International Grammar School

Year 11
Mathematics
Assessment Task 3
2015

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- In Questions 11 - 14, show relevant mathematical reasoning and/or calculations

Total marks – 70

Section I

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section.

Section II

60 marks

- Attempt Questions 11-14
- Start each question in a new writing booklet
- Write your name on the front cover of each booklet to be handed in
- If you do not attempt a question, submit a blank booklet marked with your name and "N/A" on the front cover
- Allow about 1 hour 45 minutes for this section

DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

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Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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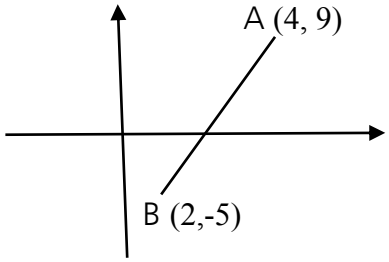
1 A is the point (4, 9) and B is the point (2, -5).

Which of the following gives the gradient of AB:



(A) 7

(B) -7

(C) 2

(D) 5

2 Find θ if $\sin\theta = \frac{\sqrt{3}}{2}$ where $0 \le \theta \le 360^0$

(A) $\theta = 60^0$ and 120^0

(B) $\theta = 60^0$ and 240^0

(C) $\theta = 240^0$ and 300^0

(D) $\theta = 120^0$ and 240^0

3 What is the exact value of $\cos 135^\circ + \operatorname{cosec} 60^\circ$?

(A) $\dfrac{2\sqrt{2}-\sqrt{3}}{\sqrt{6}}$

(B) $\dfrac{2\sqrt{2}-1}{\sqrt{2}}$

(C) $\dfrac{2\sqrt{2}+\sqrt{3}}{\sqrt{6}}$

(D) $\dfrac{2\sqrt{2}+1}{\sqrt{2}}$

4 Calculate the size of each interior angle of a regular polygon with 20 sides.

(A) 200°

(B) 180°

(C) 162°

(D) 18°

5 Solve $(2^x)^2 = 8$

(A) $x = 2$

(B) $x = 4$

(C) $x = 2\sqrt{2}$

(D) $x = \dfrac{3}{2}$

6 Find $\frac{dy}{dx}$ if $y = \frac{2}{x^4}$

(A) $\frac{-8}{x^5}$

(B) $\frac{-8}{x^3}$

(C) $\frac{-10}{x^8}$

(D) $\frac{-6}{x}$

7 Find $f'(2)$ if $f(x) = (3x - 4)^3$

(A) 72

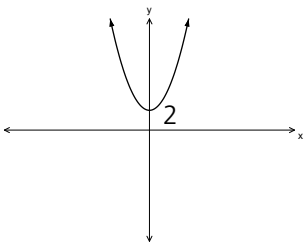
(B) 36

(C) 8

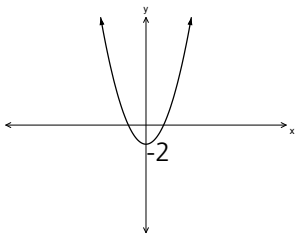
(D) 27

8 Which of the following graphs best represents $y = (x+2)^2$

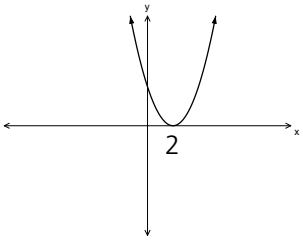
(A)



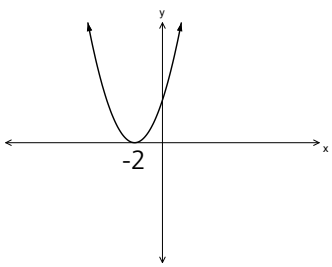
(C)



(B)



(D)



9 Let $f(x) = \sqrt{4 - x^2}$. What is the range of $f(x)$?

(A) $-2 \le y \le 2$

(B) $0 \le y \le 4$

(C) $0 \le y \le 2$

(D) $0 \le x \le 2$

10 The tangent to $y = x^2 + x$ at $x = a$ is parallel to the line $y = 5 - 3x$.
The value of a is:

(A) –2

(B) -3

(C) 1

(D) $\frac{3}{2}$

End of Section 1

Section II

60 marks
Attempt Questions 11–14
Allow about 1 hour 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 11–14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) **Marks**

(a) By rationalising the denominator, express $\frac{8}{3-\sqrt{5}}$ in the form $a + b\sqrt{5}$. 2

(b) The fuel tank on my new car was 40% full. Jiya added 28 litres and then found it was 75% full. How much fuel does the tank hold? 1

(c) Solve $\frac{3x-2}{5} = \frac{x}{4}$ 2

(d) Simplify $\frac{a+b}{4x-2y} \times \frac{2x-y}{a^2-b^2}$ 3

(e) Solve the inequality $|2x+3| < 9$ and show the solutions on a number line. 3

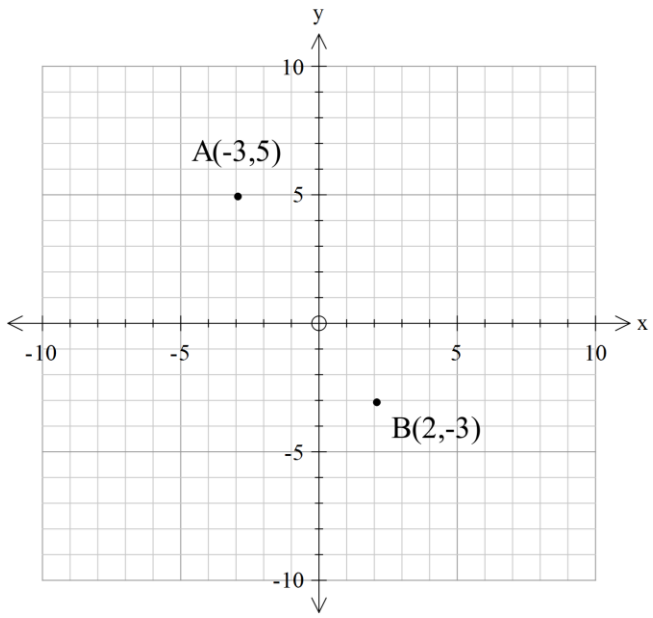
(f) Solve the equations $y = 5x - 1$ and $y = x^2 + 3x$ simultaneously. 3

(g) Evaluate $|30 - 12 \times 3| - |-30 + 12 \times 3|$ 1

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet **Marks**

(a) An interval AB is formed by the points A(–3, 5) and B(2, –3) on the number plane shown below:



(i) Find the exact length of the interval AB 1

(ii) Find the gradient of the interval AB. 1

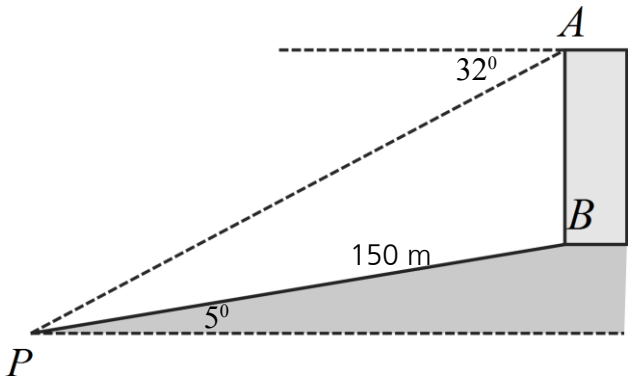
(iii) Show that the equation of the line AB is $8x + 5y - 1 = 0$. 2

(iv) Find the equation of the line through C(5, 8) perpendicular to AB. 2

(v) Calculate the perpendicular distance of the point C from the line AB. 2

(vi) Calculate the area of the triangle ABC. 1

(b) A point P is at the foot of the hill which is inclined at 5° to the horizontal. The base B, of a tower AB, is situated 150 metres up the incline of the hill from P. From the top of the tower the angle of depression of the point P is 32°.



(i) Find the size of $\angle APB$ and of $\angle PAB$. 2

(ii) Find the height of the tower, AB, correct to the nearest metre. 2

(c) Simplify $\dfrac{1}{\text{cosec}^2\theta}+\dfrac{1}{\sec^2\theta}+\dfrac{1}{\cot^2\theta}$ 2

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet **Marks**

(a) Sketch the following graphs and clearly label the vertex, asymptotes (if any) and the points where the graph cuts the x and y-axes.

(i) $y = x^2 + 2x - 3$ 2

(ii) $y = \dfrac{3}{x+2}$ 2

(b) Use the definition $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$ to find the derivative of $f(x) = x^2 - 5x$. 2

(c) Differentiate the following functions:

(i) $f(x) = 5 - 7x + 13x^4$ 1

(ii) $f(x) = \dfrac{2x-3}{3x-2}$ 2

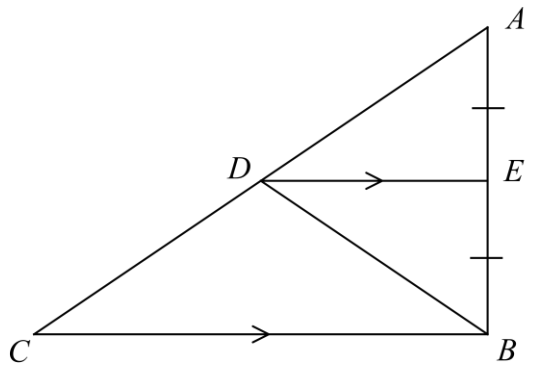
(iii) $f(x) = (3x^2 - 1)(x-1)^3$ 3

(d) Find the equation of the tangent to the curve $y = 2x^2 - 4x + 5$ at the point where $x = 3$. 3

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet **Marks**

(a) In the diagram below ΔABC has a right angle at B. Point E is the midpoint of AB. Point D lies on AC and DE is parallel to BC.



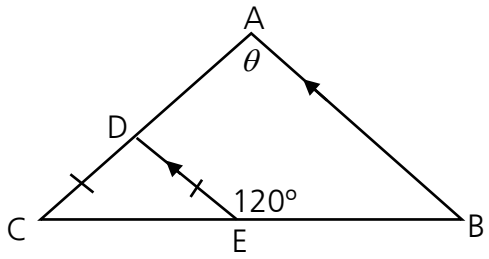
Not to scale

(i) Explain why $\angle AED$ is a right angle. 1

(ii) Prove that $\Delta AED \equiv \Delta BED$ 2

(iii) Show that $BD = DC$ 3

(b) In the diagram shown $DE \parallel AB$ and $CD = DE$

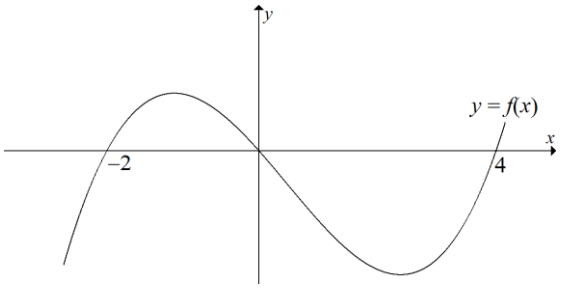


Find $\angle\theta$ giving reasons. 3

(c) Find the centre and radius of the circle $x^2 + y^2 - 2x + 4y + 1 = 0$. 2

(d) Determine if the function $f(x) = \dfrac{x}{x^2 - 1}$ is odd or even. 2

(e) Copy the graph of $y = f(x)$ shown below in your answer booklet and draw a sketch of the gradient function, $y = f'(x)$ on the same diagram. 2



End of Examination

Student Name: ……

2015 Year 11 Assessment Task 3

Mathematics

Section 1 Multiple-Choice Answer Sheet

1	A ⬭	B ⬭	C ⬭	D ⬭
2	A ⬭	B ⬭	C ⬭	D ⬭
3	A ⬭	B ⬭	C ⬭	D ⬭
4	A ⬭	B ⬭	C ⬭	D ⬭
5	A ⬭	B ⬭	C ⬭	D ⬭
6	A ⬭	B ⬭	C ⬭	D ⬭
7	A ⬭	B ⬭	C ⬭	D ⬭
8	A ⬭	B ⬭	C ⬭	D ⬭
9	A ⬭	B ⬭	C ⬭	D ⬭
10	A ⬭	B ⬭	C ⬭	D ⬭

YEAR 11 AT3 2015 SOLUTIONS

SECTION 1

1. A 2. A 3. A 4. C 5. D 6. A

7. B 8 D 9. C 10. A

SECTION 2

a) $\frac{8}{3-\sqrt{5}} = \frac{8}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}}$ ✓

$= \frac{24+8\sqrt{5}}{9-5}$

$= \frac{8(3+\sqrt{5})}{4}$

$= \frac{2(3+\sqrt{5})}{}$ or $6+2\sqrt{5}$ ✓

b) Let tank hold x L

$\therefore 0.4x + 28 = 0.75x$

$0.35x = 28$

$x = 80$ L ✓

c) $\frac{3x-2}{5} = \frac{x}{4}$

$20\left(\frac{3x-2}{5}\right) = \left(\frac{x}{4}\right)20$

$12x - 8 = 5x$ ✓ correct progress to step 3.

$7x = 8$

$x = \frac{8}{7}$ ✓

d) $\frac{a+b}{4x-2y} \times \frac{2x-y}{a^2-b^2}$

$= \frac{(a+b)}{2(2x-y)} \times \frac{(2x-y)}{(a+b)(a-b)}$ ✓ correct factorisation ✓ correct factors ✓ correct simplifying

$= \frac{1}{2(a+b)}$ ✓

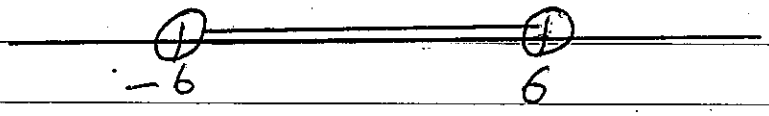
e) $|2x+3| < 9$

$2x+3 < 9$ OR $-(2x+3) < 9$

$x < 6$ OR $2x+3 > -9$

✓ $2x > -12$

$x > -6$ ✓



✓

f) $y = 5x - 1$

$y = x^2 + 3x$

$\therefore x^2 + 3x = 5x - 1$ ✓

$x^2 - 2x + 1 = 0$

$(x-1)^2 = 0$ ✓

$x - 1 = 0$

$x = 1 \Rightarrow y = 5(1) - 1 = 4$

$\therefore (1, 4)$ ✓ both x & y values.

g) $|30 - 12 \times 3| - |-30 + 12 \times 3|$

$= |30 - 36| - |-30 + 36|$

$= |-6| - |6|$

$= 6 - 6$

$= 0$ ✓

Question 12

a) (i) $AB = \sqrt{(-3-2)^2 + (5--3)^2}$

$= \sqrt{89}$ ✓

(ii) $m_{AB} = \frac{-3-5}{2--3}$

$= -\frac{8}{5}$ ✓

(iii) $m_{AB} = -\frac{8}{5}$ $B(2,-3)$

$y--3 = -\frac{8}{5}(x-2)$ ✓

$5(y+3) = -8(x-2)$

$5y+15 = -8x+16$

$8x+5y-1=0$ as required. ✓

(iv) $m_{AB} = -\frac{8}{5} \Rightarrow m_1 = \frac{5}{8}$ $C(5,8)$

$\therefore y-8 = \frac{5}{8}(x-5)$ ✓

$8y-64 = 5x-25$

$5x-8y+39=0$ ✓

(v) AB : $8x + 5y - 1 = 0$ $\quad C(5, 8)$

$$d_1 = \frac{|ax + by + c|}{\sqrt{a^2 + b^2}}$$

$$= \frac{|(8)(5) + (5)(8) - (-1)|}{\sqrt{8^2 + 5^2}} \quad \checkmark$$

$$= \frac{79}{\sqrt{89}} \quad \checkmark$$

(vi) $A = \frac{1}{2} \times \frac{79}{\sqrt{89}} \times \sqrt{89}$

$$= \frac{79}{2} \text{ u}^2 \quad \checkmark$$

b) (i)

$\angle APB = 32 - 5 = 27^\circ$

$\angle PAB = 90 - 32 = 58^\circ$ ✓ Both correct.

(ii) $\frac{h}{\sin 27} = \frac{150}{\sin 58}$ ✓ COA acceptable.

$$h = \frac{150 \sin 27}{\sin 27}$$

$= 80 \text{ m}$ (correct to nearest m) ✓

c) $\frac{1}{\operatorname{cosec}^2\theta} + \frac{1}{\sec^2\theta} + \frac{1}{\cot^2\theta}$

$= \sin^2\theta + \cos^2\theta + \tan^2\theta$

$= 1 + \tan^2\theta$ ✓

$= \sec^2\theta$ ✓

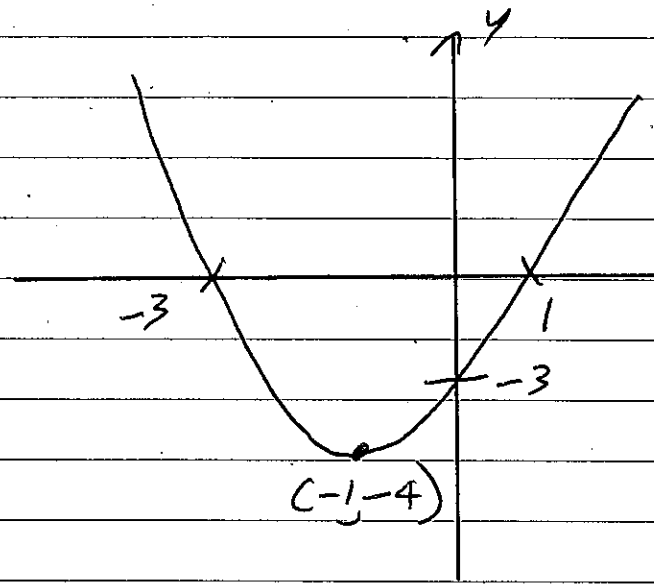
(correct either step 2 or 3.)

Question 13

a) (i) $y = x^2 + 2x - 3$

$= (x+1)^2 - 4$

$\therefore V(-1, -4) \quad (0, -3), (1, 0) (-3, 0)$



✓ correct shape and vertex.

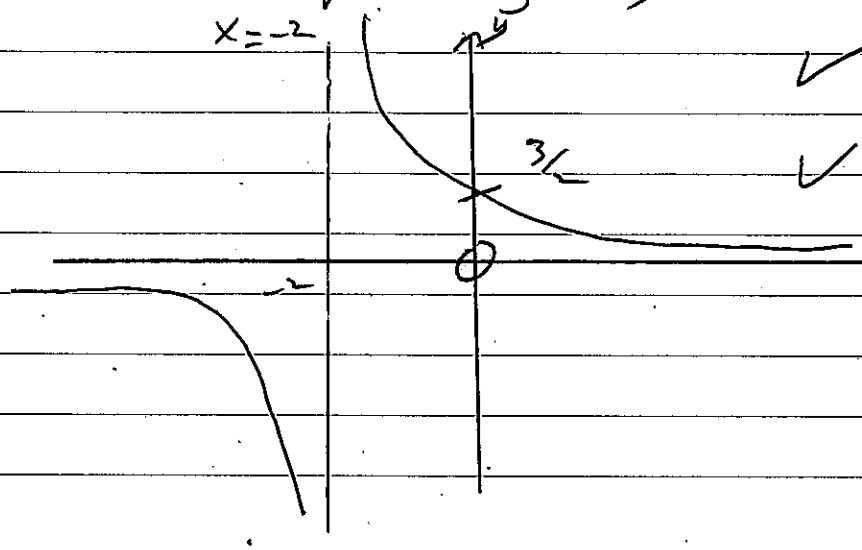
✓ all intercepts correct.

(ii) $y = \dfrac{3}{x+2} \Rightarrow y(x+2) = 3$

$\therefore$ Vertical asymptote : $x = -2$

horizontal " : $y = 0$

y-intercept $(0, 3/2)$



✓

✓ all intercepts and asymptotes correct.

b) $f(x) = x^2 - 5x$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - 5(x+h) - (x^2 - 5x)}{h}$$ ✓

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 5x - 5h - x^2 + 5x}{h}$$

$$= \lim_{h \to 0} \frac{h(2x + h - 5)}{h}$$

$$= 2x - 5$$ ✓

no mark if this method is not used.

c) $f(x) = 5 - 7x + 13x^4$

(i) $f'(x) = -7 + 52x^3$ ✓

(ii) $f(x) = \frac{2x-3}{3x-2}$

$$f'(x) = \frac{2(3x-2) - (3)(2x-3)}{(3x-2)^2}$$ (quotient rule)

$$= \frac{6x - 4 - 6x + 9}{(3x-2)^2}$$ ok if product rule is applied ✓

$$= \frac{5}{(3x-2)^2}$$ ✓

c) (iii) $f(x) = (3x^2 - 1)(x-1)^3$

Let $u = 3x^2 - 1 \Rightarrow u' = 6x$

and $v = (x-1)^3 \Rightarrow v' = 3(x-1)^2$ ✓ u' / v' correct.

$\therefore f'(x) = (3x^2-1)(3)(x-1)^2 + (x-1)^3(6x)$ ✓

$= 3(x-1)^2[3x^2 - 1 + 2x(x-1)]$ ✓

$= 3(x-1)^2(5x^2 - 2x - 1)$

---

d) $y = 2x^2 - 4x + 5$

$x = 3 \Rightarrow y = 2(3)^2 - 4(3) + 5 = 11$

$m_T = \frac{dy}{dx} = 4x - 4$ ✓ correct point and gradient.

$x = 3 \Rightarrow m_T = 4(3) - 4 = 8$

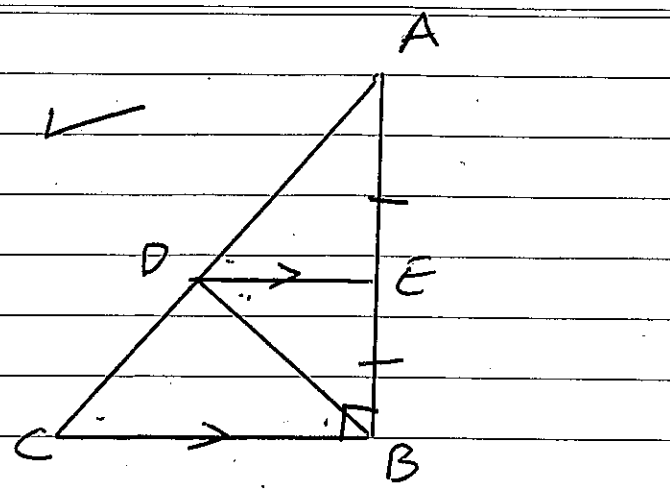
$m_T = 8 \quad (3, 11)$

$\therefore$ equation of tangent: $y - 11 = 8(x-3)$ ✓

$y - 11 = 8x - 24$

$8x - y - 13 = 0$ ✓

Question 14



a) (i) $\angle AED = 90°$ ✓
corresponding $\angle$s DE // CB

(ii) In $\triangle AED$ and $\triangle BED$

$\angle AED = \angle BED$
$= 90°$ (AEB is a straight line)

ED is common ✓
AE = BE (given)

$\therefore \triangle AED \equiv \triangle BED$ (SAS) ✓

(iii) To show BD = DC

BD = AD (corresponding sides of $\equiv \triangle$s)

$\angle EDB = \angle DBC$ (alternate $\angle$s DE // CB)
$\angle ADE = \angle ACB$ (corresponding $\angle$s DE // CB)

But $\angle EDB = \angle EDA$ (matching $\angle$s of $\equiv \triangle$s)

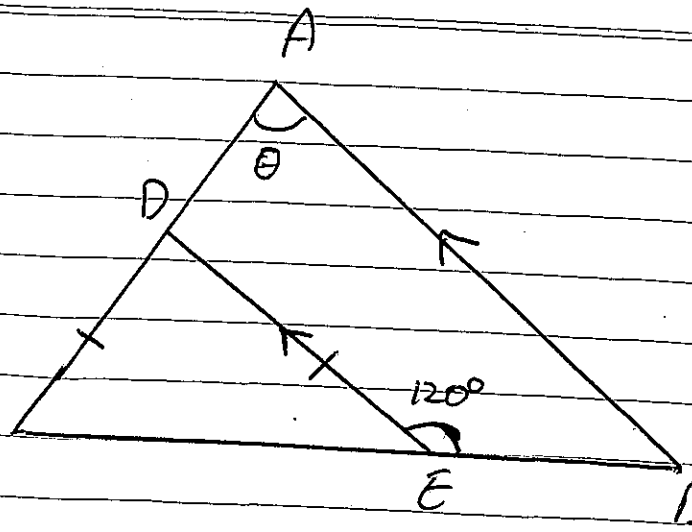
$\therefore \angle DBC = \angle ACB$

$\therefore$ $\triangle DBC$ is isosceles (base $\angle$s equal)

$\therefore CD = BD = AD$

$\therefore BD = AD$ as required.

b)



$\angle DEC = 60°$ (straight $\angle$s)

$\angle ACB = 60°$ (base $\angle$s of isosceles $\triangle$) ✓

$\therefore \angle CDE = 180 - 120$ ($\angle$ sum of $\triangle$) ✓

$= 60°$

$\therefore \theta = 60°$ (corresponding $\angle$s AB // DE) ✓

c) $x^2 - 2x + y^2 + 4y = -1$

$(x-1)^2 + (y+2)^2 = -1 + 1 + 4$

$= 4$ ✓ some working.

$\therefore C(1, -2)$ Radius $= 2$ ✓

d) $f(x) = \dfrac{x}{x^2 - 1}$

$f(-x) = \dfrac{-x}{(-x)^2 - 1} = -\dfrac{x}{x^2 - 1}$

$-f(x) = -\dfrac{x}{x^2 - 1}$

$\therefore f(-x) = -f(x)$ Hence odd function.

Question 14

e)