Trinity Grammar School

Mathematics Department

2016

TRIAL EXAMINATION
HSC ASSESSMENT TASK 4

Year 12
Mathematics

BOSTES Student Number
(Year 12 only)

Class Teacher:

………………………………………………………………

Name:
(Year 10 or 11 only)

………………………………………………………………

Do NOT write solutions on this question paper. Any working on this question paper will NOT be marked.

Date of examination:

Thursday, 25 August 2016

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- BOSTES approved calculators for this course may be used
- A Reference Sheet for this course is supplied
- In Questions **11 – 16**, show all relevant mathematical reasoning and/or calculations
- Write your BOSTES Student Number (Year 12 HSC) or Name (Year 10 or 11) **and** your Class teacher on the question paper **and** on any answer sheets or writing booklets used to write your responses to the questions submitted
- If you do not attempt a question you **must** submit an answer sheet **or** writing booklet for that question clearly indicating **N/A** and your Student Number or Name.
- **HSC Assessment Weighting:** 40%

Total marks – 100

Section I

10 marks

- Attempt all Questions
- Allow about **15** minutes for this section

Section II

90 marks

- Attempt all Questions
- Allow about **2** hours **45** minutes for this section

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Section I **10 marks**

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section
- Shade the correct response on the multiple-choice answer sheet for Questions 1 – 10
- Each question is worth 1 mark

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1 Let $p = \dfrac{2\cos x}{\sqrt{y}}$ where $x = 45°$, $y = 8192$. The value of p in scientific form correct to three significant figures is:

(A) 1.560×10^{-2}

(B) 1.56×10^{-2}

(C) 1.562×10^{-2}

(D) 15.6×10^{-2}

2 The graph of a function $f(x) = a(x-h)^2 + k$ intersects the y-axis at (0, 5) and has its vertex at the point (4, 13). The value of a, of h and of k are respectively:

(A) $a = -\dfrac{1}{2},\ h = -4,\ k = -13$

(B) $a = -\dfrac{1}{2},\ h = -4,\ k = 13$

(C) $a = -\dfrac{1}{2},\ h = 4,\ k = -13$

(D) $a = -\dfrac{1}{2},\ h = 4,\ k = 13$

3 If $g(x) = \log_e(3x-4)$, the slope of the tangent to $y = g(x)$ at the point where $x = 2$ is:

(A) $-\dfrac{3}{2}$

(B) $-\dfrac{2}{3}$

(C) $\dfrac{2}{3}$

(D) $\dfrac{3}{2}$

4 Rudolf has been given \$500 by his mother for his 18th birthday. Rudolf plans to immediately deposit this amount in a bank which offers an interest rate of 6.00% p.a., compounded quarterly, for three years.

Calculate the interest earned from the bank.

(A) \$95.51

(B) \$97.81

(C) \$131.24

(D) \$134.49

5 Evaluate the arithmetic series: $S_{46} = -7 + (-9) + (-11) + ... + (-97)$.

(A) –2800

(B) –2392

(C) –2295

(D) –2184

6 Evaluate: $\int_0^{\frac{\pi}{8}} \sin 8x \; dx$

(A) –2

(B) $-\frac{1}{4}$

(C) 0

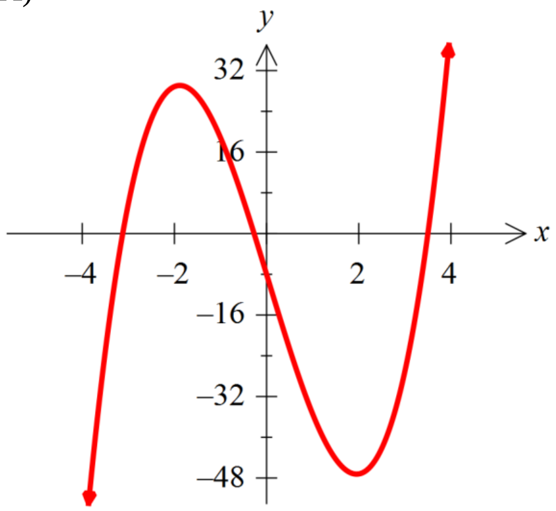
(D) $\frac{1}{4}$

7 A function $y = f(x)$ is differentiable on the interval $-\infty < x < \infty$ and has the following properties.

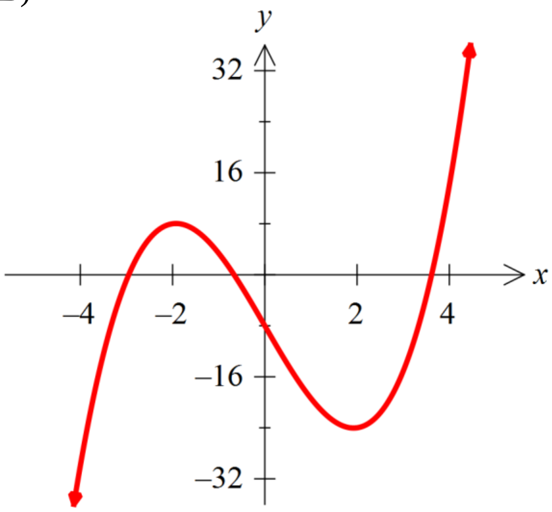
x	$y = f(x)$	**Derivatives**
$x < 2$		$y' > 0,\ y'' < 0$
-2	8	$y' = 0,\ y'' < 0$
$-2 < x < 0$		$y' < 0,\ y'' < 0$
0	-8	$y' < 0,\ y'' = 0$
$0 < x < 2$		$y' < 0,\ y'' > 0$
2	-24	$y' = 0,\ y'' > 0$
$x > 2$		$y' > 0,\ y'' > 0$

A possible graph of $y = f(x)$ could be:

(A)



(B)



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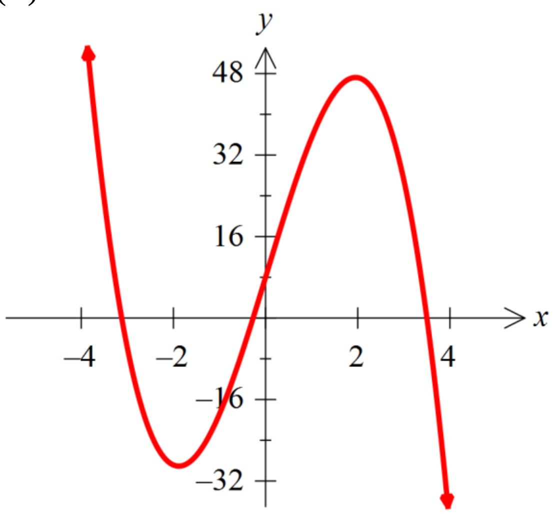
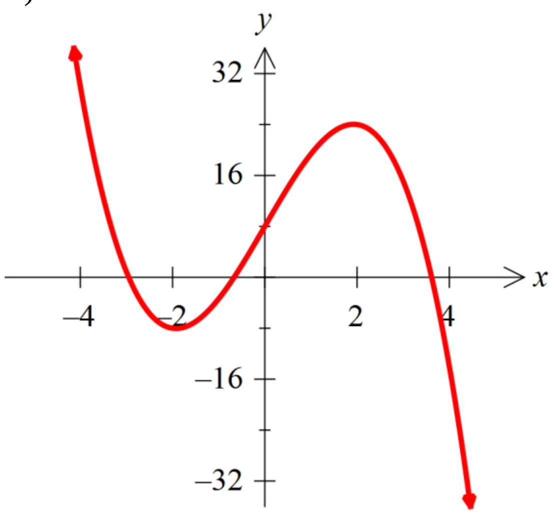


(D)



8 Solve for x if $\log_2 x + \log_2(x-2) = 3$.

(A) $x = -4$ only

(B) $x = -2$ only

(C) $x = 4$ only

(D) $x = -2$ and $x = 4$

9 When a camera flash goes off, the batteries immediately begin to recharge the flash's capacitor, which stores electric charge Q, given by $Q(t) = Q_0\left(1 - e^{-\frac{t}{4}}\right)$. The maximum charge capacity is Q_0 and t is time measured in seconds.

How long does it take to recharge the capacitor to 90% of capacity?

(A) $-4\log_e\left(\frac{1}{10}\right)$ seconds

(B) $-4\log_e\left(\frac{9}{10}\right)$ seconds

(C) $-4\log_e\left(\frac{4}{9}\right)$ seconds

(D) $-4\log_e 10$ seconds

10 For what real values of c does the curve $f(x) = 5x^3 + cx^2 + 10x$ have a local maximum stationary point **and** a local minimum stationary point?

(A) $c < -5\sqrt{6}$ *or* $c > 5\sqrt{6}$

(B) $-5\sqrt{6} \le c \le 5\sqrt{6}$

(C) $-5\sqrt{6} < c < 5\sqrt{6}$

(D) $c \le -5\sqrt{6}$ *or* $c \ge 5\sqrt{6}$

Section II 90 marks

- Attempt Questions 11 – 16
- Allow about 2 hours and 45 minutes for this section
- Answer each question in a SEPARATE writing booklet or answer sheet. Extra writing booklets are available.
- In Questions 11 – 16, your responses should include all relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet

(a) Solve for x if: $3-(2+4x)=\frac{x}{2}$ **2**

(b) Write down the domain of the function $f(x)=\ln(x+6)$. **1**

(c) Each day Stanley trains for a 10 km race. On the first day he runs 1000 m and then increases the distance by 250 m on each subsequent day.

(i) On which day does Stanley run a distance of 10 km in training? **2**

(ii) Find the total distance Stanley will have run in training by the end of the day calculated in (c)(i), giving your answer in kilometres. **2**

(d) Solve the equation $\tan x=-\frac{1}{\sqrt{3}}$, for $0\le x\le 2\pi$. **2**

(e) Calculate the size of each interior angle of a **regular** octagon? **1**

(f) A particle moves along a straight line so that its velocity, v ms^{-1}, at time t seconds is given by $v=240+20t-10t^2$, for $0\le t\le 6$.

(i) Find the value of t when the speed of the particle is greatest. **2**

(ii) Find the acceleration of the particle when its speed is zero. **3**

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet

(a) Let $f(x) = |x-1|$ and $g(x) = |2x-1|$.

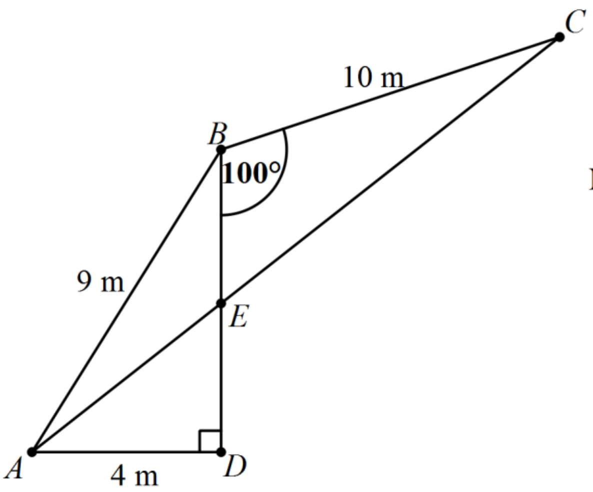
(i) Sketch, **on the same number plane**, the graphs of $y = f(x)$ **and** $y = g(x)$ **2**

(ii) Hence or otherwise, solve $f(x) \geq g(x)$. **2**

(b) Use Simpson's rule with **three** function values to estimate the value of $\int_0^4 \frac{dx}{1+\sqrt{x}}$. **2**

Give your answer correct to two decimal places.

(c) In the diagram below, AD = 4 m, AB = 9 m, BC = 10 m, $\angle BDA = 90°$ and $\angle DBC = 100°$. The point E is the intersection of intervals AC and BD.



Not to scale

(i) Using ΔADB, find the size of $\angle ABC$, correct to one decimal place. **2**

(ii) Hence, using the Cosine rule, find the length of AC, correct to the nearest integer. **2**

(iii) Using the Sine rule, find the size of $\angle BAE$, correct to the nearest degree. **2**

(d) Use the fact that $1 + \tan^2 x = \sec^2 x$, evaluate $\int_0^{\frac{\pi}{3}} \tan^2 x \ dx$. **3**

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet

(a) Evaluate the infinite geometric series: $\sum_{n=1}^{\infty} \frac{5}{6^n}$ **2**

(b) A parabola is defined by the equation $y^2 - 4y + 4 = 24x$.

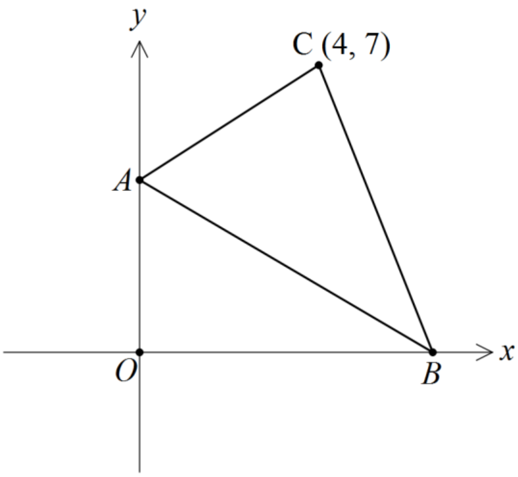
(i) Write down the coordinates of the vertex. **1**

(ii) State the coordinates of the focus. **1**

(iii) Write down the equation of the directrix. **1**

(iv) Sketch the graph of the parabola showing the features found in part (b) (i), (ii) and (iii). **2**

(c) The diagram shows triangle *ABC*. Point *C* has coordinates (4, 7) and the equation of line *AB* is $x + 2y = 8$.



Not to scale

Copy this diagram into your writing booklet.

(i) Find the exact length of the interval *AB* (in the form $a\sqrt{b}$, where *a* and *b* are integers). **2**

The point *D* lies on *AB* such that *CD* is perpendicular to *AB*.

(ii) Find the equation of the line *CD*. **2**

(iii) Find the coordinates of the point *D*. **2**

(iv) Hence or otherwise, find the area of triangle *ABC*. **2**

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet

(a) For the function $f(x)=\dfrac{1}{4}x^3-\dfrac{1}{2}x^2-8x+1$,

(i) find $f'(x)$ **and** $f''(x)$; **2**

(ii) find the coordinates of the stationary points **and** determine their nature; **3**

(iii) sketch the graph of $y=f(x)$, showing the features found in (ii); **2**

(iv) for what value(s) of x is $f(x)$ concave up. **1**

(b) The population (P) of a bacteria culture is increasing at a rate determined by $\dfrac{dP}{dt}=kt,\ t\geq 0$ (hours). Initially P = 11 000. After four hours has passed the population of the bacteria culture has tripled.

(i) Verify that $P=P_0e^{kt}$ satisfies the equation $\dfrac{dP}{dt}=kP$. **1**

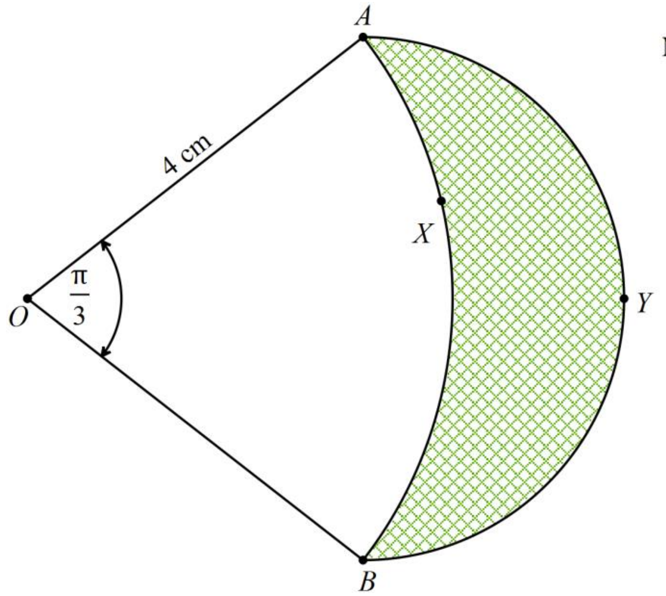
(ii) Write down the value of P_0 . **1**

(iii) Find the size of the bacteria culture after exactly 36 hours has passed. **3**

Question 14 continues on page 11

Question 14 (continued)

(c)



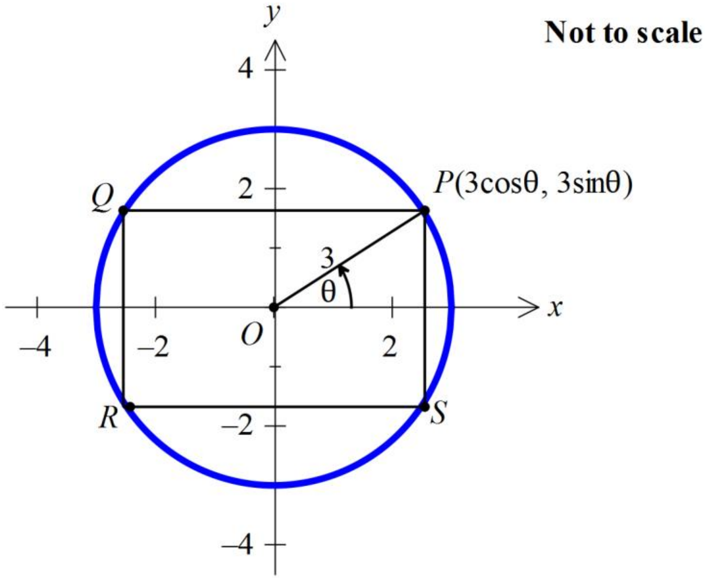
In the diagram, AYB is a semicircle with AB as diameter and $OAXB$ is a sector of a circle with centre O and radius 4 cm. Angle AOB is $\frac{\pi}{3}$ radians.

Find the area of the shaded region. **2**

End of Question 14

Question 15 (15 marks) Use a SEPARATE writing booklet

(a) A rectangle $PQRS$ is inscribed in a circle of radius 3 cm and centre O, as shown below. All vertices of the rectangle lie on the circumference of the circle.



The point $P(3\cos\theta, 3\sin\theta)$ is a vertex of the rectangle and also lies on the circle.

The angle between interval OP and the x-axis is θ radians, where $0 \le \theta \le \frac{\pi}{2}$.

Let the area of rectangle $PQRS$ be A.

(i) Show that $A = 36\sin\theta\cos\theta$. **2**

(ii) Use the product rule to show $\frac{dA}{d\theta} = 36(1 - 2\sin^2\theta)$. **2**

(iii) Hence, find the exact value of θ which maximises the area of the rectangle. *You do not need to test for maxima.* **2**

(b) The acceleration a ms^{-2}, of a particle at time t seconds is given by $a = 3\cos 2t$, for $t \ge 0$. The particle is at rest when $t = \frac{\pi}{4}$.

(i) Write down the **next two** occasions (after $t = \frac{\pi}{4}$) when the particle is again at rest. **2**

(ii) Find the distance travelled (in metres), in the first $\frac{\pi}{2}$ seconds. Give your answer exactly in terms of π. **3**

Question 15 continues on page 13

(c) The depth, $h(t)$ metres, of water at the entrance to a harbour at t hours after midnight on a particular day is given by

$$h(t) = 8 + 4\sin\left(\frac{\pi t}{6}\right), \text{ for } 0 \le t \le 24.$$

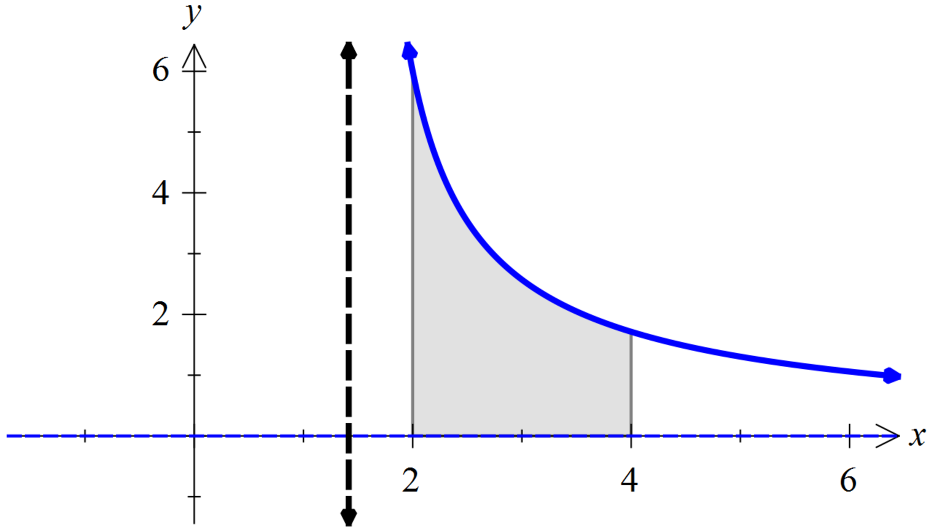
(i) Write down the amplitude of $h(t)$. **1**

(ii) Calculate the period of $h(t)$. **1**

(iii) Sketch the graph of $y = h(t)$ in the interval $0 \le t \le 15$. **2**

End of Question 15

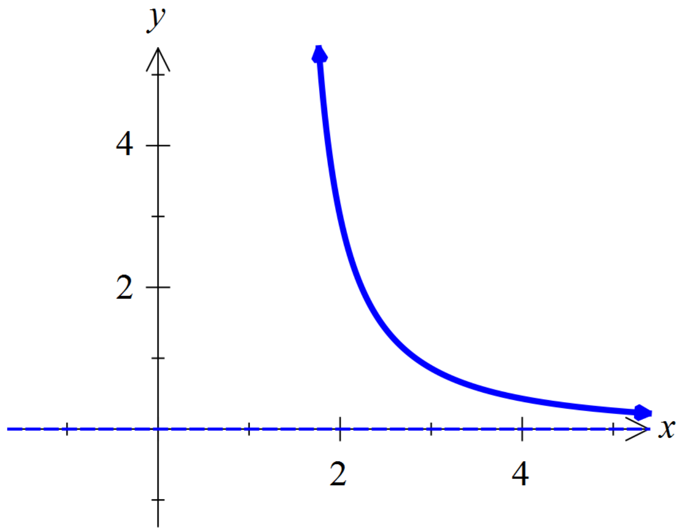
Question 16 (15 marks) Use a SEPARATE writing booklet



(a) The diagram above represents the graph of $y = \dfrac{6x}{x^2 - 2}$ for $x > \sqrt{2}$. **2**

Find the area of the shaded region leaving your answer in the form $a \ln b$ where a and b are positive integers.

(b)



The diagram above represents the graph of $y = \dfrac{6}{x^2 - 2}$ for $x > \sqrt{2}$. **3**

The region enclosed by the arc of the given curve from $y = 1$ to $y = 3$ and the y-axis, is rotated through 360° about the y-axis.

Find the volume of the solid formed. Leave your answer in terms of π.

Question 16 continues on page 15

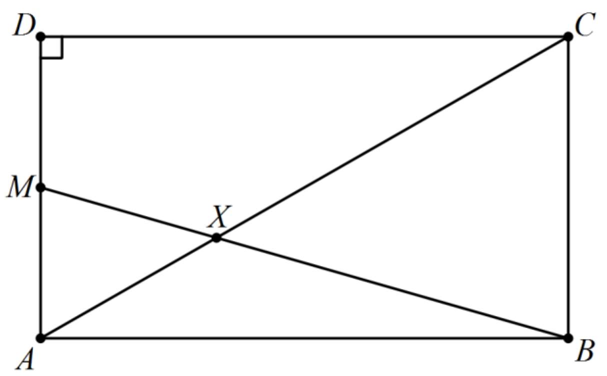
(c) Janet has taken out a loan to pay for a well-earned holiday of \$250 000. The interest is calculated monthly at the rate of 6% p.a., compounded monthly. Janet agrees to repay the loan in full, at the end of each year, with interest, in ten equal annual instalments of \$$M$. Let \$ A_n be the amount owing at the end of the n^{th} year just after the n^{th} repayment.

(i) Show that an expression for A_2 in terms of M is given by $A_2 = 250\,000(1.005)^{24} - M(1+1.005^{12})$. **2**

(ii) Write down a similar expression for A_{10}. **1**

(iii) Calculate the value of M, correct to the nearest dollar. **1**

(d)



Not to scale

In the diagram, $ABCD$ is a rectangle and $\dfrac{AD}{AB} = \dfrac{1}{2}$.

The point M is the midpoint of AD. The line BM meets diagonal AC at X.

Copy this diagram into your writing booklet.

(i) Prove that $\Delta AXM \,|||\, \Delta BXC$. **2**

(ii) Prove that $\dfrac{CX}{AC} = \dfrac{2}{3}$. **2**

(iii) Prove that $\left(\dfrac{CX}{AB}\right)^2 = \dfrac{5}{9}$. **2**

End of Question 16
End of paper

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Suggested Answers and Marking Scheme.

BOSTES Student Number
(Year 12 only)

Class Teacher: EH.

Name:
(Year 10 or 11 only)

Trinity Grammar School
Mathematics Department

2016
TRIAL EXAMINATION
HSC ASSESSMENT TASK 4

Year 12
Mathematics

Do NOT write solutions on this question paper. Any working on this question paper will NOT be marked.

MC

Date of examination:

Thursday, 25 August 2016

Total marks – 100

1. B
2. D
3. D
4. B
5. B
6. D
7. B
8. C
9. A
10. A

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Section I

10 marks

- Attempt all Questions
- Allow about **15** minutes for this section

Section II

90 marks

- Attempt all Questions
- Allow about **2** hours **45** minutes for this section

Q11	Q12	Q13	Q14	Q15	Q16
HB	BL	MD	PK	JW	MM

Section I **10 marks**

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- Each question is worth 1 mark

---

1 Let $p=\dfrac{2\cos x}{\sqrt{y}}$ where $x=45°$, $y=8192$. The value of p in scientific form correct to three significant figures is:

$$\frac{2\cos 45°}{\sqrt{8192}} = \frac{1}{64} = 0.015625 \text{ (exactly).}$$

(A) 1.560×10^{-2}

(B) 1.56×10^{-2}

(C) 1.562×10^{-2}

(D) 15.6×10^{-2}

2 The graph of a function $f(x)=a(x-h)^2+k$ intersects the y-axis at (0, 5) and has its vertex at the point (4, 13). The value of a, of h and of k are respectively:

$(h,k)=$ Vertex $=(4,13)$

sub $(0,5) \Rightarrow 5=a(0-4)^2+13$

$5=16a+13$

$16a=-8$

$a=-\frac{1}{2}$

(A) $a=-\frac{1}{2},\ h=-4,\ k=-13$

(B) $a=-\frac{1}{2},\ h=-4,\ k=13$

(C) $a=-\frac{1}{2},\ h=4,\ k=-13$

(D) $a=-\frac{1}{2},\ h=4,\ k=13$

3 If $g(x)=\log_e(3x-4)$, the slope of the tangent to $y=g(x)$ at the point where $x=2$ is:

$g'(x)=\frac{3}{3x-4}$ $\left(D: x>\frac{4}{3}\right)$

$g'(2)=\frac{3}{6-4}$

$=\frac{3}{2}$

(A) $-\frac{3}{2}$

(B) $-\frac{2}{3}$

(C) $\frac{2}{3}$

(D) $\frac{3}{2}$

4 Rudolf has been given \$500 by his mother for his 18th birthday. Rudolf plans to immediately deposit this amount in a bank which offers an interest rate of 6.00% p.a., compounded quarterly, for three years.

Calculate the interest earned from the bank.

$6\% \text{ p.a} = \frac{6\%}{4} \text{ p.m}$

$A = P(1+r)^n$

(A) \$95.51

(B) \$97.81

(C) \$131.24

(D) \$134.49

$= \frac{0.06}{4}$

$= 0.015$

$n = 3 \times 4 = 12$

$\therefore I = 500(1 \cdot 015)^{12} - 500$

$= \$97.81.$

5 Evaluate the arithmetic series: $S_{46} = -7+(-9)+(-11)+...+(-97)$.

$S_n = \frac{n}{2}[a+l]$

(A) −2800

(B) −2392

(C) −2295

(D) −2184

$n = 46$

$a = -7$

$l = -97$

$\therefore S_{46} = \frac{46}{2}[-7-97]$

$= -2392.$

6 Evaluate: $\int_0^{\frac{\pi}{8}} \sin 8x \; dx$

$= -\frac{1}{8}[\cos 8x]_0^{\pi/8}$

$= -\frac{1}{8}[\cos \pi - \cos 0]$

$= -\frac{1}{8}[-1-1]$

$= -\frac{1}{8} \times -2$

$= \frac{1}{4}.$

(A) −2

(B) $-\frac{1}{4}$

(C) 0

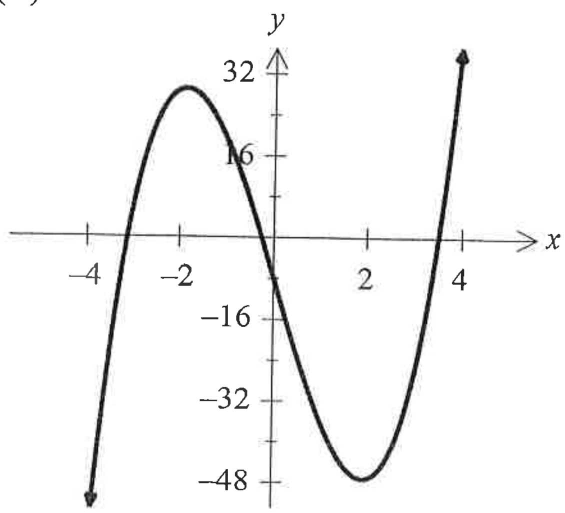
(D) $\frac{1}{4}$

7 A function $y = f(x)$ is differentiable on the interval $-\infty < x < \infty$ and has the following properties.

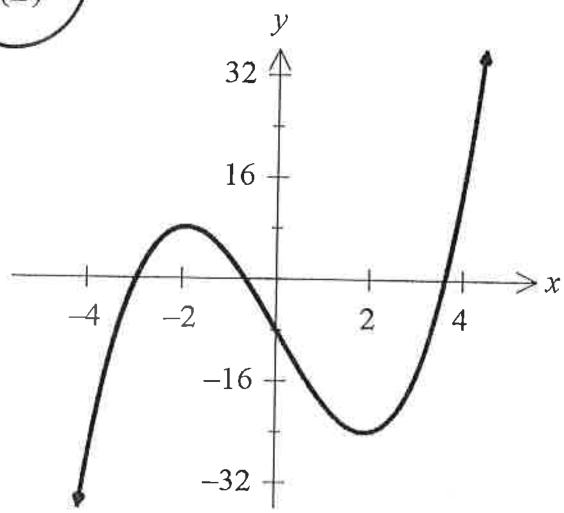
x	$y = f(x)$	**Derivatives**
$x < 2$		$y' > 0,\ y'' < 0$
-2	8	$y' = 0,\ y'' < 0$
$-2 < x < 0$		$y' < 0,\ y'' < 0$
0	-8	$y' < 0,\ y'' = 0$
$0 < x < 2$		$y' < 0,\ y'' > 0$
2	-24	$y' = 0,\ y'' > 0$
$x > 2$		$y' > 0,\ y'' > 0$

A possible graph of $y = f(x)$ could be:

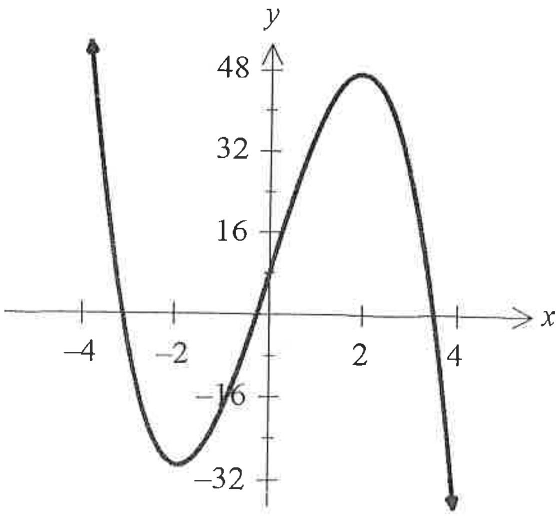
(A)



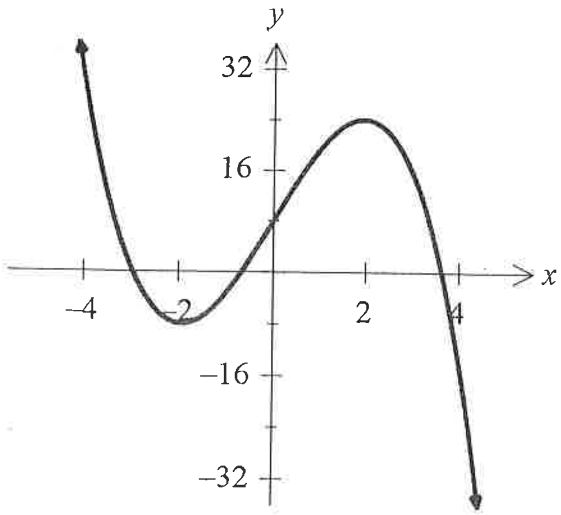
(B)



(C)



(D)



D: $x>2$

8 Solve for x if $\log_2 x + \log_2(x-2) = 3$.

(A) $x = -4$

(B) $x = -2$

(C) $x = 4$

(D) $x = 2,\ x = 4$

$\log_2 x(x-2) = 3$

$8 = x(x-2)$

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$x = 4,\ x = -2$

but $x > 2$ (domain)

9 When a camera flash goes off, the batteries immediately begin to recharge the flash's capacitor, which stores electric charge Q, given by $Q(t) = Q_0\left(1 - e^{-\frac{t}{4}}\right)$. The maximum charge capacity is Q_0 and t is time measured in seconds.

How long does it take to recharge the capacitor to 90% of capacity?

(A) $-4\log_e\left(\frac{1}{10}\right)$ seconds

(B) $-4\log_e\left(\frac{9}{10}\right)$ seconds

(C) $-4\log_e\left(\frac{4}{9}\right)$ seconds

(D) $-4\log_e 10$ seconds

$0.9 = 1 - e^{-t/4}$

$-\frac{t}{4} = \ln\frac{1}{10}$

$-t = 4\ln\left(\frac{1}{10}\right)$

$\therefore t = -4\ln\frac{1}{10}$.

10 For what values of c does the curve $f(x) = 5x^3 + cx^2 + 10x$ have a local maximum stationary point **and** a local minimum stationary point?

(A) $c < -5\sqrt{6}$ or $c > 5\sqrt{6}$

(B) $-5\sqrt{6} \le c \le 5\sqrt{6}$

(C) $-5\sqrt{6} < c < 5\sqrt{6}$

(D) $c \le -5\sqrt{6}$ or $c \ge 5\sqrt{6}$

must have <u>two</u> roots to $f'(x) = 0$ (not <u>one</u> only) $\therefore \Delta > 0$

$f'(x) = 15x^2 + 2cx + 10$

$\Delta = 4c^2 - 4(15)(10)$

ie $4c^2 - 600 > 0$

$c^2 - 150 > 0$

$(c - 5\sqrt{6})(c + 5\sqrt{6}) > 0$

Question 11:

a) $3-(2+4x)=\frac{x}{2}$

$1-4x=x/2$

$2-8x=x$

$9x=2$

$\therefore x=2/9$

b) $f(x)=\log_e(x+6)$

D: $x>-6$ (accepted $-6<x$)

c) Using $T_n=a+(n-1)d$ since the sequence is arithmetic

i) $10\,000=1000+(n-1)\times 250$

$n-1=36$

$\therefore n=37$ (37^{th} day)

ii) Want S_{37}.

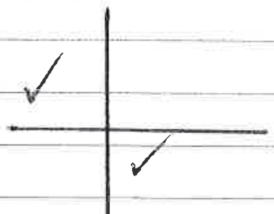
Using $S_n=\frac{n}{2}[2a+(n-1)d]$

$S_{37}=\frac{37}{2}[2\times1000+36\times250]$

$=203,500m$

$=203.5\,km$

d) $\tan x=-\frac{1}{\sqrt{3}}$



Basic angle $=\pi/6$.

$\therefore$ Solution is $x=\pi-\pi/6$ and $x=2\pi-\pi/6$

ie $x=5\pi/6$ and $x=\frac{11\pi}{6}$.

① → basic angle or 2 correct angles in degrees.

① 2 correct answers.

e) $n=8$ $\therefore$ angle $=\frac{180(8-2)}{8}$

$=135°$.

f) $v = 240 + 20t - 10t^2, \quad 0 \leq t \leq 6$

i) speed is greatest when $\frac{dv}{dt} = 0$

$\frac{dv}{dt} = 20 - 20t$

$\Rightarrow 20 = 20t$

$t = 1$ second

ii) Sub $v = 0$

$\Rightarrow 0 = 240 + 20t - 10t^2$

$\Rightarrow 24 + 2t - t^2 = 0$

ie $t^2 - 2t - 24 = 0$

$(t-6)(t+4) = 0$

$\therefore t = 6$ or $t = -4$

but $0 \leq t \leq 6$

$\therefore t = 6$ only.

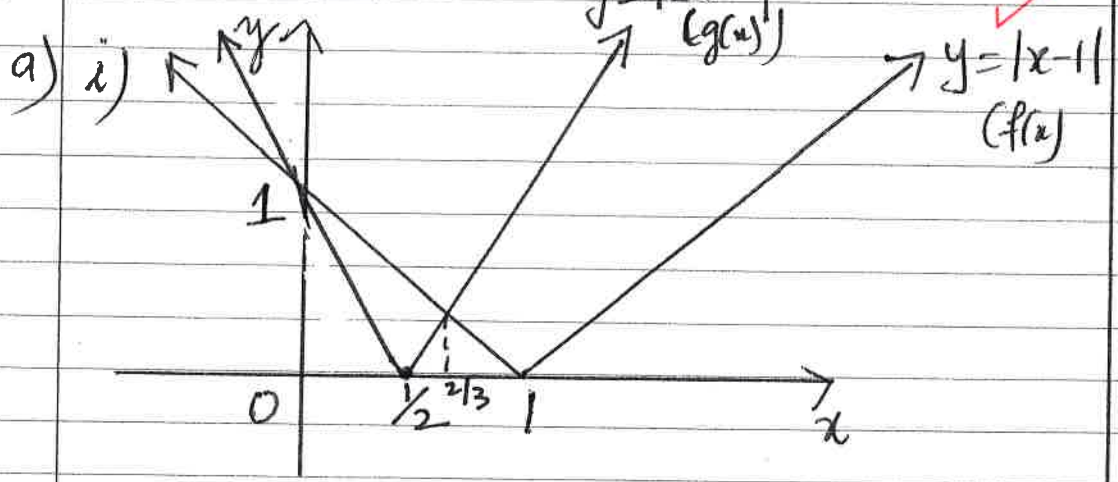
hence $a(6) = 20 - 20 \times 6$

$= -100 \text{ m/s}^2$

Marker Comments:

Question 12:

a) i)



ii) Hence: One solution is $x=0$ from above.
The other is obtained by solving

$2x-1 = 1-x$

ie $3x = 2$

$x = 2/3$

$\therefore$ $x=0$ or $x = 2/3$ are critical values.

Hence $f(x) \geq g(x)$ when $\{x : 0 \leq x \leq 2/3\}$

Otherwise (one of several ways)

$|x-1| \geq |2x-1|$

$(x-1)^2 \geq (2x-1)^2$

$(x-1)^2 - (2x-1)^2 \geq 0$

$[x-1-(2x-1)][x-1+(2x-1)] \geq 0$

$(-x)(3x-2) \geq 0$

$x(3x-2) \leq 0$

$0 \qquad 2/3 \qquad x$

$\Rightarrow \{x : 0 \leq x \leq 2/3\}$

b) $A \sim \frac{b-a}{6}\{y_1 + 4y_2 + y_3\}$ $\quad y_2 = f(\frac{x_1+x_3}{2})$

$= \frac{4-0}{6}\left[\frac{1}{1} + 4 \times \frac{1}{1+\sqrt{2}} + \frac{1}{1+\sqrt{4}}\right]$

$= 1.99$ (to 2dp, no units required)

c) i) let $\angle ABD = \theta$.

$\therefore \sin\theta = 4/9$

hence $\angle ABC = \sin^{-1}(4/9) + 100$

$= 126.4^\circ$ (1 dp).

ii) $AC^2 = 9^2 + 10^2 - 2\times 9\times 10\cos 126.4^\circ$

$\therefore AC \doteqdot 16.965\ldots$

$= 17$ (nearest integer).

iii) let $\alpha = \angle BAE$

$\therefore \dfrac{\sin\alpha}{10} = \dfrac{\sin 126.4^\circ}{17}$

$\sin\alpha = \dfrac{10\sin 126.4^\circ}{17}$

$\therefore \alpha = 28.2595\ldots$

$= 28^\circ$ (nearest degree)

d) $\displaystyle\int_0^{\pi/3} \tan^2 x\,dx = \int_0^{\pi/3} \sec^2 x - 1\,dx$

$= \left.\tan x - x\right]_0^{\pi/3}$

$= \tan\frac{\pi}{3} - \frac{\pi}{3} - \tan 0 + 0$

$= \sqrt{3} - \frac{\pi}{3}$. ($0.68 \Rightarrow$ 2dp) approximate is penalised – 1 mark)

Marker Comments.

ie: if an approximation is given in the final answer, a 1 mark penalty is applied. "Evaluate $\Rightarrow$ exact answer"

Question 13:

a) $= \sum_{n=1}^{\infty} 5 \times \left(\frac{1}{6}\right)^n$

$= \sum_{n=1}^{\infty} 5 \times \frac{1}{6}\left(\frac{1}{6}\right)^{n-1}$

$\therefore \quad a = 5/6, \quad r = 1/6$ $\qquad a = 5/6 \rightarrow ✓$

hence $S_\infty = \frac{a}{1-r}$

$= \frac{5/6}{1-1/6}$

$= 1.$ ✓

b) $y^2 - 4y + 4 = 24x$

$(y-2)^2 = 24x$

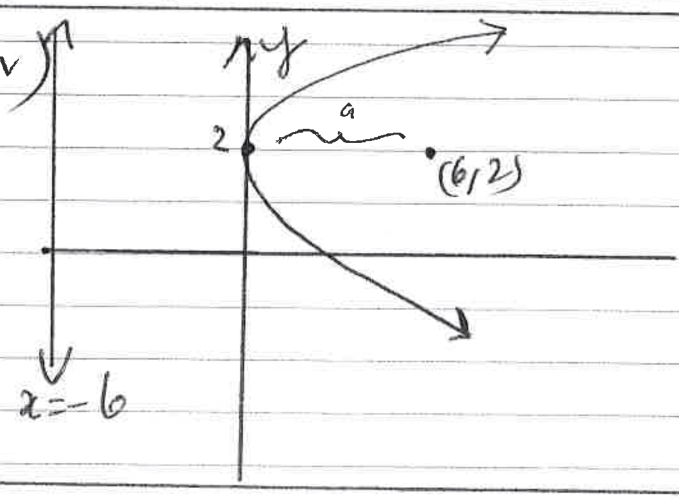
$(y-2)^2 = 4(6)x \quad \Rightarrow a = 6$

i) $V(0,2)$ ✓ must be a coordinate

ii) $S(6,2)$ ✓ must be a coordinate.

iii) $x = -6$ ✓ must be an equation.

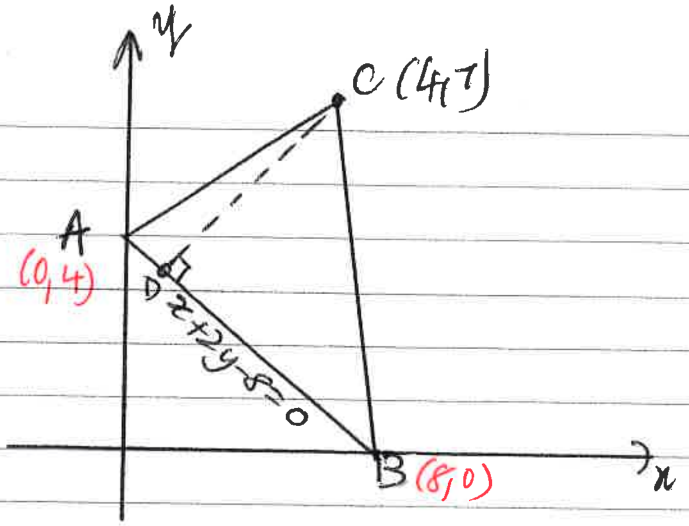
iv)



Correct shape
Correctly plotted
i), ii), iii) $\rightarrow$ (2)

(2) $\rightarrow$ from their i) ii) & iii) if incorrect from above.

c)



i) A and B are the y and x intercepts respectively.

$\therefore$ in $x+2y-8=0$

let $x=0$, $\Rightarrow$ $2y=8$

$y=4$

let $y=0$ $\Rightarrow$ $x=8$

$\therefore A=(0,4)$ and $B=(8,0)$ ✓

hence $AB=\sqrt{4^2+8^2}$ ✓

$=\sqrt{80}$

$=4\sqrt{5}$ units

---

ii) $m_{AB}=-\frac{1}{2}$

$\therefore m_{CD}=2$ ✓

Using $y-y_1=m(x-x_1)$ and $C(4,7)$

Required equation of CD is:

$y-7=2(x-4)$

$y=2x-8+7$

$y=2x-1$ [or $2x-y-1=0$] ✓

---

iii) Solving $\begin{cases} 2x-y=1 & (1) \\ x+2y=8 & (2) \end{cases}$

in (2)$\times 2$ $\Rightarrow$ $2x-y=1$ (1)

$2x+4y=16$ (3)

(3)$-$(1) $\Rightarrow$ $5y=15$

$\therefore y=3$

hence $x=2$

$\therefore D=(2,3)$

(2) → correct solution

(1) → use of a valid method obtaining at least one correct coordinate.

iv) $A = \frac{1}{2} \times CD \times AB$

$= \frac{1}{2} \times \sqrt{(7-3)^2 + (4-2)^2} \times 4\sqrt{5}$

$= \frac{1}{2} \times \sqrt{20} \times 4\sqrt{5}$

$= 20$ sq. units

Otherwise use $d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2+B^2}}$ to calculate CD.

Marker Comments:

Question 14:

a) $f(x) = \frac{1}{4}x^3 - \frac{1}{2}x^2 - 8x + 1$

i) $f'(x) = \frac{3}{4}x^2 - x - 8$ ✓

$f''(x) = \frac{3}{2}x - 1$ ✓

(1) Provides correct derivative.
(2) Provides correct derivatives

---

ii) SP's occur when $f'(x) = 0$

$\Rightarrow \frac{3}{4}x^2 - x - 8 = 0$

ie $3x^2 - 4x - 32 = 0$

$(3x+8)(x-4) = 0$

$\Rightarrow \begin{cases} x = -8/3 \\ y = \frac{91}{27} \end{cases}$ or $\begin{cases} x = 4 \\ y = -23 \end{cases}$

Nature: By the 2nd Derivative test:

$f''(-8/3) = \frac{3 \times -8/3}{2} - 1$

$= -5$

< 0 ✓

$\therefore \left(-8/3, \frac{\cancel{91}\ 379}{27}\right)$ is a local max. SP.

$f''(4) = \frac{3 \times 4}{2} - 1$

$= 4$

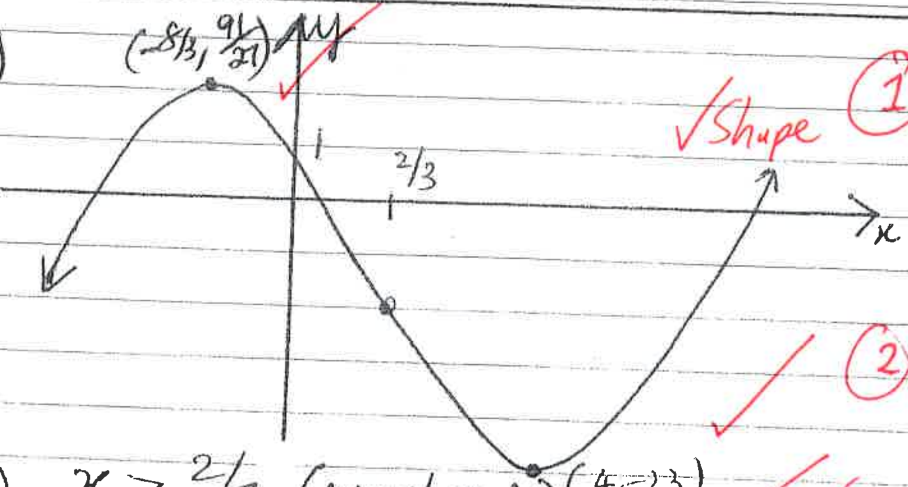
> 0 ✓

$\therefore (4, -23)$ is a local min SP.

(1) Attempts to solve $\frac{dy}{dx} = 0$, or equivalent merit.
(2) Finds coordinates of both stationary points.
(3) Provides correct solutions ✓

---

iii)



✓ ✓Shape ✓

(1) Correctly plots and labels stationary points or equivalent merit.
(2) Provides correct sketch

iv) $x > 2/3$ (accept $x \geq 2/3$) ✓

(1) Provides correct solution

b) i) Given $P = P_0 e^{kt}$

$$\frac{dP}{dt} = kP_0 e^{kt} = kP$$

1 Provides correct solution

ii) $P_0 = 11,000$.

1 Provides correct value of P_0

iii) Want $P(36)$

$P = 11000e^{kt}$

Sub $(4, 33000)$

$\Rightarrow 3 = e^{4k}$

$4k = \ln 3$

$\therefore k = \frac{1}{4}\ln 3$

1 Finds the correct value of k

$\therefore P(36) = 11000e^{36k}$

$= 11000\, e^{36[\frac{1}{4}\ln 3]}$

2 Correct substitution.

$= 11000 \times 3^9$

$= 216,513,000$

3 Provides correct solution

c) Note $\triangle AOB$ is equilateral

$\therefore AB = 4\text{cm}$ (diameter)

1 Find the value of AB

Shaded area = Area of semi-circle − Segment AXB

$= \frac{1}{2} \times \pi \times (2)^2 - \frac{1}{2} \times 4^2 (\pi/3 - \sin \pi/3)$

$= 2\pi - 8(\pi/3 - \sqrt{3}/2)$

$= \left(4\sqrt{3} - \frac{2\pi}{3}\right)$ sq. units

2 Provides correct solution

Marker Comments: 4.83 units u^2

Question 15:

a) i) $A = PQ \times PS$

$= 2 \times 3\cos\theta \times 2 \times 3\sin\theta$

$= 36\sin\theta\cos\theta$

✓ clearly identifying a product 2 correct sides of the rectangle.

ii) $\frac{dA}{d\theta} = 36[\cos\theta\cos\theta + \sin\theta(-\sin\theta)]$

✓ correct differentiation

$= 36[\cos^2\theta - \sin^2\theta]$

$= 36[1 - \sin^2\theta - \sin^2\theta]$

✓ substitution of $(1-\sin^2\theta)$.

$= 36[1 - 2\sin^2\theta]$

iii) We want $\frac{dA}{d\theta} = 0$ for maxima/minima.

$\Rightarrow 1 - 2\sin^2\theta = 0$

ie $\sin\theta = \pm\frac{1}{\sqrt{2}}$

but since $0 \le \theta \le \pi/2$ choose

$\sin\theta > 0$ ie $\sin\theta = \frac{1}{\sqrt{2}}$ only.

✓ valid method that leads to a correct answer.

Hence $\theta = \pi/4$ only. (degree measure unacceptable).

✓ with justification that $\sin\theta > 0$ in $0 < x < \pi/2$.

b) $a = 3\cos 2t$

i) Period of the motion is $\frac{2\pi}{2} = \pi$ seconds

✓ 1 valid method (e.g. by integration)

Rest at $t = \pi/4$ ∴ the next two occasions would be:

$t = \pi/4 + \pi = 5\pi/4$ seconds.

and $t = \pi/4 + 2\pi = 9\pi/4$ seconds

✓ ✓ [if bald answer given awarded 2 marks]

Correct answer from an incorrect method receives no marks.

ii) $v = \frac{3}{2}\sin 2t + c$

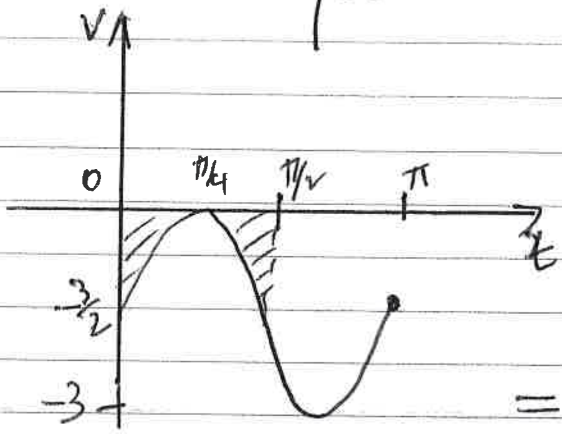
sub $(\pi/4, 0) \Rightarrow 0 = \frac{3}{2}\sin\pi/2 + c$

∴ $c = -3/2$

hence $v = \frac{3}{2}\sin 2t - \frac{3}{2} < 0$ for all t

∴ $d = \left|\int_0^{\pi/2} \frac{3}{2}\sin 2t - \frac{3}{2}\, dt\right|$

✓ Recognise $|\ |$ is required and integral ic) $d = 2\left|\int_0^{\pi/4} \frac{3}{2}\sin 2t - \frac{3}{2}\,dt\right|$



✓ → correct integration

$= 2\int_{\pi/4}^{0} \frac{3}{2}\sin 2t - \frac{3}{2}\,dt$

$= 2\left[\frac{-3}{4}\cos 2t - \frac{3t}{2}\right]_{\pi/4}^{0}$

$= 2\left[\left(\frac{-3}{4}\cos 0 - 0\right) - \left(\frac{-3}{4}\cos\frac{\pi}{2} - \frac{3\pi}{8}\right)\right]$

$= 2\left[-\frac{3}{4} + \frac{3\pi}{8}\right]$

$= \left(\frac{3\pi}{4} - \frac{3}{2}\right)$ m

✓ correct answer.

[if $\frac{3}{2} - \frac{3\pi}{4}$ is given a max of 2 marks is awarded].

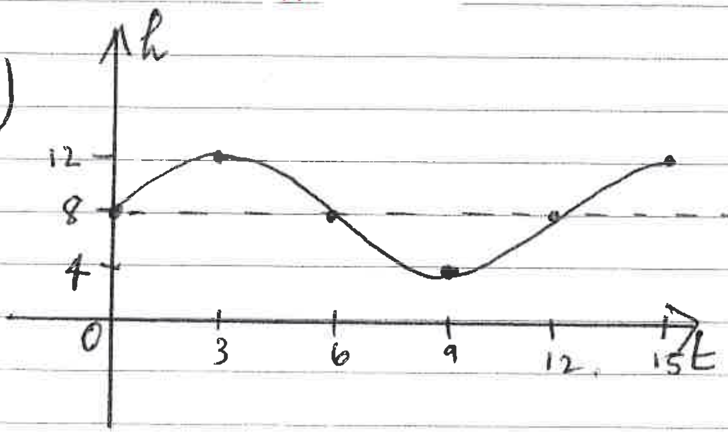
c). $h(t) = 8 + 4\sin\left(\frac{\pi}{6}t\right), \quad 0 \le t \le 24.$

i) $a = 4$ (metres) ✓

ii) $T = \frac{2\pi}{\pi/6}$

$= 12$ hours. ✓

iii)



✓ 1 shape

✓ points - position.

Marker Comments:

Question 16:

a) $\text{Area} = \int_2^4 \frac{6x}{x^2-2}\,dx$

$= 3\ln(x^2-2)\Big|_2^4$

$= 3[\ln 14 - \ln 2]$

$= 3\ln 7.$

b) Making x^2 the subject:

$yx^2 - 2y = 6$

$\therefore x^2 = \frac{6+2y}{y} \quad (y \neq 0)$

ie $x^2 = \frac{6}{y} + 2.$

$\therefore V = \pi\int_1^3 \left(\frac{6}{y}+2\right) dy$

$= \pi\left[6\ln y + 2y\right]_1^3$

$= \pi\left[6\ln 3 + 6 - 6\ln 1 - 2\right]$

$= \pi\left[6\ln 3 + 4\right]$ cubic units.

c) i) $r = 6\%$ p.a

$= \frac{0.06}{12}$

$= 0.005$

$A_1 = 250\,000(1.005)^{12} - M$

$A_2 = A_1 \times (1.005)^{12} - M$

$= [250\,000(1.005)^{12} - M](1.005)^{12} - M$

$A_2 = 250000(1.005)^{24} - M(1 + 1.005^{12})$

ii) $A_{10} = 250000(1.005)^{120}$

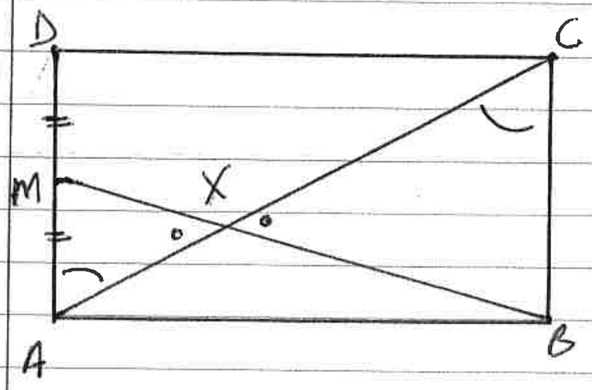
$- M(1 + 1.005^{12} + \cdots + 1.005^{108})$

iii) We want $A_{10} = 0$

$$\therefore \quad M = \frac{250000\,(1.005)^{120}\,[1.005^{12} - 1]}{[(1.005)^{120} - 1]}$$

$$= \$\,34{,}238$$ ✓

d)



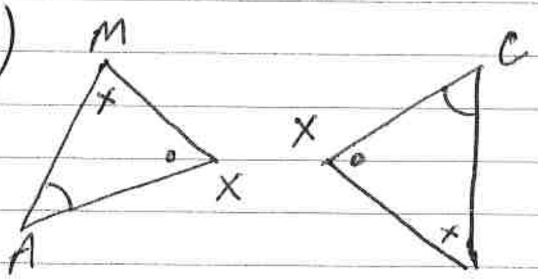
i) In ΔAXM and ΔBXC

1. $M\hat{X}A = B\hat{X}C$ (vertically opp. angles)
2. $X\hat{C}B = X\hat{A}M$ (alt. angles in parallel lines $DC \parallel AB$).

$\therefore\ \Delta AXM \,|||\, \Delta BXC$ (equiangular)

✓ → 1 correct statement (= 1 mark)
→ 3 correct lines (= 2 marks)
required.

ii)



In similar Δ's, corresponding sides are in proportion.

$$\therefore \quad \frac{CX}{AX} = \frac{CB}{AM} = \frac{2}{1}$$ ✓

$$\therefore \quad \frac{CX}{AX} = \frac{2}{1}$$

but $AX = AC - CX$

$$\therefore \quad \frac{CX}{AC - CX} = \frac{2}{1} \Rightarrow CX = 2AC - 2CX$$

ie $3CX = 2AC$ ✓

hence: $\dfrac{CX}{AC} = \dfrac{2}{3}$

iii) By Pythagoras' theorem in ΔCAB

$$AC^2 = AB^2 + CB^2$$

but $CB = \frac{1}{2}AB$ (given)

$$\therefore \quad AC^2 = AB^2 + \frac{1}{4}AB^2$$

$$AC^2 = \frac{5}{4}AB^2$$

since $AC = \dfrac{3CX}{2}$ from (ii)

then $\left(\dfrac{3CX}{2}\right)^2 = \frac{5}{4}(AB)^2$

ie $\frac{9}{4}CX^2 = \frac{5}{4}AB^2$

$$\left(\frac{CX}{AB}\right)^2 = \frac{5}{4} \times \frac{4}{9}$$

$$= 5/9$$

Marker Comments: