



TRINITY GRAMMAR SCHOOL

Mathematics Department

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(NESA Student Number | Year 12 only)

2022

Year 12

Mathematics Advanced

HSC ASSESSMENT TASK 4

Date of Assessment Task: Monday, 22 August 2022

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Erasable pens and all electronic devices (or timekeepers), unless otherwise advised, are not permitted in this Assessment
- NESA approved calculators may be used
- A reference sheet is provided
- In Questions 11 – 34, show relevant mathematical reasoning and/or calculations in the spaces provided
- Write your NESA Student Number (Year 12 HSC) or Name (Year 11) and your Class teacher on the question paper and on any answer sheets or writing booklets used to write your responses to the questions submitted
- If you do not attempt a question you must submit an answer sheet or writing booklet for that question clearly indicating N/A and your NESA Student Number or Name

Total marks: 100

Section I – 10 marks (pages 2 – 5)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 6 – 25)

- Attempt Questions 11 – 34
- Allow about 2 hours and 45 minutes for this section

- **HSC Assessment Weighting:** 30%

Section I

10 marks

Attempt Questions 1 – 10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10.

1. What are the factors of $6x^2 + x - 2$?
- (A) $(2x+1)(3x+2)$
- (B) $(2x+1)(3x-2)$
- (C) $(2x-1)(3x-2)$
- (D) $(2x-1)(3x+2)$
2. The 7th term of an arithmetic sequence is 45 and the 11th term is 77. What is the first term (a) **and** the common difference (d) ?
- (A) $a = -3$ and $d = 8$
- (B) $a = 3$ and $d = -8$
- (C) $a = 8$ and $d = -3$
- (D) $a = 8$ and $d = 3$
3. What is the derivative of $e^{x \sin 3x}$?
- (A) $e^{3x \cos 3x}$
- (B) $e^{x \sin 3x}(\sin 3x + 3x \cos 3x)$
- (C) $e^{x \sin 3x}$
- (D) $e^{x \sin 3x}(\sin 3x - 3x \cos 3x)$

- 4 A bag contains 5 red, 6 green, 4 white and 5 yellow jellybeans.

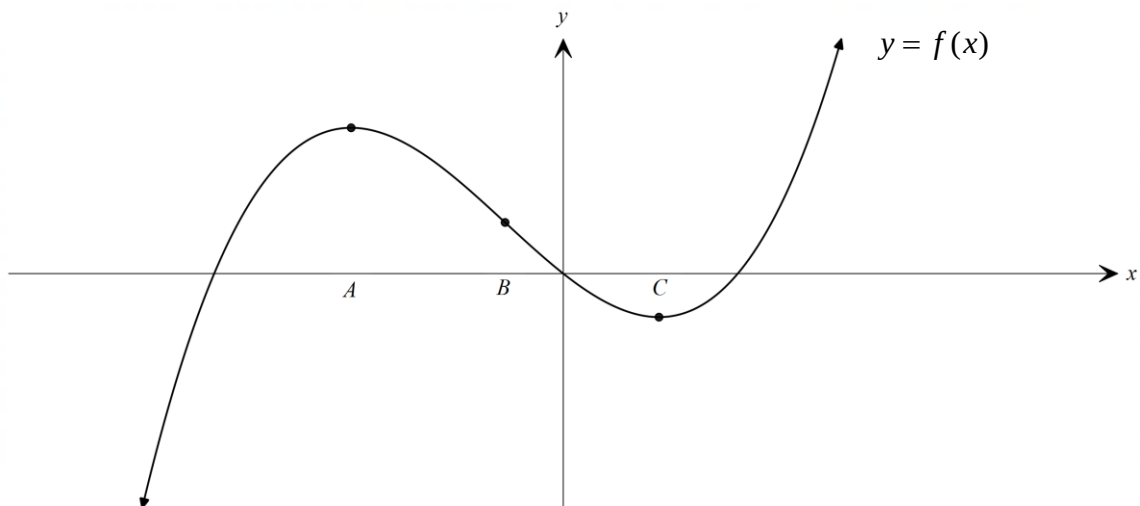
Two jellybeans are chosen, one after the other in succession at random, without replacement.

What is the probability of selecting at least 1 red jellybean?

- (A) $\frac{15}{38}$
- (B) $\frac{17}{38}$
- (C) $\frac{20}{38}$
- (D) $\frac{31}{38}$

- 5 The graph of $y = f(x)$ is shown below.

At the point where $x = A$ and $x = C$ there is a stationary point. When $x = B$ there is a point of inflection.



Which of the domains listed below is $f'(x) > 0$ **and** $f''(x) > 0$?

- (A) $(-\infty, A)$
- (B) (A, B)
- (C) (B, ∞)
- (D) (C, ∞)

- 6 Carol bought a triangular block of land with sides of length 9, 5 and 10 metres respectively. Which of the following gives the area of the block of land in square metres?
- (A) $22.5 \sin 86^\circ 11'$
- (B) $20 \sin 106^\circ 36'$
- (C) $14 \sin 86^\circ 11'$
- (D) $45 \sin 106^\circ 36'$
- 7 What is the solution of $\sin 2x = 1$ in the domain $[0, 2\pi]$?
- (A) $x = \frac{\pi}{4}, \frac{5\pi}{4}$
- (B) $x = \frac{\pi}{4}, \frac{7\pi}{4}$
- (C) $x = \frac{\pi}{2}, \frac{5\pi}{4}$
- (D) $x = \frac{\pi}{4}, \frac{3\pi}{4}$
- 8 Which of the following represents a function $f(x)$ whose graph has undergone the transformations in the following order?
- Translated left 2 units
 - Translated down 4 units
- (A) $f(x-2)-4$
- (B) $f(x+2)-4$
- (C) $f(x-2)+4$
- (D) $f(x+2)+4$

- 9 Jarred is investing \$400 every three months into an annuity which earns interest at 4% p.a. compounded quarterly. Use the table to answer the question below.

Future values of annuity of \$1					
Period	Interest Rate				
	1%	2%	3%	4%	8%
3	3.0301	3.0604	3.0909	3.1216	3.2464
6	6.1520	6.3081	6.4684	6.6330	7.3359
9	9.3685	9.7546	10.1591	10.5828	12.4876
12	12.6825	13.4121	14.1920	15.0258	18.9771
18	19.6147	21.4123	23.4144	25.6454	37.4502
24	26.9735	30.4219	34.4265	39.0826	66.7648
30	34.7849	40.5681	47.5754	56.0849	113.2832
36	43.0769	51.9944	63.2759	77.5983	187.1021

What is the amount in Jarred's account after 3 years?

- (A) \$6,010.32
 (B) \$5,073.00
 (C) \$1,248.64
 (D) \$1,212.04
- 10 What is an anti-derivative of $f'(x) = \frac{2}{3-x}$?
- (A) $f(x) = 2 \ln|3 - x|$
 (B) $f(x) = -2 \ln|3 - x|$
 (C) $f(x) = 3 \ln|3 + x|$
 (D) $f(x) = -3 \ln|3 + x|$

End of Section I

Section II

90 marks

Attempt Questions 11 – 34

Allow about 2 hours and 45 minutes for this section

Answer each question in the spaces provided. These spaces provide guidance for the expected length of responses.

In Questions 11 – 34, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (2 marks)

Solve $\sqrt{3} \sin \theta = \cos \theta$ for $-\pi \leq \theta \leq \pi$.

2

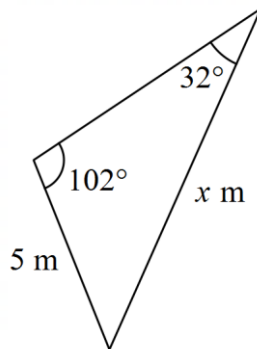
Question 12 (3 marks)

If $y = \frac{x^2}{1-3x}$, solve $\frac{dy}{dx} = 0$.

3

Question 13 (3 marks)

The diagram shows a triangle with sides of length x m and 5 m, and angles 102° and 32° .



Use the sine rule to calculate the value of x , correct to two decimal places.

3

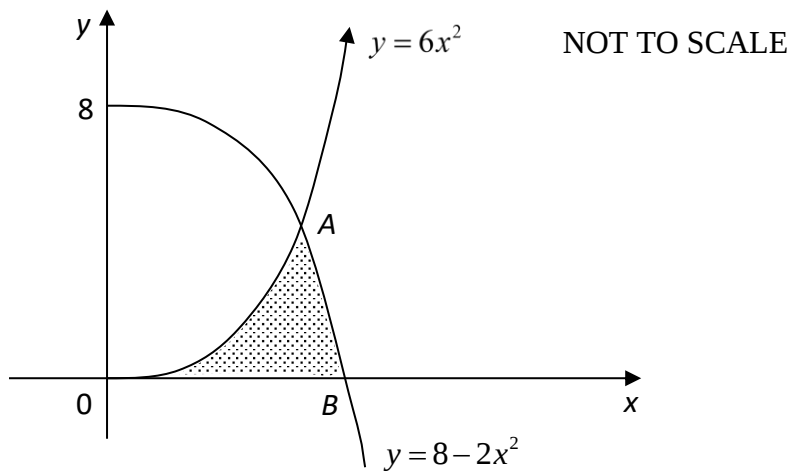
Question 14 (3 marks)

Use the formula, $\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$ where $n \neq -1$, to evaluate

$$\int_{-1}^1 3x^2(x^3 + 12)^3 dx.$$

3

Question 15 (5 Marks)



The shaded region OAB is bounded by the parabolas: $y = 6x^2$, $y = 8 - 2x^2$ and the x -axis. Point A is the intersection of the two parabolas and point B is the x -intercept of the parabola $y = 8 - 2x^2$.

- (a) Find the x -ordinates of points A and B .

2

[illegible]

(b) Find the **exact** area of the shaded region OAB .

3

Question 16 (3 marks)

The formula for differentiation by First Principles is, $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

3

Use this formula to find the derivative of $f(x) = 2x^2$.

Question 17 (3 marks)

The circle with equation $x^2 - 4x + y^2 - 6y - 3 = 0$ is reflected in the y -axis **and** then reflected in the x -axis. Find the coordinates of the centre **and** radius of the new circle. **3**

Question 18 (6 marks)

- (a) For the curve $y = 3x^3 - 9x$ find the coordinates of the two stationary points **and** determine their nature. **4**

(b) Show that the inflection point is the midpoint of the stationary points.

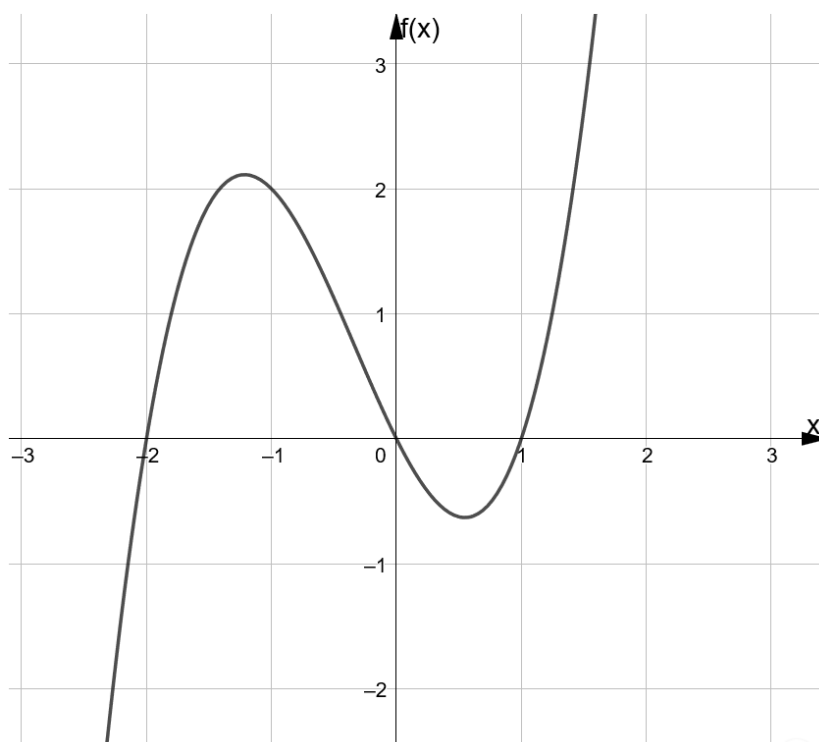
2

Question 19 (2 marks)

The graph below shows $y = f(x)$.

On the same axes, sketch $y = 2 - f(x)$.

2



Question 20 (2 marks)

A quadratic function is given by the expression $f(x) = 3x^2 + 4x + c$.

Determine the values of c if $f(x)$ has two distinct real zeroes.

2

Question 21 (3 marks)

Find the equation of the tangent to the curve $y = e^{2x}$ at the point $(1, e^2)$

3

Question 22 (5 marks)

Let two functions be defined as, $f(x) = \sqrt{4 - x}$ and $g(x) = x^2$.

(a) State, using interval notation, the domain **and** range of $f(x)$.

2

(b) Sketch the graph of $f(g(x))$.

1

(c) Hence, state, using interval notation, the domain **and** range of $f(g(x))$.

2

Question 23 (4 marks)

The sum of the first two terms of an infinite geometric series is 45.

4

The third term in the series is 12.

Show that there are two possible geometric series **and** find the first term of **each** series.

Question 24 (2 marks)

The function $f(x) = (x - 2)^3 - 2$ is translated 5 units to the right and 4 units up.

Find the equation of the transformed equation.

2

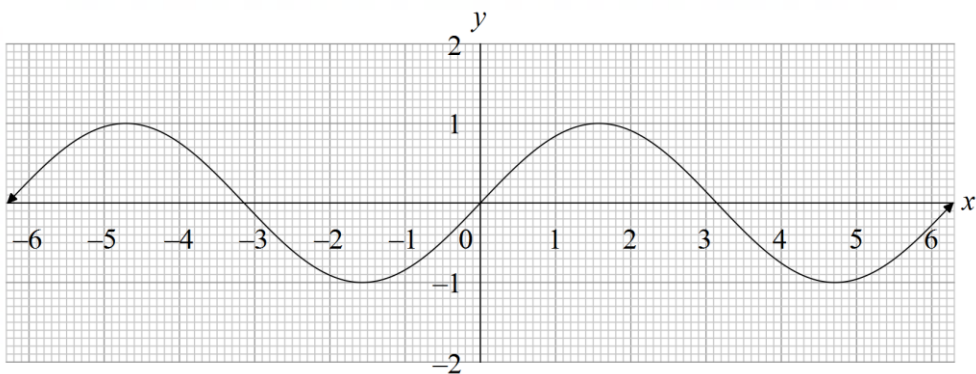
Question 25 (2 marks)

Find $\int \frac{1}{\sqrt{1-3x}} dx$.

2

Question 26 (1 mark)

The graph of $y = \sin x$ is shown in the diagram below.



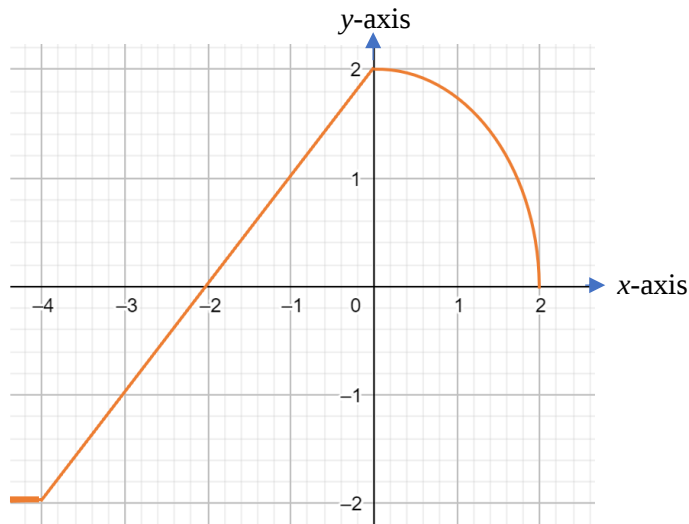
Explain why the equation $\sin x = 1 - \frac{x}{4}$ has three solutions in the domain $[0, 6]$.

1

Question 27 (3 marks)

A piece-wise function is defined as:

$$f(x) = \begin{cases} -2, & x < -4 \\ x + 2, & -4 \leq x < 0 \\ \sqrt{4 - x^2}, & 0 \leq x \leq 2 \end{cases}$$

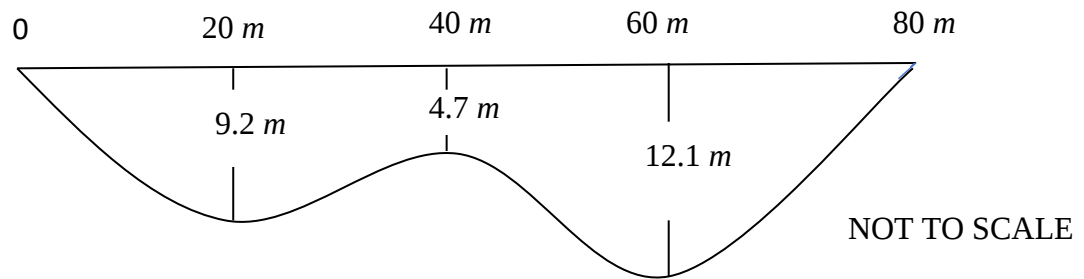


Calculate the **exact** value of $\int_{-4}^2 f(x) dx$.

3

Question 28 (4 marks)

The following diagram represents a cross-section through a river. The depth of the river is marked every 20 metres.



- (a) Use the trapezoidal rule to estimate the area of the cross-section.

2

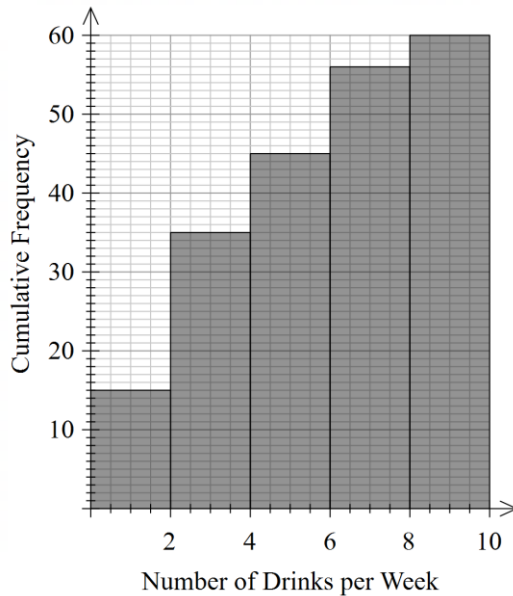
- (b) If the river is flowing at the rate of 6m/s, what is the volume of water that passes through the cross-section each hour.

2

Question 29 (5 marks)

Lizzie is a university student collecting data about her classmates' alcohol consumption. A question in her survey asks the respondent to indicate how many alcoholic drinks they typically consume each week.

The data collected is displayed in the cumulative frequency histogram.

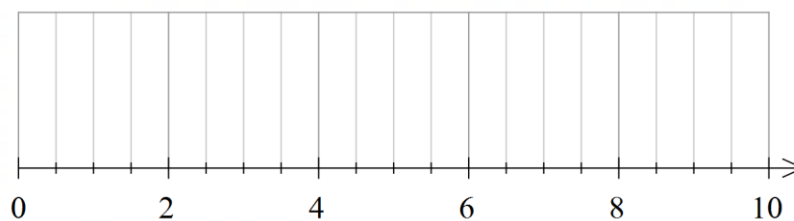


- (a) How many drinks per week do the top decile of respondents consume?

2

- (b) Construct a box and whisker plot to represent the data, **and** comment on the distribution of the data.

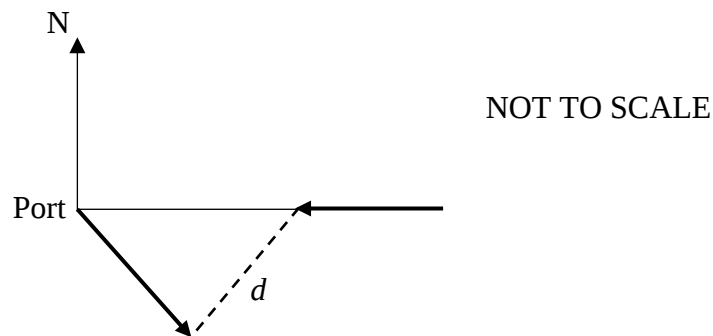
3



Question 30 (6 marks)

A fishing boat leaves a port at midday on a bearing of 150° at a speed of 10 km/h.

At the same time that the fishing boat leaves the port, a sailing boat 100 km due east of the port starts travelling towards the port at 20 km/h.



(b) Show that the distance between the two boats at t hours after midday is given by:

$$d = \sqrt{700t^2 - 5000t + 10000} \quad 3$$

(b) Determine the time, to the nearest minute, when the distance between the two boats is a minimum. 3

Question 31 (6 marks)

A particle is moving in a straight line with velocity $v = 3e^t + 6e^{-t}$ with t measured in seconds and v in ms^{-1} .

The particle begins its motion at the origin.

(a) What is the particle's initial velocity?

1

(b) Find the equation for x , the displacement of the particle in terms of time t .

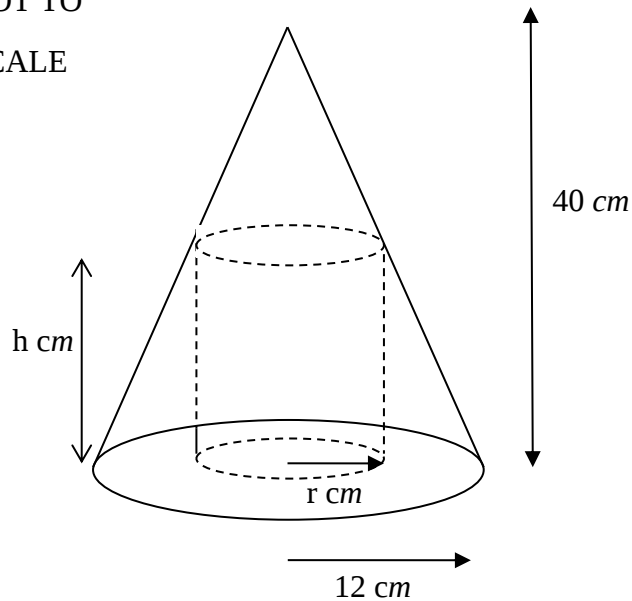
2

(c) Find the **exact** value of t when $x = 10$.

3

Question 32 (5 marks)

NOT TO
SCALE



A cylinder of radius r cm and height h cm is inscribed in a cone with base radius 12 cm and height 40 cm as shown in the above diagram.

(a) Show that $h = 40 - \frac{10r}{3}$.

2

Question 32 continues on page 22

(b) Hence determine the values of r and h for the **cylinder** to have maximum volume. 3

Question 33 (5 marks)

Max did a survey of a group of people he knew about their age and how much they earn each week. The results are shown in the table below.

Age (years) (x)	18	45	28	15	32	68
Wage (\$/week) ($W$)	715	2350	1530	438	1690	1320

- (a) Using your calculator find r , the correlation coefficient, correct to 4 decimal places, **2**
and determine the strength of correlation this data gives.

- (b) Using your calculator find the equation of the least-squares regression line in the **1**
form $W = Bx + A$, where A and B are integers.

- (c) Use your equation to estimate the earnings of a 50 year-old worker. **1**

Question 33 continues on page 24

- (d) Could your equation from part (b) be used to make valid estimates for ages greater than 68 and less than 15 years?

Validate your response with calculations and/or reasons.

1

Question 34 (7 marks)

Zoe takes out a loan to buy a new car. The loan is for \$25 000 and she plans to pay back the amount, including interest, over 5 years with equal monthly instalments of \$ M paid at the end of each month. She will be charged interest of 6% p.a. compounded monthly.

The amount that Zoe still owes immediately after making the n th monthly repayment can be determined using the recurrence relation

$$A_n = A_{n-1}(1.005) - M \quad \text{DO NOT PROVE THIS}$$

where $n = 1, 2, 3, \dots$ and $A_0 = 25\,000$

- (a) Use the recurrence relation to show that

$$A_3 = 25000(1.005)^3 - M(1 + 1.005 + 1.005^2)$$

2

(b) Hence, show that

$$A_n = A_0(1.005)^n - 200M(1.005^n - 1) \quad 3$$

(c) Hence, find the monthly repayment, M . 2

End of Section II

End of Task

Clearly identify the question number you are responding to.

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This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There is no text or other markings on the paper.



TRINITY GRAMMAR SCHOOL

Mathematics Department

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(NESA Student Number | Year 12 only)

2022

Solutions

Year 12

Mathematics Advanced

HSC ASSESSMENT TASK 4

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Total marks:
100

Section I – 10 marks (pages 2 – 5)

- Attempt Questions 1 – 10
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Section II – 90 marks (pages 6 – 25)

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- **HSC Assessment Weighting: 30%**

Multiple Choice:

1D 2A 3B 4B 5D 6A 7A 8B 9B 10B

Section II

90 marks

Attempt Questions 11 – 34

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Answer each question in the spaces provided. These spaces provide guidance for the expected length of responses.

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Question 11 (2 marks)

Solve $\sqrt{3} \sin \theta = \cos \theta$ for $-\pi \leq \theta \leq \pi$.

2

$$\frac{\sin \theta}{\cos \theta} = \frac{1}{\sqrt{3}}$$
$$\tan \theta = \frac{1}{\sqrt{3}}$$
$$\theta = \frac{\pi}{6}, -\frac{5\pi}{6}$$

Question 12 (3 marks)

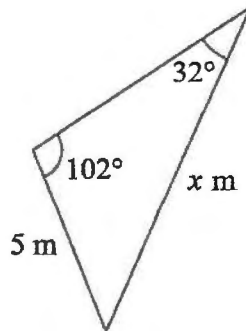
If $y = \frac{x^2}{1-3x}$, solve $\frac{dy}{dx} = 0$.

3

$$\frac{dy}{dx} = \frac{(1-3x)2x - x^2 \times -3}{(1-3x)^2}$$
$$= \frac{2x - 6x^2 + 3x^2}{(1-3x)^2}$$
$$= \frac{2x - 3x^2}{(1-3x)^2} = 0 \Rightarrow x(2-3x) = 0$$
$$\therefore x = 0, x = \frac{2}{3}$$

Question 13 (3 marks)

The diagram shows a triangle with sides of length x m and 5 m, and angles 102° and 32° .



Use the sine rule to calculate the value of x , correct to two decimal places.

3

$$\begin{aligned}\frac{x}{\sin 102} &= \frac{5}{\sin 32} \\ x &= \frac{5 \sin 102}{\sin 32} \\ &= 9.23 \text{ m}\end{aligned}$$

Question 14 (3 marks)

Use the formula, $\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$ where $n \neq -1$, to evaluate

$$\int_{-1}^1 3x^2(x^3+12)^3 dx.$$

3

$$\begin{aligned}\int_{-1}^1 3x^2(x^3+12)^3 dx &= \left[\frac{(x^3+12)^4}{4} \right]_{-1}^1 \\ &= \frac{13^4}{4} - \frac{11^4}{4} \\ &= 3480\end{aligned}$$

(b) Find the exact area of the shaded region OAB.

3

$$\begin{aligned}\text{Area} &= \int_0^1 6x^2 dx + \int_1^2 8 - 2x^2 dx \\ &= 2x^3 \Big|_0^1 + \left[8x - \frac{2}{3}x^3 \right]_1^2 \\ &= \frac{16}{3} \text{ sq units}\end{aligned}$$

Question 16 (3 marks)

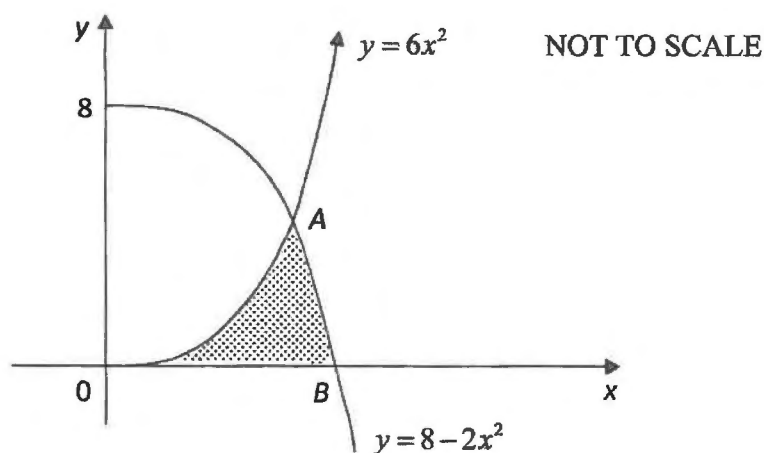
The formula for differentiation by First Principles is, $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

3

Use this formula to find the derivative of $f(x) = 2x^2$.

$$\begin{aligned}f(x+h) &= 2(x+h)^2 = 2(x^2 + 2xh + h^2) \\ &= 2x^2 + 4xh + 2h^2 \\ f'(x) &= \lim_{h \rightarrow 0} \frac{2x^2 + 4xh + 2h^2 - 2x^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{4xh + 2h^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(4x + 2h)}{h} \\ &= \lim_{h \rightarrow 0} 4x + 2h \\ &= 4x\end{aligned}$$

Question 15 (5 Marks)



The shaded region OAB is bounded by the parabolas: $y = 6x^2$, $y = 8 - 2x^2$ and the x -axis. Point A is the intersection of the two parabolas and point B is the x -intercept of the parabola $y = 8 - 2x^2$.

- (a) Find the x -ordinates of points A and B .

2

$6x^2 = 8 - 2x^2$
$8x^2 = 8$
$x = \pm 1$
$\therefore A = 1$
$8 - 2x^2 = 0$
$2x^2 = 8$
$x^2 = 4$
$x = \pm 2$
$\therefore B = 2$

Question 17 (3 marks)

The circle with equation $x^2 - 4x + y^2 - 6y - 3 = 0$ is reflected in the y -axis and then reflected in the x -axis. Find the coordinates of the centre and radius of the new circle. 3

$$\begin{aligned}(x-2)^2 - 4 + (y-3)^2 - 9 &= 3 \\(x-2)^2 + (y-3)^2 &= 16 \quad \checkmark \\ \text{Centre is } (2, 3), \text{ radius is } 4 \\ \text{Reflected in } y\text{-axis gives centre } (-2, 3), \\ \text{Reflected in } x\text{-axis gives} \\ \text{centre } (-2, -3) \text{ and radius } 4 \quad \checkmark \quad \checkmark\end{aligned}$$

Question 18 (6 marks)

- (a) For the curve $y = 3x^3 - 9x$ find the coordinates of the two stationary points and determine their nature. 4

$$\begin{aligned}\frac{dy}{dx} &= 9x^2 - 9 = 0 \text{ for stat. pts.} \\ \therefore x &= \pm 1 \\ \text{When } x = -1, y &= 3(-1)^3 - 9(-1) = 6 \\ \text{When } x = 1, y &= 3(1)^3 - 9(1) = -6 \\ \frac{d^2y}{dx^2} &= 18x = -18 < 0 \text{ when } x = -1 \\ \therefore (-1, 6) &\text{ is a max. } \checkmark \\ \frac{d^2y}{dx^2} &= 18 > 0 \text{ when } x = 1 \\ \therefore (1, -6) &\text{ is a min. } \checkmark \quad \checkmark\end{aligned}$$

(b) Show that the inflection point is the midpoint of the stationary points.

2

$$\frac{d^2y}{dx^2} = 18x = 0 \text{ for p.o.I}$$
$$\therefore x = 0 \Rightarrow (0,0)$$

Test:

x	$-\frac{1}{2}$	0	$\frac{1}{2}$
$\frac{d^2y}{dx^2}$	-9	0	9

$\therefore$ Change of concavity shows that $(0,0)$ is a pt of inflection

Midpoint of s.p. = $\left(\frac{1-1}{2}, \frac{-6+6}{2}\right)$

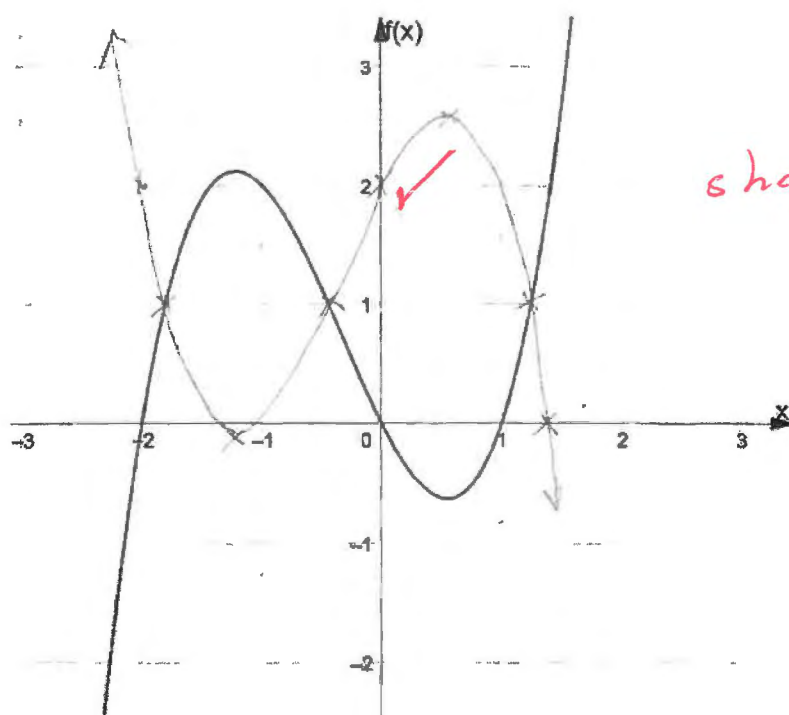
$$= (0,0)$$

Question 19 (2 marks)

The graph below shows $y = f(x)$.

On the same axes, sketch $y = 2 - f(x)$.

2



Question 20 (2 marks)

A quadratic function is given by the expression $f(x) = 3x^2 + 4x + c$.

Determine the values of c if $f(x)$ has two distinct real zeroes.

2

For two distinct roots, $\Delta > 0$
 $\therefore 4^2 - 4(3)c > 0$
 $16 - 12c > 0$ ① using discriminant
 $12c < 16$
 $c < \frac{4}{3}$ ① $c < \frac{4}{3}$
 For $c = \frac{4}{3}$, only ① in total.

Question 21 (3 marks)

Find the equation of the tangent to the curve $y = e^{2x}$ at the point $(1, e^2)$

3

$\frac{dy}{dx} = 2e^{2x} = 2e^2$ when $x=1$ ① m when $x=1$
 ① correct $\frac{dy}{dx}$
 $y - y_1 = m(x - x_1)$
 $y - e^2 = 2e^2(x - 1)$
 $y = 2e^2x - 2e^2 + e^2$
 $\therefore y = 2e^2x - e^2$ } ① mark for either.

Question 22 (5 marks)

Let two functions be defined as, $f(x) = \sqrt{4-x}$ and $g(x) = x^2$.

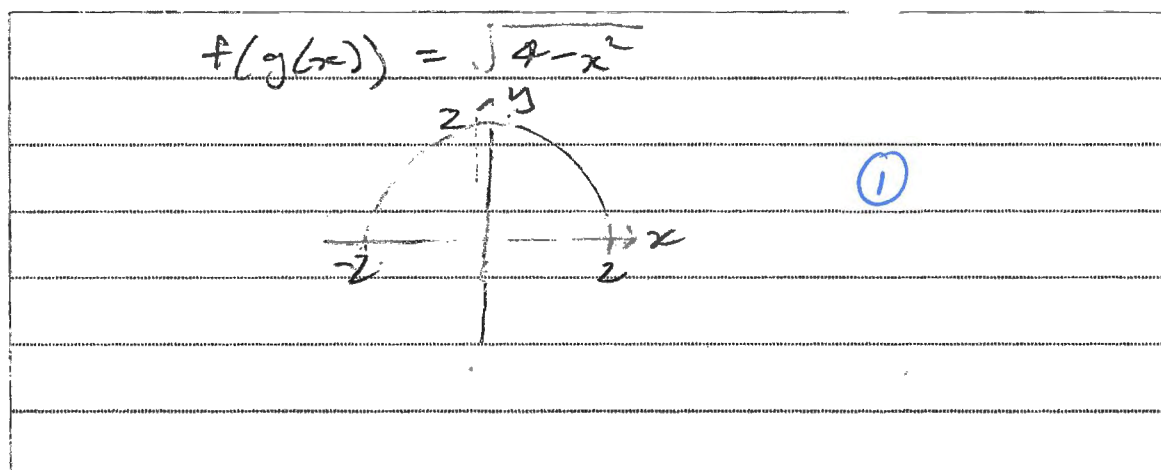
- (a) State, using interval notation, the domain **and** range of $f(x)$.

2

$$\begin{aligned} 4-x &\geq 0 \\ -x &\geq -4 \\ x &\leq 4 \\ \therefore \text{Domain is } (-\infty, 4] &\quad (1) \\ f(x) &\geq 0 \\ \therefore \text{Range is } [0, \infty) &\quad (1) \end{aligned}$$

- (b) Sketch the graph of $f(g(x))$.

1



- (c) Hence, state, using interval notation, the domain **and** range of $f(g(x))$.

2

$$\begin{aligned} \text{Domain is } [-2, 2] \\ \text{Range is } [0, 2] \end{aligned}$$

If students drew incorrect graph in (b) but Domain + Range matched (2) (Error carried)

Question 23 (4 marks)

The sum of the first two terms of an infinite geometric series is 45.

4

The third term in the series is 12.

Show that there are two possible geometric series and find the first term of each series.

$$\begin{aligned} a + ar &= 45 & \textcircled{1} \\ ar^2 &= 12 & \textcircled{2} \end{aligned} \quad \left. \vphantom{\begin{aligned} a + ar &= 45 \\ ar^2 &= 12 \end{aligned}} \right\} \textcircled{1} \text{ Either / Both of these.}$$
$$\text{From } \textcircled{1} \quad a = \frac{45}{r+1} \quad \textcircled{3}$$
$$\text{Sub } \textcircled{3} \text{ into } \textcircled{2}: \frac{45r^2}{r+1} = 12$$
$$45r^2 = 12(r+1)$$
$$45r^2 - 12r - 12 = 0$$
$$r = \frac{12 \pm \sqrt{12^2 - 4(45)(-12)}}{2(45)}$$
$$= \frac{12 \pm 48}{90}$$
$$\therefore r = \frac{2}{3} \text{ or } -\frac{2}{3}$$
$$\begin{aligned} \text{For } r = \frac{2}{3}, a &= 27 & \textcircled{1} \\ \text{For } r = -\frac{2}{3}, a &= 75 & \textcircled{1} \end{aligned} \quad \left. \vphantom{\begin{aligned} \text{For } r = \frac{2}{3}, a &= 27 \\ \text{For } r = -\frac{2}{3}, a &= 75 \end{aligned}} \right\} \text{To get these } \textcircled{1}$$

Question 24 (2 marks)

The function $f(x) = (x-2)^3 - 2$ is translated 5 units to the right and 4 units up.

Find the equation of the transformed equation.

2

$$\begin{aligned} y &= (x-2-5)^3 - 2 + 4 \\ &= (x-7)^3 + 2 \end{aligned}$$

$\textcircled{1} \quad \textcircled{1}$

Question 25 (2 marks)

Find $\int \frac{1}{\sqrt{1-3x}} dx$.

2

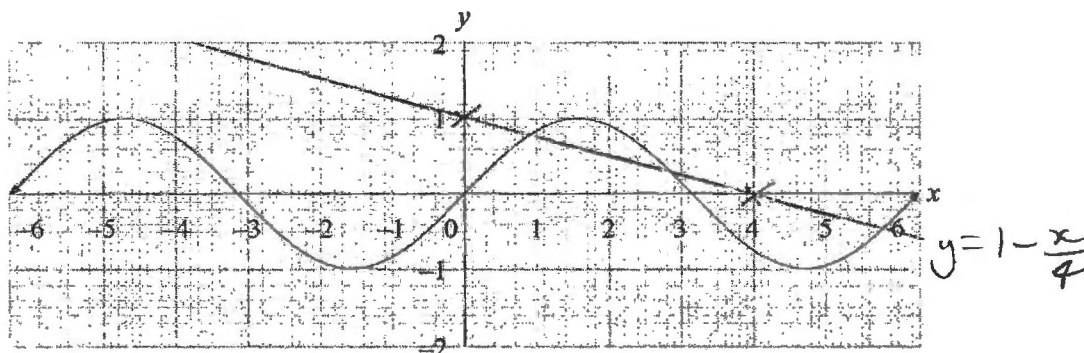
$$\begin{aligned} \int \frac{1}{\sqrt{1-3x}} dx &= \int (1-3x)^{-\frac{1}{2}} dx \\ &= \frac{(1-3x)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C \\ &= \frac{-2\sqrt{1-3x}}{3} + C \end{aligned}$$

Don't forget + C !

② Correct integral (with/without +C)
① if you forgot to divide by -3.

Question 26 (1 mark)

The graph of $y = \sin x$ is shown in the diagram below.



Explain why the equation $\sin x = 1 - \frac{x}{4}$ has three solutions in the domain $[0, 6]$.

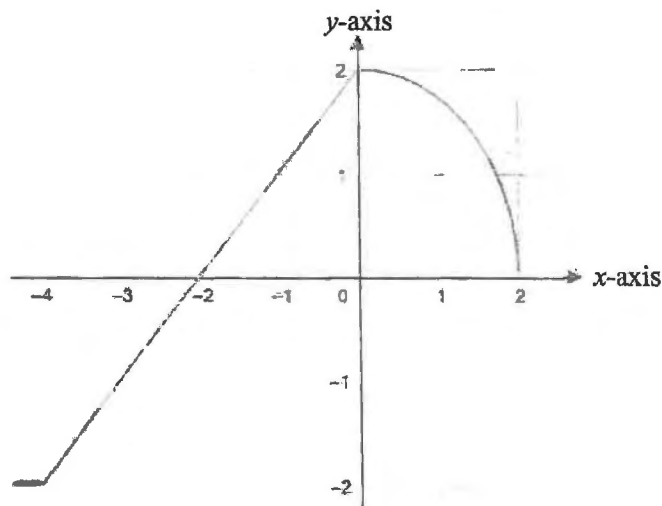
1

The line $y = 1 - \frac{x}{4}$ intersects the graph of $y = \sin x$ three times. Must have intersects

Question 27 (3 marks)

A piece-wise function is defined as:

$$f(x) = \begin{cases} -2, & x < -4 \\ x+2, & -4 \leq x < 0 \\ \sqrt{4-x^2}, & 0 \leq x \leq 2 \end{cases}$$



Calculate the exact value of $\int_{-4}^2 f(x) dx$.

3

$$\begin{aligned} \int_{-4}^2 f(x) dx &= \int_{-4}^0 f(x) dx + \int_0^2 f(x) dx \\ &= 0 + \frac{1}{4} \pi \times 2^2 \\ &= \pi \end{aligned}$$

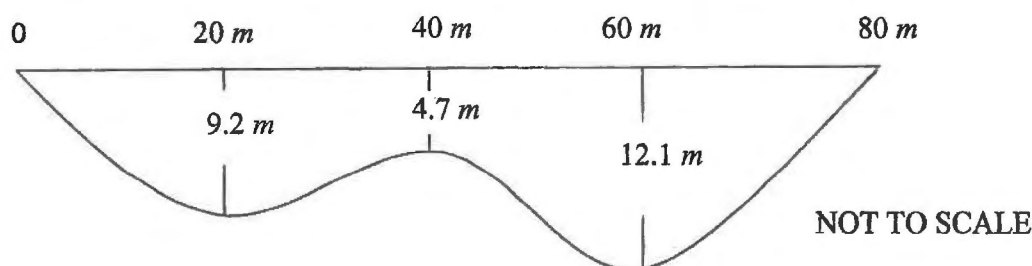
① Integration of $\int_{-4}^0 x+2 dx$

① = 0

① π units² as final or within working.

Question 28 (4 marks)

The following diagram represents a cross-section through a river. The depth of the river is marked every 20 metres.



- (a) Use the trapezoidal rule to estimate the area of the cross-section.

2

$$A = \frac{20}{2} [0 + 0 + 2(9.2 + 4.7 + 12.1)] \quad \textcircled{1}$$

$$= 520 \text{ m}^2 \quad \textcircled{1}$$

Some students got $\textcircled{1}$ for appropriate working with only one substitution / computation error.

- (b) If the river is flowing at the rate of 6m/s, what is the volume of water that passes through the cross-section each hour.

2

$$6 \text{ m/s} = 6 \times 3600 \\ = 21600 \text{ m/h} \quad \textcircled{1}$$

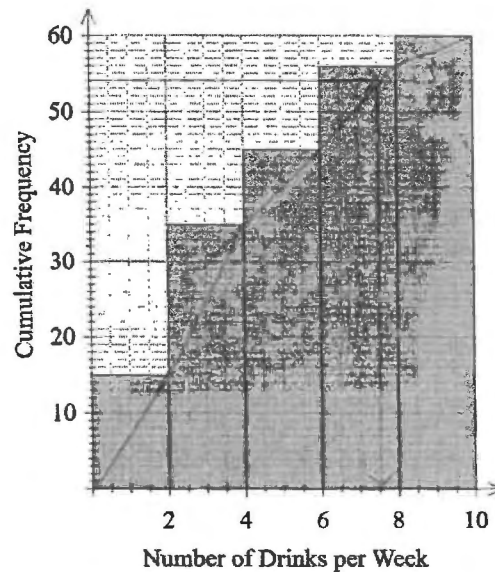
$$\text{Volume} = 520 \times 21600 \quad \textcircled{1} \\ = 11\,232\,000 \text{ m}^3$$

EC from part a)

Question 29 (5 marks)

Lizzie is a university student collecting data about her classmates' alcohol consumption. A question in her survey asks the respondent to indicate how many alcoholic drinks they typically consume each week.

The data collected is displayed in the cumulative frequency histogram.



① correct polygon .

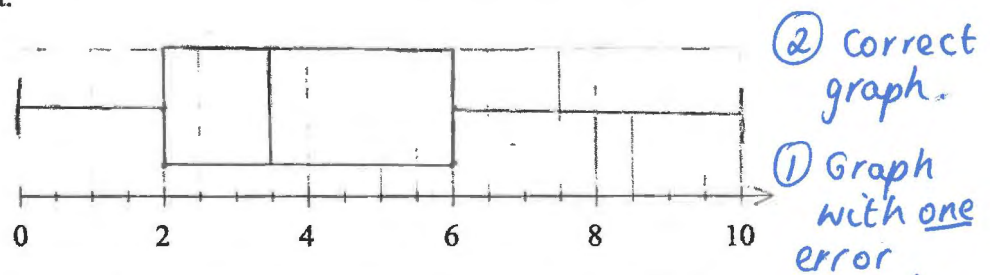
- (a) How many drinks per week do the top decile of respondents consume?

2

10% of 60 = 6
Using the ogive drawn above,
the top 10% drink 7.5 to 10 drinks per week
①

- (b) Construct a box and whisker plot to represent the data, and comment on the distribution of the data.

3

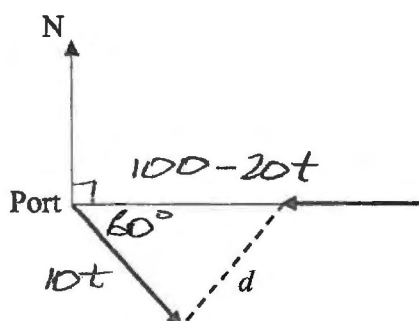


The distribution is positively skewed. ①

Question 30 (6 marks)

A fishing boat leaves a port at midday on a bearing of 150° at a speed of 10 km/h.

At the same time that the fishing boat leaves the port, a sailing boat 100 km due east of the port starts travelling towards the port at 20 km/h.



NOT TO SCALE

① for $100 - 20t$ or $10t$ seen

(b) Show that the distance between the two boats at t hours after midday is given by:

$$d = \sqrt{700t^2 - 5000t + 10000}$$

3

$$\begin{aligned} d^2 &= (10t)^2 + (100 - 20t)^2 - 2(10t)(100 - 20t)\cos 60^\circ && \text{① cosine rule} \\ &= 100t^2 + 10000 - 4000t + 400t^2 - 1000t + 200t^2 \\ &= 700t^2 - 5000t + 10000 && \text{① simplification} \end{aligned}$$

$$\therefore d = \sqrt{700t^2 - 5000t + 10000}$$

(b) Determine the time, to the nearest minute, when the distance between the two boats is a minimum. 3

① derivative = 0

$$\begin{aligned} \frac{d(d^2)}{dt} &= 1400t - 5000 = 0 \text{ for min} \\ 1400t &= 5000 && \text{①} \\ t &= \frac{25}{7} \text{ h} \\ &= 3 \text{ h } 34 \text{ min} && \text{①} \end{aligned}$$

Question 31 (6 marks)

A particle is moving in a straight line with velocity $v = 3e^t + 6e^{-t}$ with t measured in seconds and v in ms^{-1} .

The particle begins its motion at the origin.

- (a) What is the particle's initial velocity?

1

$$\begin{aligned}\text{When } t=0, v &= 3e^0 + 6e^0 \\ &= 3 + 6 \\ &= 9 \text{ m/s} \quad (1)\end{aligned}$$

- (b) Find the equation for x , the displacement of the particle in terms of time t .

2

$$\begin{aligned}x &= \int v dt \\ &= 3e^t + 6e^{-t} + c \quad (1) \\ \text{When } t=0, x &= 0 \\ 0 &= 3 - 6 + c \\ c &= 3 \quad (1) \\ \therefore x &= 3e^t - 6e^{-t} + 3\end{aligned}$$

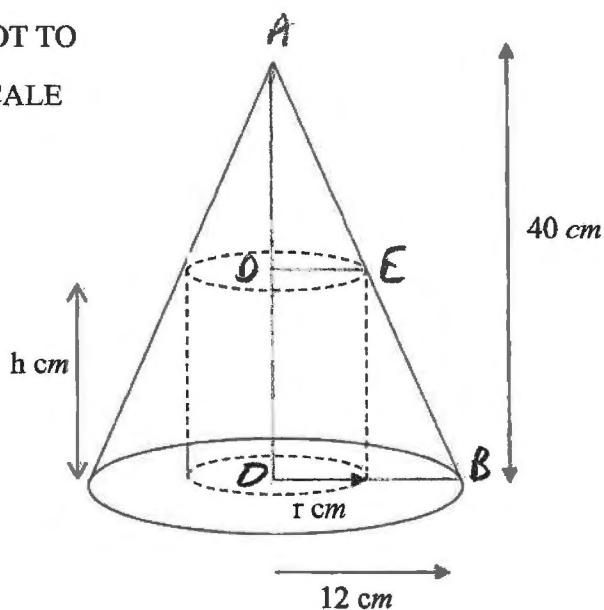
- (c) Find the exact value of t when $x = 10$.

3

$$\begin{aligned}10 &= 3e^t - 6e^{-t} + 3 \quad (1) \text{ fr allowed} \\ 7 &= 3e^t - \frac{6}{e^t} \\ 3e^{2t} - 7e^t - 6 &= 0 \\ \text{Let } u &= e^t \therefore 3u^2 - 7u - 6 = 0 \quad (1) \\ (3u+2)(u-3) &= 0 \\ u &= -\frac{2}{3}, 3 \therefore e^t = 3 \Rightarrow t = \ln 3 \quad (1)\end{aligned}$$

Question 32 (5 marks)

NOT TO
SCALE



A cylinder of radius r cm and height h cm is inscribed in a cone with base radius 12 cm and height 40 cm as shown in the above diagram.

- (a) Show that $h = 40 - \frac{10r}{3}$.

2

$$\frac{AO}{AO + OB} = \frac{DE}{OB}$$

$$\frac{40-h}{40} = \frac{r}{12} \quad (1)$$

$$12(40-h) = 40r$$

$$480 - 12h = 40r \quad (1)$$

$$12h = 480 - 40r$$

$$h = \frac{480 - 40r}{12} \quad (1)$$

$$\therefore h = 40 - \frac{10r}{3}$$

Question 32 continues on page 22

(b) Hence determine the values of r and h for the cylinder to have maximum volume. 3

$$V = \pi r^2 h$$

$$= \pi r^2 \left(40 - \frac{10r}{3}\right) \quad (1)$$

$$= 40\pi r^2 - \frac{10\pi r^3}{3}$$

$$\frac{dV}{dr} = 80\pi r - 10\pi r^2 = 0 \quad (2)$$

$$10\pi r(8 - r) = 0$$

$$\therefore r = 8$$

$$\frac{d^2V}{dr^2} = 80\pi - 20\pi r < 0 \text{ when } r = 8 \quad (3)$$

$$\therefore \text{max volume when } r = 8, h = \frac{40}{3}$$

(1)

(1)

Question 33 (5 marks)

Max did a survey of a group of people he knew about their age and how much they earn each week. The results are shown in the table below.

Age (years) (x)	18	45	28	15	32	68
Wage (\$/week) (W)	715	2350	1530	438	1690	1320

- (a) Using your calculator find r , the correlation coefficient, correct to 4 decimal places, and determine the strength of correlation this data gives. 2

$r = 0.5263$ ①
This shows a weak (positive) correlation
① ↑ not necessary

- (b) Using your calculator find the equation of the least-squares regression line in the form $W = Bx + A$, where A and B are integers. 1

$W = 18x + 706$ ①

- (c) Use your equation to estimate the earnings of a 50 year-old worker. 1

$W = 18(50) + 706$ ①
$= \$1606$

Question 33 continues on page 24

- (d) Could your equation from part (b) be used to make valid estimates for ages greater than 68 and less than 15 years?

Validate your response with calculations and/or reasons.

1

No. The equation is only valid for interpolation between 15 and 68.

Question 34 (7 marks)

Zoe takes out a loan to buy a new car. The loan is for \$25 000 and she plans to pay back the amount, including interest, over 5 years with equal monthly instalments of \$ M paid at the end of each month. She will be charged interest of 6% p.a. compounded monthly.

The amount that Zoe still owes immediately after making the n th monthly repayment can be determined using the recurrence relation

$$A_n = A_{n-1}(1.005) - M \quad \text{DO NOT PROVE THIS}$$

where $n = 1, 2, 3, \dots$ and $A_0 = 25\,000$

- (a) Use the recurrence relation to show that

$$A_3 = 25000(1.005)^3 - M(1 + 1.005 + 1.005^2)$$

2

$A_1 = 25000(1.005) - m$
$A_2 = [25000(1.005) - m]1.005 - m$
$= 25000(1.005)^2 - 1.005m - m$
$A_3 = [25000(1.005)^2 - 1.005m - m]1.005 - m$
$= 25000(1.005)^3 - 1.005^2m - 1.005m - m$
$= 25000(1.005)^3 - m(1 + 1.005 + 1.005^2)$

(b) Hence, show that

$$A_n = A_0(1.005)^n - 200M(1.005^n - 1)$$

3

Given that $A_0 = 25000$

$$A_n = A_0(1.005)^n - m(1 + 1.005 + \dots + 1.005^{n-1})$$

Now $1 + 1.005 + \dots + 1.005^{n-1} = \frac{1(1.005^n - 1)}{1.005 - 1}$

$$= 200(1.005^n - 1)$$
$$\therefore A_n = A_0(1.005)^n - 200M(1.005^n - 1)$$

(c) Hence, find the monthly repayment, M .

2

$n = 60$

$A_{60} = 25000(1.005)^{60} - 200M(1.005^{60} - 1) = 0$

$$\therefore 200M(1.005^{60} - 1) = 25000(1.005^{60} - 1)$$
$$M = \frac{25000(1.005^{60} - 1)}{(1.005^{60} - 1)}$$
$$= 483.32$$

The monthly repayment is \$483.32

End of Section II

End of Task