

Students' Name: _____ Teacher: _____



Saint Ignatius' College, Riverview Mathematics Assessment Task 2019

Year 11
Mathematics
Task 3
PRELIMINARY COURSE EXAMINATION
SEPTEMBER 2019

General Instructions:

- **Reading time:** 5 Minutes
- **Time Allowed:** 120 Minutes
- Write using blue or black pen only
- Board approved calculators may be used
- Attempt all questions in the space provided on the paper
- Write **your name** and **your teacher's name** in the positions indicated
- Marks may not be awarded for missing or carelessly arranged working

Teachers:

Mr R Maxwell	REM
Mr P Collins	PPC
Dr M Furtado	MXF
Mr J Newey	JPN
Ms K Mullan	KXM

Topics Examined:

Question 1: Algebra, Equations and Inequalities
Question 2: Functions and Further Functions
Question 3: Trigonometry
Question 4: Calculus
Question 5: Probability
Question 6: Exponential and Logarithmic Functions
Question 7: Trigonometric Functions

Total

84 Marks

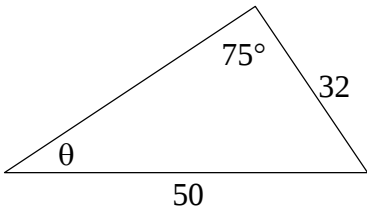
Question 1 Algebra Equations and Inequalities**Total 12 Marks*****Please use a separate writing booklet.***

a)	If $x = \left(\frac{1}{3}\right)^4$ and $y = \frac{3}{2}$ evaluate xy^3 as a single fraction expressed in lowest terms.	2
b)	Find the value of a and b if $\frac{\sqrt{3}}{3\sqrt{2}} = \frac{a\sqrt{6}}{b}$	2
c)	Expand and simplify $\left(x + \frac{1}{x}\right)^2$	1
d)	Solve the following equation $ 3x - 2 - 7 = 0$	3
e)	The area of a circle is given by the formula $A = \pi r^2$ Find the value of r correct to two significant figures if $A = 150$ sq cm.	1
f)	Solve the following inequality and express the solution on a number line $-2 < 3x - 1 \leq 11$	3

Question 2 Functions and Further Functions**Total 12 Marks***Please use a separate writing booklet.*

a)	Given $h(x) = x^2 + 2x + 3$, find $h(x-1)$	1
b)	Find the value of p such that $px^2 + 3x - 4 < 0$ for all values of x .	2
c)	Find the centre and radius of the circle with equation $x^2 + 4x + y^2 + 6y = 0$	2
d)	State the domain and range of $f(x) = (x-1)^2 + 3$	2
e)	Make a neat accurate sketch of the following function. Indicate ANY intercepts on the sketch. $f(x) = \frac{2}{x+1}$	3
f)	If $f(x) = x^2 + x - 6$ and $g(x) = x^3 - 1$ find: i) The y intercept of $y = f(x) - g(x)$ ii) The leading term of $y = f(x)g(x)$	2

Question 3 Trigonometry**Total 12 Marks***Please use a separate writing booklet.*

a)	 Find the value of θ to the nearest minute.	2
b)	The angle subtended at the centre of a circle by an arc of length $\frac{3\pi}{2}$ cm is $\frac{\pi}{5}$. Find the exact length of the radius of the circle.	2
c)	Fiona runs south from her home for a distance of 4.5 km. She then turns west and runs a further 5.1 km. What is the bearing (to the nearest degree) of her home from where she is now ?	3
d)	ABCD is a rectangle with AB and CD each 8.5 cm in length. The diagonal BD meets the side DC creating an angle of $\angle BDC$ equal to 54° . Find the area of the rectangle correct to one decimal place.	2
e)	From a point O the point P bears 120° and is 12.3 km away. The point Q is 15.2 km South West of O. i) Mark the relative positions of O, P, Q on a sketch. ii) Calculate the distance PQ in kilometres (rounded off correct to one decimal place)	3

Question 4 Calculus**Total 12 Marks***Please use a separate writing booklet.*

a)	If $f(x) = x^2 - 3x + 5$ find: $f(x+h) - f(x)$	2
b)	Differentiate the following. i) $y = x\sqrt{x}$ ii) $y = x(x+1)^4$	4
c)	Find the equation of the normal to the curve $y = 2\sqrt{x+1}$ at the point where $x = 8$. (Leave your equation in general form)	3
d)	A circular metal disc is heated. As the temperature increases the disc expands. The area of the disc (in sq cm) is given by the formula $A = 4t^2 + 2t + 10$ where t is the time in minutes since the heating of the disc began. i) Find the initial area of the disc. ii) At what rate is the disc expanding after three minutes.	3

Question 5 Probability**Total 12 Marks***Please use a separate writing booklet.*

a)	A jar contains two red lollies and one green lolly of the same size and shape. If two lollies are selected at random, what is the probability that they are both red? Note: The first lolly is NOT replaced before the second one is selected.	2										
b)	In a drawer there are four socks. Two are red and two are white. Two socks are taken out at random. What is the probability that two of the same colour are selected?	2										
c)	The College has three printing machines. The probability that any one of them will break down at any time is 6.5%. Find the probability that at least one will be broken down at a particular time. (Answer as a percentage correct to one decimal place)	2										
d)	A newsagent in a town sells a local morning paper and an evening paper. A survey of the people in the town is conducted to study their buying habits. The results are shown on the graph below. <table><caption>Bar Chart Data</caption><thead><tr><th>Buying Habit</th><th>Percentage</th></tr></thead><tbody><tr><td>Buy morning paper only</td><td>20</td></tr><tr><td>Buy evening paper only</td><td>10</td></tr><tr><td>Buy both papers</td><td>30</td></tr><tr><td>Buy neither paper</td><td>40</td></tr></tbody></table> A person from the town is selected at random. What is the probability that this person buys an evening paper?	Buying Habit	Percentage	Buy morning paper only	20	Buy evening paper only	10	Buy both papers	30	Buy neither paper	40	2
Buying Habit	Percentage											
Buy morning paper only	20											
Buy evening paper only	10											
Buy both papers	30											
Buy neither paper	40											
e)	There is a daily flight from Jacintaville to Neilopolis. The probability of the flight departing on time, given that the weather is clear is 0.7 and the probability that the flight departs on time in poor weather is 0.5. In July the probability that the weather is clear is 0.4. Find the probability that on a particular day in July i) The flight from Jacintaville departs on time. ii) The weather is fine in Jacintaville, given that the flight departs on time.	2 2										

Question 6 Exponential and Logarithmic Functions**Total 12 Marks***Please use a separate writing booklet.*

a)	Differentiate with respect to x . $(e^x + 1)^3$	2
b)	Find the value of m . $\log_x 48 = \log_x m + 3\log_x 2$	2
c)	Solve the equation correct to four significant figures. $100 = 5e^{x+1}$	2
d)	Find the equation of the TANGENT to the curve $y = e^{2x+1}$ at the point where $x = 1$. (leave your equation in general form AND in terms of e)	3
e)	The population of rabbits living inside the grounds of Riverview after t years can be calculated using $P = 100 + 3e^{0.5t}$ i) How many rabbits will be living at Riverview in ten years? ii) How many years until the population of rabbits reaches 5000? (Answer to the nearest year)	1 2

Question 7 Trigonometric Functions**Total 12 Marks***Please use a separate writing booklet.*

a)	Find the exact value of $1 - \cos^2 \frac{\pi}{6}$.	1
b)	If $\cos \theta = \frac{2}{5}$ and $\tan \theta < 0$, find the exact value of $\sec \theta + \sin \theta$.	3
c)	Prove the following $1 + \tan^2 \theta = \frac{1}{1 - \sin^2 \theta}$	3
d)	Solve for $[0, \pi]$: $1 + \sin^2 x - \cos x = 0$	3
e)	For $[0, 2\pi]$, sketch the graph of. $y = 3\sin x + 1$	2

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SOLUTIONS

QUESTION ONE

$$\begin{aligned} \text{a) } xy^3 &= \left(\frac{1}{3}\right)^4 \times \left(\frac{3}{2}\right)^3 \\ &= \frac{3^3}{3^4 \cdot 2^3} \\ &= \frac{1}{24} \end{aligned}$$

$$\begin{aligned} \text{b) } \frac{\sqrt{3}}{3\sqrt{2}} &= \frac{\sqrt{3}}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\ &= \frac{\sqrt{6}}{6} \end{aligned}$$

$$\therefore a = 1 \quad b = 6.$$

$$\begin{aligned} \text{c) } \left(x + \frac{1}{x}\right)^2 &= x^2 + 2x \cdot \frac{1}{x} + \frac{1}{x^2} \\ &= x^2 + 2 + \frac{1}{x^2} \end{aligned}$$

$$\text{d) } |3x - 2| - 7 = 0$$

$$|3x - 2| = 7$$

$$3x - 2 = 7 \quad \text{or} \quad 3x - 2 = -7$$

$$3x = 9$$

$$x = 3$$

$$3x = -5$$

$$x = -\frac{5}{3}$$

e) $A = \pi r^2$

$$150 = \pi r^2$$

$$r^2 = \frac{150}{\pi}$$

$$r = 6.9$$

f) $-2 < 3x - 1 \leq 11$

$$-1 < 3x \leq 12$$

$$-\frac{1}{3} < x \leq 4$$



QUESTION TWO

a) $h(x) = x^2 + 2x + 3$

$$\begin{aligned}h(x-1) &= (x-1)^2 + 2(x-1) + 3 \\&= x^2 - 2x + 1 + 2x - 2 + 3 \\&= x^2 + 2\end{aligned}$$

b) $px^2 + 3x - 4 < 0$

$$\Delta < 0 \quad p < 0$$

$$\Delta = 9 - 4p(-4)$$

$$= 9 + 16p$$

$$9 + 16p < 0$$

$$16p < -9$$

$$p < -\frac{9}{16}$$

$$\therefore \Delta < 0 \\ p < 0$$

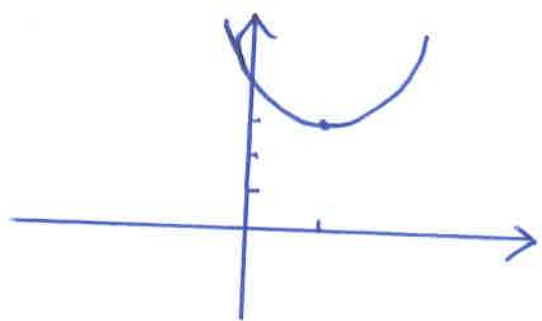
c) $x^2 + 4x + y^2 + 6y = 0$

$$x^2 + 4x + 4 + y^2 + 6y + 9 = 13$$

$$(x+2)^2 + (y+3)^2 = 13$$

$$c(-2, -3) \quad R = \sqrt{13}$$

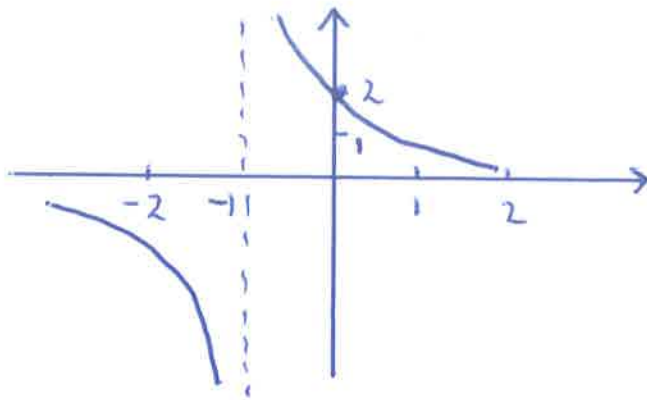
d) $f(x) = (x-1)^2 + 3$



$$D: \text{all real } x \\ [-\infty, \infty]$$

$$R: y \geq 3 \\ [3, \infty]$$

e) $f(x) = \frac{2}{x+1}$



f) $f(x) = x^2 + x - 6$
 $g(x) = x^3 - 1$

i) $f(x) - g(x) = x^2 + x - 6 - (x^3 - 1)$
 $= -x^3 + x^2 + x - 5$

y int = -5

ii) leading term x^5

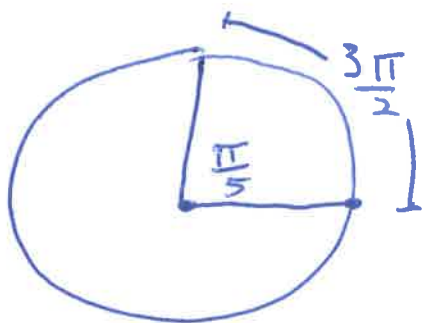
QUESTION THREE

$$a) \frac{\sin \theta}{32} = \frac{\sin 75}{50}$$

$$\sin \theta = \frac{\sin 75}{50} \times 32$$

$$\theta = 38^{\circ} 11'$$

b)

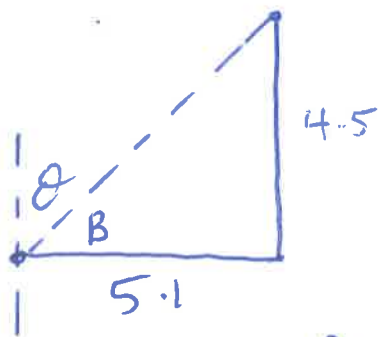


$$l = r\theta$$

$$\frac{3\pi}{2} = r \cdot \frac{\pi}{5}$$

$$r = \frac{15}{2} \text{ cm}$$

c)



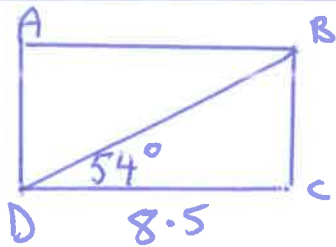
$$\tan B = \frac{4.5}{5.1}$$

$$B = 41^{\circ} 25'$$

$$\therefore \theta = 48^{\circ} 35'$$

$$\text{Bearing} = 049^{\circ}$$

d)



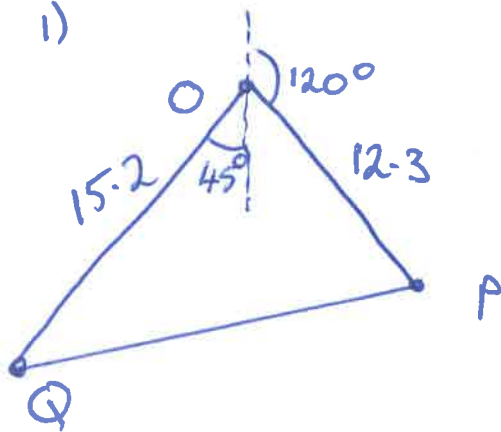
$$\frac{BC}{8.5} = \tan 54$$

$$BC = 11.7$$

$$A = 8.5 \times 11.7 \\ = 99.5 \text{ cm}^2$$

e)

i)



$$\text{ii) } \angle POQ = 60 + 45 \\ = 105^\circ$$

$$PQ^2 = 15.2^2 + 12.3^2 - 2 \cdot 15.2 \cdot 12.3 \cos 105$$

$$PQ = 21.9 \text{ km.}$$

QUESTION FOUR

a) $f(x) = x^2 - 3x + 5$

$$f(x+h) = (x+h)^2 - 3(x+h) + 5$$

$$= x^2 + 2xh + h^2 - 3x - 3h + 5$$

$$f(x+h) - f(x)$$

$$= x^2 + 2xh + h^2 - 3x - 3h + 5 - [x^2 - 3x + 5]$$

$$= 2xh + h^2 - 3h$$

b) i) $\frac{d}{dx} x\sqrt{x} = \frac{d}{dx} x^{3/2}$

$$= \frac{3}{2} x^{1/2}$$

$$= \frac{3\sqrt{x}}{2}$$

ii) $y = x(x+1)^4$

$$y' = x \cdot 4(x+1)^3 + (x+1)^4$$

$$= (x+1)^3(4x + x+1)$$

$$= (x+1)^3(5x+1)$$

c) $y = 2\sqrt{x+1}$

$$= 2(x+1)^{1/2}$$

$$y' = (x+1)^{-1/2}$$

$$= \frac{1}{\sqrt{x+1}}$$

$$\text{at } x = 8 \quad M_T = \frac{1}{\sqrt{9}} \quad y = 2\sqrt{9} \\ = \frac{1}{3} \quad = 6$$

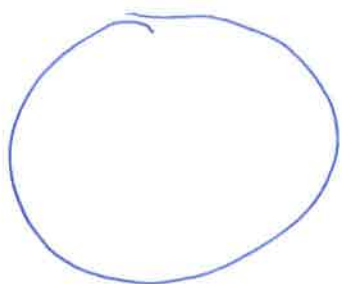
$$M_N = -3$$

$$y - 6 = -3(x - 8)$$

$$y - 6 = -3x + 24$$

$$3x + y - 30 = 0$$

d)



$$A = 4t^2 + 2t + 10$$

$$i) \quad t = 0 \quad A = 10 \text{ cm}^2.$$

$$ii) \quad \frac{dA}{dt} = 8t + 2$$

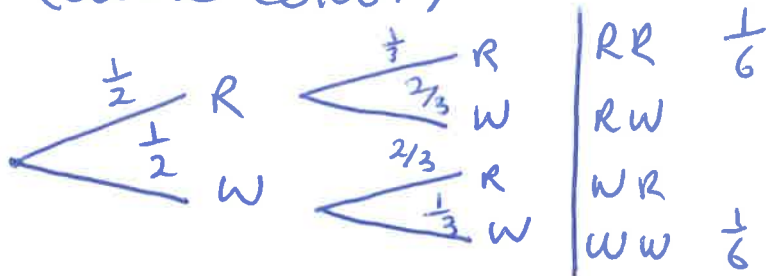
$$\text{at } t = 3$$

$$\frac{dA}{dt} = 26 \text{ cm}^2 / \text{min}$$

QUESTION FIVE

a)  $P(RR) = \frac{2}{3} \times \frac{1}{2}$
 $= \frac{1}{3}$

b) $P(\text{same colour})$



$\therefore P(\text{same}) = \frac{1}{3}$

c) $P(\text{at least one}) = 1 - P(\text{none})$

$= 1 - (0.935)^3$

$= 0.18259$

$\therefore P(e) = 18.3\%$

d) Total people = 100%

$P(\text{evening only}) = 10\%$

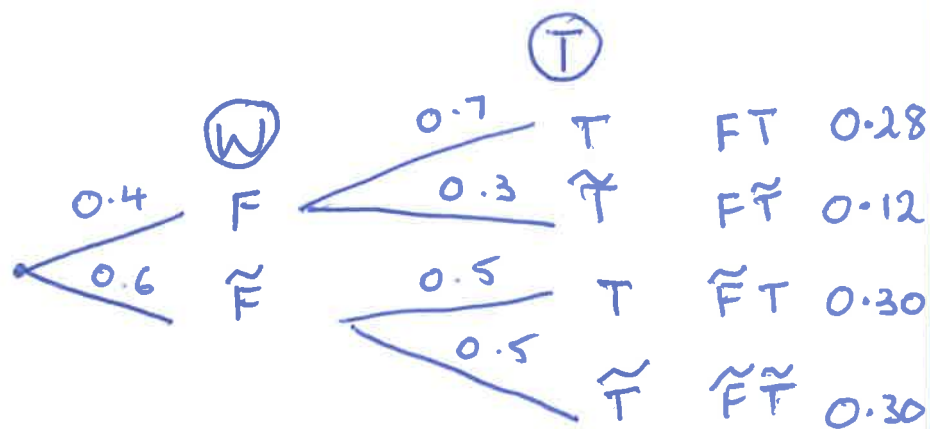
$P(\text{both}) = 30\%$

$\therefore P(\text{buys an evening paper}) = 40\%$

$= 0.4$

$= \frac{2}{5}$

e) On time T
 Not on time $\tilde{T}$
 Fine weather F
 Not FINE $\tilde{F}$



$$\begin{aligned} \text{i) } P(e) &= 0.28 + 0.30 \\ &= 0.58 \end{aligned}$$

$$\begin{aligned} \text{ii) } P(F|T) &= \frac{P(F \cap T)}{P(T)} \\ &= \frac{0.28}{0.58} \\ &= 0.48 \end{aligned}$$

Question Six

$$\begin{aligned} \text{a) } \frac{d}{dx} (e^x + 1)^3 &= 3(e^x + 1)^2 \cdot e^x \\ &= 3e^x (e^x + 1)^2 \end{aligned}$$

$$\begin{aligned} \text{b) } \log_x 48 &= \log_x M + 3 \log_x 2 \\ &= \log_x M + \log_x 8 \\ &= \log_x 8M \\ \therefore 8M &= 48 \\ M &= 6 \end{aligned}$$

$$\begin{aligned} \text{c) } 100 &= 5e^{x+1} \\ e^{x+1} &= 20 \\ \ln e^{x+1} &= \ln 20 \\ x+1 &= \ln 20 \\ x &= 1.996 \end{aligned}$$

$$\begin{aligned} \text{d) } y &= e^{2x+1} \\ y' &= 2e^{2x+1} \\ \text{at } x=1 \quad M_T &= 2e^3 \quad y = e^3 \\ y - e^3 &= 2e^3(x-1) \\ y &= 2e^3x - e^3 \\ 0 &= 2e^3x - y - e^3 \end{aligned}$$

$$e) \quad P = 100 + 3e^{0.5t}$$

$$i) \quad t = 10$$

$$P = 100 + 3e^5$$
$$= 545 \text{ rabbits}$$

$$ii) \quad 5000 = 100 + 3e^{0.5t}$$
$$4900 = 3e^{0.5t}$$

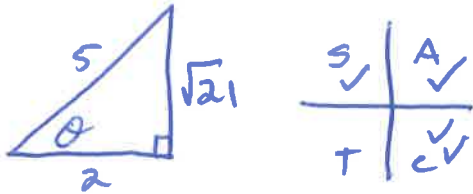
$$\ln\left(\frac{4900}{3}\right) = 0.5t$$

$$\therefore t = 14.7$$
$$= 15 \text{ yrs.}$$

QUESTION SEVEN.

$$\begin{aligned} \text{a) } 1 - \cos^2 \frac{\pi}{6} &= 1 - \frac{3}{4} \\ &= \frac{1}{4} \end{aligned}$$

$$\text{b) } \cos \theta = \frac{2}{5} \quad \tan \theta < 0$$



$$\begin{aligned} \sec \theta + \sin \theta &= \frac{5}{2} + \frac{-\sqrt{21}}{5} \\ &= \frac{25 - 2\sqrt{21}}{10} \end{aligned}$$

$$\text{c) Prove } 1 + \tan^2 \theta = \frac{1}{1 - \sin^2 \theta}$$

$$\begin{aligned} \text{LHS} &= 1 + \tan^2 \theta \\ &= 1 + \frac{\sin^2 \theta}{\cos^2 \theta} \\ &= \frac{\cos^2 \theta + \sin^2 \theta}{\cos^2 \theta} \\ &= \frac{1}{1 - \sin^2 \theta} \\ &= \text{RHS} \end{aligned}$$

$$d) 1 + \sin^2 x - \cos x = 0$$

$$1 + (1 - \cos^2 x) - \cos x = 0$$

$$2 - \cos^2 x - \cos x = 0$$

$$\cos^2 x + \cos x - 2 = 0$$

$$(\cos x - 1)(\cos x + 2) = 0$$

$$\therefore \cos x = 1 \quad \cos x = -2$$

No soln

$$x = 0$$

$$e) y = 3 \sin x + 1$$

