International Grammar School

Year 11
Mathematics
Assessment Task 3
2018

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black pen
- NESA approved calculators may be used
- In Questions 11 - 14, show relevant mathematical reasoning and/or calculations
- A reference sheet has been provided

Total marks – 70

Section I

10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section.

Section II

60 marks

- Attempt Questions 11-14
- Start each question in a new writing booklet
- Write your name on the front cover of each writing booklet
- If you do not attempt a question, submit a blank booklet marked with your name and "N/A" on the front cover
- Allow about 1 hour 45 minutes for this section

DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

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Section I
10 marks
Attempt Questions 1-10

Use the multiple choice answer sheet for Questions 1-10.

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1 Which of the following is the expansion of $(2a-b)^2$?

(A) $4a^2-b^2$

(B) $4a^2-4ab+b^2$

(C) $4a^2+4ab+b^2$

(D) $2a^2-2ab+b^2$

2 Calculate $\sqrt{9.96\times10^{-4}}$ correct to 2 significant figures.

(A) 31.5×10^{-3}

(B) 3.2×10^{-2}

(C) 31.6×10^{-3}

(D) 0.03

3 Simplify the expression $\sqrt{12}-\sqrt{50}+\sqrt{3}-\sqrt{8}$.

(A) $3\sqrt{3}-3\sqrt{2}$

(B) $3\sqrt{3}-7\sqrt{2}$

(C) $\sqrt{43}$

(D) $-5\sqrt{6}$

4 What are the solutions to the inequality $|2x-1|>5$?

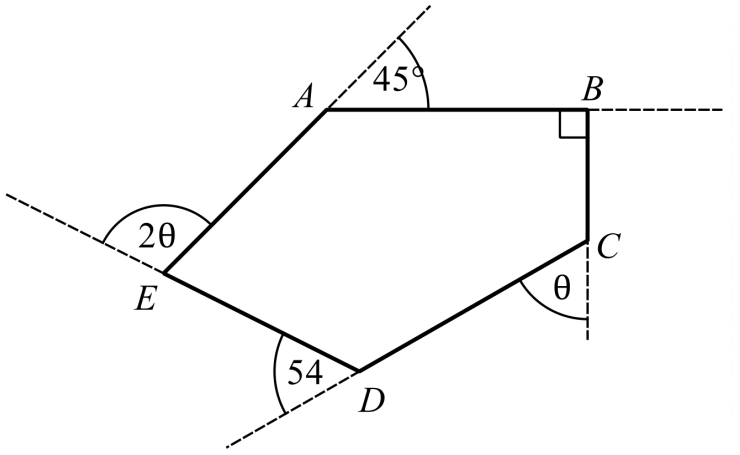
(A) $-2<x<3$

(B) $-3<x<3$

(C) $x<-3$ or $x>3$

(D) $x<-2$ or $x>3$

5 The diagram shows an irregular pentagon. Some of the interior and exterior angle sizes have been marked on the diagram.



NOT TO SCALE

Find the value of θ.

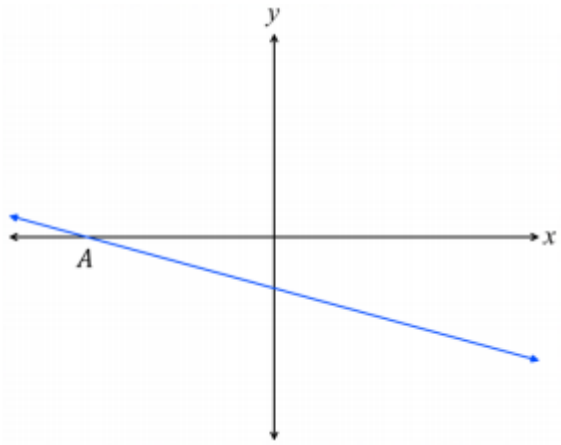
(A) 171°

(B) 87°

(C) 56°

(D) 57°

6 The line $2x+4y+8=0$ cuts the x-axis at A.



The coordinates of A are:

(A) $(-4,0)$

(B) $(0,-4)$

(C) $(-2,0)$

(D) $(0,-2)$

7 The domain of the function $f(x)=\sqrt{9-x^2}$ is:

(A) $-9 \le x \le 9$

(B) $-3 \le x \le 3$

(C) $0 \le y \le 3$

(D) $x \ge 0$

8 Which of the following is an expression for $\dfrac{1-\sin^2 x}{\cot x}$?

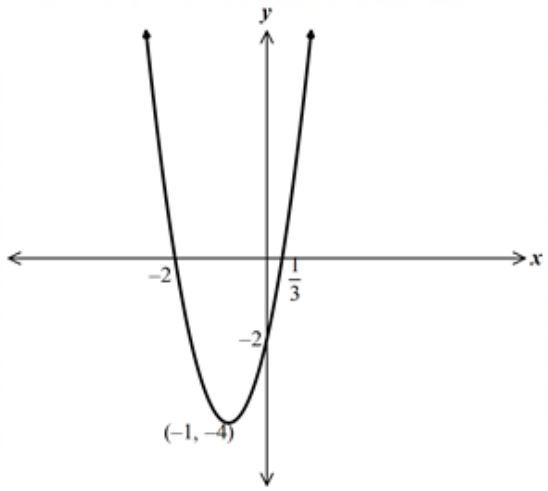
(A) $\cos^2 x$

(B) $\tan x$

(C) $\sin x \cos x$

(D) $\tan x - \sin x \cos x$

9 Which of the equations below describes the parabola in the diagram?



(A) $y = (3x-1)(x+2)$

(B) $y = (3x+1)(x-2)$

(C) $y = (x-3)(x+2)$

(D) $y = (x-1)(x-4)$

10 Find $\frac{d}{dx}\left(3x^4 - 7x^{-3}\right)$

(A) $3x^3 - 7x^{-2}$

(B) $12x^3 - \frac{21}{x^2}$

(C) $12x^3 + \frac{21}{x^4}$

(D) $12x^3 - \frac{21}{x^4}$

End of Section I

Section II
60 marks
Attempt Questions 11-14

Answer each question in separate writing booklets. Extra writing booklets are available.

In Questions 11–14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) **Marks**

(a) Solve each of the following equations.

(i) $2^{x-1} = 4$ **1**

(ii) $2x^2 + 7x + 3 = 0$ **2**

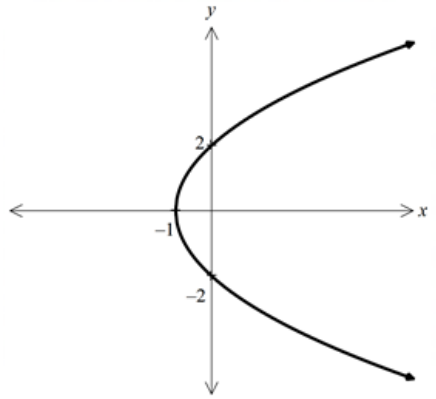
(iii) $|4x + 3| = 5x$ **3**

(b) Express with a rational denominator $\dfrac{\sqrt{2}}{2+\sqrt{3}}$ **2**

(c) Solve $2\sin\theta = -1$ in the domain $0^\circ \le \theta \le 360^\circ$ **2**

(d) Prove that $\tan\theta\operatorname{cosec}\theta = \sec\theta$ **2**

(e) For the graph shown below:



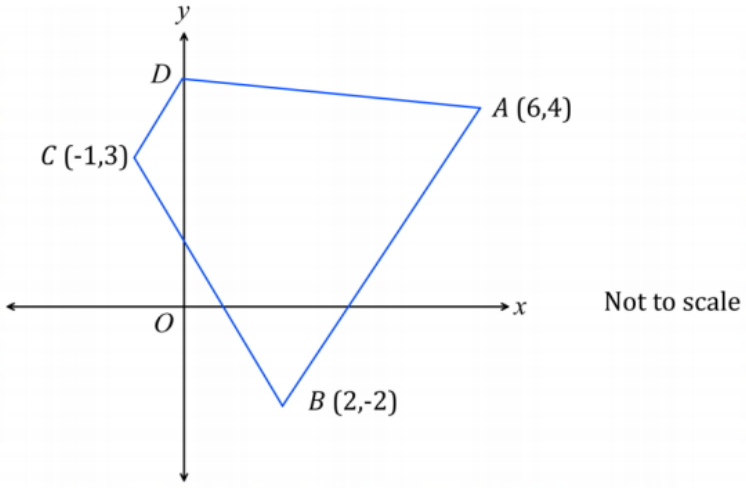
(i) Is this a graph of a function? Justify your answer. **1**

(ii) State the domain and range. **2**

END OF QUESTION 11

Question 12 (15 marks) **Marks**

(a) The points $A(6, 4)$, $B(2, -2)$ and $C(-1, 3)$ are plotted on a number plane. The point D lies on the y-axis such that AB is parallel to CD.



(i) Find the exact length of AB. **1**

(ii) Find the gradient of AB. **1**

(iii) Show that the equation of the line AB is $3x-2y-10=0$ **2**

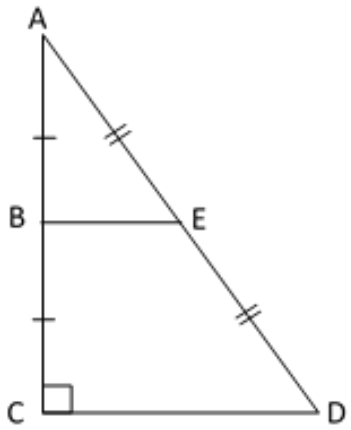
(vi) Find the perpendicular distance from point C to AB. **2**

(b) State the equation of a circle with centre (0,2) and radius of 3 units. **2**

(c) For the function $f(x)=\begin{cases}-2 \text{ for } x<0\\ x^3 \text{ for } x\geq 0\end{cases}$, find $f(-1)-f(2)$. **2**

QUESTION 12 CONTINUES ON PAGE 10

(d) The diagram shows ΔACD, right angled at C. B and E are the midpoints of AC and AD, respectively.



(i) Prove that ΔACD and ΔABE are similar. **2**

(ii) Hence, or otherwise, prove that BE is parallel to CD **1**

(iii) If AB = 4 cm, BE = 3 cm, and AE = 5 cm, find the area of the trapezium $BCDE$. **2**

END OF QUESTION 12

Question 13 (15 marks) **Marks**

(a) A, B, C are 3 co-linear points of which, A is the point (-3, 2) and B (2, 4) is the midpoint of AC. Find the coordinates of C. **2**

(b) Given the function $f(x) = x^2 - 7x + 6$

(i) Find $f(x+h)$ **1**

(ii) Find $f(x+h) - f(x)$ **1**

(iii) Hence show that $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = 2x - 7$ **1**

(c) Evaluate $\lim_{x \to 5} \frac{x^2 - 25}{x - 5}$ **2**

(d) Differentiate the following.

(i) $6x^2 - x + 2$ **1**

(ii) $3\sqrt{x}$ **1**

(iii) $(x^4 - 2x)^3$ **2**

(iv) $x^2(x-4)^5$ **2**

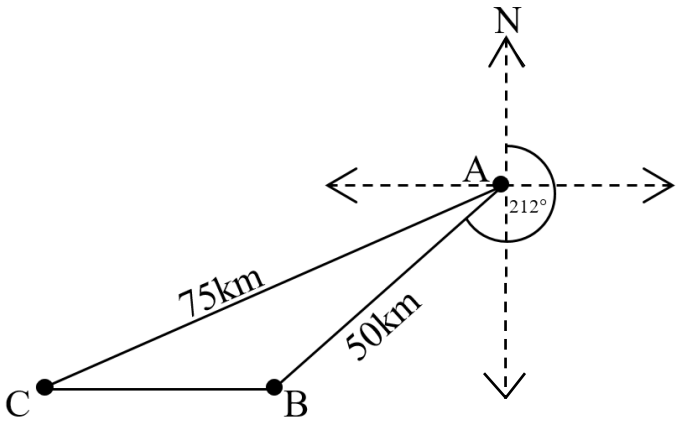
(v) $\frac{2x}{7 - 3x}$ **2**

END OF QUESTION 13

Question 14 (15 marks) **Marks**

(a) Find the equation of the normal to the curve $y = 3x^2 - 2$ at the point on the curve where $x = 1$. **3**

(b) A ship sailed 50 kilometres from point A to point B on a bearing of 212°.
From point B, it sailed due west to reach point C.
The distance AC is 75 kilometres.



(i) Show that $\sin C = 50 \times \dfrac{\sin 122}{75}$. **3**

(ii) Hence, find the bearing of point C from point A. **2**
(Give your answer correct to the nearest degree.)

(c) For the equations $y = |x+1|$ and $y = -x^2 + x + 2$,

(i) Sketch the graph of $y = |x+1|$ in your writing booklet. Show all important features. **2**

(ii) On the same axes as (i), sketch the graph of $y = -x^2 + x + 2$. Show where the curve cuts the x and y axes and the coordinates of its vertex. **3**

(iii) Shade the region bound by $y < |x+1|$ and $y < -x^2 + x + 2$. **2**

END OF EXAMINATION

Student Name: ……

2018 Year 11 Assessment Task 3

Mathematics

Section I Multiple-Choice Answer Sheet

1	A ⬭	B ⬭	C ⬭	D ⬭
2	A ⬭	B ⬭	C ⬭	D ⬭
3	A ⬭	B ⬭	C ⬭	D ⬭
4	A ⬭	B ⬭	C ⬭	D ⬭
5	A ⬭	B ⬭	C ⬭	D ⬭
6	A ⬭	B ⬭	C ⬭	D ⬭
7	A ⬭	B ⬭	C ⬭	D ⬭
8	A ⬭	B ⬭	C ⬭	D ⬭
9	A ⬭	B ⬭	C ⬭	D ⬭
10	A ⬭	B ⬭	C ⬭	D ⬭

Question 11

(a). (i) $2^{x-1} = 4$

$2^{x-1} = 2^2$

$x - 1 = 2$

$x = 3$ (1)

(ii) $2x^2 + 7x + 3 = 0$

P: 6 S: 7 f: (6, 1)

$2x^2 + 6x + x + 3 = 0$

$2x(x + 3) + (x + 3) = 0$

$(x + 3)(2x + 1) = 0$

$x + 3 = 0$ $\quad 2x + 1 = 0$

$x = -3$ $\quad x = -\frac{1}{2}$ (1) (1)

(iii) $|4x + 3| = 5x$

case 1: $4x + 3 \geq 0$

$4x + 3 = 5x$

$x = 3$ (1)

Test

$|4 \times 3 + 3| = 5 \times 3$

$15 = 15$

LHS = RHS

case 2: $4x + 3 < 0$

$-4x - 3 = 5x$

$-3 = 9x$

$x = -\frac{1}{3}$ (1)

Test

$|4 \times -\frac{1}{3} + 3| = 5 \times -\frac{1}{3}$

$\frac{5}{3} \neq -\frac{5}{3}$

LHS $\neq$ RHS.

$x = 3$ (1)

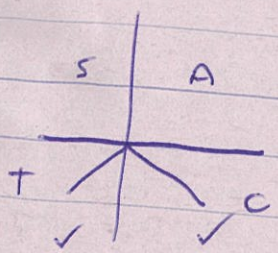
(b) $\frac{\sqrt{2}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$ (1) $= \frac{\sqrt{2}(2-\sqrt{3})}{4-3}$

$= 2\sqrt{2} - \sqrt{6}$ (1)

(c) $2\sin\theta = -1$

$\sin\theta = \frac{-1}{2}$

Basic angle $= 30°$



In Q_3

$\theta = 180 + 30$

$\theta = 210°$ (1)

In Q_4

$\theta = 360 - 30$

$\theta = 330°$ (1)

$\therefore \theta = 210°, 330°$

) $\tan\theta \operatorname{cosec}\theta = \sec\theta$

LHS:

$\frac{\sin\theta}{\cos\theta} \times \frac{1}{\sin\theta}$ (1)

$\frac{1}{\cos\theta}$

$\sec\theta$ (1)

∴ LHS = RHS

(e)(i) No, does not pass straight line test.
More than one y-value for each x-value. (1)

(ii) Domain $\{x \geq -1, \text{ all real } x\}$ (1)

Range $\{\text{all real } y\}$ (1)

Question 12.

(a) (i) $\sqrt{52} = 2\sqrt{13}$ (1) (either or)

(ii) $M_{AB} = \frac{3}{2}$ (1)

(iii) $y - y_1 = m(x - x_1)$

$y - 4 = \frac{3}{2}(x - 6)$

$2y - 8 = 3(x - 6)$ (1)

$2y - 8 = 3x - 18$

$3x - 2y + 10 = 0$ (1)

(iv) $d_\perp = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

$d_\perp = \frac{|(3x - 1) + (-2 \times 3) + 10|}{\sqrt{3^2 + (-2)^2}}$ (1)

$d_\perp = \frac{1}{\sqrt{13}}$ or 0.28 or $\frac{\sqrt{13}}{13}$ (1)

$(x - a)^2 + (y - b)^2 = r^2$

$(x - 0)^2 + (y - 2)^2 = 9$ (1)

$x^2 + (y - 2)^2 = 9$ (1)

$f(x) = \begin{cases} -2, & x < 0 \\ x^3, & x \geq 0 \end{cases}$

$f(-1) - f(2)$

$f(-1) = -2$

$f(2) = 2^3$ (1)

$= 8$

$f(-1) - f(2) = -2 - 8$

$= -10$ (1)

(d) (i) In $\triangle ACD$ and $\triangle ABE$,

Let $AB = x$, $AE = y$

$AC = 2x$, $AD = 2y$.

$$\frac{AB}{AC} = \frac{x}{2x}$$

$$\frac{AB}{AC} = \frac{1}{2}$$

$$\frac{AE}{AD} = \frac{y}{2y}$$

$$\frac{AE}{AD} = \frac{1}{2}$$

(1) sides in proportion.

$\angle BAE = \angle CAD$ (common angle) (1) angle.

$\therefore \triangle ACD \,|||\, \triangle ABE$

(2 sides in proportion and equal angle)

(ii) Since $\triangle ACD \,|||\, \triangle ABE$,

$\angle AEB = \angle ADC$ (matching angles) (1)

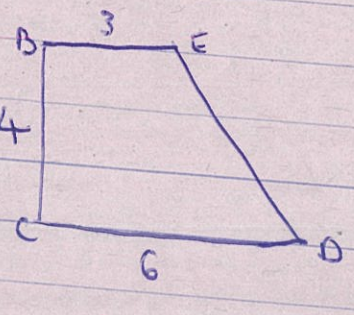
$\angle AEB$ and $\angle ADC$ are corresponding.

$\therefore BE \parallel CD$ (corresponding angles equal on parallel lines)

(iii) $A = \frac{h}{2}(a+b)$

$A = \frac{4}{2}(3+6)$ (1)

$A = 18\,cm^2$ (1)



Question 13

(a) $M_{(x,y)} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$M_{(2,4)} = \left(\frac{x-3}{2}, \frac{y+2}{2}\right)$

$\frac{x-3}{2} = 2$ $\qquad$ $\frac{y+2}{2} = 4$

$x - 3 = 4$ $\qquad$ $y + 2 = 8$

$x = 7$ $\qquad$ $y = 6$

$\therefore C(7, 6)$

(b)(i) $f(x+h) = (x+h)^2 - 7(x+h) + 6$

$= x^2 + 2xh + h^2 - 7x - 7h + 6$] either or. (1)

(ii) $f(x+h) - f(x) = x^2 + 2xh + h^2 - 7x - 7h + 6 - x^2 + 7x - 6$] (1)

$= 2xh + h^2 - 7h$

(iii) $\lim_{h \to 0} \frac{2xh + h^2 - 7h}{h}$

$= \lim_{h \to 0} \frac{h(2x + h - 7)}{h}$

$\lim_{h \to 0} 2x + h - 7$

sub $h = 0$ (1)

$= 2x - 7$

$\lim_{x \to 5} \frac{x^2 - 25}{x - 5} =$

$= \lim_{x \to 5} \frac{(x-5)(x+5)}{x-5}$ (1)

$\lim_{x \to 5} x + 5$

sub $x = 5$ (1)

$= 10$

(d)(i) $\frac{d}{dx} = 12x - 1$ (1)

(ii) $\frac{d}{dx} = \frac{3}{2}x^{-1/2}$ or $\frac{3}{2\sqrt{x}}$ (1)

(iii) $\frac{d}{dx} = 3(x^4 - 2x)^2 \times (4x^3 - 2)$ (1) (1)

$= 3(4x^3 - 2)(x^4 - 2x)^2$

(iv) $y' = uv' + vu'$

Let $u = x^2$ $\quad v = (x-4)^5$

$u' = 2x$ $\quad v' = 5(x-4)^4 \times 1$

$y' = 5x^2(x-4)^4 + 2x(x-4)^5$ (1)

$y' = x(x-4)^4(5x + 2(x-4))$

$y' = x(x-4)^4(5x + 2x - 8)$

$y' = x(x-4)^4(7x - 8)$ (1) fully factorised.

(v) $y' = \frac{vu' - uv'}{v^2}$

Let $u = 2x$ $\quad v = 7 - 3x$

$u' = 2$ $\quad v' = -3$

$y' = \frac{2(7-3x) + 6x}{(7-3x)^2}$ (1)

$y' = \frac{14 - 6x + 6x}{(7-3x)^2}$

$y' = \frac{14}{(7-3x)^2}$ (1) fully simplified.

Question 14.

(a) $y = 3x^2 - 2$

$\frac{dy}{dx} = 6x$

sub $x = 1$

$\frac{dy}{dx} = 6$

$m_{tangent} = 6$

$m_{normal} = \frac{-1}{6}$ (1)

If $x = 1$, $y = 3 \times (1)^2 - 2$

$y = 1$ (1)

$y - y_1 = m(x - x_1)$

$y - 1 = \frac{-1}{6}(x - 1)$

$6y - 6 = -x + 1$ or $y = \frac{-x + 7}{6}$

$x + 6y - 7 = 0$ (1)

(b)(i) $\frac{\sin C}{c} = \frac{\sin B}{b}$

(2) finding 122° with reasoning.

$\frac{\sin C}{50} = \frac{\sin 122}{75}$

$\sin C = 50 \times \frac{\sin 122}{75}$ (1)

(ii) $C = \sin^{-1}\left(50 \times \frac{\sin 122}{75}\right)$

$C = 34°$

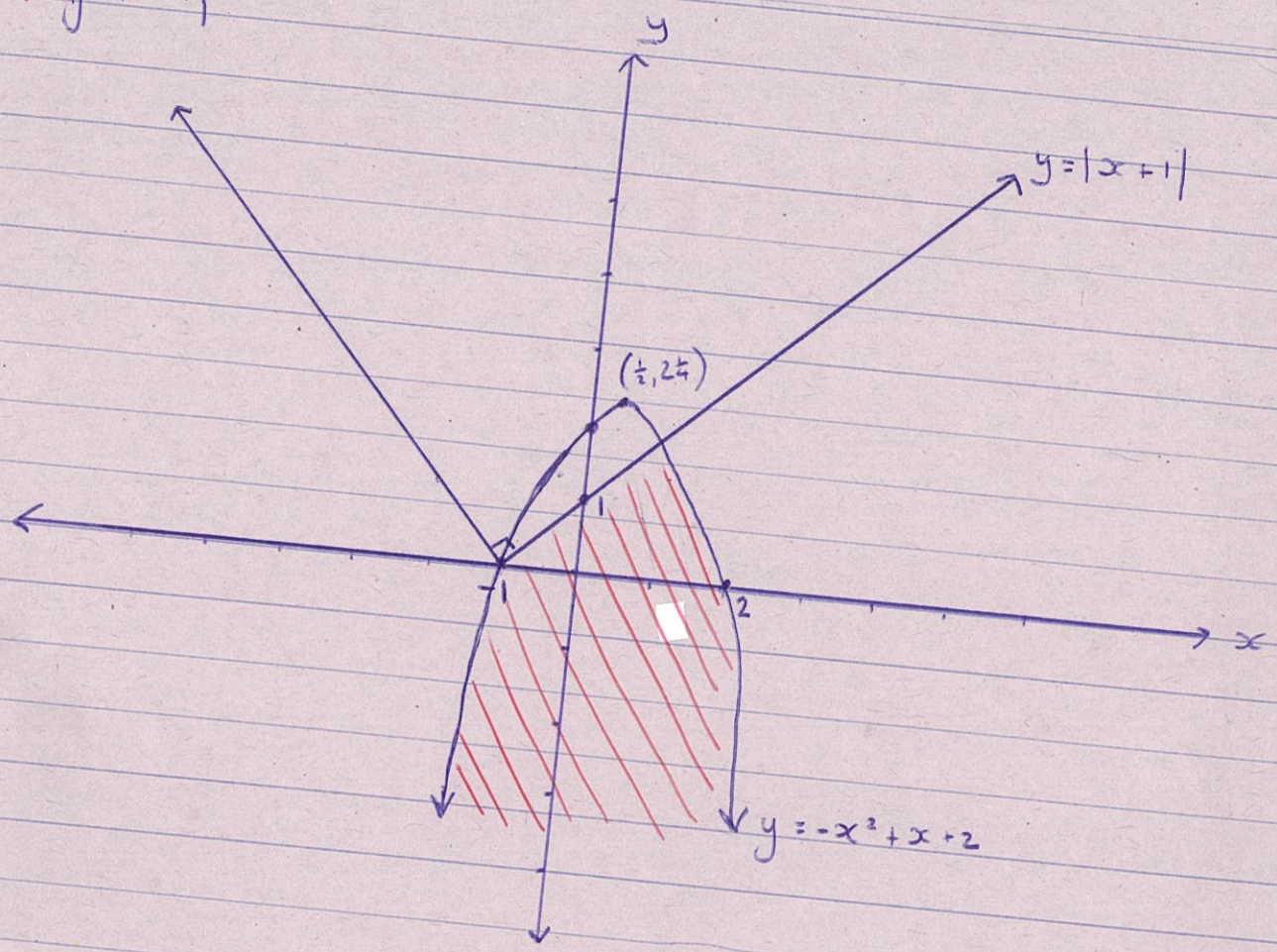
$\angle CAB = 180 - 34 - 122$ (1) (angle sum triangle)

$\angle CAB = 24°$

$\therefore$ Bearing $= 212 + 24$

$= 236°$ (1)

(c)(i) $y = |x+1|$



① x. y intercepts
② correct shape.

(ii) $y = -x^2 + x + 2$

x - intercept

$0 = -x^2 + x + 2$

$x^2 - x - 2 = 0$

$(x+1)(x-2) = 0$

$x = -1, \quad x = 2.$

$(-1, 0) \quad (2, 0)$

y - intercept.

$y = 2.$

$(0, 2)$

(iii) Test (0, 0)

$0 < |0+1|$

$0 < 1$

True

$0 < 0 + 0 + 2$

$0 < 2$

True

Vertex

$x = \frac{-b}{2a}$

$x = \frac{-1}{2 \times 1}$

$x = +\frac{1}{2}, \quad y = 2\frac{1}{4}$