



TRINITY GRAMMAR SCHOOL

Mathematics Department

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(NESA Student Number)

2023

Year 12

Mathematics Advanced

TRIAL EXAMINATION

ASSESSMENT TASK 4

Date of Assessment Task: Monday, 21 August 2023

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Erasable pens and all electronic devices (or timekeepers), unless otherwise advised, are **not** permitted in this Assessment
- Calculators approved by NESA may be used
- A reference sheet is provided with your paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks:
100

• *Write your NESA Student Number at the top of this page*

Section I – 10 marks (pages 2 – 4)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 5 – 29)

- Attempt Questions 11 – 28
- Allow about 2 hours and 45 minutes for this section

- **Year 12 HSC Assessment Weighting: 30%**

Section I

10 marks

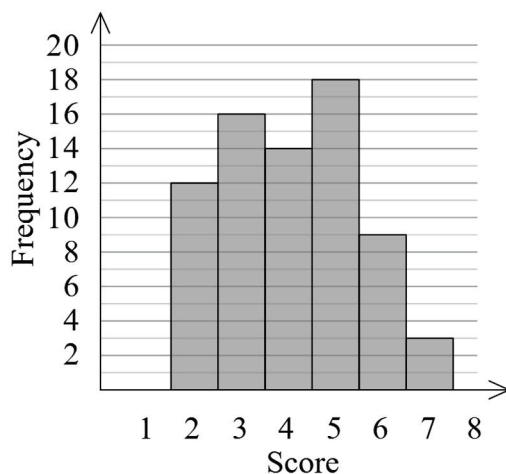
Attempt Questions 1–10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

- 1 The first three terms of an arithmetic sequence are 487, 474 and 461.
What is the 23rd term of this sequence?
- A. 20
B. 188
C. 201
D. 7912
- 2 What amount must be invested now at 4.2% per annum, compounded annually, so that in five years the amount will have grown to \$100 000?
- A. \$17320
B. \$81407
C. \$95969
D. \$122840
- 3 What is the population standard deviation (σ) of the following data, correct to two decimal places?
- 4.5, 6.2, 7.3, 7.4, 9.0
- A. 1.49
B. 1.66
C. 6.88
D. 7.30

- 4 What is the mean score of the data represented in this frequency histogram, correct to one decimal place?



- A. 4.1
 B. 4.5
 C. 5.0
 D. 12.0
- 5 $f(x)$ is an even function in the domain $-a \leq x \leq a$. Given that $\int_0^a f(x)dx = b$.
 What is the value of $\int_{-a}^a f(x)dx$?

- A. $-2b$
 B. 0
 C. $-b$
 D. $2b$

- 6 The graph of a function $f(x) = \sqrt{x}$ is dilated horizontally by a factor of $\frac{1}{3}$ and then translated 4 units to the left. What is the equation of the new graph?

- A. $f(x) = \sqrt{\frac{1}{3}(x+4)}$
 B. $f(x) = \sqrt{\frac{1}{3}(x-4)}$
 C. $f(x) = \sqrt{3(x+4)}$
 D. $f(x) = \sqrt{3(x-4)}$

- 7 What is the derivative of e^{x^3-1} ?
- A. $3x^2e^{x^3-1}$
 B. $3e^{x^3-1}$
 C. $3x^2e^{3x^2}$
 D. $(x^3-1)e^{x^3-1}$
- 8 Which of the following is equivalent to $\operatorname{cosec}^2\theta - 1$?
- A. $\cos^2\theta$
 B. $\cot^2\theta$
 C. $\sec^2\theta$
 D. $\sin^2\theta$
- 9 The function $f(x)$ is continuous and differentiable at $x = a$. What is the equation of the normal to the function $f(x)$ at the point where $x = a$?
- A. $y = f'(a)(x - a) + f(a)$
 B. $y = -\frac{1}{f'(a)}(x - a) + a$
 C. $y = -\frac{1}{f(a)}(x - a) + a$
 D. $y = -\frac{1}{f'(a)}(x - a) + f(a)$
- 10 For events A and B from a sample space, $P(A | B) = \frac{7}{8}$ and $P(B) = \frac{4}{5}$.
 What is $P(A' \cap B)$ equal to?
- A. $\frac{1}{10}$
 B. $\frac{1}{8}$
 C. $\frac{1}{5}$
 D. $\frac{7}{10}$

End of Section I

Section II

90 marks

Attempt Questions 11-28

Allow about 2 hours and 45 minutes for this section.

Answer the question in the spaces provided. These spaces provide guidance for the expected length of response.

In Questions 11-28 your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (2 marks)

Solve $|2x + 7| = 25$.

2

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Please turn over

Question 12 (3 marks)

In a geometric series, $T_3 = 144$ and $T_4 = 108$.

- (a) Write down the value of the common ratio, r .

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- (b) Calculate the limiting sum of the geometric series.

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Question 13 (4 marks)

Find the value(s) of k for which the expression $(k - 6)x^2 + 8x + k$ (where $k \neq 6$) has exactly one real root.

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Please turn over

Question 14 (6 marks)

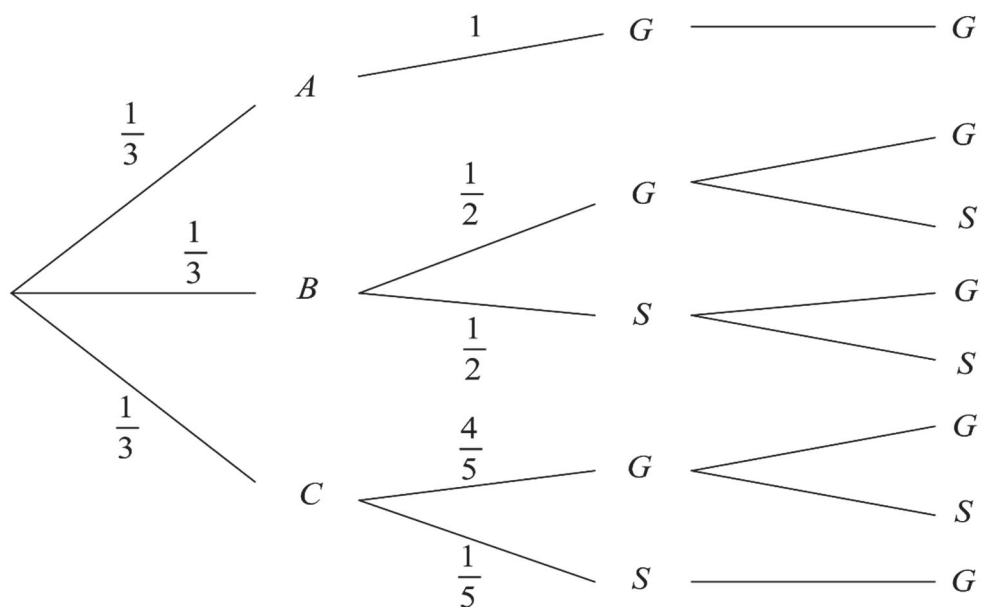
Three boxes A , B , and C , contain gold (G) and silver (S) coins.

- Box A contains 3 gold coins.
- Box B contains 2 gold coins and 2 silver coins.
- Box C contains 4 gold coins and 1 silver coin.

A box is selected at random, and then two coins are selected out of the box, without replacement.

- (a) **Complete the probability tree diagram below** by writing the missing probabilities on the remaining branches.

2



- (b) Calculate the probability of selecting two gold coins.

2

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- (c) Calculate the probability that Box *C* was selected, given that both coins chosen were gold.

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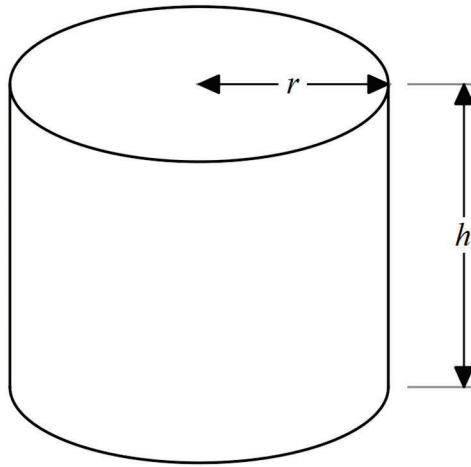
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End of Question 14

Question 15 (8 marks)

The figure below shows a **closed** cylindrical can of tuna, of radius r cm and height h cm.



- (a) The volume of the can is 330 cm^3 .

Show that the surface area of the can is given by the equation: $A = 2\pi r^2 + \frac{660}{r}$

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- (b) Determine the minimum surface area of the can **and** show that it is the minimum surface area. Give your answer in cm^2 to the nearest integer.

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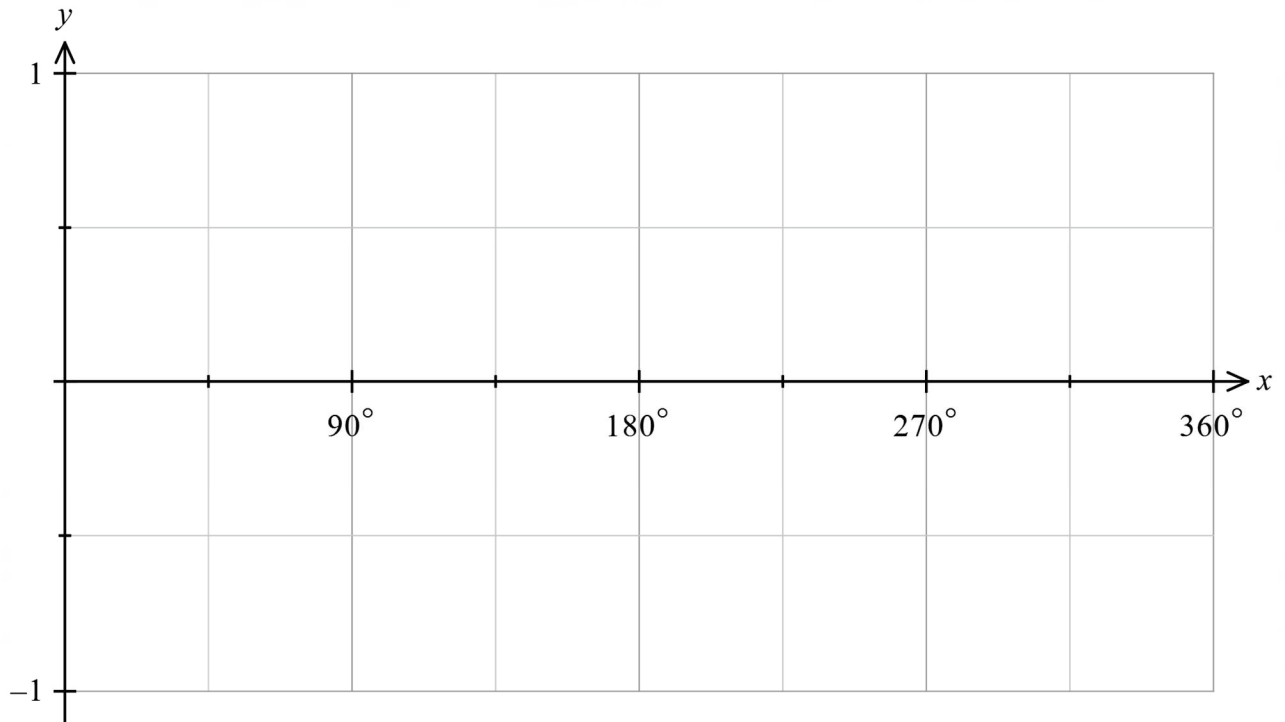
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End of Question 15

Question 16 (5 marks)

- (a) Sketch the graph of $y = \frac{1}{2}\sin(2x)^\circ$ on the number plane below for $0^\circ \leq x \leq 360^\circ$. **2**



- (b) Solve the equation $\frac{1}{2}\sin(2x)^\circ = -\frac{1}{4}$ for $0^\circ \leq x \leq 360^\circ$. **3**

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Question 17 (4 marks)

Solve the equation $\log_3(8t) - 4\log_3(t) = 3$.

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Question 18 (7 marks)

Given that $f(x) = 8 + 4x - 2x^2 - x^3$.

- (a) Write down $f'(x)$.

1

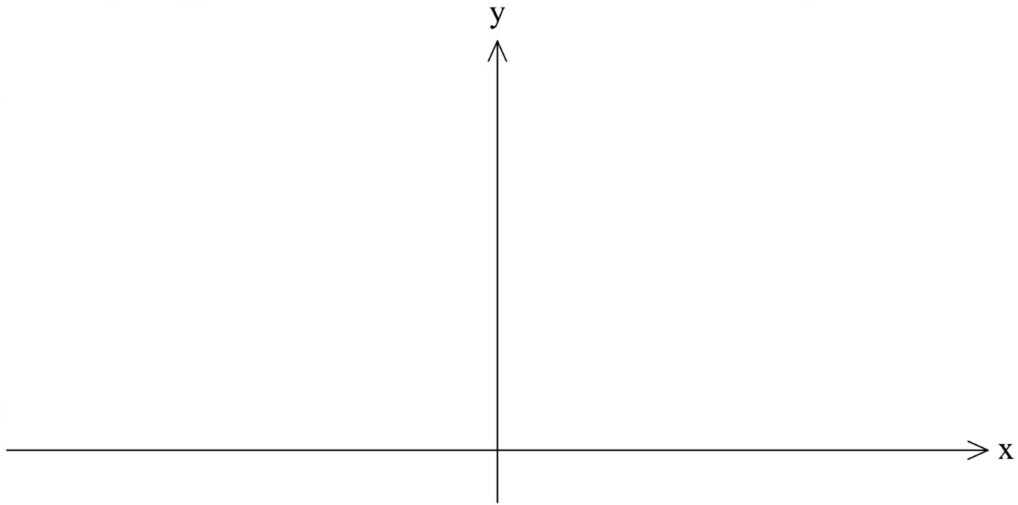
- (b) Find the coordinates of the stationary points **and** determine their nature.

2

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- (c) Sketch the graph of $y=f(x)$ on the number plane below.

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- (d) Find the exact area of the shaded region bounded by the curve $y=f(x)$ and the x -axis.

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End of Question 18

Question 19 (8 marks)

Differentiate the following functions with respect to x .

(a) $y = (2x - 3)^5$

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(b) $y = x\cos(3x)$

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(c) $y = \frac{e^x}{x^e}$

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Questions 11 – 19 are worth 47 marks in total
End of Question 19

Question 20 (3 marks)

Let $y = \ln(\sqrt{x})$.

Show that $\frac{d^2y}{dx^2} < 0$ for all $x > 0$. **3**

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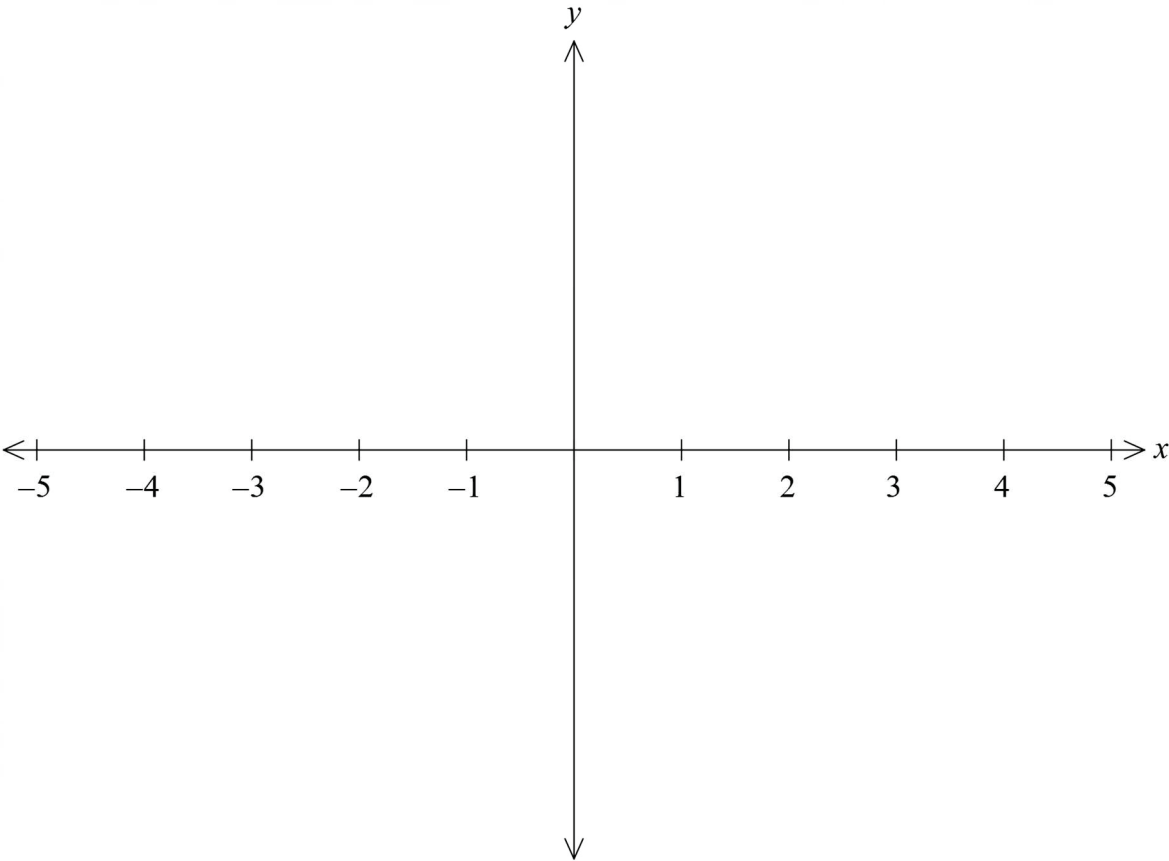
Question 21 (4 marks)

In the table below, the function $f(x)$ has the following properties:

x	-2	-1	0	1	2	3
$f(x)$	0	$-$	0	$+$	undefined	0
$f'(x)$	$-$	0	0	$+$	undefined	$+$
$f''(x)$	$+$	$+$	0	$+$	undefined	$-$

Sketch a possible graph of $y = f(x)$ on the axis below that satisfies all the properties in the table above.

4



Please turn over

Question 22 (4 marks)

Evaluate p and q in the discrete probability distribution table below, given that $E(X) = 4.5$ **4**

x	1	3	5	7	9
$P(X=x)$	0.2	p	0.2	q	0.15

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Question 23 (5 marks)

(a) Find $\int (3x + 1)^2 dx$ **1**

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(b) Evaluate $\int_0^1 \frac{4}{2x + 3} dx$ **2**

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(c) Evaluate $\int_0^4 \sqrt{16 - x^2} dx$ **2**

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Question 24 (4 marks)

The velocity of an object is given by:

$$v = \sin(t) - 10t$$

where v is the velocity in metres per second and t is the time in seconds.

- (a) Write an expression for the acceleration of the object in terms of time, t .

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- (b) Find the displacement of the object when $t = 3.7$, given that the initial displacement is 7 metres.

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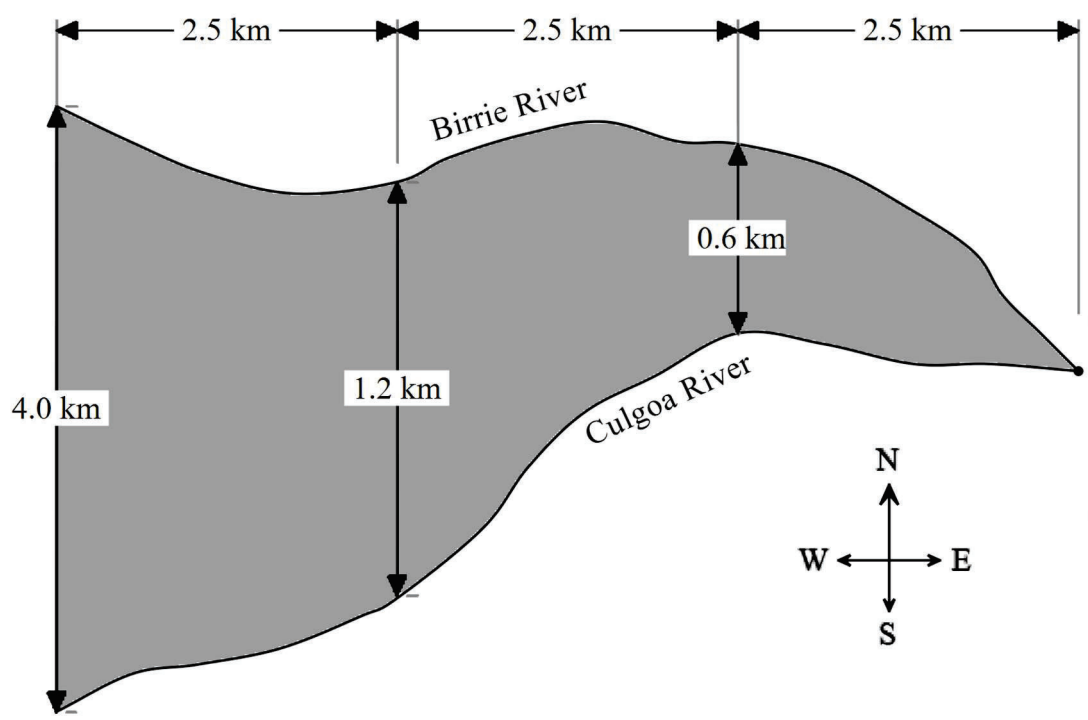
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Question 25 (3 marks)

A National Park is being created where the Birrie River and the Culgoa River meet. A surveyor measures the north-south distance between the rivers at 2.5 km intervals. The western boundary of the National Park runs due north and is exactly 7.5 km west of the point where the rivers meet. This information is shown in the diagram below.



Use three applications of the trapezoidal rule to find an approximation of the area of the National Park.

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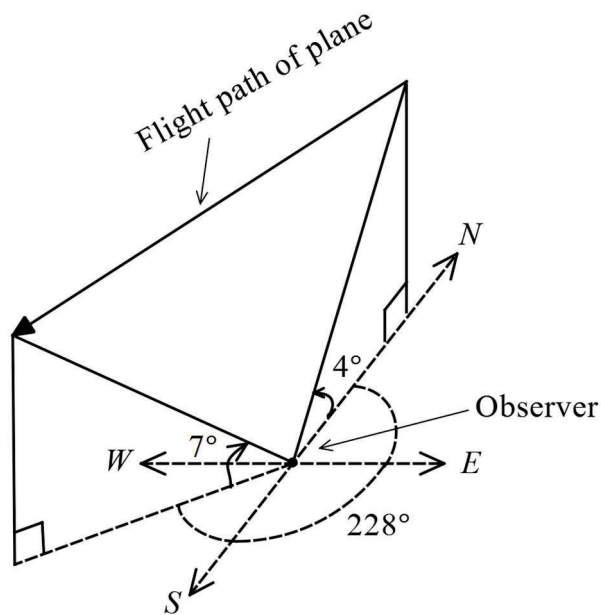
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Question 26 (7 marks)

A stationary observer spots a plane due north at an angle of elevation of 4° . One-and-a-half minutes later, they observe the plane at a bearing of 228°T at an angle of elevation of 7° .

The plane is travelling at a constant speed of 240 km/h and is also maintaining a constant altitude and direction.



- (a) Show that the plane is flying at an altitude of 290 m (correct to the nearest metre).

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(b) As a bearing, what direction is the plane travelling?

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Question 27 (8 marks)

A Chief Executive Officer (CEO) introduces a new policy that aims to increase the proportion of female employees in a company. The number of male (M) and female (F) employees in the company over the first 6 years (t) since the implementation of the policy is recorded below.

t	0	1	2	3	4	5	6
F	120	138	198	225	245	256	250
M	380	552	602	675	980	1024	750

- (a) Using technology, calculate Pearson's correlation coefficient for the number of female employees and the time since the policy was implemented.
Give your answer to 3 decimal places.

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- (b) Describe the strength of the association between the number of female employees and time since the policy was implemented.

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- (c) Determine the equation of the least-squares regression line that relates the number of female employees to the number of years since the policy was implemented.
Give all values correct to the nearest integer.

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- (d) Using your regression equation from part (c), estimate how many years it will take for the number of female employees to reach 500 **and** explain why this might not be reliable.

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- (e) The CEO is reviewing the policy. Given that the percentage of females is given by $0.00287 - 0.0006t$ and $r = 0.1146$, evaluate the effectiveness of the policy at increasing the percentage of female employees.

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End of Question 27

Question 28 (5 marks)

At the start of August, Tyrone opens a bank account and makes a deposit of \$1000 into the same account. At the start of each subsequent month, he makes a deposit of 2% more than the previous month's deposit.

At the end of every month, the bank pays interest of 0.5% (per month) on the balance of the account.

Let A_n = balance in the account at the end of month n (including principal + interest)

(a) Show that $A_2 = \$1\,000(1.005)^2 + \$1\,000(1.02)(1.005)$

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- (b) Find the balance in the account at the end of the 50th month, correct to the nearest dollar. 3

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End of Question 28

End of paper

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TRINITY GRAMMAR SCHOOL

Mathematics Department

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(NESA Student Number)

2023

Year 12

Mathematics Advanced

TRIAL EXAMINATION

ASSESSMENT TASK 4

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Total marks:
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Section I – 10 marks (pages 2 – 4)

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Section II – 90 marks (pages 5 – 29)

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- **Year 12 HSC Assessment Weighting: 30%**

Section I

10 marks

Attempt Questions 1–10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

- 1 The first three terms of an arithmetic sequence are 487, 474 and 461.
What is the 23rd term of this sequence?

A. 20
B. 188
☒ C. 201
D. 7912

$$a = 487$$

$$d = -13$$

$$n = 23$$

$$\begin{aligned} T_{23} &= 487 + (23-1) \times -13 \\ &= 201 \end{aligned}$$

- 2 What amount must be invested now at 4.2% per annum, compounded annually, so that in five years the amount will have grown to \$100 000?

A. \$17320
☒ B. \$81407
C. \$95969
D. \$122840

$$A = P(1+r)^n$$

$$100000 = P(1+0.042)^5$$

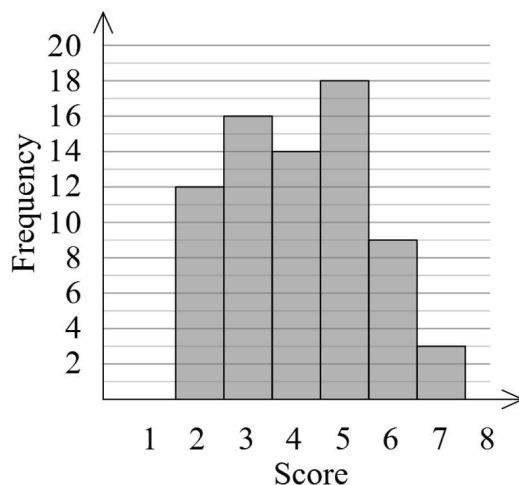
$$P = \frac{100000}{(1.042)^5}$$

- 3 What is the population standard deviation (σ) of the following data, correct to two decimal places?

4.5, 6.2, 7.3, 7.4, 9.0

☒ A. 1.49
B. 1.66
C. 6.88
D. 7.30

- 4 What is the mean score of the data represented in this frequency histogram, correct to one decimal place?



- A. 4.1
 B. 4.5
 C. 5.0
 D. 12.0
- 5 $f(x)$ is an even function in the domain $-a \leq x \leq a$. Given that $\int_0^a f(x) dx = b$.

What is the value of $\int_{-a}^a f(x) dx$?

- A. $-2b$
 B. 0
 C. $-b$
 D. $2b$
- 6 The graph of a function $f(x) = \sqrt{x}$ is dilated horizontally by a factor of $\frac{1}{3}$ and then translated 4 units to the left. What is the equation of the new graph?

A. $f(x) = \sqrt{\frac{1}{3}(x+4)}$

B. $f(x) = \sqrt{\frac{1}{3}(x-4)}$

C. $f(x) = \sqrt{3(x+4)}$

D. $f(x) = \sqrt{3(x-4)}$

horizontal dilation: SF = $\frac{1}{3}$

$a = 3$

horizontal translation: $b = 4$

7 What is the derivative of e^{x^3-1} ?

- A. $3x^2 e^{x^3-1}$
- B. $3e^{x^3-1}$
- C. $3x^2 e^{3x^2}$
- D. $(x^3-1)e^{x^3-1}$

8 Which of the following is equivalent to $\operatorname{cosec}^2 \theta - 1$?

- A. $\cos^2 \theta$
- B. $\cot^2 \theta$
- C. $\sec^2 \theta$
- D. $\sin^2 \theta$

$$\begin{aligned} \operatorname{cosec}^2 \theta - 1 &= \frac{1}{\sin^2 \theta} - \frac{\sin^2 \theta}{\sin^2 \theta} \\ &= \frac{\cos^2 \theta}{\sin^2 \theta} = \cot^2 \theta \end{aligned}$$

9 The function $f(x)$ is continuous and differentiable at $x = a$. What is the equation of the normal to the function $f(x)$ at the point where $x = a$?

- A. $y = f'(a)(x-a) + f(a)$
- B. $y = -\frac{1}{f'(a)}(x-a) + a$
- C. $y = -\frac{1}{f(a)}(x-a) + a$
- D. $y = -\frac{1}{f'(a)}(x-a) + f(a)$

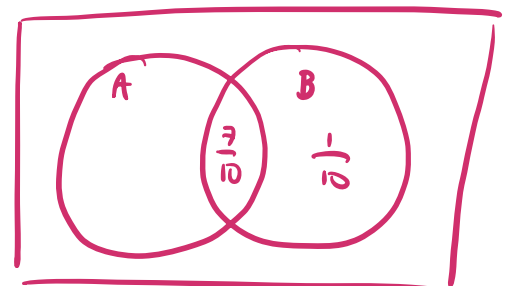
$$\begin{aligned} m &= f'(a) \\ m_n &= -\frac{1}{f'(a)} \\ y - y_1 &= m(x - x_1) \\ y - f(a) &= -\frac{1}{f'(a)}(x - a) \end{aligned}$$

10 For events A and B from a sample space, $P(A | B) = \frac{7}{8}$ and $P(B) = \frac{4}{5}$.

What is $P(A' \cap B)$ equal to?

- A. $\frac{1}{10}$
- B. $\frac{1}{8}$
- C. $\frac{1}{5}$
- D. $\frac{7}{10}$

$$\begin{aligned} P(A|B) &= \frac{P(A \cap B)}{P(B)} \\ \frac{7}{8} &= \frac{P(A \cap B)}{4/5} \\ P(A \cap B) &= \frac{7}{10} \end{aligned}$$



End of Section I

Section II

90 marks

Attempt Questions 11-28

Allow about 2 hours and 45 minutes for this section.

Answer the question in the spaces provided. These spaces provide guidance for the expected length of response.

In Questions 11-28 your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (2 marks)

Solve $|2x + 7| = 25$.

2

$$2x + 7 = 25$$

$$2x + 7 = -25$$

$$2x = 18$$

$$2x = -32$$

$$x = 9$$

$$x = -16$$

① answer

① answer

Please turn over

Question 12 (3 marks)

In a geometric series, $T_3 = 144$ and $T_4 = 108$.

- (a) Write down the value of the common ratio, r .

1

$$r = \frac{108}{144} = 0.75 \quad \textcircled{1} \text{ correct } r \text{ value}$$

- (b) Calculate the limiting sum of the geometric series.

2

$$a = 144 \div (0.75)^2$$

$$= 256$$

$$S = \frac{a}{1-r}$$

① calculates a
correctly from r

$$= \frac{256}{1-0.75}$$

$$= 1024$$

① correct
final answer

Question 13 (4 marks)

Find the value(s) of k for which the expression $(k-6)x^2 + 8x + k$ (where $k \neq 6$) has exactly one real root.

4

$$b^2 - 4ac = 0 \quad \textcircled{1} \text{ identifies discriminant} = 0$$

$$8^2 - 4(k-6) \times k = 0 \quad \textcircled{1} \text{ sub into discriminant and expands brackets correctly}$$

$$64 - 4k^2 + 24k = 0$$

$$k^2 - 6k - 16 = 0$$

$$P : -16$$

$$(k-8)(k+2) = 0$$

$$S : -6$$

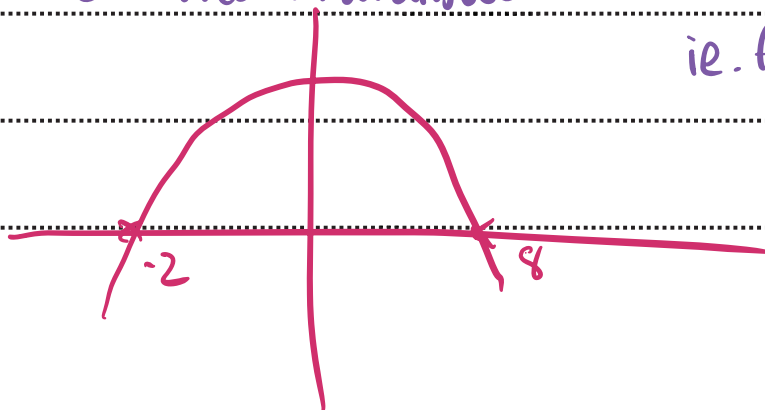
$$F : -8, 2$$

$$k = 8, k = -2$$

$\textcircled{1}$ valid attempt to solve quadratic

$\textcircled{1}$ correct final answer

ie. factorise, quad form



Please turn over

Question 14 (6 marks)

Three boxes A , B , and C , contain gold (G) and silver (S) coins.

- Box A contains 3 gold coins.
- Box B contains 2 gold coins and 2 silver coins.
- Box C contains 4 gold coins and 1 silver coin.

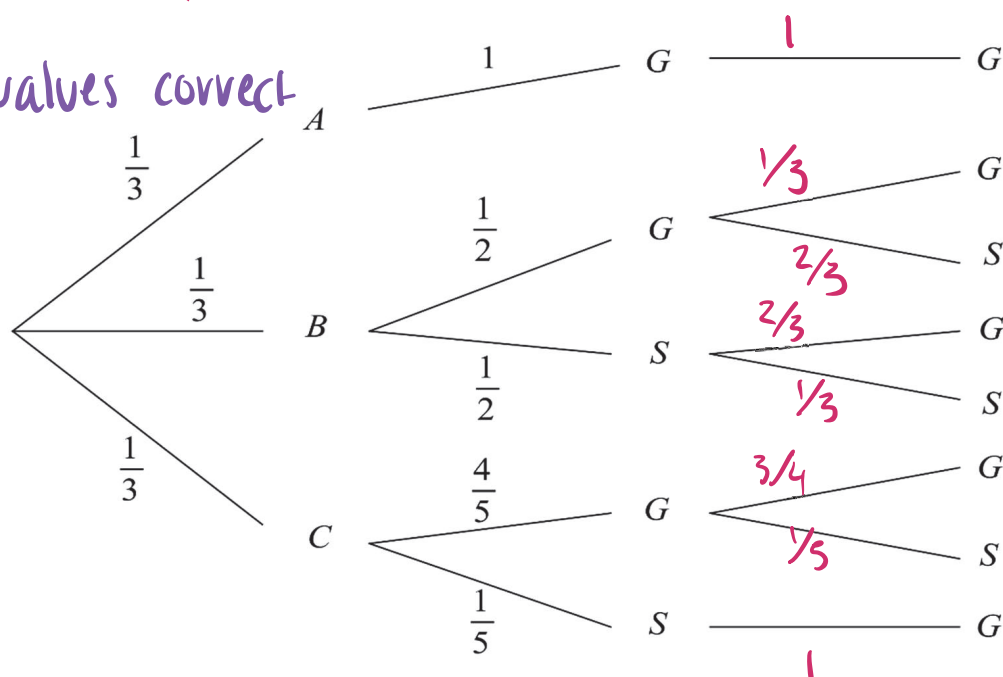
A box is selected at random, and then two coins are selected out of the box, without replacement.

- (a) Complete the probability tree diagram below by writing the missing probabilities on the remaining branches.

2

① at least 4 values correct

① all values correct



- (b) Calculate the probability of selecting two gold coins.

2

$$P(GG) = \left(\frac{1}{3} \times 1 \times 1\right) + \left(\frac{1}{3} \times \frac{2}{4} \times \frac{1}{3}\right) + \left(\frac{1}{3} \times \frac{4}{5} \times \frac{3}{4}\right)$$

$$= \frac{53}{90} \text{ or } 0.59$$

① demonstrates multiplication of multi-stage probability OR correctly identifies the 3 branches and adds

① correct answer

- (c) Calculate the probability that Box C was selected, given that both coins chosen were gold.

2

$$P(C | GG) = \frac{P(C \cap GG)}{P(GG)}$$

$$P(GG)$$

① identifies correct

conditional probability
formula

$$= \frac{(\frac{1}{3} \times \frac{4}{5} \times \frac{3}{4})}{53/90}$$

$$53/90$$

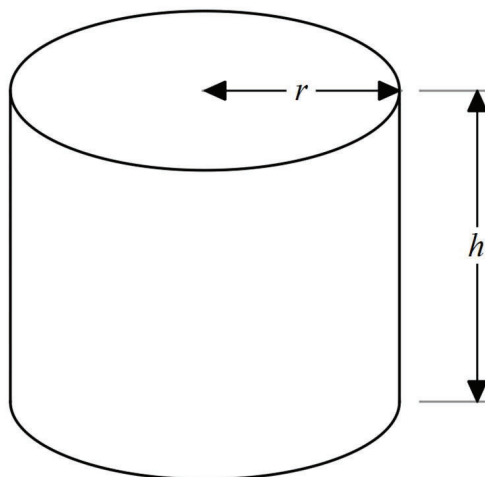
$$= 0.34 \text{ OR } 18/53$$

① correct final answer

End of Question 14

Question 15 (8 marks)

The figure below shows a **closed** cylindrical can of tuna, of radius r cm and height h cm.



- (a) The volume of the can is 330 cm^3 .

Show that the surface area of the can is given by the equation: $A = 2\pi r^2 + \frac{660}{r}$

3

$$V = 330$$

$$\pi r^2 h = 330$$

① set up equation for volume of cylinder

$$h = \frac{330}{\pi r^2}$$

① develops a correct relationship between h and r

$$A = 2\pi r^2 + 2\pi r h$$

$$= 2\pi r^2 + 2\pi r \left(\frac{330}{\pi r^2} \right)$$

① substitutes expression into formula for SA correctly

$$= 2\pi r^2 + \frac{660}{r}$$

- (b) Determine the minimum surface area of the can **and** show that it is the minimum surface area. Give your answer in cm^2 to the nearest integer.

5

$$A = 2\pi r^2 + 660r^{-1}$$

$$\frac{dA}{dr} = 4\pi r - 660r^{-2} \quad \textcircled{1} \text{ differentiates correctly}$$

$$\frac{dA}{dr} = 0 \quad 0 = 4\pi r - \frac{660}{r^2} \quad \textcircled{1} \text{ sets derivative} = 0$$

$$\frac{660}{r^2} = 4\pi r$$

$$660 = 4\pi r^3$$

$$r^3 = \frac{660}{4\pi}$$

$$r = 3.745 \text{ cm} \quad \textcircled{1} \text{ solves to find } r \text{ correctly}$$

$$\frac{d^2A}{dr^2} = 4\pi + \frac{1320}{r^3}$$

$\textcircled{1}$ correct/valid method to determine minimum

$$= 4\pi + \frac{1320}{3.745^3}$$

$$= 37.7 > 0$$

$\therefore \text{min}$

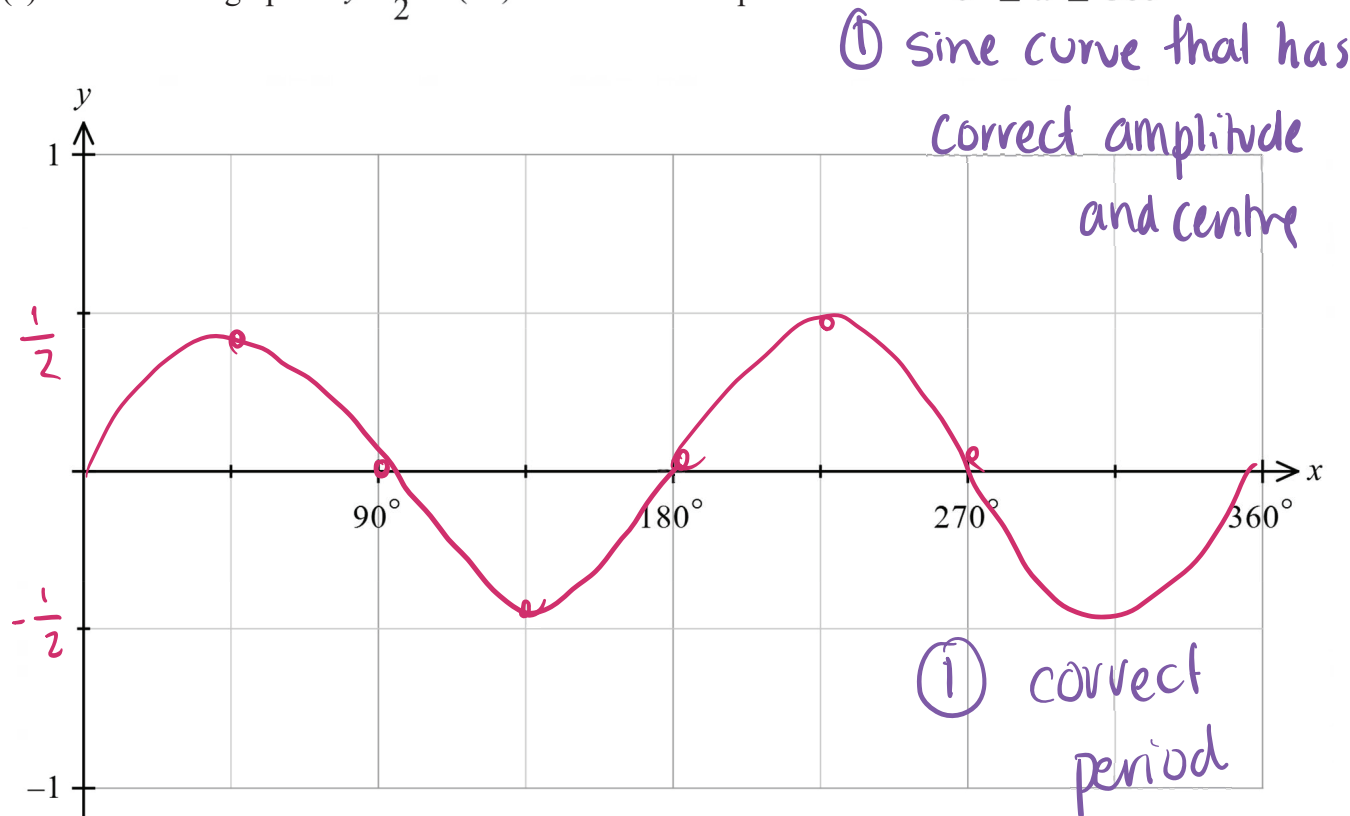
$$\text{min SA} = 2\pi(3.745)^2 + 660(3.745)^{-1}$$

$$= 264 \text{ cm}^2 \quad \textcircled{1} \text{ correct minimum SA}$$

End of Question 15

Question 16 (5 marks)

- (a) Sketch the graph of $y = \frac{1}{2}\sin(2x)^\circ$ on the number plane below for $0^\circ \leq x \leq 360^\circ$. 2



- (b) Solve the equation $\frac{1}{2}\sin(2x)^\circ = -\frac{1}{4}$ for $0^\circ \leq x \leq 360^\circ$. 3

$$\sin 2x = -\frac{1}{2} \quad 0 \leq 2x \leq 720$$

$$2x = \sin^{-1}\left(-\frac{1}{2}\right)$$

① correct base angle ✓ | ✓

$$= 30$$

30° for 2x

$$2x = 210, 330, 570, 690 \quad \text{① any 2 of correct angles}$$

$$x = 105^\circ, 165^\circ, 285^\circ, 345^\circ \quad \text{for } 2x \text{ eg. } 210^\circ, 330^\circ$$

① all answers correct for x

Question 17 (4 marks)

Solve the equation $\log_3(8t) - 4\log_3(t) = 3$.

4

$$\log_3 8t - \log_3 t^4 = 3 \log_3 3 \quad \textcircled{1} \text{ uses power log law correctly}$$

$$\log_3 \left(\frac{8t}{t^4} \right) = \log_3 27$$

$$\textcircled{1} \text{ uses division log law correctly}$$

$$\frac{8}{t^3} = 27$$

$$t^3 = \frac{8}{27} \quad \textcircled{1} \text{ uses definition of log law correctly ie. this line or equiv}$$

$$t = \frac{2}{3}$$

$$\textcircled{1} \text{ correct final answer}$$

Question 18 (7 marks)

Given that $f(x) = 8 + 4x - 2x^2 - x^3$.

- (a) Write down $f'(x)$.

1

$$f'(x) = 4 - 4x - 3x^2 \quad \textcircled{1} \text{ correct answer}$$

- (b) Find the coordinates of the stationary points **and** determine their nature.

2

$$-3x^2 - 4x + 4 = 0 \quad \text{SP : } y' = 0$$

$$x = \frac{4 \pm \sqrt{16 - 4(-3 \times 4)}}{-6}$$

$$= \frac{2}{3} \text{ OR } -2$$

$$f\left(\frac{2}{3}\right) = 8 + 4\left(\frac{2}{3}\right) - 2\left(\frac{2}{3}\right)^2 - \left(\frac{2}{3}\right)^3$$
$$= 9^{13}/27$$

① x coordinates

$$f(-2) = 8 + 4(-2) - 2(-2)^2 - (-2)^3 \quad \text{for S.p}$$
$$= 0$$

$$f''(x) = -6x - 4$$

$$f''\left(\frac{2}{3}\right) = -8 < 0 \quad \therefore \text{max } \left(\frac{2}{3}, 9^{13}/27\right)$$

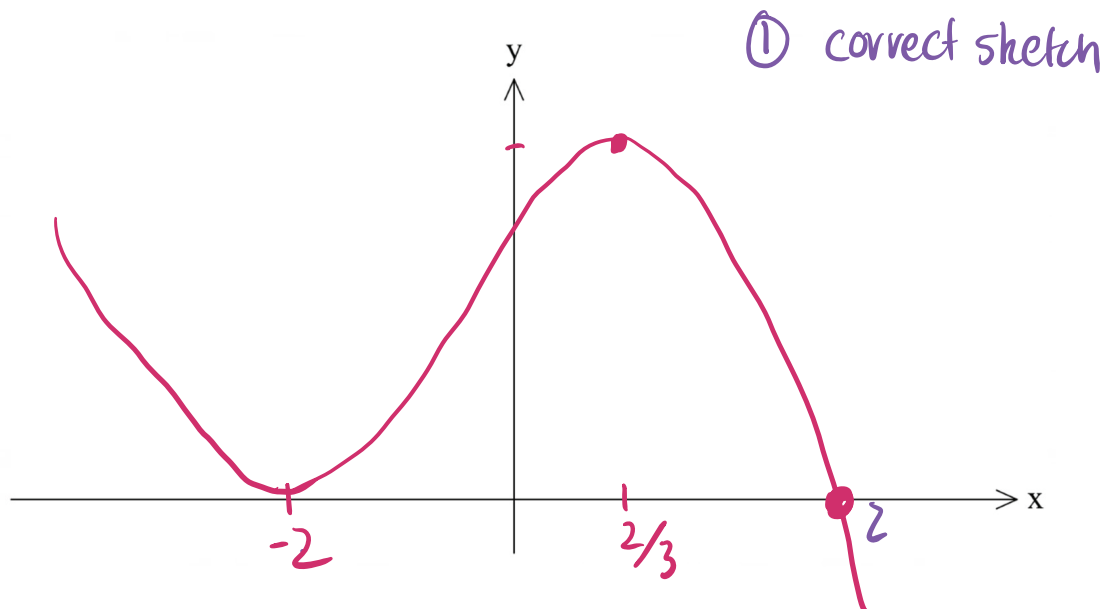
$$f''(-2) = 8 > 0 \quad \therefore \text{min } (-2, 0)$$

① correct coordinates

+ nature determined

- (c) Sketch the graph of $y=f(x)$ on the number plane below.

1



- (d) Find the exact area of the shaded region bounded by the curve $y=f(x)$ and the x -axis.

3

$$x\text{-int: } 4(2+x) - x^2(2+x) = 0$$

$$(2+x)(4-x^2) = 0$$

$$\therefore x = -2, -2, 2$$

① determines boundaries

$$\int_{-2}^2 (8 + 4x - 2x^2 - x^3) dx$$

$$\left[8x + 2x^2 - \frac{2}{3}x^3 - \frac{x^4}{4} \right]_{-2}^2$$

① integrates correctly

$$(16 + 8 - \frac{16}{3} - 4) - (-16 + 8 + \frac{16}{3} - 4)$$

$$= \frac{64}{3} \text{ units}^2$$

① substitutes correctly leading to correct final answer

End of Question 18

Question 19 (8 marks)

Differentiate the following functions with respect to x .

(a) $y = (2x - 3)^5$

2

$$y' = 10(2x-3)^4$$

① multiplies by the derivative of the inside function

① correctly differentiates power function

(b) $y = x \cos(3x)$

3

$$u = x \quad v = \cos 3x \quad \text{① evidence of product rule}$$

$$u' = 1 \quad v' = -3 \sin 3x \quad \text{① differentiates } \cos(3x) \text{ correctly}$$

$$y' = u'v + v'u \quad \text{① correct final answer}$$

$$y' = \cos 3x - 3x \sin 3x$$

(c) $y = \frac{e^x}{x^e}$

3

$$u = e^x$$

$$v = x^e$$

$$u' = e^x \quad \textcircled{1}$$

$$v' = e x^{e-1} \quad \textcircled{1}$$

differentiates
 e^x correctly

differentiates
 x^e correctly

$$y' = \frac{v u' - u v'}{v^2}$$

OR x^{-e}

$$= \frac{x^e e^x - e^x e x^{e-1}}{(x^e)^2}$$

$\textcircled{1}$ correct
derivative

(quotient / product)

Questions 11 – 19 are worth 47 marks in total
End of Question 19

Question 20 (3 marks)

Let $y = \ln(\sqrt{x})$.

Show that $\frac{d^2y}{dx^2} < 0$ for all $x > 0$.

3

$$y = \ln(x^{1/2})$$

$$\frac{dy}{dx} = \frac{\frac{1}{2} x^{-1/2}}{x^{1/2}}$$

① first derivative

① second derivative

$$\frac{d^2y}{dx^2} = \frac{-\frac{1}{4} x^{-1} - \frac{1}{4} x^{-1}}{x}$$

① concluding statement

$$u = \frac{1}{2} x^{-1/2} \quad \times \quad v = x^{1/2}$$

$$= \frac{-\frac{1}{2} x^{-1}}{x}$$

$$u' = -\frac{1}{4} x^{-3/2} \quad v' = \frac{1}{2} x^{-1/2}$$

$$= -\frac{1}{2x^2} \quad \text{but } x^2 > 0$$

$$\therefore -\frac{1}{2x^2} < 0$$

OR

$$y = \ln(x^{1/2})$$

$$\frac{dy}{dx} = \frac{\frac{1}{2} x^{-1/2}}{x^{1/2}}$$

$$\frac{dy}{dx} = \frac{1}{2x}$$

$$\frac{dy}{dx} = \frac{1}{2} x^{-1}$$

① simplifies using index laws
(this line or equivalent)

$$\frac{d^2y}{dx^2} = -\frac{1}{2} x^{-2}$$

$$\text{but } x^2 > 0$$

① first derivative

$$\text{① concluding statement } = -\frac{1}{2x^2} \quad \therefore -\frac{1}{2x^2} < 0$$

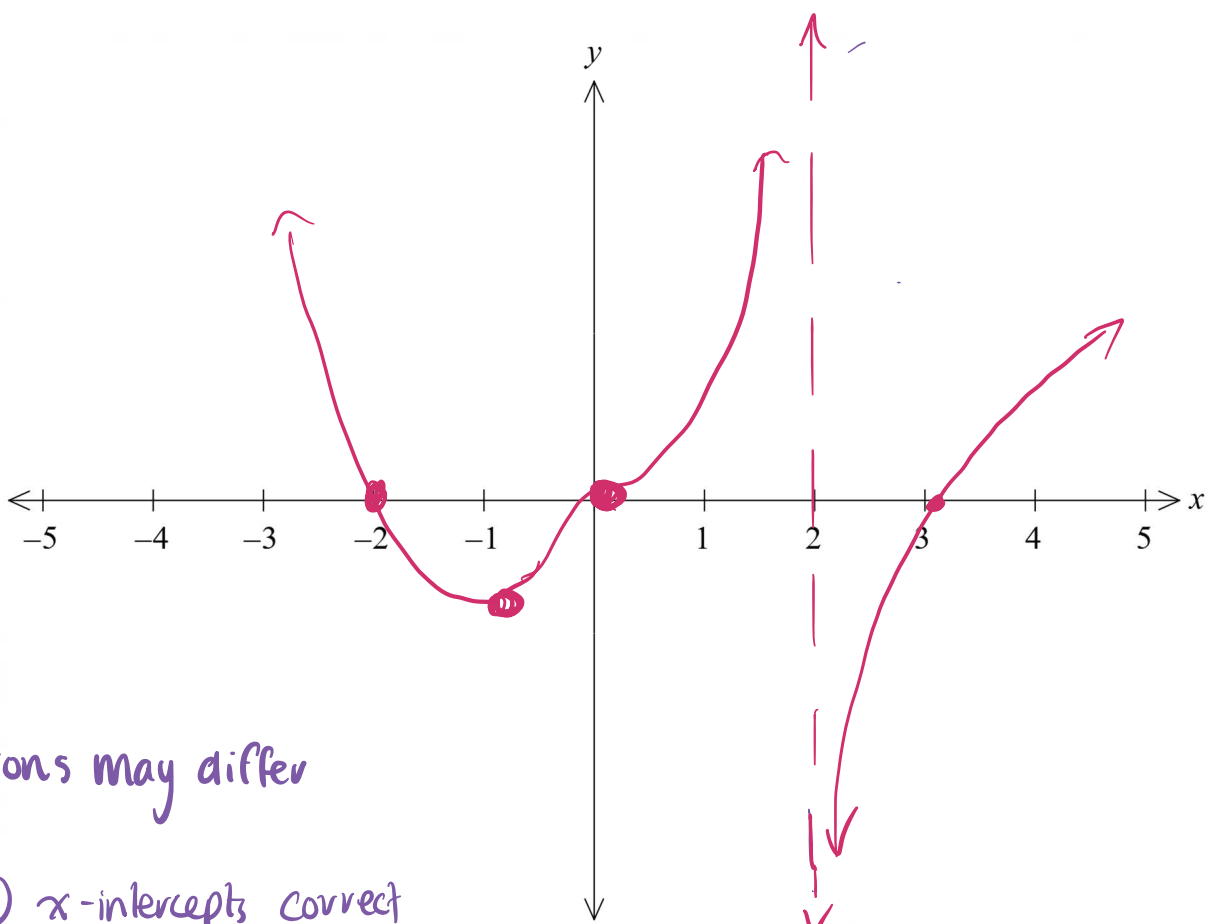
Question 21 (4 marks)

In the table below, the function $f(x)$ has the following properties:

x	-2	-1	0	1	2	3
$f(x)$	0	-	0	+	undefined	0
$f'(x)$	-	0	0	+	undefined	+
$f''(x)$	+	+	0	+	undefined	-

Sketch a possible graph of $y = f(x)$ on the axis below that satisfies all the properties in the table above.

4



solutions may differ

- ① x -intercepts correct
- ① horizontal point of inflection @ $x=0$
- ① local minimum @ $x=-1$
- ① correct behaviour around vertical asymptote ($x>0$).

Please turn over

Question 22 (4 marks)

Evaluate p and q in the discrete probability distribution table below, given that $E(X) = 4.5$ 4

x	1	3	5	7	9
$P(X=x)$	0.2	p	0.2	q	0.15

$$0.2 + p + 0.2 + q + 0.15 = 1$$

① set up eqn
for probability
= 1

$$p + q = 0.45$$

①

$$E(X) = 4.5$$

① set up eqn
for $E(X)$

$$(1 \times 0.2) + (3p) + (5 \times 0.2) + 7q + (9 \times 0.15) = 4.5$$

$$3p + 7q = 1.95$$

2

$$\textcircled{1} \times 3$$

$$3p + 3q = 1.35 \quad \textcircled{3}$$

$$\textcircled{2} - \textcircled{3}$$

① valid method for
solving simultaneous
eqns (elimination/
substitution)

$$4q = 0.6$$

$$q = 0.15$$

$$p = 0.3$$

① correct p and q

Question 23 (5 marks)

(a) Find $\int (3x+1)^2 dx$

1

$$\frac{1}{3 \times 3} (3x+1)^3 + C = \frac{1}{9} (3x+1)^3 + C$$

① correct integral

OR $\int 9x^2 + 6x + 1 dx$

(b) Evaluate $\int_0^1 \frac{4}{2x+3} dx = 3x^3 + 3x^2 + x + C$

2

$$= 2 \int_0^1 \frac{2}{2x+3} dx$$

$$= 2 [\ln(2x+3)]_0^1$$

① integrates correctly

$$= 2 [\ln 5 - \ln 3]$$

$$= 2 \ln\left(\frac{5}{3}\right)$$

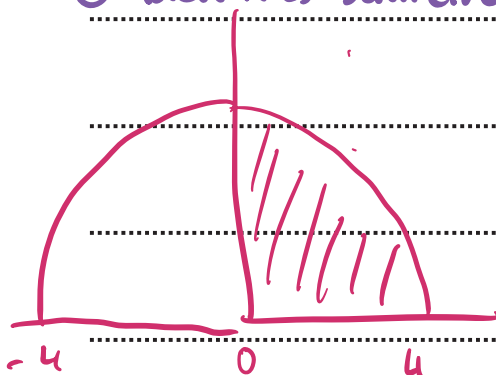
① correct answer

(c) Evaluate $\int_0^4 \sqrt{16-x^2} dx$

2

① identifies semicircle

$$\frac{\pi r^2}{4} = \frac{\pi \times 4^2}{4}$$



$$= 4\pi$$

① correct answer

Question 24 (4 marks)

The velocity of an object is given by:

$$v = \sin(t) - 10t$$

where v is the velocity in metres per second and t is the time in seconds.

- (a) Write an expression for the acceleration of the object in terms of time, t .

1

$$a = \cos(t) - 10 \quad \textcircled{1} \text{ correct answer}$$

- (b) Find the displacement of the object when $t = 3.7$, given that the initial displacement is 7 metres.

3

$$x = -\cos t - 5t^2 + C \quad \textcircled{1} \text{ integrates } v \text{ correctly to get } x$$

$$7 = -\cos(0) - 5(0)^2 + C$$

$$C = 8 \quad \textcircled{1} \text{ determines } C$$

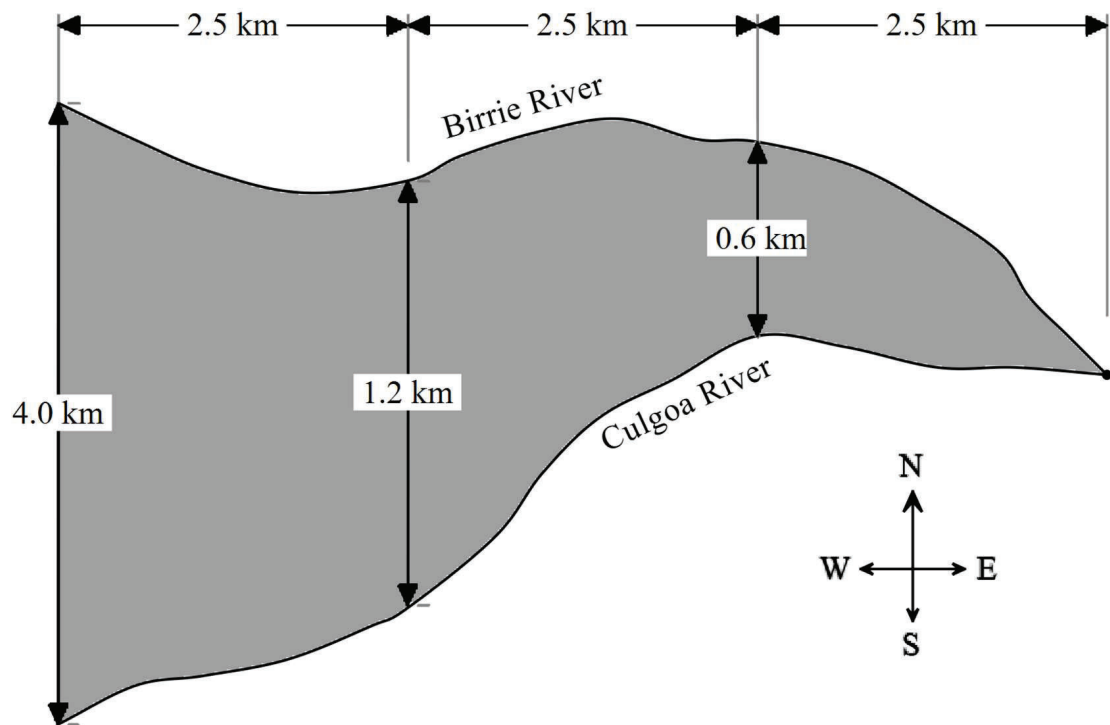
$$x = -\cos t - 5t^2 + 8 \quad \text{when } t = 3.7$$

$$= -\cos(3.7) - 5(3.7)^2 + 8$$

$$= -59.6 \text{ m} \quad \textcircled{1} \text{ correct answer}$$

Question 25 (3 marks)

A National Park is being created where the Birrie River and the Culgoa River meet. A surveyor measures the north-south distance between the rivers at 2.5 km intervals. The western boundary of the National Park runs due north and is exactly 7.5 km west of the point where the rivers meet. This information is shown in the diagram below.



Use three applications of the trapezoidal rule to find an approximation of the area of the National Park.

3

① correct width

$$A \approx \frac{7.5}{2 \times 3} [4 + 0 + 2(1.2 + 0.6)]$$

① correct inside brackets

$$= \frac{7.5}{6} [4 + 0 + 2(1.2 + 0.6)]$$

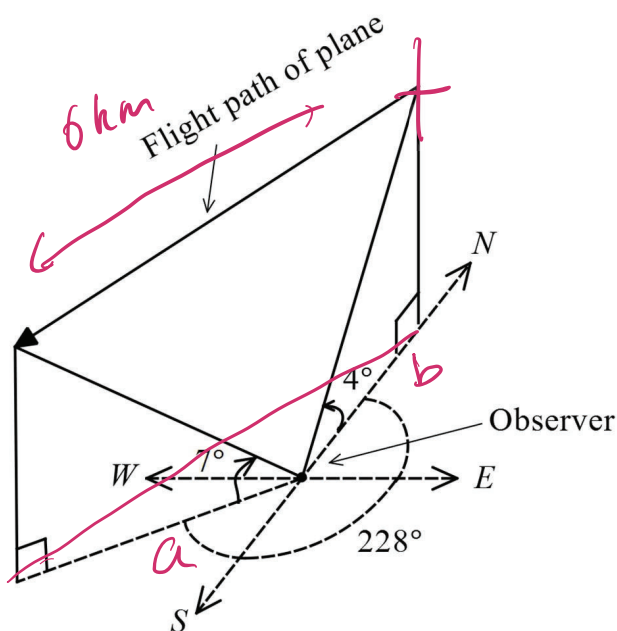
$$= 9.5 \text{ km}^2$$

① correct answer

Question 26 (7 marks)

A stationary observer spots a plane due north at an angle of elevation of 4° . One-and-a-half minutes later, they observe the plane at a bearing of 228°T at an angle of elevation of 7° .

The plane is travelling at a constant speed of 240 km/h and is also maintaining a constant altitude and direction.

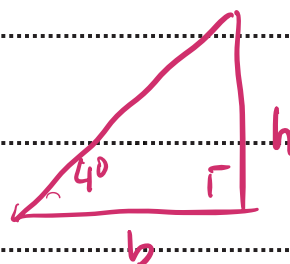
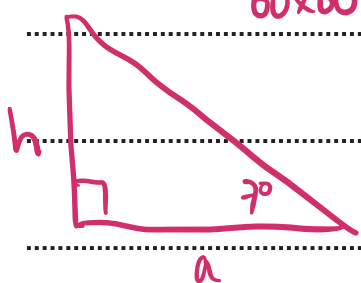


- (a) Show that the plane is flying at an altitude of 290 m (correct to the nearest metre).

4

$$s \times t = d$$

$$90 \times \frac{240}{60 \times 60} = 6 \text{ ① distance}$$



$$\tan 7^\circ = \frac{h}{a}$$

$$\tan 4^\circ = \frac{h}{b}$$

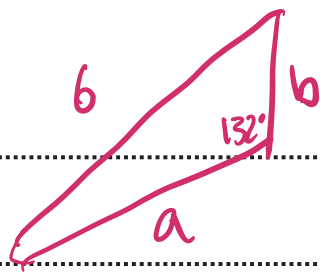
$$b = \frac{h}{\tan 4^\circ}$$

$$a = \frac{h}{\tan 7^\circ}$$

$$b = h \cot 4^\circ$$

$$a = h \cot 7^\circ$$

① writes correct trig eqn for one triangle



$$c^2 = a^2 + b^2 - 2ab \cos C \quad \textcircled{1} \text{ substitutes into cosine formula}$$

$$6^2 = h^2 \cot^2 7^\circ + h^2 \cot^2 4^\circ - 2(h \cot 7^\circ h \cot 4^\circ \times \cos 132^\circ)$$

$$6^2 = h^2 (\cot^2 7^\circ + \cot^2 4^\circ - 2 \cot 7^\circ \cot 4^\circ \cos 132^\circ)$$

$$36 = h^2 \times 426.706 \dots$$

$$h^2 = 0.084 \dots$$

① simplifies, divides by 426.706... and square roots.

$$h = 0.290 \text{ km}$$

$$= 290 \text{ m}$$

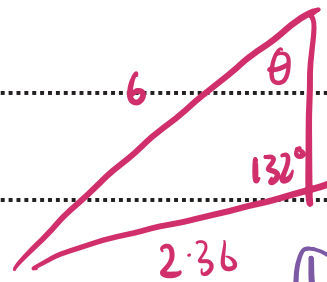
(b) As a bearing, what direction is the plane travelling?

3

$$a = h \cot 7^\circ$$

$$= 0.290 \times \cot 7^\circ$$

$$= 2.36$$



① evaluates length

$$\frac{\sin \theta}{2.36} = \frac{\sin 132^\circ}{6}$$

① substitutes of unknown side into sine/cosine rule

$$\theta = \sin^{-1} \left(\frac{2.36 \times \sin 132^\circ}{6} \right)$$

$$= 16.99 \dots = 17^\circ$$

$$\text{bearing: } 180 + 17$$

$$= 197^\circ \text{ T}$$

① correct bearing

Question 27 (8 marks)

A Chief Executive Officer (CEO) introduces a new policy that aims to increase the proportion of female employees in a company. The number of male (M) and female (F) employees in the company over the first 6 years (t) since the implementation of the policy is recorded below.

t	0	1	2	3	4	5	6
F	120	138	198	225	245	256	250
M	380	552	602	675	980	1024	750

- (a) Using technology, calculate Pearson's correlation coefficient for the number of female employees and the time since the policy was implemented.
Give your answer to 3 decimal places.

1

$r = 0.938$ ① correct answer

- (b) Describe the strength of the association between the number of female employees and time since the policy was implemented.

1

strong association
① describes correctly
eg strong

- (c) Determine the equation of the least-squares regression line that relates the number of female employees to the number of years since the policy was implemented.
Give all values correct to the nearest integer.

2

$A = 132.46 \rightarrow 132$
 $B = 24$ ① correct A OR B
 $F = 132 + 24t$ ① correct equation

- (d) Using your regression equation from part (c), estimate how many years it will take for the number of female employees to reach 500 **and** explain why this might not be reliable.

3

$$500 = 132 + 24t \quad \textcircled{1} \text{ substitutes } F \text{ into}$$

eqn

$$368 = 24t$$

$$t = 15.3 \text{ years}$$

$$t \approx 16 \text{ years}$$

$\textcircled{1}$ correct t years

either

not accurate as uses extrapolation $\textcircled{1}$ recognises

extrapolation or

equivalent

- (e) The CEO is reviewing the policy. Given that the percentage of females is given by $0.00287 - 0.0006t$ and $r = 0.1146$, evaluate the effectiveness of the policy at increasing the percentage of female employees.

1

$\textcircled{1}$ evaluates the effectiveness

policy is not effective as % of females

decreases (negative gradient) OR

no correlation (r value is low)

End of Question 27

Question 28 (5 marks)

At the start of August, Tyrone opens a bank account and makes a deposit of \$1000 into the same account. At the start of each subsequent month, he makes a deposit of 2% more than the previous month's deposit.

At the end of every month, the bank pays interest of 0.5% (per month) on the balance of the account.

Let A_n = balance in the account at the end of month n (including principal + interest)

(a) Show that $A_2 = \$1\,000(1.005)^2 + \$1\,000(1.02)(1.005)$

2

$$A_1 = 1000(1.005)$$

$$A_2 = A_1(1.005) + 1000(1.02)(1.005)$$

$$= 1000(1.005)^2 + 1000(1.02)(1.005)$$

① evidence of setting up A_1
ie. $1000(1.005)$

① demonstrates substitution of
 A_1 into valid recursive relationship

- (b) Find the balance in the account at the end of the 50th month, correct to the nearest dollar. 3

$$A_3 = A_2 (1.005) + 1000 (1.02)^2 (1.005)$$

$$= 1000 [1.005 (1.02)^2 + (1.005)^2 (1.02) \cancel{(1.005)^3}]$$

$$A_{50} = 1000 [1.005 (1.02)^{49} + (1.005)^2 (1.02)^{48} + \dots + (1.005)^{50}]$$

$$GP = \text{ where } a = 1.005 (1.02)^{49} \quad r = \frac{1.005}{1.02}$$

$$A_{50} = 1000 \left[\frac{a(r^n - 1)}{r - 1} \right]$$

$$= 1000 \left[\frac{1.005 (1.02)^{49} \left(\left(\frac{1.005}{1.02} \right)^{50} - 1 \right)}{\left(\frac{1.005}{1.02} - 1 \right)} \right]$$

$$= \$94360$$

① valid expression for amount at end of any month
ie. A_3 or A_{50}

① a or r correct for GP

① correct answer

End of Question 28

End of paper