\sin 20°}$$

$$AD\approx 6.04\,cm\,(2\,dp)$$

$$\therefore A=\frac{1}{2}\times 6.04\times 21.1\times \sin 52°$$

$$\approx 50\,\text{cm}^2 \text{ (nearest cm}^2\text{)}$$

(d)

(i) $u=x^2-2 \qquad v=x^2+2$ **2**

$u'=2x \qquad v'=2x$

$$\frac{d}{dx}\left(\frac{x^2-2}{x^2+2}\right)=\frac{2x\left(x^2+2\right)-2x\left(x^2-2\right)}{\left(x^2+2\right)^2}$$

$$=\frac{8x}{\left(x^2+2\right)^2}$$

(ii) $$\int_2^4 \frac{x}{\left(x^2+2\right)^2}\,dx=\frac{1}{8}\int_2^4 \frac{8x}{\left(x^2+2\right)^2}\,dx$$ **3**

$$=\frac{1}{8}\left[\frac{x^2-2}{x^2+2}\right]_2^4 \qquad \text{(from i.)}$$

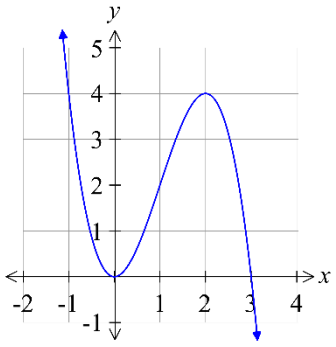
$$=\frac{1}{8}\left(\frac{16-2}{16+2}-\frac{4-2}{4+2}\right)$$

$$=\frac{1}{18}$$

Question 14

14(a) (i)	$f(x)=x^2(3-x)=3x^2-x^3$	2 Marks: Correct answer.

	Stationary points $f'(x)=0$ $6x-3x^2=0$ $3x(2-x)=0$ $x=0,\ x=2$ Stationary points are $(0,0)$ and $(2,4)$. $f''(x)=6-6x$ At $(0,0)$, $f''(0)=6>0$, Minimum stationary point At $(2,4)$, $f''(1)=-6<0$, Maximum stationary point	1 Mark: Finds one of the stationary points or recognises $6x-3x^2=0$.

14(a) (ii)	*x*-intercepts ($y=0$) $x^2(3-x)=0$ $x=0,\ x=3$ Max (2,4) Min (0,0)  	2 Marks: Correct answer. 1 Mark: Makes some progress towards sketching the curve.
14(a) (iii)	$f'(x)=6x-3x^2$ At the point $P(1,2)$ $f'(1)=6\times1-3\times1^2=3$ Point slope formula $y-y_1=m(x-x_1)$ $y-2=3(x-1)$ $y=3x-1$ or $3x-y-1=0$	1 Mark: Correct answer
14(b) (i)	$\alpha+\beta=-\frac{b}{a}=-\frac{-3}{2}=\frac{3}{2}$	1 Mark: Correct answer
14(b) (ii)	$\alpha\beta=\frac{c}{a}=\frac{6}{2}=3$	1 Mark: Correct answer
14(b) (iii)	$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ $=\left(\frac{3}{2}\right)^2-2\times3=-\frac{15}{4}$	1 Mark: Correct answer
14(c)	The object comes to rest when $\dot{x}=0$ $x=3e^{-2t}+10e^{-t}+4t$ $\dot{x}=-6e^{-2t}-10e^{-t}+4$ $=-2(3e^{-2t}+5e^{-t}-2)$ Let $m=e^{-t}$ $-2(3m^2+5m-2)=0$ $-2(3m-1)(m+2)=0$	3 Marks: Correct answer. 2 Marks: Finds and factorises a a quadratic equation. 1 Mark: Correctly differentiates *x*.

	Hence $3m-1=0$ or $m+2=0$ $m=\frac{1}{3}$ $\quad$ $m=-2$ $e^{-t}=\frac{1}{3}$ $\quad$ $e^{-t}=-2$ (no solution) Therefore $t=-\log_e \frac{1}{3}$ $=\log_e 3$ $\approx 1.0986...$	

(d)

(i) $\sqrt{\sin x}=1$ **1**

$\sin x=1$

$x=\frac{\pi}{2}$

Coords are $\left(\frac{\pi}{2},1\right)$

(ii) $V=\pi\int_0^{\frac{\pi}{2}} y^2 dx$ **3**

$V=\pi\int_0^{\frac{\pi}{2}} 1^2 dx-\pi\int_0^{\frac{\pi}{2}}\left(\sqrt{\sin x}\right)^2 dx$

$=\pi\int_0^{\frac{\pi}{2}} 1\, dx-\pi\int_0^{\frac{\pi}{2}} \sin x\, dx$

$=\pi\left[x\right]_0^{\frac{\pi}{2}}+\pi\left[\cos x\right]_0^{\frac{\pi}{2}}$

$=\pi\left(\frac{\pi}{2}-0\right)+\pi(0-1)$

$=\frac{\pi^2}{2}-\pi$

$=\frac{\pi^2-2\pi}{2}$

Question 15

(a)

$2^x = 5$

$\log_e 2^x = \log_e 5$

$x \log_e 2 = \log_e 5$

$x = \dfrac{\log_e 5}{\log_e 2} = 2.3219...$

16(b) (i)	Initially t = 0 and $R = 8000$ $R = R_0 e^{-kt}$ $8000 = R_0 e^{-k\times 0}$ $R_0 = 8000$ Also t = 1 and $R = 7000$ $7000 = 8000 e^{-k\times 1}$ $e^{-k} = \dfrac{7000}{8000}$ $-k = \log_e \dfrac{7}{8}$ $k = -\log_e \dfrac{7}{8} = 0.13353139...$	2 Marks: Correct answer. 1 Mark: Finds the correct value for R_0 or k.

16(b) (ii)	We need to find *R* when $t = 10$ $R = 8000e^{\log_e \frac{7}{8} \times 10}$ $= 2104.604609... \approx 2105$ bq	1 Mark: Correct answer.
16(b) (iii)	We need to find *t* when $R = 50$. $50 = 8000e^{-k \times t}$ $e^{-kt} = \frac{1}{160}$ $-kt = \log_e \frac{1}{160}$ $t = -\frac{1}{k}\log_e \frac{1}{160}$ $= \log_e \frac{1}{160} \div \log_e \frac{7}{8}$ $= 38.0073458.. \approx 38$ years	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution.

(c) (i) In ΔOAB and ΔOCD **3**

$OA = OC$ (radii of a circle are equal)

$OB = CD$ (given)

$\angle OBA = \angle ODC$ (given)

$\therefore \Delta OAB$ and ΔOCD (RHS)

(ii) $\left(2\sqrt{3}\right)^2 = OD^2 + 4^2$ **1**

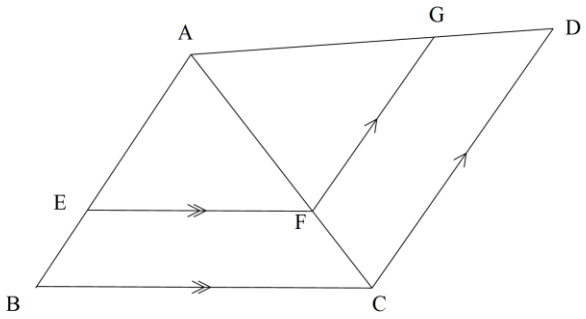
$OD = 6$ cm

$BD = 6 - 4 = 2$ cm

(iii) $\frac{FB}{CD} = \frac{OB}{OD}$ **2**

$FB = \frac{4 \times 4}{6} = 2\frac{2}{3}$ cm

$A = \frac{1}{2} \times 2 \times \left(2\frac{2}{3} + 4\right) = 6\frac{2}{3}$ cm^2

	Solution	Marks	Allocation of marks
(a)	 AE/AB = AF/AC (ratios of intercepts) AF/AC = AG/AD (ratios of intercepts) ∴ AE/AB = AG/AD (equating ratios)	2	1 for use of ratios of intercepts 1 for conclusion

Question 16

(a)	(i) $P = \$650000 \quad r = 5.4 \div 100 \div 12 = 0.0045$ $A = P(1+r)^n - M$ $A_1 = 650000(1.0045)^1 - M$ $A_2 = A_1(1.0045)^1 - M$ $A_2 = [650000(1.0045)^1 - M](1.0045) - M$ $A_2 = 650000(1.0045)^2 - M(1.0045) - M$ $A_2 = 650000(1.0045)^2 - M[1 + 1.0045]$ $A_3 = (650000(1.0045)^2 - M[1 + 1.0045])(1.0045) - M$ $A_3 = 650000(1.0045)^3 - M[1 + 1.0045 + 1.0045^2]$ . . . $A_n = 650000(1.0045)^n - M\left[1 + 1.0045 + ... + 1.0045^{n-1}\right]$	2	1 for setting up initial terms as examples 1 for following pattern to establish required formula

	(ii) $\text{Months} = 30 \times 12 = 360 \text{ repayments}$ $A_{360} = 0 \text{ (loan repaid)}$ $A_n = 650000(1.0045)^n - M\left[1 + 1.0045 + ... + 1.0045^{n-1}\right]$ $0 = 650000(1.0045)^n - M\left[1 + 1.0045 + ... + 1.0045^{n-1}\right]$ $M\left[1 + 1.0045 + ... + 1.0045^{n-1}\right] = 650000(1.0045)^n$ $M = \dfrac{650000(1.0045)^n}{1 + 1.0045 + ... + 1.0045^{n-1}}$ The denominator is a geometric series with $a = 1$, $r = 1.0045$ and $n = 360$ $S_n = \dfrac{a(r^n - 1)}{r}$ $S_{360} = \dfrac{1((1.0045)^{360} - 1)}{0.0045}$ $S_{360} = \dfrac{(1.0045)^{360} - 1}{0.0045}$ $\therefore M = \dfrac{(650000(1.0045)^{360}) \times 0.0045}{(1.0045)^{360} - 1}$ $M = \$3649.95$	2	1 for expression for M 1 for substituting into sum of series and finding M (can use rounded answer for S_n)
	(iii) $A_n = 650000(1.0045)^n - 5000S_n$ $A_n = \$0 \text{ paid off}$ $5000S_n = 650000(1.0045)^n$ $5000\left[\dfrac{(1.0045)^n - 1}{0.0045}\right] = 650000(1.0045)^n$ $5000(1.0045)^n - 5000 = 2925(1.0045)^n$ $5000(1.0045)^n - 2925(1.0045)^n = 5000$ $(1.0045)^n[5000 - 2925] = 5000$ $(1.0045)^n = \dfrac{5000}{2075}$ $\ln\left(1.0045^n\right) = \ln\left[\dfrac{5000}{2075}\right]$ $n \ln(1.0045) = \ln\left[\dfrac{5000}{2075}\right]$ $n = \dfrac{\ln\left[\dfrac{5000}{2075}\right]}{\ln 1.0045}$ $n = 195.88$ $= 196 \text{ months}$	2	1 for using sum to establish equation 1 for solving to find n

	(iv) Total of loan over 30 years 360 × $3 649.95 = $1 313 982 Total of loan by paying $5000/month 196 × $5 000 = $980 000 Interest Saving" $1 313 982 - $980 000 = $333 982	1	1 for answer

(b)

(i) 2 marks

Outcomes Assessed: **P4, H4**

Targeted Performance Bands: **3-4**

Criteria	Marks
• Correctly shows the expression for the Area	2
• Finds the correct expression for the height of the mattress	1

Sample Answer

$$V_{mattress} = 1.8xh$$

$$\therefore 1.8xh = 0.504$$

$$h = \frac{0.28}{x}$$

$$A_{blanket} = 1.8(x + 2h)$$

$$= 1.8\left(x + 2\left(\frac{0.28}{x}\right)\right)$$

$$= 1.8\left(x + \frac{0.56}{x}\right)$$

(ii) 3 marks

Outcomes Assessed: **H4, H5**

Targeted Performance Bands: **4-5**

Criteria	Marks
• Gives the correct solution	3
• Justifies the correct value of *x* to attain a minimum	2
• Finds the correct value of *x*	1

Sample Answer is on the next page…

$A = 1.8x + 1.008x^{-1}$

$A' = 1.8 - 1.008x^{-2}$

$A' = 0$

$1.8 = \dfrac{1.008}{x^2}$

$x^2 = 0.56$

$x = \sqrt{0.56} \quad (x > 0)$

$A'' = \dfrac{2.016}{x^3}$

when $x = \sqrt{0.56} \qquad A'' = 4.810702354$

$\therefore A'' > 0$ minimum

$$\therefore \text{Minimum Area} = 1.8\left(\sqrt{0.56} + \frac{0.56}{\sqrt{0.56}}\right)$$

$$\approx 2.69\,\text{m}^2 \text{ (correct to 2 decimal places)}$$

(c)	$\dfrac{10^{3n} \times 25^{n+2}}{8^n} = 1$ LHS $= \dfrac{(10^3)^n \times (5^2)^{n+2}}{(2^3)^n}$ $= \dfrac{(1000)^n \times (5^2)^{n+2}}{(2^3)^n}$ $= \dfrac{(2^3 \times 5^3)^n \times 5^{2n+4}}{(2^{3n})}$ $= \dfrac{(2^{3n} \times 5^{3n}) \times (5^2)^{n+2}}{(2^{3n})}$ $= 5^{3n} \times 5^{2n+4}$ $\therefore 5^{5n+4} = 1$ $5^0 = 1$ $\therefore 5n + 4 = 1$ $n = -\dfrac{4}{5}$	3	1 for reducing 10, 25 and 8 1 for collecting powers of 2 and of 5 1 for solving for *n*