

Year 11

Mathematics Advanced

Assessment Task 3

2019

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations.

Total marks – 70

Section I

10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section.

Section II

60 marks

- Attempt Questions 11-16
- Allow about 1 hour 45 minutes for this section

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Section I

10 marks

Attempt Questions 1-10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

1 Which of the following is equivalent to $\frac{3}{\sqrt{2}}$ with a rational denominator?

A. $\frac{\sqrt{6}}{4}$

B. $\frac{3}{2}$

C. $\frac{3\sqrt{2}}{2}$

D. $\frac{3\sqrt{2}}{4}$

2 Which of the following is the closest to $\frac{2\pi}{7}$ in degrees?

A. 0.9°

B. 0.016°

C. 51.4°

D. 161.6°

3 What is the value of $f(-1)$ if $f(x) = x^3 - x^2$?

A. -2

B. -1

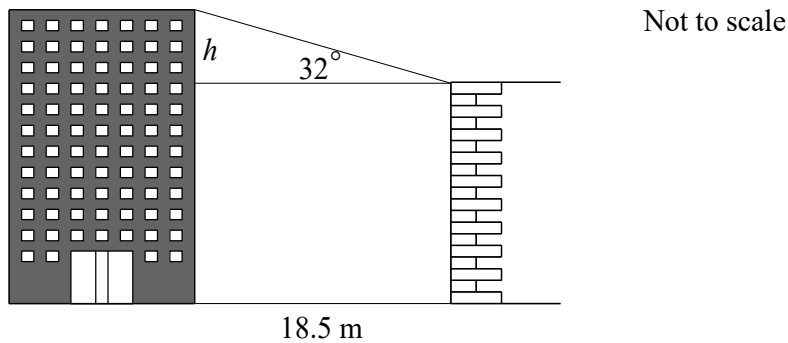
C. 0

D. 2

4 What is the domain of the function $f(x) = \frac{x}{\sqrt{x+5}}$?

- A. $(-5, \infty)$
- B. $[-5, \infty)$
- C. $(-\infty, -5)$
- D. $(-\infty, -5]$

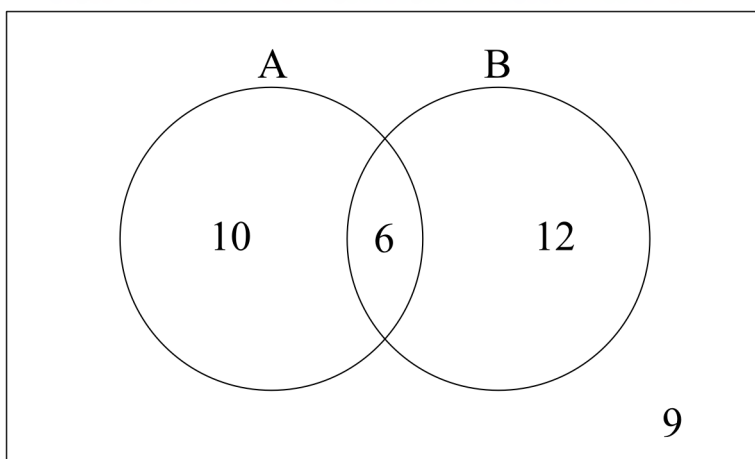
5 The two buildings below are standing on level ground. The horizontal distance between the buildings is 18.5 metres and the angle of elevation from the top of the smaller building to the top of the larger building is 32° .



What is the difference in height (h) between the buildings?

- A. 9.8m
 - B. 11.6m
 - C. 15.7m
 - D. 29.6m
- 6 What is the derivative of $3x^5 - 4x^2 + 2$?
- A. $15x^4 - 8x + 2$
 - B. $15x^5 - 8x^2 + 2$
 - C. $15x^4 - 8x$
 - D. $15x^5 - 8x^2$

- 7 Consider the Venn diagram below.



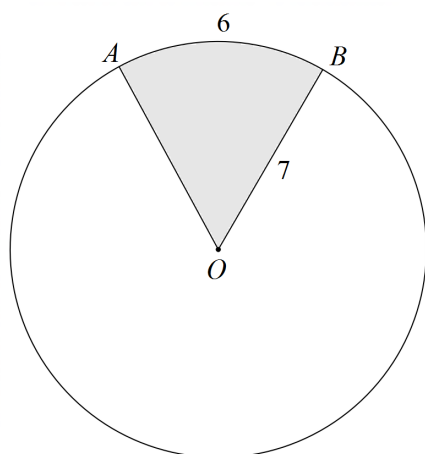
According to the diagram, what is $P(A|B)$?

- A. $\frac{1}{3}$
 - B. $\frac{5}{6}$
 - C. $\frac{28}{37}$
 - D. $\frac{3}{5}$
- 8 The function $f(x)$ is an even function, and the function $g(x)$ is an odd function.

Which of the following is a true statement?

- A. $g(f(x))$ is neither odd nor even.
- B. $g(f(x))$ is odd.
- C. $f(g(x))$ is even.
- D. $f(g(x))$ is odd.

- 9 The circle centred at O has a radius of 7. Arc AB has length 6, as shown in the diagram.



NOT TO
SCALE

What is the area of the shaded sector OAB ?

- A. 21π units²
- B. 21 units²
- C. $\frac{343\pi}{12}$ units²
- D. $\frac{343}{12}$ units²
- 10 Of the following, which function is not differentiable at $x = 0$?
- A. $f(x) = |x + 1|$
- B. $f(x) = x^2$
- C. $f(x) = \frac{x}{x^2 + 1}$
- D. $f(x) = |x|$

END OF SECTION I

Section II

60 Marks

Allow about 1 hour and 45 minutes for this section

Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses.

Your responses should include relevant mathematical reasoning and/or calculations.

Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering

Question 11 (10 Marks)

- (a) Solve $|x + 2| = 6$ **2**

- (b) The equation of a circle is given by $(x - 2)^2 + (y + 3)^2 = 16$. **1**
State the centre and radius of the circle.

- (c) Using the following sets,

$$A = \{1, 2, 3, 4, 5\} \text{ and } B = \{3, 4, 5, 6, 7\}$$

List the elements of:

- (i) $A \cap B$ **1**

- (ii) $A \cup B$ **1**

- (d) For the function $y = x^2 - 2x - 8$:
- (i) State the x and y intercepts.

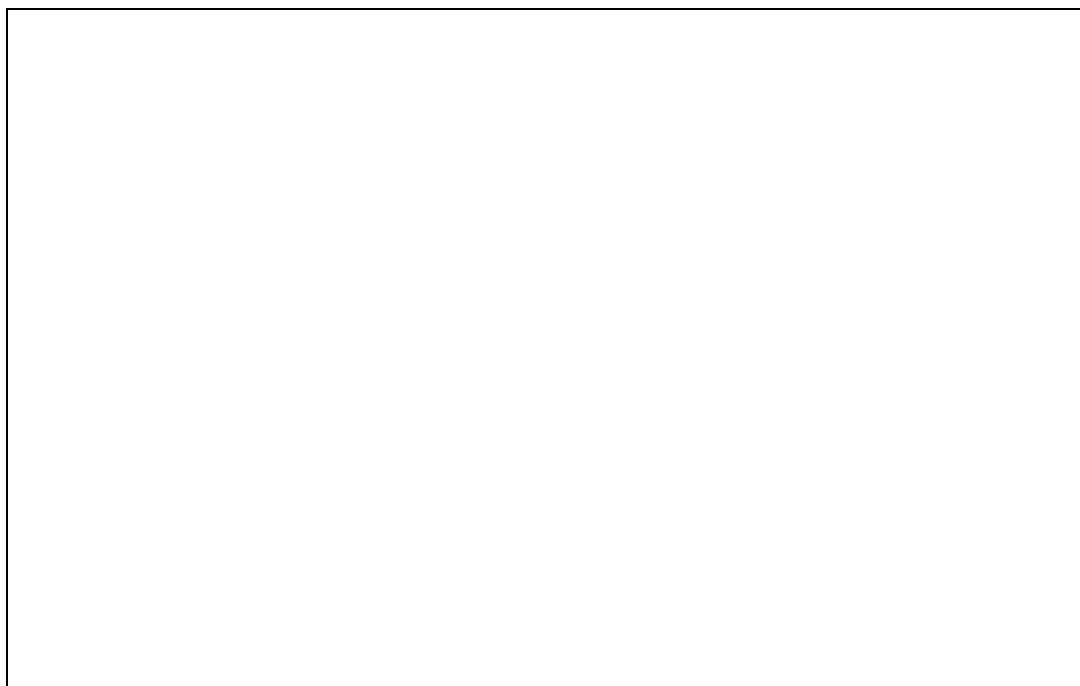
2

- (ii) Show that the coordinates of the vertex are $(1, -9)$.

2

- (iii) Sketch the function showing all important features.

1



END OF QUESTION 11

Question 12 (10 Marks)

(a) Find the exact value of $\cos(135^\circ)$

1

(b) Differentiate the following:

(i) $y = x^{-2}$

1

(ii) $y = (2x - 3)^8$

2

(iii) $y = 2\sqrt{x}$

2

(c) Differentiate $y = x(x+1)^3$ using the product rule.

2

2

END OF QUESTION 12

Question 13 (10 Marks)

- (a) Given that $\cos\theta = -\frac{12}{13}$ and $\sin\theta > 0$, find the value of $\operatorname{cosec}\theta$.

2

- (b) For the curve $f(x) = x^2 - 4x$,

- (i) Find the gradient of the tangent at the point $(1, -3)$.

1

- (ii) Find the gradient of the normal at the point $(1, -3)$.

1

- (iii) Show that the equation of the normal to the curve at the point $(1, -3)$ is $x - 2y - 7 = 0$.

2

- (c) The displacement, x metres, from the origin at time, t seconds, of a particle moving in a straight line is given by $x = t^3 - 6t^2 + 5t + 1$.

(i) Find an expression for the velocity as a function of time.

1

(ii) Find an expression for the acceleration as a function of time.

1

(iii) Find the particle's position when acceleration is zero.

2

END OF QUESTION 13

Question 14 (10 Marks)

(a) Two functions are defined as follows:

2

$$h(x) = x^3 \quad \text{and}$$

$$j(x) = \frac{2x-1}{3}$$

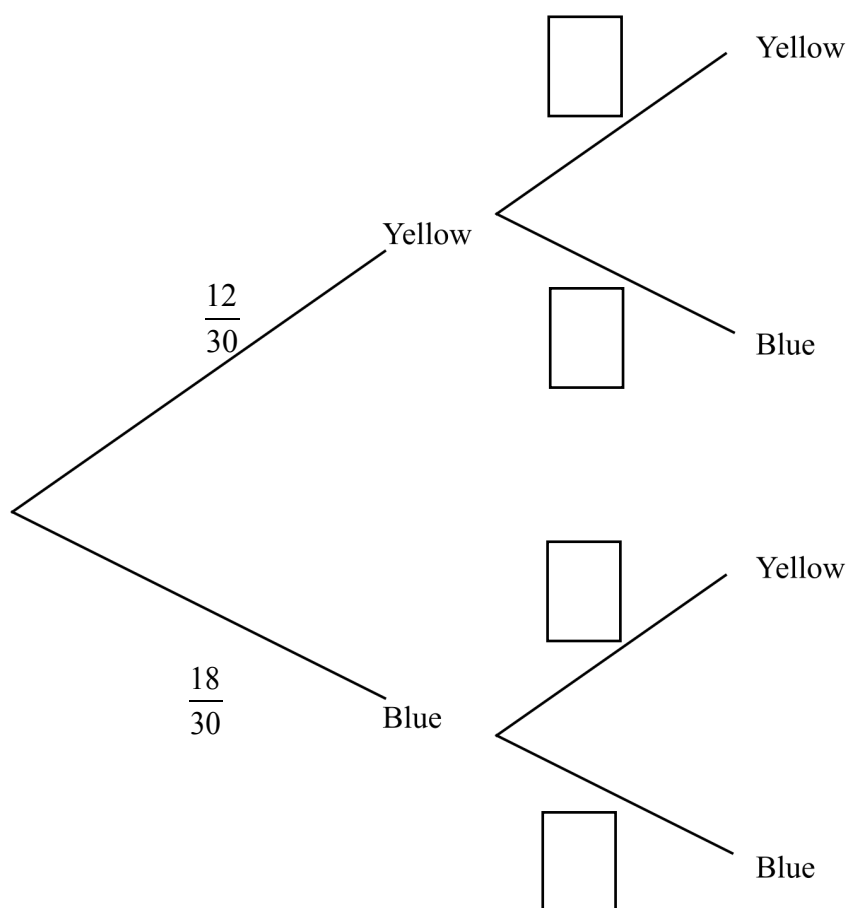
Find the value of $h(j(5))$.

QUESTION 14 CONTINUES

- (b) A barrel contains 18 blue marbles and 12 yellow marbles.

Mike draws two marbles, with no replacement, from the barrel.

A probability tree diagram has been started to model this situation.



- (i) Complete the tree by writing the remaining probabilities in the blank boxes. **1**

- (ii) What is the probability that exactly one of the two marbles is blue? **1**

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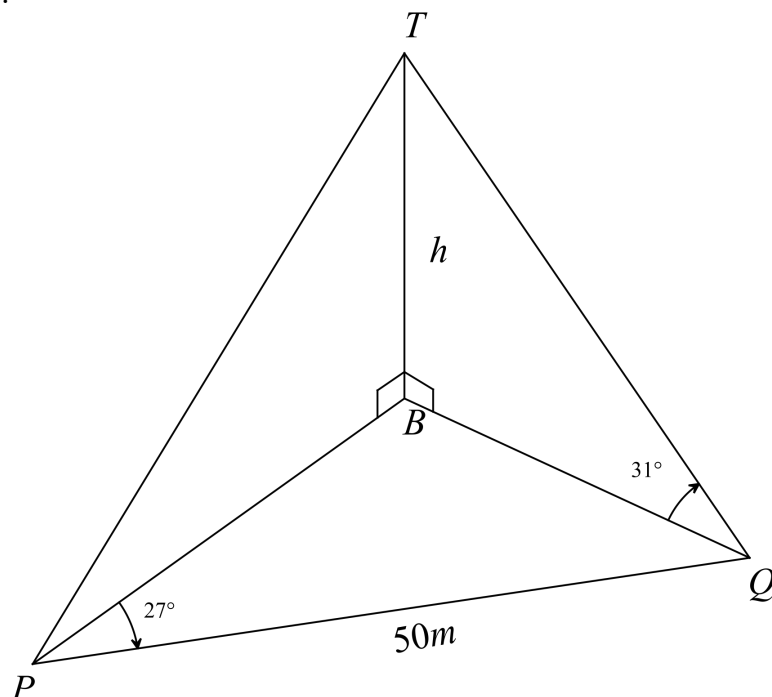
- (iii) What is the probability that at least one of the two marbles is blue? **1**

.....

.....

.....

- (c) A tree, BT , is due north of an observer at P and due west of an observer at Q . The two observers are 50m apart and the bearing of Q from P is 27° . The angle of elevation of T from Q is 31° .



- (i) Show that $BQ = 50\sin 27^\circ$ 1

- (ii) Hence find the height of the tree, correct to the nearest metre. 2

- (iii) Find the angle of elevation of T from P , correct to the nearest degree. 2

END OF QUESTION 14

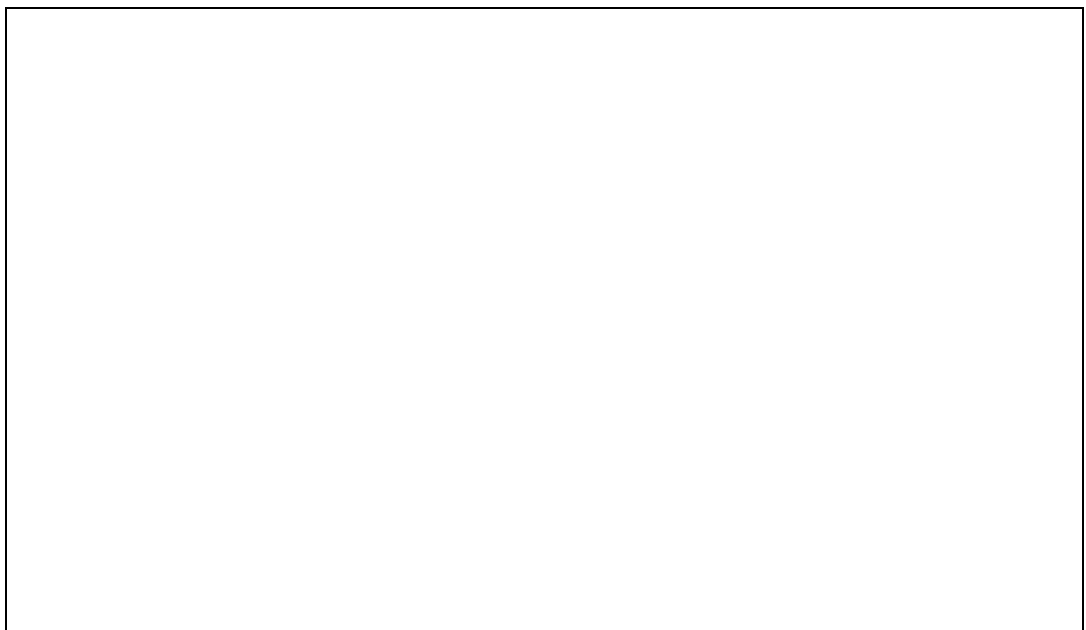
Question 15 (10 Marks)

(a) The volume, V , of a gas varies **inversely** with the pressure, P , it is under.

- (i) If the volume of the gas is 120cm^3 when it is under a pressure of 40 kg/cm^3 , find an equation that relates the variables V and P . **2**

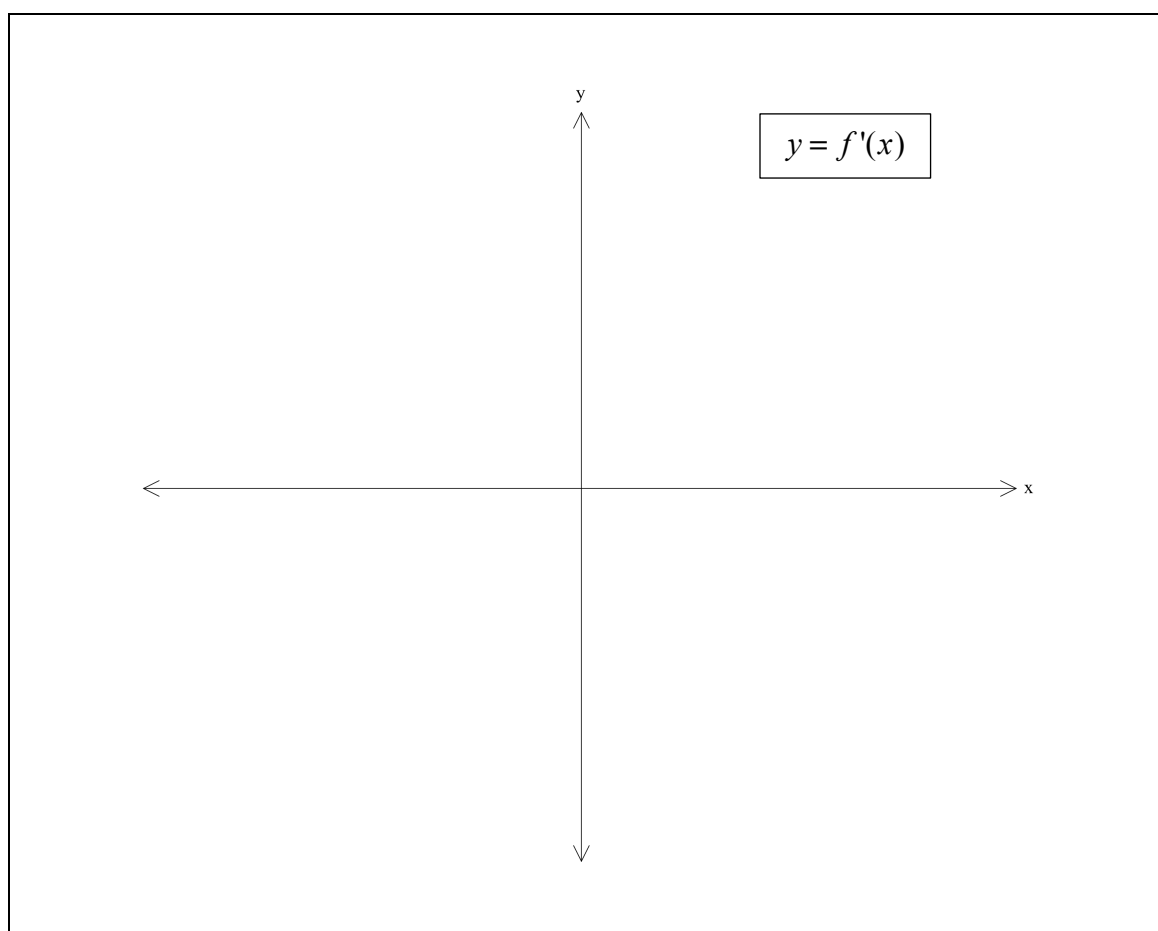
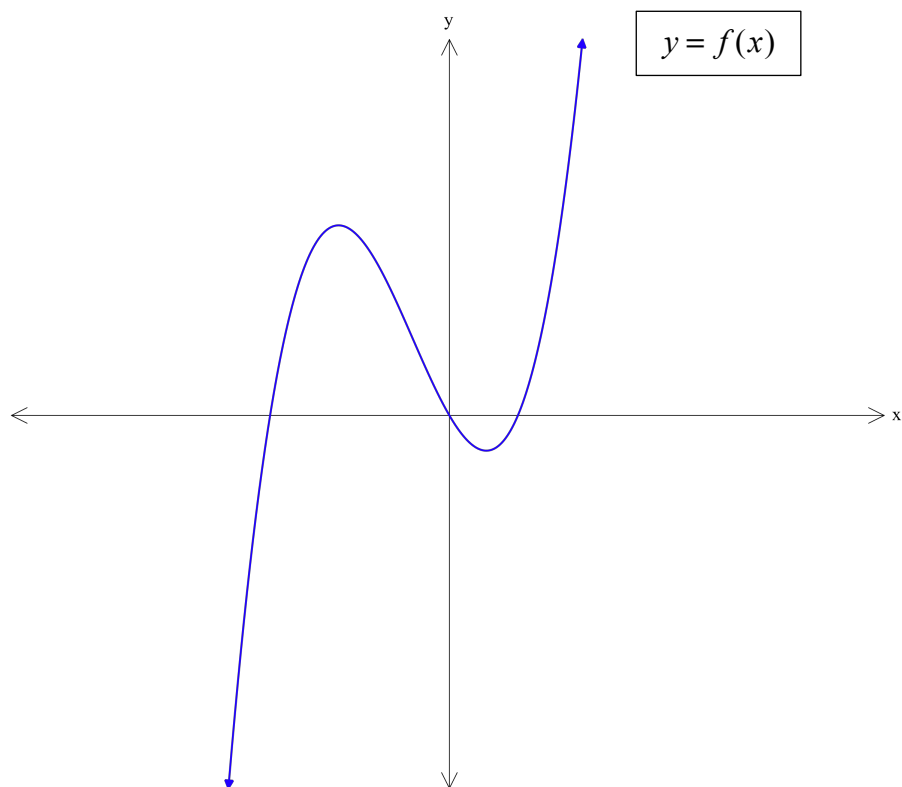
- (ii) What amount of pressure is needed so that the volume of the gas is 80 cm^3 ? **1**

- (iii) Draw a graph of the relationship between V and P , using P as the independent variable and V as the dependent variable, where $P \geq 0$. Label the axes. **2**



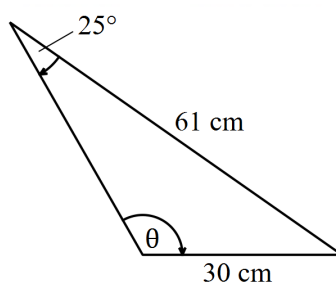
(b) Sketch the gradient function of $y = f(x)$ on the separate set of axes below.

2



(c) In the following triangle, θ is an **obtuse** angle.

3



Find the value of θ to the nearest minute.

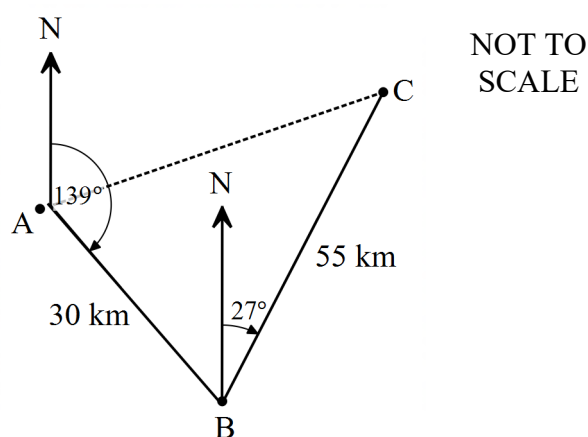
END OF QUESTION 15

Question 16 (10 Marks)

(a) Solve $4^{x-2} = 2^{3x-4}$

2

- (b) A boat sails 30 kilometres from Island A to Island B on a bearing of 139° . It then sails 55 kilometres to Island C on a bearing of 27° from Island B.



- (i) Show that $\angle ABC = 68^\circ$.

1

- (ii) Find the distance the boat must sail to return to Island A from Island C. Answer correct to 2 decimal places.

2

(c) Prove $\frac{1}{\sin^2 \theta} + \frac{1}{\cos^2 \theta} = \sec^2 \theta \operatorname{cosec}^2 \theta$.

2

(d) Two regular dice are rolled.

(i) Find the probability that a five appears on at least one of the two dice.

1

(ii) Given that a five appears on at least one of the dice, find the probability that the sum of the uppermost faces of the two dice is greater than 9.

2

END OF EXAMINATION

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B. -1

C. 0

D. 2

4 What is the domain of the function $f(x) = \frac{x}{\sqrt{x+5}}$?

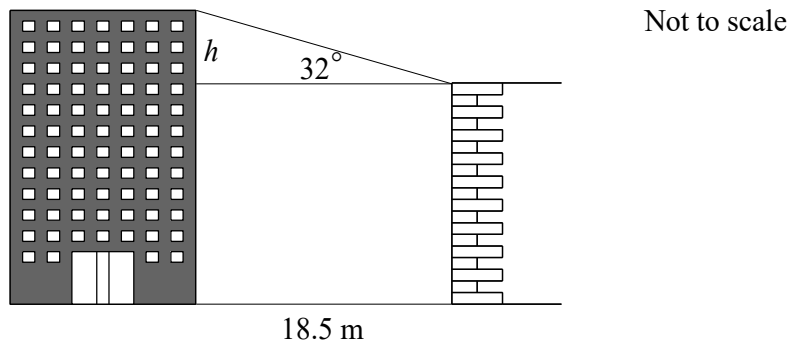
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5 The two buildings below are standing on level ground. The horizontal distance between the buildings is 18.5 metres and the angle of elevation from the top of the smaller building to the top of the larger building is 32° .



What is the difference in height (h) between the buildings?

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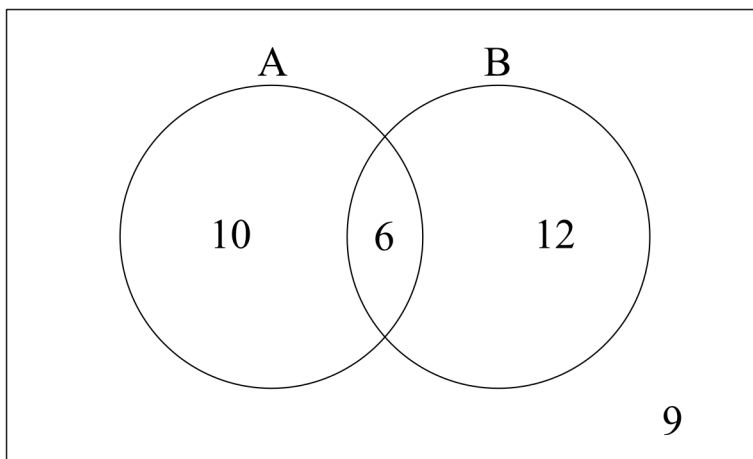
A. $15x^4 - 8x + 2$

B. $15x^5 - 8x^2 + 2$

C. $15x^4 - 8x$

D. $15x^5 - 8x^2$

- 7 Consider the Venn diagram below.



According to the diagram, what is $P(A|B)$?

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Which of the following is a true statement?

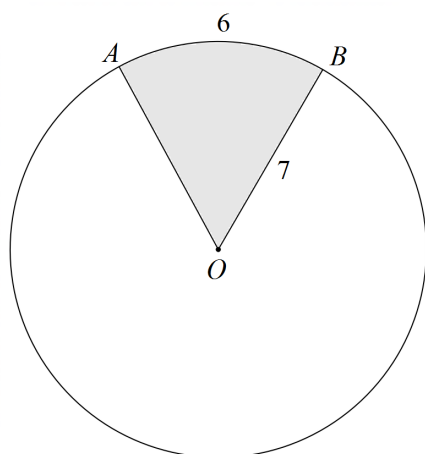
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What is the area of the shaded sector OAB ?

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- B. $f(x) = x^2$
- C. $f(x) = \frac{x}{x^2 + 1}$
- D. $f(x) = |x|$

END OF SECTION I

Section II

60 Marks

Allow about 1 hour and 45 minutes for this section

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Question 11 (10 Marks)

- (a) Solve $|x + 2| = 6$ 2

$$x + 2 = 6$$

$$x = 4$$

or

$$-(x + 2) = 6$$

$$x = -8$$

- (b) The equation of a circle is given by $(x - 2)^2 + (y + 3)^2 = 16$. 1
State the centre and radius of the circle.

$$\text{Centre} = (2, -3)$$

$$\text{Radius} = 4$$

- (c) Using the following sets,

$$A = \{1, 2, 3, 4, 5\} \text{ and } B = \{3, 4, 5, 6, 7\}$$

List the elements of:

- (i) $A \cap B$ 1

$$\{3, 4, 5\}$$

- (ii) $A \cup B$ 1

$$\{1, 2, 3, 4, 5, 6, 7\}$$

(d) For the function $y = x^2 - 2x - 8$:

(i) State the x and y intercepts.

2

y – int *ercepts*

$(0, -8)$

x – int *ercepts*

$(4, 0)$ and $(-2, 0)$

(ii) Show that the coordinates of the vertex are $(1, -9)$.

2

$$x = \frac{-b}{2a}$$

$$x = \frac{-(-2)}{2}$$

$$x = 1$$

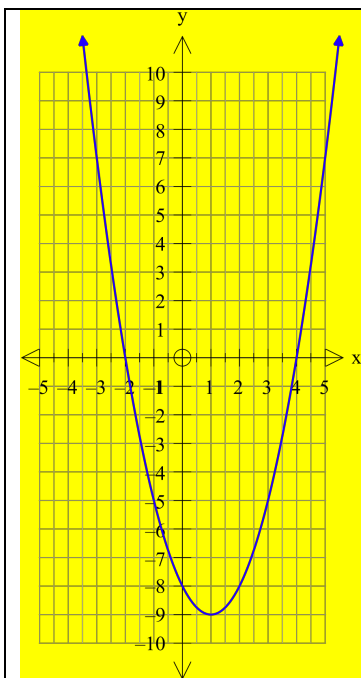
$$y = (1)^2 - 2 \times (1) - 8$$

$$y = -9$$

$$\therefore \text{vertex} = (1, -9)$$

(iii) Sketch the function showing all important features.

1



END OF QUESTION 11

Question 12 (10 Marks)

- (a) Find the exact value of $\cos(135^\circ)$

1

$$\begin{aligned}\cos(135^\circ) &= -\cos(45^\circ) \\ &= -\frac{1}{\sqrt{2}}\end{aligned}$$

- (b) Differentiate the following:

(i) $y = x^{-2}$

1

$$\begin{aligned}y' &= -2x^{-3} \\ \text{or} \\ y' &= \frac{-2}{x^3}\end{aligned}$$

(ii) $y = (2x-3)^8$

2

$$y' = 16(2x-3)^7$$

(iii) $y = 2\sqrt{x}$

2

$$\begin{aligned}y &= 2\left(x^{\frac{1}{2}}\right) \\ y' &= x^{-\frac{1}{2}} \\ \text{or} \\ y' &= \frac{1}{\sqrt{x}}\end{aligned}$$

- (c) Differentiate $y = x(x+1)^3$ using the product rule.

2

$$\begin{aligned}u &= x \\ u' &= 1 \\ v &= (x+1)^3 \\ v' &= 3(x+1)^2 \\ y' &= uv' + vu' \\ y' &= 3x(x+1)^2 + (x+1)^3 \\ \text{or} \\ y' &= (4x+1)(x+1)^2\end{aligned}$$

- (d) Differentiate $y = \frac{x+3}{x-2}$ using the quotient rule.

$$u = x + 3$$

$$u' = 1$$

$$v = x - 2$$

$$v' = 1$$

$$y' = \frac{vu' - uv'}{v^2}$$

$$y' = \frac{(x-2) - (x+3)}{(x-2)^2}$$

$$y' = \frac{-5}{(x-2)^2}$$

END OF QUESTION 12

Question 13 (10 Marks)

- (a) Given that $\cos\theta = -\frac{12}{13}$ and $\sin\theta > 0$, find the value of $\operatorname{cosec}\theta$. 2

In quadrant 2, $\sin\theta > 0$ and $\cos\theta < 0$.

$$\text{opposite} = \sqrt{13^2 - 12^2}$$

$$\text{opposite} = 5$$

$$\sin\theta = \frac{5}{13}$$

$$\therefore \operatorname{cosec}\theta = \frac{13}{5}$$

- (b) For the curve $f(x) = x^2 - 4x$,

- (i) Find the gradient of the tangent at the point $(1, -3)$. 1

$$f'(x) = 2x - 4$$

$$f'(1) = -2$$

- (ii) Find the gradient of the normal at the point $(1, -3)$. 1

$$m_T \times m_N = -1$$

$$\therefore m_N = \frac{1}{2}$$

- (iii) Show that the equation of the normal to the curve at the point $(1, -3)$ is $x - 2y - 7 = 0$. 2

$$y - y_1 = m(x - x_1)$$

$$y + 3 = \frac{1}{2}(x - 1)$$

$$2y + 6 = x - 1$$

$$\therefore x - 2y - 7 = 0$$

(c) The displacement, x metres, from the origin at time, t seconds, of a particle moving in a straight line is given by $x = t^3 - 6t^2 + 5t + 1$.

(i) Find an expression for the velocity as a function of time. 1

$$v = 3t^2 - 12t + 5$$

(ii) Find an expression for the acceleration as a function of time. 1

$$a = 6t - 12$$

(iii) Find the particle's position when acceleration is zero. 2

$$a = 0$$

$$6t - 12 = 0$$

$$t = 2$$

$$x = (2)^3 - 6 \times (2)^2 + 5 \times (2) + 1$$

$$x = -5m$$

END OF QUESTION 13

Question 14 (10 Marks)

(a) Two functions are defined as follows:

2

$$h(x) = x^3 \quad \text{and}$$

$$j(x) = \frac{2x-1}{3}$$

Find the value of $h(j(5))$.

$$j(5) = \frac{2 \times (5) - 1}{3}$$

$$j(5) = 3$$

$$h(j(5)) = 3^3$$

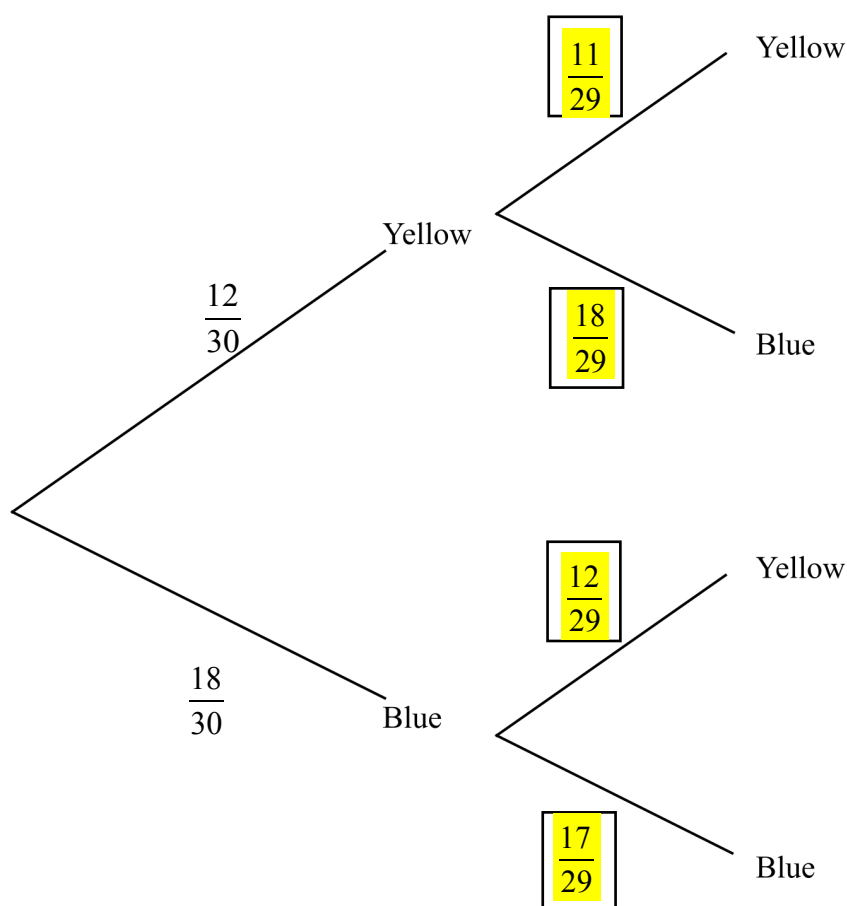
$$h(j(5)) = 27$$

QUESTION 14 CONTINUES

- (b) A barrel contains 18 blue marbles and 12 yellow marbles.

Mike draws two marbles, with no replacement, from the barrel.

A probability tree diagram has been started to model this situation.



- (i) Complete the tree by writing the remaining probabilities in the blank boxes. 1

- (ii) What is the probability that exactly one of the two marbles is blue? 1

$$P(YB) + P(BY) = \left(\frac{12}{30} \times \frac{18}{29} \right) + \left(\frac{18}{30} \times \frac{12}{29} \right)$$

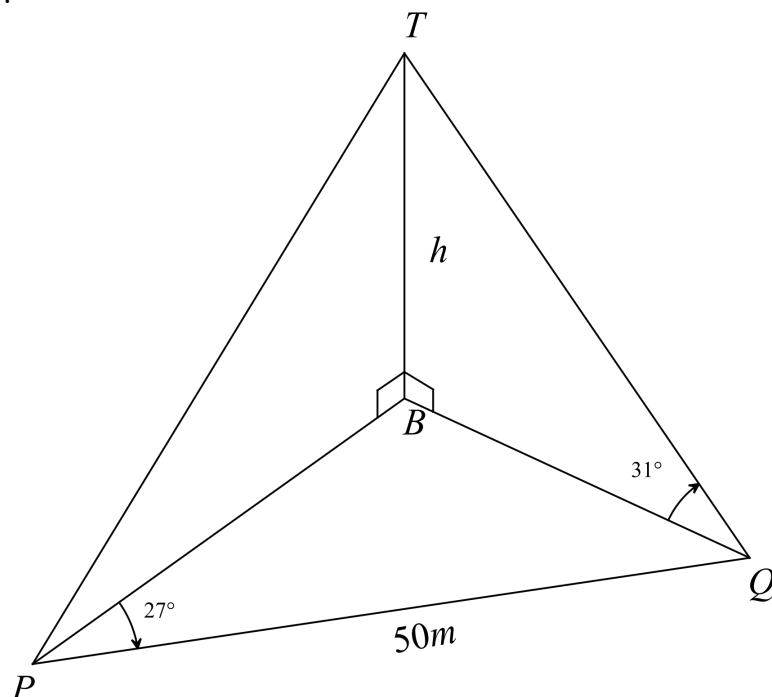
$$= \frac{72}{145}$$

- (iii) What is the probability that at least one of the two marbles is blue? 1

$$1 - P(YY) = 1 - \left(\frac{12}{30} \times \frac{11}{29} \right)$$

$$= \frac{123}{145}$$

- (c) A tree, BT , is due north of an observer at P and due west of an observer at Q . The two observers are 50m apart and the bearing of Q from P is 27° . The angle of elevation of T from Q is 31° .



- (i) Show that $BQ = 50 \sin 27^\circ$

1

$$\sin(27^\circ) = \frac{BQ}{50}$$

$$\therefore BQ = 50 \sin(27^\circ)$$

- (ii) Hence find the height of the tree, correct to the nearest metre.

2

$$\tan(31^\circ) = \frac{h}{50 \sin(27^\circ)}$$

$$h = 50 \sin(27^\circ) \times \tan(31^\circ)$$

$$h = 14\text{m}$$

- (iii) Find the angle of elevation of T from P , correct to the nearest degree.

2

$$\tan \theta = \frac{14}{PB}$$

$$\tan \theta = \frac{14}{50 \cos(27^\circ)}$$

$$\theta = 17^\circ$$

END OF QUESTION 14

Question 15 (10 Marks)

(a) The volume, V , of a gas varies **inversely** with the pressure, P , it is under.

- (i) If the volume of the gas is 120cm^3 when it is under a pressure of 40 kg/cm^3 , find an equation that relates the variables V and P . 2

$$V = \frac{k}{P}$$

sub $V = 120$ and $P = 40$

$$120 = \frac{k}{40}$$

$$\therefore k = 4800$$

$$\therefore V = \frac{4800}{P}$$

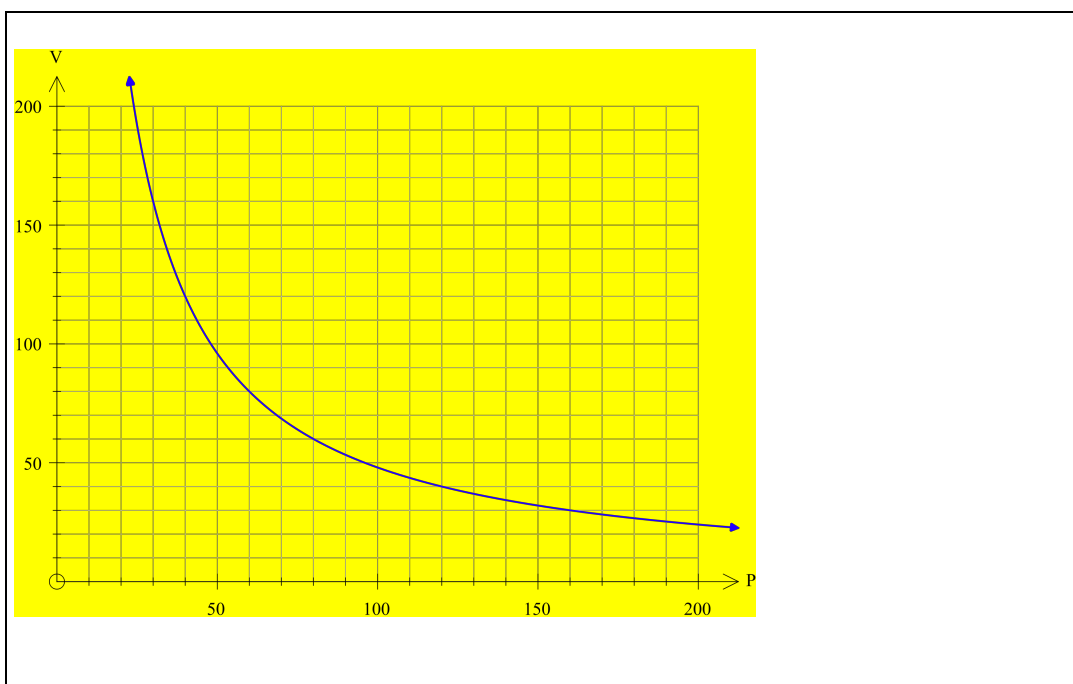
- (ii) What amount of pressure is needed so that the volume of the gas is 80 cm^3 ? 1

sub $V = 80$

$$80 = \frac{4800}{P}$$

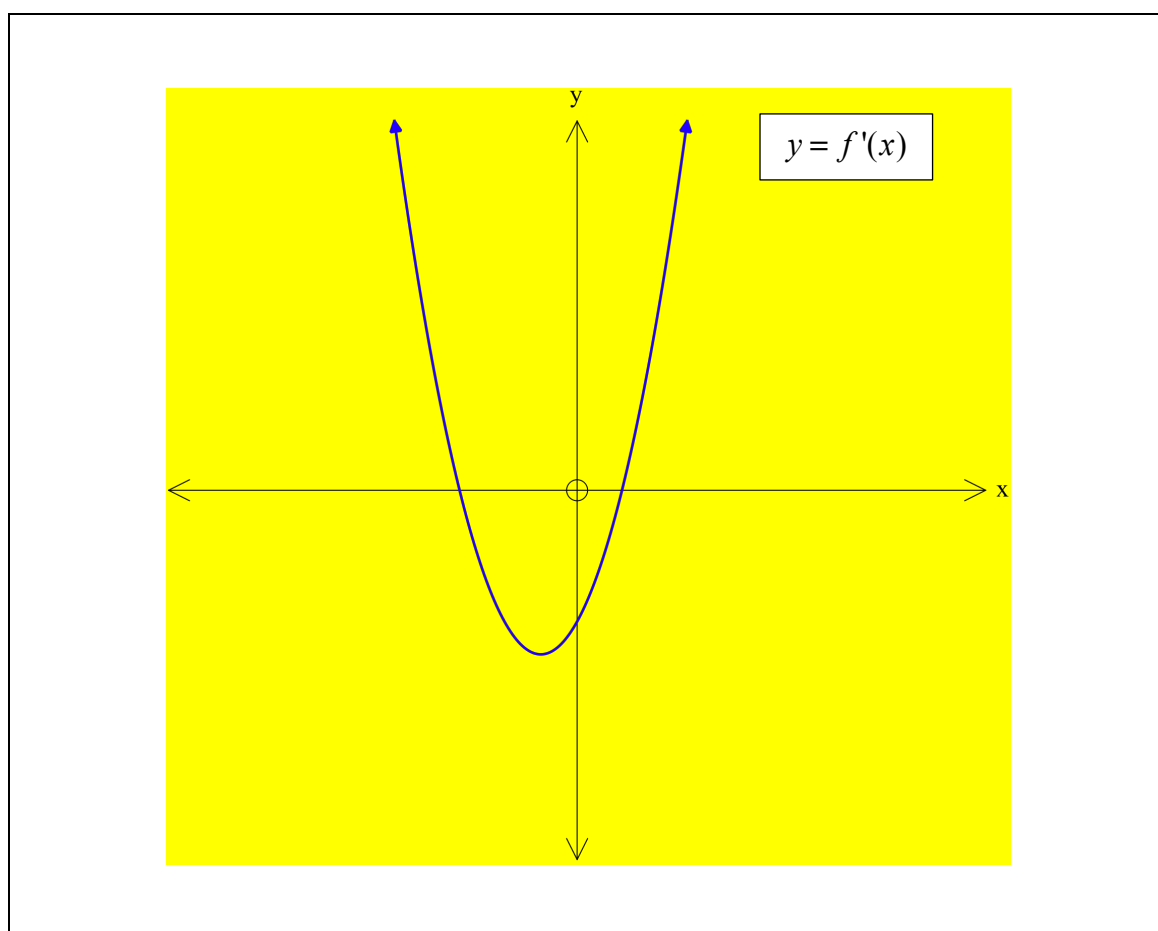
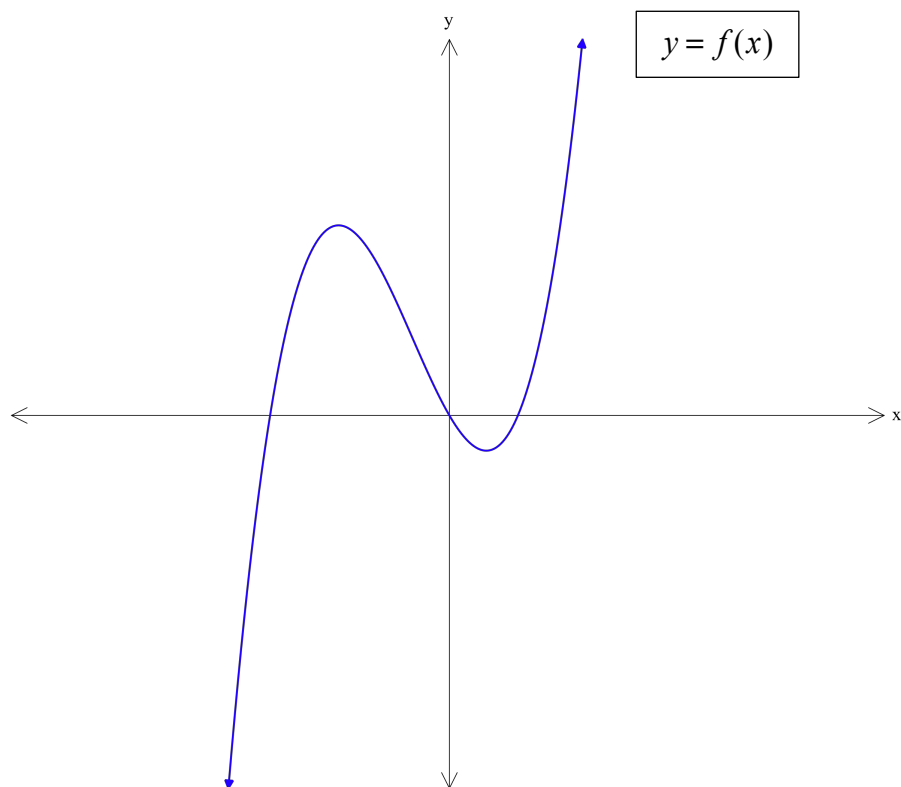
$$\therefore P = 60\text{ kg / cm}^3$$

- (iii) Draw a graph of the relationship between V and P , using P as the independent variable and V as the dependent variable, where $P \geq 0$. Label the axes. 2



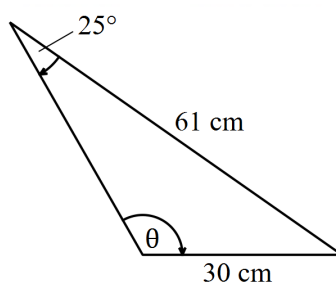
(b) Sketch the gradient function of $y = f(x)$ on the separate set of axes below.

2



(c) In the following triangle, θ is an **obtuse** angle.

3



Find the value of θ to the nearest minute.

$$\frac{\sin \theta}{61} = \frac{\sin 25^\circ}{30}$$
$$\sin \theta = \frac{61 \times \sin 25^\circ}{30}$$
$$\theta = 59^\circ$$

since θ is obtuse,

$$\theta = 180 - 59$$
$$\theta = 120^\circ 46'$$

END OF QUESTION 15

Question 16 (10 Marks)

(a) Solve $4^{x-2} = 2^{3x-4}$

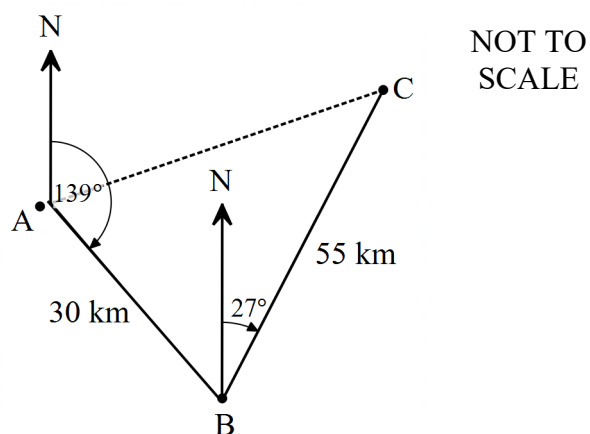
2

$$2^{2(x-2)} = 2^{3x-4}$$

$$2x - 4 = 3x - 4$$

$$\therefore x = 0$$

- (b) A boat sails 30 kilometres from Island A to Island B on a bearing of 139° . It then sails 55 kilometres to Island C on a bearing of 27° from Island B.



- (i) Show that $\angle ABC = 68^\circ$.

1

$$\angle ABC = 41^\circ + 27^\circ$$

(cointerior angles on parallel lines are supplementary)

- (ii) Find the distance the boat must sail to return to Island A from Island C. Answer correct to 2 decimal places.

2

$$c^2 = 30^2 + 55^2 - 2 \times 30 \times 55 \times \cos 68^\circ$$

$$c = 51.85 \text{ km}$$

(c) Prove $\frac{1}{\sin^2 \theta} + \frac{1}{\cos^2 \theta} = \sec^2 \theta \operatorname{cosec}^2 \theta$.

2

$$\begin{aligned} LHS &= \\ &= \frac{\cos^2 \theta + \sin^2 \theta}{\sin^2 \theta \cos^2 \theta} \\ &= \frac{1}{\sin^2 \theta \cos^2 \theta} \\ &= \frac{1}{\sin^2 \theta} \times \frac{1}{\cos^2 \theta} \\ &= \sec^2 \theta \operatorname{cosec}^2 \theta \\ \therefore LHS &= RHS \end{aligned}$$

(d) Two regular dice are rolled.

(i) Find the probability that a five appears on at least one of the two dice.

1

$$\begin{aligned} &P(5\bar{5}) + P(\bar{5}5) + P(55) \\ &= \left(\frac{1}{6} \times \frac{5}{6}\right) + \left(\frac{5}{6} \times \frac{1}{6}\right) + \left(\frac{1}{6} \times \frac{1}{6}\right) \\ &= \frac{11}{36} \end{aligned}$$

(ii) Given that a five appears on at least one of the dice, find the probability that the sum of the uppermost faces of the two dice is greater than 9.

2

11 options of a 5 appearing at least on one dice.

$$P(5,5) + P(5,6) + P(6,5)$$

There are 3 options of a sum greater than 9.

$$\therefore P(E) = \frac{3}{11}$$

Or

+	1	2	3	4	5	6
1						
2						
3						
4						
5					10	11
6					11	

END OF EXAMINATION