Students' Name: _____Teacher:_____

Saint Ignatius' College, Riverview
Mathematics Assessment Task
2018

Year 11
Mathematics
Task 3
PRELIMINARY COURSE EXAMINATION
Date: MONDAY 24th SEPTEMBER 2018

General Instructions:

- **Reading time: 5 Minutes**
- **Time Allowed: 120 Minutes**
- Write using blue or black pen only
- Board approved calculators may be used
- Attempt all questions in the space provided on the paper
- Write **your name** and **your teacher's name** in the positions indicated
- Marks may not be awarded for missing or carelessly arranged working

Teachers:

Mr R Maxwell	REM
Mr P Collins	PPC
Dr M Furtado	MXF
Ms F Yates	FEY
Ms H Haxby	HBH
Ms K Mullan	KXM

Topics Examined:

- Question 1 - Basic Arithmetic, Algebra and
- Surds
- Question 2 - Equations
- Question 3 - Geometry
- Question 4 - Functions and Graphs
- Question 5 - Trigonometry
- Question 6 - Linear Functions
- Question 7 - Introduction to Calculus

_____

Total **84 Marks**

Question 1 **Basic Arithmetic, Algebra and Surds** **Total 12 Marks**

Please use a separate writing booklet.

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a. Find the value of $\dfrac{2\pi^2 + 3.14}{\sqrt{\pi}}$ correct to 3 significant figures. **1**

b. Simplify $\sqrt{18} - \sqrt{8}$, in simplest surd form. **1**

c. If $V = \dfrac{4}{3}\pi r^3$ find r, when $V=20.6$, to 2 significant figures. **2**

d. Simplify $\dfrac{a-4}{a^3-64}$. **2**

e. Solve $2 - 3x \leq 8$, and graph the solution on a number line. **2**

f. Factorise $25 - (x-1)^2$. **2**

g. If $x = \sqrt{5} + 2$, find b when $x + \dfrac{1}{x} = b\sqrt{5}$. **2**

Question 2 **Equations** **Total 12 Marks**

Please use a separate writing booklet.

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a. Solve simultaneously $3x + 2y = 7$ and $3x - 2y = 11$. **2**

b. Solve the equation $\frac{x+3}{2} - \frac{2x-1}{5} = 1$. **2**

c. Solve $2x^2 + 7x + 3 = 0$. **2**

d. Solve $2^{3x-1} = \frac{1}{4}$. **2**

e. Solve $(x-3)(x-4) \geq 20$. **2**

f. Solve $|2x-3| = 3x - 1$. **2**

Question 3 **Geometry** **Total 12 Marks**

Please use a separate writing booklet.

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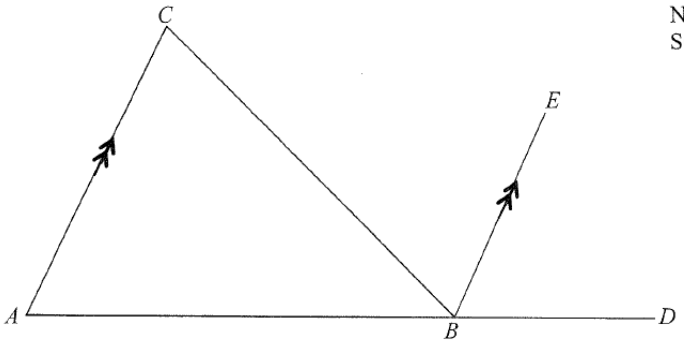
a. A regular polygon is such that the size of each interior angle is 9 times the size of each exterior angle. How many sides does the polygon have? **2**

b. In the diagram, the side AB of $\Delta\ ABC$ is produced to D. BE is parallel to AC and BE bisects $\angle CBD$. Copy the diagram and show that $AB = BC$.

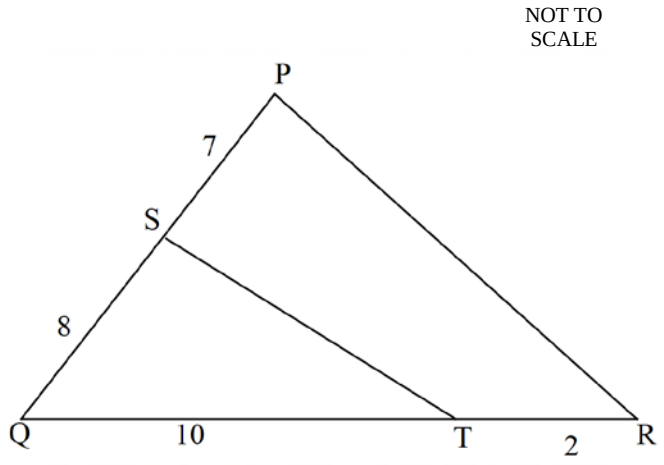


(i) Copy the diagram on your answer booklet **1**

(ii) Show that $AB=BC$. (Giving Reasons) **3**

(iii) If $\angle EBD = 50°$. Find the measure of $\angle CBA$. **1**

c. Given the diagram below:



(i) Prove that triangle QST is similar to triangle QPR. **3**

(ii) Hence find the length of PR, given that ST=6. **2**

Question 4 **Functions and Graphs** **Total 12 Marks**

Please use a separate writing booklet.

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a. Find the $\lim_{x \to 2} \frac{x^4 - 16}{4x - 8}$. **2**

b. Consider the function $f(x) = 2x^4 + 3x^2 + 4$. Show algebraically that the function $f(x)$ is either odd, even or neither. **2**

c. Sketch the graph $y = |x + 4|$.
Showing all intercepts on the *x* and *y* axis. **2**

d. (i) Sketch the function $y = -\sqrt{9 - x^2}$.
Showing all intercepts on the *x* and *y* axis. **2**

(ii) State the range of this function. **1**

e. (i) Sketch the graph of the function $f(x) = \sqrt{x} - 2$.
Showing all intercepts on the *x* and *y* axis. **2**

(ii) State the domain of this function. **1**

Question 5 **Trigonometry** **Total 12 Marks**

Please use a separate writing booklet.

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a. Show that $\frac{cot\theta}{cosec\theta} = cos\theta$. **2**

b. Show that $cos\theta + tan\theta\, sin\theta = sec\theta$. **2**

c. Solve $sin^2\theta - 1 = 0$. $0° \leq \theta \leq 360°$. **2**

d. Given $sin\theta = -sin\, 50°$ for $0° \leq \theta \leq 360°$. Find the values of θ. **2**

e. In Δ ABC, $BC = 19cm$, $AC = 22cm$ and $\angle ABC = 55°$.

(i) Find the size of $\angle CAB$ to the nearest degree. **2**

(ii) Hence find the area of ΔABC, to the nearest square centimetre. **2**

Question 6 **Linear Functions** **Total 12 Marks**

Please use a separate writing booklet.

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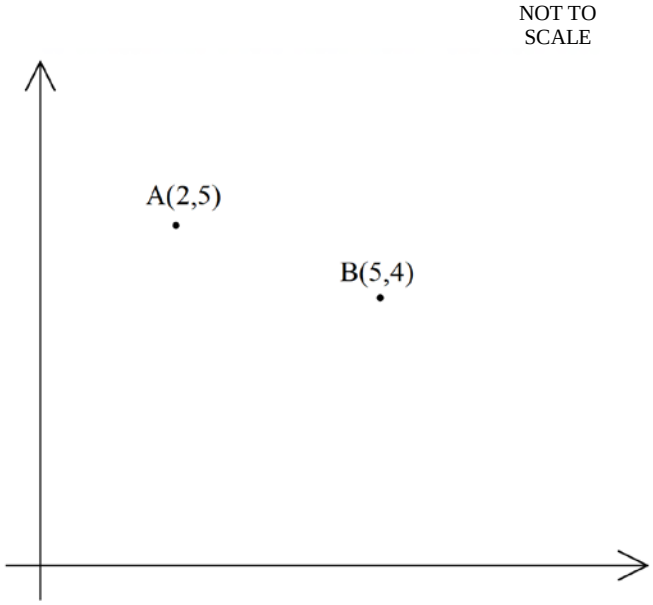
a. The diagram below shows the points $A(2,5)$ and $B(5,4)$.



Not to scale

(i) Find the coordinates of M the mid-point of AB. **1**

(ii) Show that the equation of the line AB is $x + 3y = 17$. **2**

(iii) Show that the equation of the perpendicular bisector of AB is $3x - y = 6$. **2**

(iv) The perpendicular bisector of AB cuts the *x-axis* at C.
Find the coordinates of C. **1**

(v) Find the area of ΔABC. **2**

b. What is the value of k such that the lines $k^2x - 4y = 0$ and $y = \frac{2}{k}x$,
are perpendicular to one another? **2**

c. Find the exact value of y, given that the points (2, 3) and (1, y)
make an angle of $60°$ with the x axis. **2**

Question 7 **Introduction to Calculus** **Total 12 Marks**

Please use a separate writing booklet.

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a. Find the derivatives of the following functions:

(i) $y = \frac{1}{3}x^{\frac{3}{2}}$. **1**

(ii) $y = (3x + 1)^4$ **1**

(iii) $y = x^2(4x + 1)^3$. (Answer to be fully factorised and simplified) **2**

b. Find the coordinates of the points at which the curve $y = x^4 + 1$, has a tangent with gradient 4? **2**

c. Using first principles **ONLY** differentiate the function $f(x) = 2x^2 - x$. **3**

d. (i) Find the gradient of the tangent to the curve $y = \frac{3x}{x-2}$ at the point on the curve where $x = 3$. **1**

(ii) Hence find the equation of the normal to the given curve. **2**

SOLUTIONS

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Solutions | Comments

Question 1

(a) 12.9

(b) $3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$

(c) $\pi r^3 = 20.6$

$r^3 = \frac{20.6}{\pi}$

$r = 1.9$ units.

(d) $\frac{(a-4)}{(a-4)(a^2+4a+16)}$

$= \frac{1}{(a^2+4a+16)}$

(e) $2x^2+7x+3=0$

$(2x+1)(x+3)=0 \quad \therefore x = -\frac{1}{2}, x = -3$

(f) $5^2 - (x-1)^2 = (5-x+1)(5+x-1)$

$= (6-x)(4+x)$

or $= -(x+4)(x-6)$

(g) $\sqrt{5}+2+\frac{1}{\sqrt{5}+2} = \sqrt{5}+2+\frac{(\sqrt{5}-2)}{(\sqrt{5}+2)(\sqrt{5}-2)}$

$= \frac{\sqrt{5}+2+\sqrt{5}-2}{5-4} = 2\sqrt{5} = b\sqrt{5}$

$\therefore b = 2$

Solutions | Comments

Question 2

(a) $3x+2y=7 \quad -1$

$3x-2y=11 \quad -2$

$\oplus$

$\therefore 6x = 18$

$x = 3$

Sub $x=3$ in 1: $3(3)+2y=7$

$2y=-2 \qquad y=-1$

$\therefore x=3, \; y=-1$

(b) $5(x+3)-2(2x-1)=10$

$5x+15-4x+2=10$

$x+17=10$

$x=-7$

(c) $2^{3x-1} = 2^{-2}$

$\therefore 3x-1=-2$

$3x=-1$

$x=\frac{-1}{3}$

(d) $2x-3=3x-1$ and $2x-3=-3x+1$

$-x=2 \qquad 5x=4$

$x=-2 \qquad x=\frac{4}{5}$

Check: $3(-2)-1=-7<0 \qquad 3\left(\frac{4}{5}\right)-1=\frac{7}{5}>0$

$\therefore$ Not a Soln. $\qquad \therefore x=\frac{4}{5}$

Solutions	Comments
Question 2 (cont)	
(e) $2-3x \leq 8$	
$-3x \leq 6$	
$3x \geq -6$	
$x \geq -2$	
	
(f) $(x-3)(x-4) \geq 20$	
$x^2-7x+12-20 \geq 0$	
$x^2-7x-8 \geq 0$	
$(x-8)(x+1) \geq 0$	
	
$\therefore x \leq -1 \qquad x \geq 8$	

Solutions | Comments

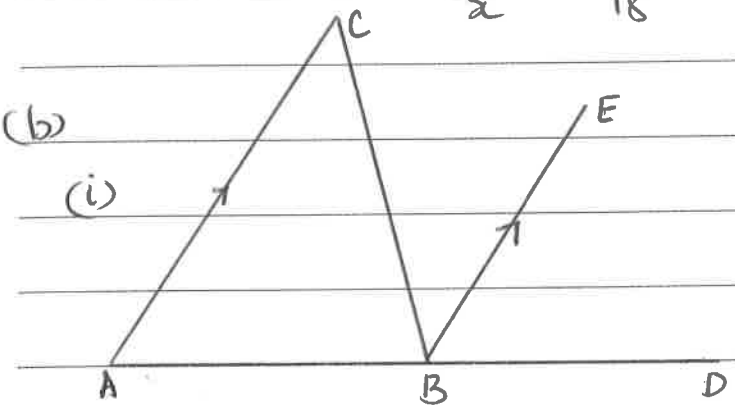
Question 3

(a) $9x$ / x

$\therefore 10x = 180^\circ$ $\quad x = 18^\circ$ = Ext. $\angle$

$\therefore$ No. of sides $= \frac{360}{x} = \frac{360}{18} = 20$

(b)



(i)

(ii)

	Reason
1. $\angle DBE = \angle EBC$	(Given) BE bisects $\angle CBD$
2. $\angle BCA = \angle EBC$	Alternate $\angle$'s on parallel lines AC and BE are equal.
3. $\angle BAC = \angle DBE$	Corresponding $\angle$'s on parallel lines Ac and BE are equal.

$\therefore \angle BCA = \angle BAC$

$\therefore$ AB = BC (sides opposite equal $\angle$'s are equal) in $\triangle ABC$

(iii) $\angle EBD = 50^\circ$

$\therefore \angle CBA = 80$ (No Reason Reqd.)

Solutions | Comments

Question 3 (cont)

(c) (i) Prove $\triangle QST \;|||\; \triangle QPR$.

1. $\angle SQT = \angle PQR$ (A) Common Angle

2. $\frac{QT}{QP} = \frac{QS}{QR}$ as

$\frac{10}{15} = \frac{8}{12} = \frac{2}{3}$ $\therefore$

$\therefore$ Sides Corresponding to the 2 $\triangle$'s are proportional.

3. $\therefore \triangle QST \;|||\; \triangle QRP$

(Two triangles are similar if the ratios of two pairs of corresponding sides are equal and the angles included by these sides are equal)

(ii) $\therefore$ The ~~tri~~ third sides are also proportional. ie.

$\frac{QT}{QP} = \frac{QS}{QR} = \frac{ST}{PR} = \frac{2}{3}$

$\therefore$ Given $ST = 6$

$\frac{ST}{PR} = \frac{2}{3}$

$PR = \frac{6 \times 3}{2} = 9.$

Solutions | Comments

Question 4

(a) $\lim_{x \to 2} \frac{x^4 - 16}{4x - 8}$

$= \lim_{x \to 2} \frac{(x^2-4)(x^2+4)}{4(x-2)}$

$= \lim_{x \to 2} \frac{(x-2)(x+2)(x^2+4)}{4(x-2)}$

$= \frac{(2+2)(4+4)}{4}$

$= \frac{4 \times 8}{4} = 8$

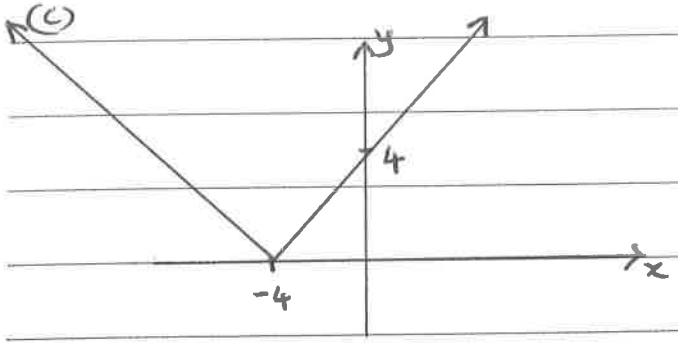
(b) $f(x) = 2x^4 + 3x^2 + 4$

$f(-x) = 2(-x)^4 + 3(-x)^2 + 4$

$= 2x^4 + 3x^2 + 4$

$= f(x)$ $\therefore$ EVEN fn.

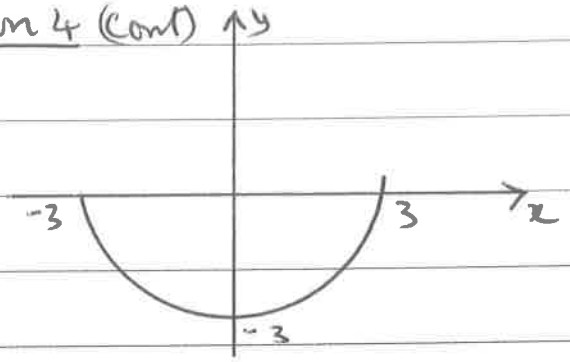
(c)



Solutions | Comments

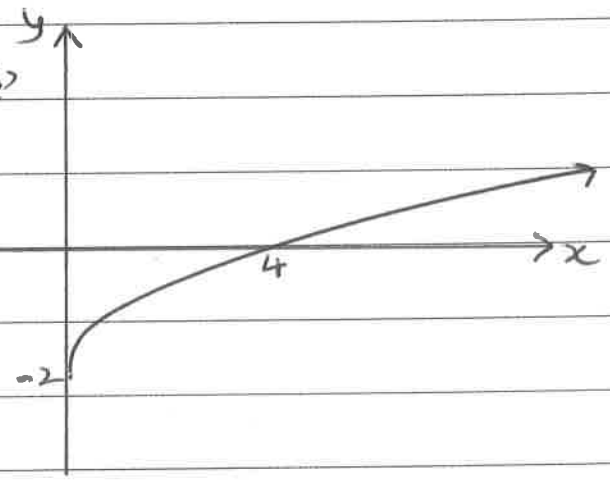
Question 4 (cont)

(d) (i)



(ii) Range: $-3 \leq y \leq 0$

(e) (i)



(ii) Domain: $x \geq 0$

Solutions | Comments

Question 5:

(a) $\dfrac{\dfrac{\cos\theta}{\sin\theta}}{\dfrac{1}{\sin\theta}} = \dfrac{\cos\theta}{\sin\theta} \times \dfrac{\sin\theta}{1} = \cos\theta$

(b) $\cos\theta + \dfrac{\sin\theta}{\cos\theta} \times \sin\theta$

$\dfrac{\cos^2\theta + \sin^2\theta}{\cos\theta} = \dfrac{1}{\cos\theta} = \sec\theta$

(c) $\sin^2 x - 1 = 0$

$(\sin x - 1)(\sin x + 1) = 0$

$\therefore \sin x = 1 \qquad \sin x = -1$

$x = 90^\circ \qquad x = 270^\circ$

(d) $\sin\theta = -\sin 50$

$\theta = 50^\circ$

S	A
T	C
$180+\theta$ ✓	$360-\theta$ ✓

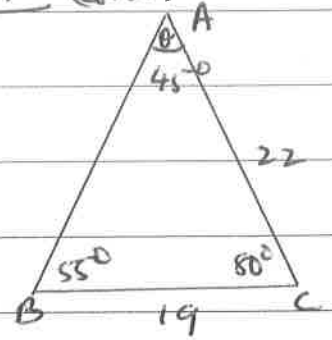
$\therefore \theta = 180+50 = 230^\circ$

or $\theta = 360-50 = 310^\circ$

Solutions | Comments

Question 5 (cont)

(c)



(i) $\therefore \frac{\sin\theta}{19} = \frac{\sin 55}{22}$

$\sin\theta = \frac{19 \times \sin 55}{22} = 0.707\ldots$

$\theta = 45^\circ$ (nearest degree)

(ii) Area of ΔABC

$= \frac{1}{2} ab \sin\theta$

$= \frac{1}{2} \times 19 \times 22 \times \sin 80$

$= 205.82\ldots$

$= 206$ sq. cms.

Solutions | Comments

Question 6

(i) $M = \left[\frac{2+5}{2}, \frac{5+4}{2}\right] = \left(\frac{7}{2}, \frac{9}{2}\right)$

(ii) $m_{AB} = \frac{4-5}{5-2} = -\frac{1}{3}$

∴ Equation of AB

$y - 5 = -\frac{1}{3}(x-2)$

$3y - 15 = -x + 2$

$x + 3y = 17$ or $x + 3y - 17 = 0$

(iii) Equation of ⊥ar bisector of AB

$m_{AB} = -\frac{1}{3}$ ∴ m of ⊥ is 3

∴ Equation

$y - \frac{9}{2} = 3\left(x - \frac{7}{2}\right)$

$2y - 9 = 3(2x - 7)$

$2y - 9 = 6x - 21$

$6x - 2y = 12$

$3x - y = 6$ or $3x - y - 6 = 0$

(iv) $y = 0$ in $3x - y - 6 = 0$

∴ $3x = 6$ $x = 2$ $C(2, 0)$

(v) Area of $\triangle ABC = \frac{1}{2} \times b \times h$

$d_{MC} = \frac{|x + 3y - 17|}{\sqrt{1+9}} = \frac{|2 - 17|}{\sqrt{10}} = \frac{15}{\sqrt{10}}$

$d_{AB} = \sqrt{(5-2)^2 + (4-5)^2} = \sqrt{10}$

$A = \frac{1}{2} \times \frac{15}{\sqrt{10}} \times \sqrt{15} = 7.5u^2$

Solutions	Comments
Question 6 (cont)	
(b) $m_1 \times m_2 = -1$	
$m_1 = \frac{k^2}{4} \qquad m_2 = \frac{2}{k}$	
$\therefore \frac{k^2}{4} \times \frac{2}{k} = -1$	
$\frac{k}{2} = -1$	
$k = -2$	
(c) $\tan\theta = \frac{y-3}{1-2}$	
$\sqrt{3} = \frac{y-3}{-1}$	
$-\sqrt{3} = y-3$	
$y = 3-\sqrt{3}$	

Solutions Question 7 Comments

a)i) $y = \frac{1}{3}x^{\frac{3}{2}}$

$y' = \frac{1}{3} \times \frac{3}{2} \times x^{\frac{1}{2}}$

$= \frac{1}{2}x^{\frac{1}{2}}$

(ii) $y = (3x+1)^4$

$y' = 4(3x+1)^3 \times 3$

$= 12(3x+1)^3$

(iii) $y = x^2(4x+1)^3$ $u = x^2$ $v = (4x+1)^3$

$u' = 2x$ $v' = 3 \times 4(4x+1)^2$

$u' = 2x$ + $= 12(4x+1)^2$

$y' = 2x(4x+1)^3 + 12x^2(4x+1)^2$

$= 2x(4x+1)^2[4x+1+6x]$

$= 2x(4x+1)^2(10x+1)$

(b) $y = x^4 + 1$

$y' = 4x^3 = 4$

$x^3 = 1$

$x = 1$, $y = 1+1 = 2$

∴ Coords: $(1, 2)$

Solutions | Comments.

Question 7: Cont.

(c) $f(x) = 2x^2 - x$

$f(x+h) = 2(x+h)^2 - (x+h)$

$= 2x^2 + 4xh + 2h^2 - x - h$

$f(x+h) - f(x) = 4xh + 2h^2 - h$

$\therefore \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = f'(x)$

$= \lim_{h \to 0} \frac{4xh + 2h^2 - h}{h}$

$= \lim_{h \to 0} \frac{h(4x + 2h - 1)}{h}$

$= 4x + 2h - 1$

Let $h = 0$

$= 4x + 0 - 1$

$= 4x - 1$

(d)(i) $y = \frac{3x}{x-2}$ $\quad u = 3x \quad v = x - 2$

$u' = 3 \quad v' = 1$

$\therefore y' = \frac{3(x-2) - 3x}{(x-2)^2}$

at $x = 3$, $m_t = \frac{-6}{1} = -6$

$m_n = \frac{1}{6}$, $x = 3$, $y = 9$

(ii) $\therefore$ Eqn. of normal:

$y - 9 = \frac{1}{6}(x - 3)$

$6y - 54 = x - 3$

$x - 6y + 51 = 0$.