International Grammar School

2021 **ASSESSMENT TASK 3**

Mathematics Advanced

General Instructions

- Reading time − 5 minutes
- Working time – 1 hour 30 minutes
- Write on A4 lined paper using black pen
- The spaces provide guidance for the expected length of response.
- Calculators approved by NESA may be used
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total Marks:

50

Section I – 10 Marks

- Attempt questions 1 – 10
- Allow about 15 minutes for this section.

Section II – 40 Marks

- Attempt all questions
- Allow about 1 hour and 15 minutes for this section.

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Write your answers on A4 lined paper.

1 What is the solution to the equation $4\sqrt{x}-9=0$?

A. $x=\frac{18}{8}$

B. $x=\frac{81}{16}$

C. $x=\frac{3}{2}$

D. $x=6$

2 Which of the following is true for the function $f(x)=\frac{3}{x^2}-2$?

A. Even function

B. Odd function

C. Zero function

D. Neither an even, odd or zero function

3 Given that $\log_a 4=0.420$ and $\log_a 5=0.488$. What is the value of $\log_a 100$?

A. 0.410

B. 0.664

C. 1.396

D. 2.908

4 Two events, A and B, are independent, and their respective probabilities of occurring, $P(A)$ and $P(B)$, are both non-zero.

Which of the following does NOT give the probability of events A and B both occurring?

A. $P(A) \times P(B \mid A)$

B. $P(A \cap B)$

C. $P(A) \times P(B)$

D. $P(A) + P(B)$

5 What is the exact value of $\sin 135° + \operatorname{cosec} 60°$?

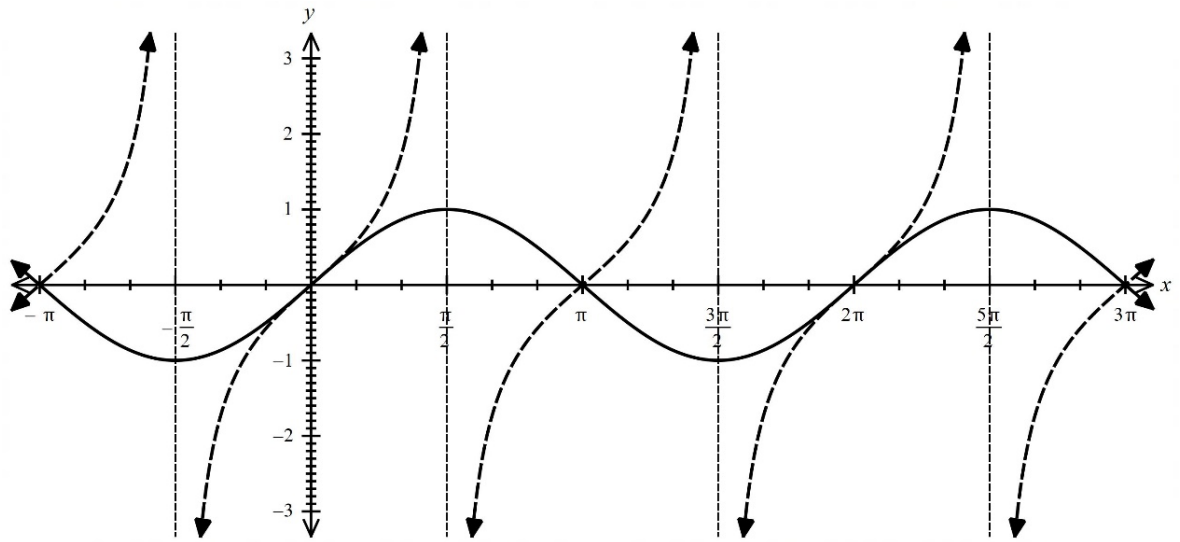
A. $\dfrac{2\sqrt{2} - \sqrt{3}}{\sqrt{6}}$

B. $\dfrac{2\sqrt{2} - 1}{\sqrt{2}}$

C. $\dfrac{2\sqrt{2} + \sqrt{3}}{\sqrt{6}}$

D. $\dfrac{2\sqrt{2} + 1}{\sqrt{2}}$

6 The graphs of two trigonometric functions are shown on the number plane below.



Which two functions are shown?

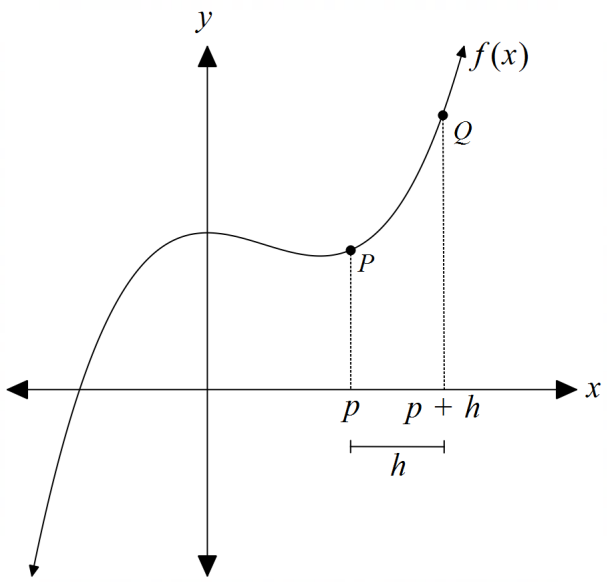
A. $y = \cos x$ and $y = \sin x$

B. $y = \cos x$ and $y = \tan x$

C. $y = \sin x$ and $y = \tan x$

D. $y = \sin x$ and $y = \sec x$

7 The graph of the function $f(x)$ is shown below, with points P and Q on the graph. The coordinates of P are $(p, f(p))$, and point Q's x-coordinate is h to the right of p, hence Q's coordinates are $(p+h, f(p+h))$.



Which of the following would give the gradient of the tangent at point P?

A. $\lim_{h\to 0} \dfrac{f(p) - f(p+h)}{h}$

B. $\lim_{h\to 0} \dfrac{f(p-h) + f(p)}{h}$

C. $\lim_{h\to 0} \dfrac{f(p+h) - f(p)}{h}$

D. $\lim_{h\to 0} \dfrac{f(p-h) - f(p)}{h}$

8 What is the value of $|3 - 5|$?

A. 8

B. 2

C. -8

D. -2

9 What is the exact value of $\cot\left(\frac{5\pi}{3}\right)$?

A. $\sqrt{3}$

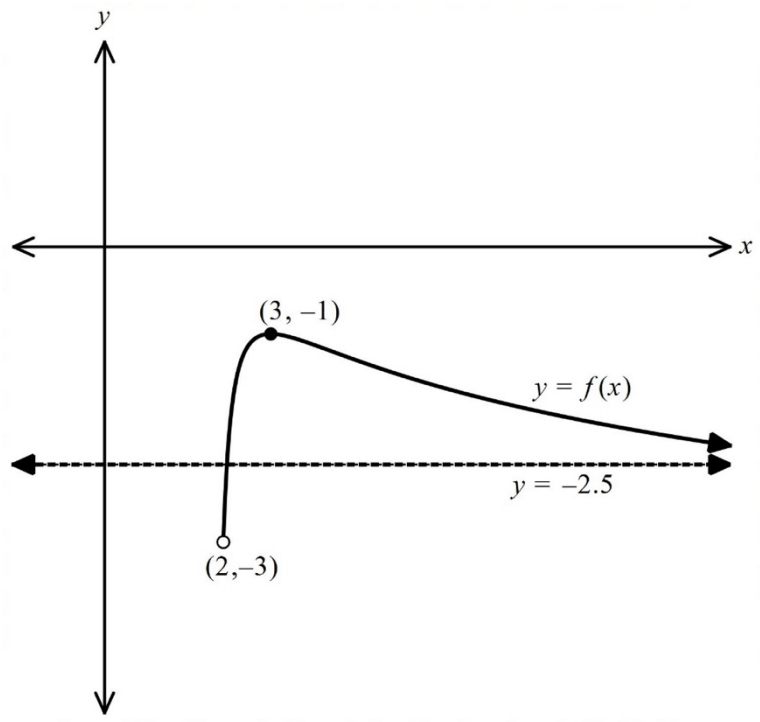
B. $-\sqrt{3}$

C. $\frac{1}{\sqrt{3}}$

D. $-\frac{1}{\sqrt{3}}$

10 The graph of a function $y = f(x)$ is shown, below.

It has an asymptote at $y = -2.5$.



Which statement correctly describes its domain and range.

A. Domain : [2, ∞) ; Range : [–3, –1]

B. Domain : (–3, –1] ; Range : (2, ∞)

C. Domain : [2, ∞) ; Range : [–3, –1)

D. Domain : (2, ∞) ; Range : [–3, –1]

END OF SECTION I

2021 **YEAR 11 ASSESSMENT TASK 3**

Mathematics Advanced
Section II Answer Booklet

Section II

40 Marks
Attempt Questions 11 – 23
Allow about 1 hour and 15 minutes for this section

Instructions

- Answer the questions on A4 lined paper.
- The spaces provide guidance for the expected length of responses.
- Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (1 mark) **1**

Simplify $\dfrac{xy^5}{\left(x^{-2}y^3\right)^4}$ and express your answer with positive indices.

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Question 12 (2 marks) **2**

Given $\sin\theta = \frac{7}{12}$ for an acute angle θ, find the exact value of $\cos\theta$.

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Question 13 (2 marks) **2**

Solve the equation

$$5\log_{10} x = 3 + 2\log_{10} x$$

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Question 14 (4 marks)

Find the following derivatives and simplify your answer.

(a) $\frac{d}{dx}\left[e^{4x+3}(3x-1)\right]$ **2**

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(b) $\frac{d}{dx}\left(\frac{x}{3x^3-2}\right)$ **2**

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Question 15 (2 marks) **2**

The lengths of the sides in ΔABC are AB = 15 cm, AC = 10 cm and BC = 8 cm. Find the size of the largest angle in ΔABC. Answer to the nearest minute.

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Question 16 (3 marks) **3**

Prove the identity

$$\cot x + \tan x = \sec x \operatorname{cosec} x$$

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Question 17 (2 marks) **2**

Draw a sketch of the circle with equation $x^2 - 2x + y^2 + 6y + 6 = 0$

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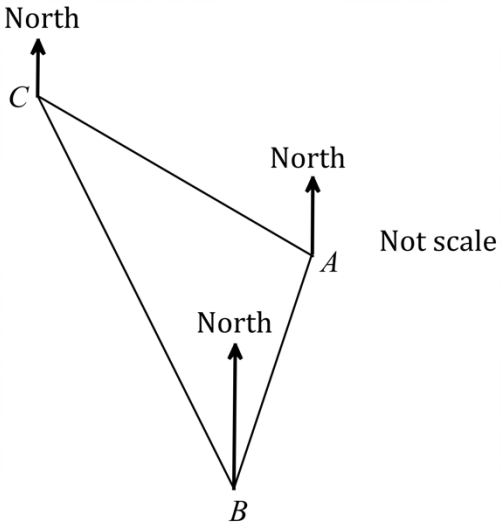
Question 18 (4 marks)

Lucy walks 4.2km from her house at A on a true bearing of 185°T to the park at B.
She then walks on a true bearing of 349°T to the shop at C.
Lucy knows she must walk 5.2 km from shop *C* to her house at A.



(a) Copy the diagram onto your writing paper and explain why $\angle ABC = 16°$. **1**

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(b) Hence or otherwise find the bearing from C to A. **3**

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Question 19 (3 marks) **3**

Find the equation of the normal to the curve of $f(x) = -2x^3 + 5x^2 - 3x$ at the point where $x = 2$.

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Question 20 (2 marks) **2**

On a given day, the probability that it is cloudy is 0.25, the probability that it is windy is 0.7, and the probability that it is both cloudy and windy is 0.15.

What is the probability that it is cloudy or windy (or both)?

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Question 21 (7 marks)

The number of fish, N, in a hatchery at time t months, is given by the formula: $N = N_0 e^{0.322t}$, where N_0 is a constant.

Initially there were 1000 fish in the hatchery.

(a) Find the number of fish in the hatchery at the end of 8 months? Answer correct to the nearest hundred. **3**

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(b) At the end of which month will the fish population first exceed 50 000? **2**

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(c) Find the rate of increase in the number of fish at the end of six months. Answer correct to the nearest hundred fish per month. **2**

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Question 22 (3 marks)

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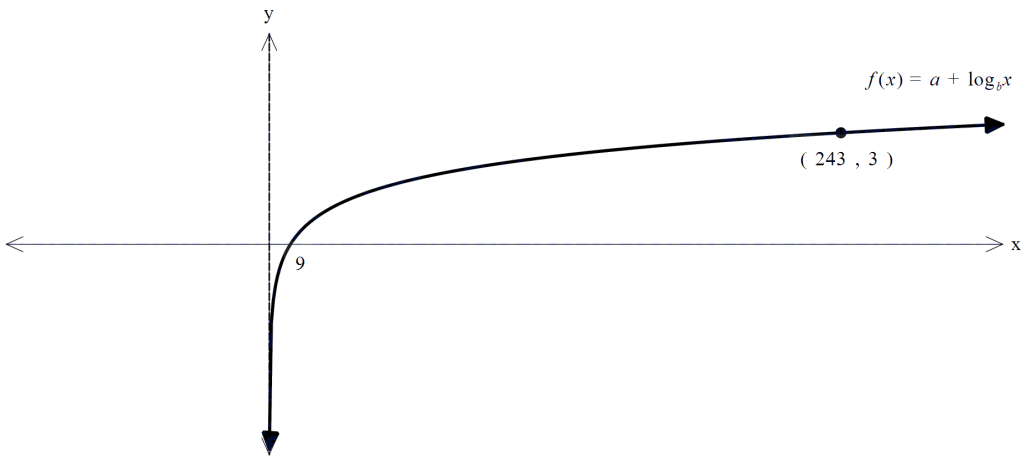
For the function $f(x) = a + \log_b x$, the x-intercept is 9 and $f(x)$ passes through the point (243, 3) as shown below.



Find the values of a and b.

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Question 23 (5 marks)

The displacement from the origin (x metres) of a particle travelling in a straight line at a time (t seconds) is given by:

$$x = 3t^4 - 16t^3 + 18t^2 \text{, where } t \geq 0.$$

(a) Find the velocity of the particle after 2 seconds. **1**

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(b) Find the times when the particle is stationary. **1**

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(c) Find the acceleration of the particle after 2 seconds. **1**

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(d) Determine the time(s) when zero acceleration is acting on the particle. **2**

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END OF EXAMINATION

International Grammar School

2021 **ASSESSMENT TASK 3**

Mathematics Advanced Solutions

General Instructions

- Reading time − 10 minutes
- Working time – 1 hour 30 mins
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided with this question paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total Marks:

Section I – 10 Marks

- Attempt questions 1 – 10
- Allow about 15 minutes for this section.

Section II – 40 Marks

- Attempt all questions
- Allow about 1 hour and 15 minutes for this section.

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

1 What is the solution to the equation $4\sqrt{x} - 9 = 0$?

A. $x = \frac{18}{8}$

B. $x = \frac{81}{16}$

C. $x = \frac{3}{2}$

D. $x = 6$

2 Which of the following is true for the function $f(x) = \frac{3}{x^2} - 2$?

A. Even function

B. Odd function

C. Zero function

D. Neither an even, odd or zero function

3 Given that $\log_a 4 = 0.420$ and $\log_a 5 = 0.488$. What is the value of $\log_a 100$?

A. 0.410

B. 0.664

C. 1.396

D. 2.908

4 Two events, A and B, are independent, and their respective probabilities of occurring, P(A) and P(B), are both non-zero.

Which of the following does NOT give the probability of events A and B both occurring?

A. $P(A) \times P(B|A)$

B. $P(A \cap B)$

C. $P(A) \times P(B)$

D. $P(A) + P(B)$

5 What is the exact value of $\sin 135° + \operatorname{cosec} 60°$?

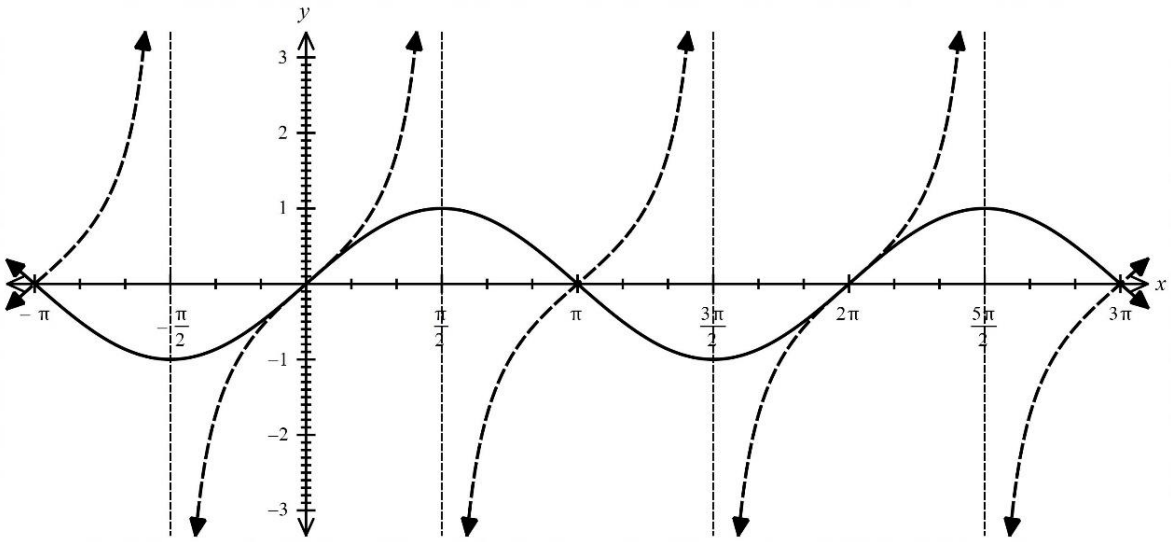
A. $\frac{2\sqrt{2}-\sqrt{3}}{\sqrt{6}}$

B. $\frac{2\sqrt{2}-1}{\sqrt{2}}$

C. $\frac{2\sqrt{2}+\sqrt{3}}{\sqrt{6}}$

D. $\frac{2\sqrt{2}+1}{\sqrt{2}}$

6 The graphs of two trigonometric functions are shown on the number plane below.



Which two functions are shown?

A. $y = \cos x$ and $y = \sin x$

B. $y = \cos x$ and $y = \tan x$

C. $y = \sin x$ and $y = \tan x$

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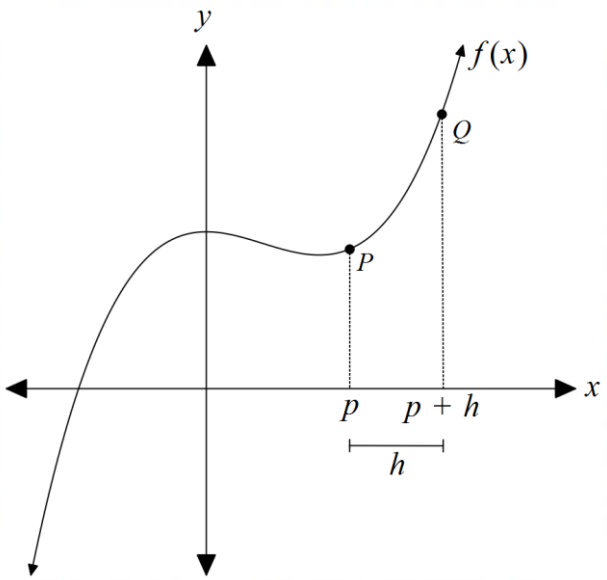
7 The graph of the function $f(x)$ is shown below, with points P and Q on the graph. The coordinates of P are $(p, f(p))$, and point Q's x-coordinate is h to the right of p, hence Q's coordinates are $(p+h, f(p+h))$.



Which of the following would give the gradient of the tangent at point P?

A. $\lim_{h\to 0} \dfrac{f(p)-f(p+h)}{h}$

B. $\lim_{h\to 0} \dfrac{f(p-h)+f(p)}{h}$

C. $\lim_{h\to 0} \dfrac{f(p+h)-f(p)}{h}$

D. $\lim_{h\to 0} \dfrac{f(p-h)-f(p)}{h}$

8 What is the value of $|3-5|$?

A. 8

B. 2

C. -8

D. -2

9 What is the exact value of $\cot\left(\frac{5}{3}\pi\right)$

A. $\sqrt{3}$

B. $-\sqrt{3}$

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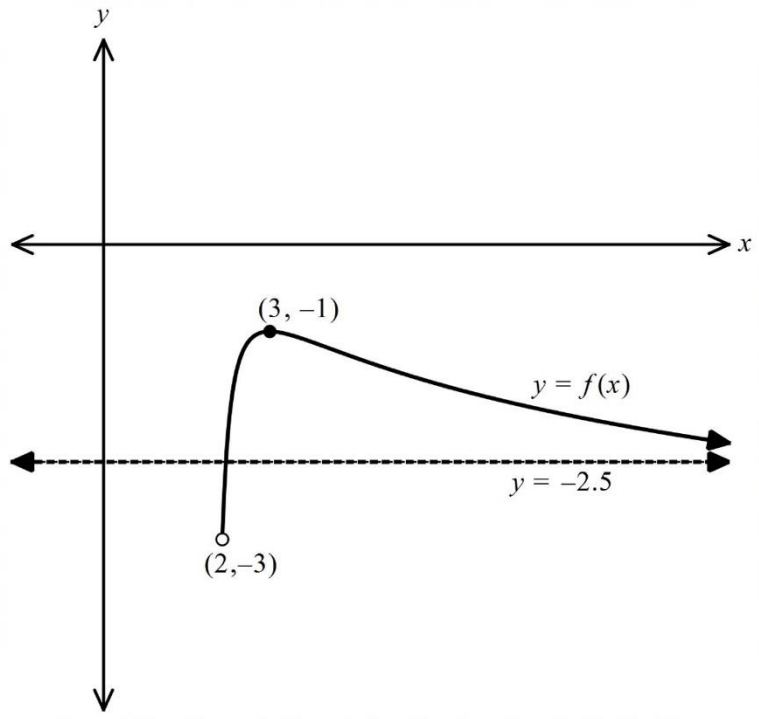
It has an asymptote at $y = -2.5$.



Which statement correctly describes its domain and range.

A. Domain : [2, ∞) ; Range : [–3, –1]

B. Domain : (–3, –1] ; Range : (2, ∞)

C. Domain : [2, ∞] ; Range : [–3, –1)

D. Domain : (2, ∞) ; Range : (–3, –1]

END OF SECTION I

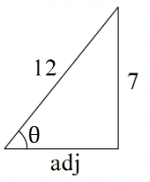
2021 **ASSESSMENT TASK 3**

Mathematics Advanced
Section II Answer Booklet

40 Marks
Attempt all questions
Allow about 1 hour and 15 minutes for this section

Instructions

- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Question 11		**1**
	Simplify $\frac{xy^5}{(x^{-2}y^3)^4}$ and express your answer with positive indices.	
	$\frac{xy^5}{x^{-8}y^{12}}$ $= \frac{x^9}{y^7}$	1Mark: Correct answer
Question 12		**2**
	Given $\sin\theta = \frac{7}{12}$ for an acute angle θ, find the exact value of $\cos\theta$.	
	$\sin\theta = \frac{7}{12} = \frac{\text{opp}}{\text{hyp}}$  Using Pythagoras' to find the adjacent side. $a^2 + 7^2 = 12^2$ $a = \sqrt{12^2 - 7^2} = \sqrt{95}$ $\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{95}}{12}$	2Marks: Correct answer 1 Mark: Uses Pythagoras
Question 13		**2**
	Solve the equation $5\log_{10}x = 3 + 2\log_{10}x$	
	$5\log_{10}x = 3 + 2\log_{10}x$ $3\log_{10}x = 3$ $\log_{10}x = 1$ $\therefore\ x = 10$	2Marks: Correct answer. 1 Mark: Uses the logarithmic laws correctly

Question 14

Find the following derivatives and simplify your answer.

(a) $\frac{d}{dx}[e^{4x+3}(3x-1)]$ — **2**

$$\frac{d}{dx}[e^{4x+3}(3x-1)] = 4e^{4x+3}(3x-1) + e^{4x+3}(3) = 4e^{4x+3}(3x-1) + 3e^{4x+3}$$

2Marks: Correct answer

1 Mark: Attempts first step

(b) $\frac{d}{dx}\frac{x}{(3x^3-2x)^4}$ — **2**

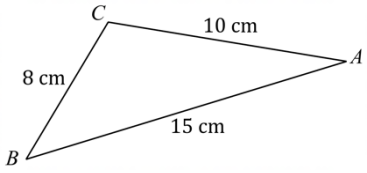
$$\frac{d}{dx}\frac{x}{(3x^3-2x)^4} = \frac{1(3x^3-2x)^4 - 4(3x^3-2x)^3(9x^2-2)(x)}{(3x^3-2x)^8} = \frac{(3x^3-2x)-4x(9x^2-2)}{(3x^3-2x)^5} = \frac{3x^3-2x-36x^3+8x}{(3x^3-2x)^5} = \frac{-33x^3+6x}{(3x^3-2x)^5}$$

2Marks: Correct answer

1 Mark: Attempts first step

Question 15 — **2**

The lengths of the sides in ΔABC are AB = 15 cm, AC = 10 cm and BC = 8 cm. Find the size of the largest angle in ΔABC. Answer to the nearest minute.



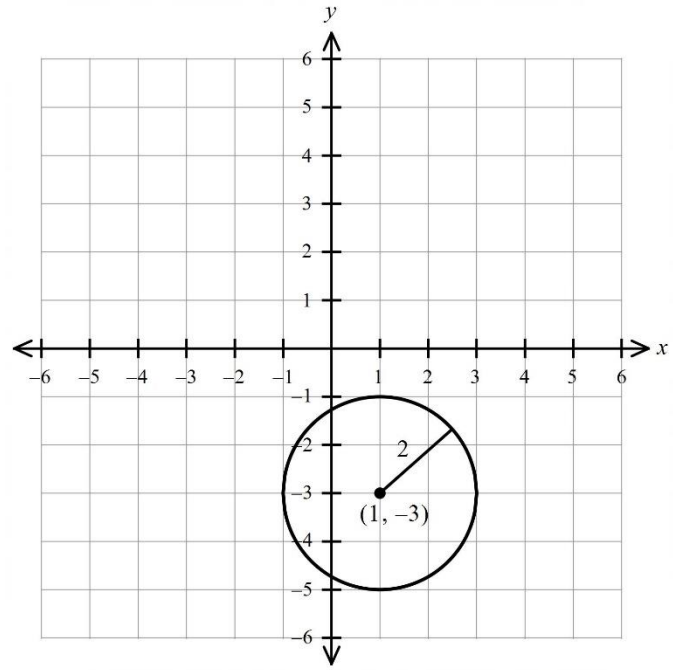
The largest angle is opposite the largest side ($\angle BCA$)

$$\cos\angle BCA = \frac{8^2 + 10^2 - 15^2}{2 \times 8 \times 10}$$
$$\angle BCA = 112.4111\ldots$$
$$\approx 112°25'$$

$\therefore$ Largest angle is 112°25'.

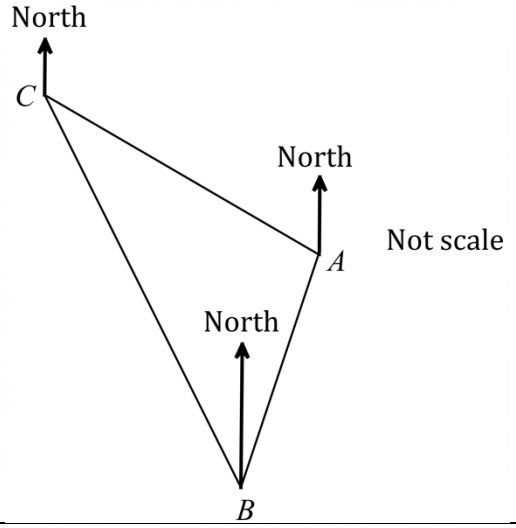
2marks: Correct answer.

1 mark: Uses the cosine rule with one correct value.

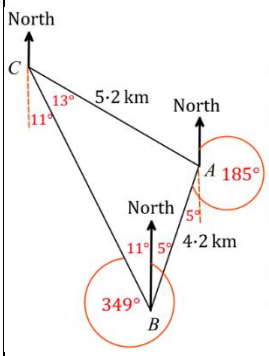
Question 16		**3**
	Prove the identity $\cot(x) + \tan(x) = \sec(x)\,\text{cosec}(x)$.	
	$\text{LHS} = \cot(x) + \tan(x) = \frac{\cos(x)}{\sin(x)} + \frac{\sin(x)}{\cos(x)}$ (identity for cot and tan) $= \frac{\cos(x)}{\sin(x)} \times \frac{\cos(x)}{\cos(x)} + \frac{\sin(x)}{\cos(x)} \times \frac{\sin(x)}{\sin(x)}$ (get fractions with the same denominator) $= \frac{\cos^2(x)}{\sin(x)\cos(x)} + \frac{\sin^2(x)}{\sin(x)\cos(x)}$ (simplifying) $= \frac{\cos^2(x)+\sin^2(x)}{\sin(x)\cos(x)}$ (adding the fractions) $= \frac{1}{\sin(x)\cos(x)}$ (Pythagorean identity) $= \frac{1}{\sin(x)} \times \frac{1}{\cos(x)}$ (separating into the product of two fractions) $= \text{cosec}(x)\,\sec(x) = \text{RHS}$ (identity for cosec and sec)	3Marks: Correct answer 2Marks: Makes significant progress towards the solution 1 Mark: Attempts first step
Question 17		**2**
	Draw a sketch of the circle with equation $x^2 - 2x + y^2 + 6y + 6 = 0$.	
	$x^2 - 2x + y^2 + 6y + 6 = 0$ $x^2 - 2x + \quad y^2 + 6y \quad = -6$ $x^2 - 2x + 1 + y^2 + 6y + 9 = -6 + 10 = 4$ $(x-1)^2 + (y+3)^2 = 4$ Circle centre $(1, -3)$ radius $= 2$ 	2 marks for correct graph 1 mark if centre or radius is correct or equivalent merit

Question 18

Lucy walks 4.2km from her house at *A* on a true bearing of 185°T to the park at *B*.
She then walks on a true bearing of 349°T to the shop at *C*.
Lucy knows she must walk 5.2 km from shop *C* to her house at *A*.



(a) Show that $\angle ABC = 16°$. **1**



$$\angle ACB = (185° - 180°) + (360° - 349°)$$
$$= 5° + 11°$$
$$= 16°$$

1 Mark: Correct answer.

(b) Hence or otherwise find the bearing from *C* to *A*. **3**

Use the sine rule

$$\frac{\sin\angle BCA}{4.2} = \frac{\sin 16°}{5.2}$$

$$\sin\angle BCA = \frac{4.2 \times \sin 16°}{5.2}$$

$$\angle BCA = 12.8635\ldots$$

$$\approx 13°$$

$$\text{Bearing} = 180° - 13° - 11°$$

$$= 156°$$

$\therefore$ Bearing from *C* to *A* is 156°.

3Marks: Correct answer.

2Marks: Finds $\angle BCA$.

1 Mark: Uses the sine rule

Question 19		**3**
	Find the equation of the normal to the curve of $f(x) = -2x^3 + 5x^2 - 3x$ at the point where $x = 2$.	
	Finding the point of tangency: $f(2) = -2(2)^3 + 5(2)^2 - 3(2) = -2$ Finding the gradient of the tangent: $f'(x) = -6x^2 + 10x - 3$ $f'(2) = -6(2)^2 + 10(2) - 3 = -7$ Finding the equation of the normal: Gradient is the negative reciprocal of -7, so $\frac{1}{7}$ $y - (-2) = \frac{1}{7}(x - 2)$ $y + 2 = \frac{1}{7}x - \frac{2}{7}$ $y = \frac{1}{7}x - \frac{16}{7}$ $x - 7y - 16 = 0$	3Marks: Correct answer. 2Marks: Correctly finds one value
Question 20		**2**
	On a given day, the probability that it is cloudy is 0.25, the probability that it is windy is 0.7, and the probability that it is both cloudy and windy is 0.15. What is the probability that it is cloudy or windy (or both)?	
	Probability of A or B (see Reference Sheet): $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $0.25 + 0.7 - 0.15 = 0.8$	2Marks: Correct answer. 1 Mark: Finds one of the values.

Question 21

The number of fish, N, in a hatchery at time t months, is given by the formula:

$N = N_0 e^{0.322t}$ where N_0 is a constant.

Initially there were 1000 fish in the hatchery.

(a) Find the number of fish in the hatchery at the end of 8 months? Answer correct to the nearest hundred. **3**

$$N = N_0 e^{0.322t}$$

At $t = 0$, $N = 1000$ $\therefore N_0 = 1000$

To find N when $t = 8$

$$N = N_0 e^{0.322t}$$

$$= 1000e^{0.322\times 8}$$

$$= 13\ 144.4550\ ...$$

$$\approx 13\ 100$$

$\therefore$ There are 13 100 fish in the hatchery after 8 months.

3 Marks: Correct answer.

2 Marks: Makes significant progress towards the solution

1 Mark: Finds $N_0 = 1000$

(b) At the end of which month will the fish population exceed 50 000? **2**

$N = 50\ 000$

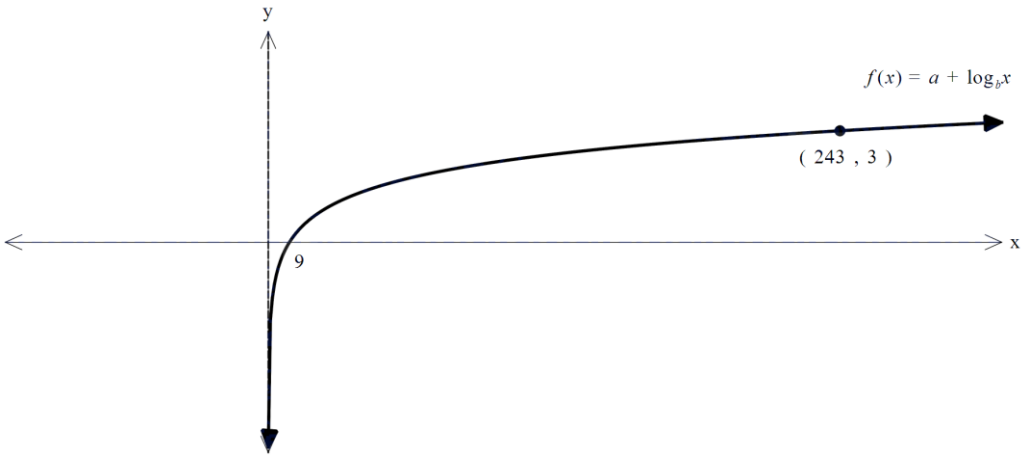
$$50\ 000 < 1000e^{0.322t}$$
$$e^{0.322t} > 50$$
$$0.322t > \ln 50$$
$$t > \frac{\ln 50}{0.322}$$
$$> 12.1491\ ...$$

$\therefore$ At the end of the 13th month the fish population exceeds 50 000

2 Marks: Correct answer.

1 Mark: Shows some understanding.

(c) Find the rate of increase in the number of fish at the end of six months. Answer correct to the nearest hundred fish per month. **2**

	Rate of increase is $\frac{dN}{dt}$ $\frac{dN}{dt} = 0.322 \times 1000e^{0.322t}$ $= 0.322 \times 1000e^{0.322\times6}$ $= 2222.8635 \ldots$ ≈ 2200 fish per month	2 Marks: Correct answer. 1 Mark: Recognises $\frac{dN}{dt}$ as the rate of increase.
Question 22		
	For the function $f(x) = a + \log_b x$, the x-intercept is 9 and $f(x)$ passes through the point (243, 3) as shown below.  Find the values of a and b.	
	$0 = a + \log_b 9 \ldots.(1)$ $3 = a + \log_b 243 \ldots.(2)$ In (2), $3 = a + \log_b(27 \times 9)$ $3 = a + \log_b 9 + \log_b 27$ $3 = 0 + \log_b 27$ (from (1)) $b = 3$ $0 = a + \log_3 9$ $a = -2$	3Marks: Correct answer. 2Marks: Correctly finds one value 1Mark: attempts simultaneous equations

Question 23			
	The displacement from the origin (x metres) of a particle travelling in a straight line at a time (t seconds) is given by: $x = 3t^4 - 16t^3 + 18t^2$, where $t \geq 0$.		
	(a)	Find the velocity of the particle at time t = 2 seconds.	**1**
	$x = 3t^4 - 16t^3 + 18t^2$ Velocity $\dot{x} = 12t^3 - 48t^2 + 36t$ When $t = 2$ $\dot{x} = 12(2)^3 - 48(2)^2 + 36(2)$ $= -24$ m/s		1Mark: Correct answer
	(b)	Find the time(s) when the particle is stationary.	**1**
	When $\dot{x} = 0$ $12t^3 - 48t^2 + 36t = 0$ $12(t^3 - 4t + 3) = 0$ $12t(t-1)(t-3) = 0$ Stationary when $t = 0$, $t = 1$ and $t = 3$.		1Mark: Correct answer
	(c)	Find the acceleration of the particle at time t = 2 seconds.	**1**
	Acceleration $\ddot{x} = 36t^2 - 96t + 36$ When $t = 2$ $\ddot{x} = 36(2)^2 - 96(2) + 36 = -12\text{m/s}^2$		1Mark: Correct answer

	(d)	Determine when there is zero acceleration acting on the particle.	**2**
	$\ddot{x} = 12(3t^2 - 8t + 3)$ If $\ddot{x} = 0$ then $t = \frac{- -8 \pm \sqrt{(-8)^2 - 4(3)(3)}}{2(3)}$ $t = \frac{8 \pm \sqrt{28}}{6}$ $t = 0.45\text{s}$ or 2.22s		1Mark: Correct answer
	END OF EXAMINATION		