Name:_____ Teacher's Code : PPC MXF REM HBH JPN

Saint Ignatius' College, Riverview
Mathematics Assessment Task
2020

Year 11
Mathematics Advanced
Task 4
Preliminary HSC Exam
Date: Monday 21st September 2020

General Instructions:

- **Time Allowed: 2 hours (+5 minutes reading)**
- Write using blue or black pen only
- Board approved calculators may be used
- Attempt all questions on separate answer sheets
- Write **your name** and circle **your teacher's code** in the positions indicated
- Marks may be deducted for missing or carelessly arranged working

Teachers :

- Mr P Collins **PPC**
- Dr M Furtado **MXF**
- Mr Maxwell **REM**
- Ms Haxby **HBH**
- Mr Newey **JPN**

Topics Examined:

Section I – 10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II – 70 marks

- Attempt all questions 11-34
- Allow about 1 hour and 45 minutes for this section

Total **80 Marks**

Section I

10 marks
Attempt questions 1 - 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for questions 1-10

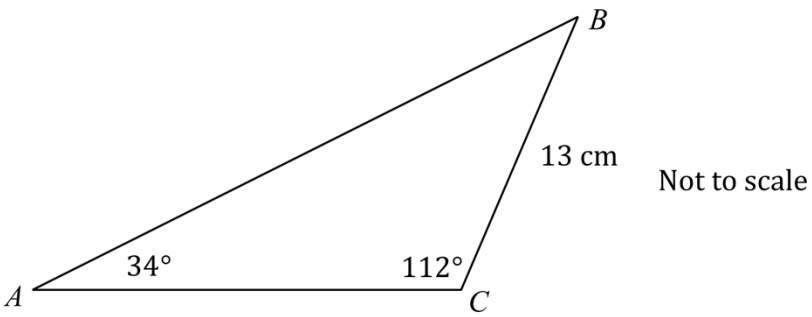
1. Given $f(x) = x^2(x-2)$. What is the value of $f'(1)$?

(A) –2
(B) –1
(C) 1
(D) 7

2. What is the value of *x* if $\log_5(x^2+16) = 2$?

(A) ± 3
(B) ± 4
(C) –4
(D) 3

3.



What is the length of *AB*, correct to one decimal place?

(A) 13.0 cm
(B) 20.9 cm
(C) 21.6 cm
(D) 40.6 cm

4. Which of the following is *not* a one-to-one function?

(A) $y = 3x^2 - 3x, x \geq 1$
(B) $y = \sqrt{3 - x^2}$
(C) $y = 3x^2, x \geq 0$
(D) $y = 3x$

5. The line $5x - y - 4 = 0$ passes through the point (k, 26). What is the value of k?

(A) 3
(B) 4
(C) 5
(D) 6

6. Which of the following statements about the graph of the function $y = 2\sin 4x$ is correct?

(A) The amplitude is 2 and the period is $\frac{\pi}{4}$
(B) The amplitude is 2 and the period is $\frac{\pi}{2}$
(C) The amplitude is 4 and the period is $\frac{\pi}{2}$
(D) The amplitude is 4 and the period is π

7. What is the derivative of $x^{-2} - \frac{1}{x}$?

(A) $\frac{-2}{x} - \frac{1}{x^2}$
(B) $\frac{-2}{x} + \frac{1}{x^2}$
(C) $\frac{-2}{x^3} - \frac{1}{x^2}$
(D) $\frac{-2}{x^3} + \frac{1}{x^2}$

8. Leo owns five blue and seven red ties. He chooses a tie at random for himself and puts it on. He then chooses another tie at random, from the remaining ties, and gives it to his brother. What is the probability that both of the ties, are the same colour?

(A) $\frac{1}{2}$
(B) $\frac{5}{33}$
(C) $\frac{31}{66}$
(D) $\frac{31}{72}$

9. Which of the following expressions is equal to $4 + 4\cot^2 x$?

(A) $4\sec^2 x$

(B) $4\operatorname{cosec}^2 x$

(C) $\cot^2 x$

(D) $8\cot^2 x$

10. If $\frac{4}{x-3} + \frac{2}{x} = 1, \quad x > 0$ then x is equal to:

(A) $\frac{9 \pm \sqrt{57}}{2}$

(B) $\frac{9 + \sqrt{57}}{2}$

(C) 2

(D) 7

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Section II **70 marks**
Attempt all questions (Question 11 – 31)
Allow about 1 hour and 45 minutes for this section

Answer the questions in the spaces provided. Your responses should include relevant mathematical reasoning and/or calculations.
Extra writing space is provided at the back of each section of the examination paper.

Algebraic Techniques (6 Marks)

Marks

Question 11

(a) Write in index form $\frac{1}{\sqrt[3]{x^2}}$ **1**

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(b) Express $\frac{3x+2}{x+1} - \frac{1}{x(x+1)}$ as a single fraction in simplest form. **3**

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Question 12

Express $(2\sqrt{3}+1)(2-\sqrt{3})$ in the form $a+b\sqrt{3}$, and hence find the value of a and b. **2**

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Introductory Calculus (14 Marks)

Question 13

Differentiate with respect to x.

(a) $y = 4x^{-3}$ **1**

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(b) $y = \sqrt{5 + x^2}$ **2**

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(c) $f(x) = \dfrac{3x^2 - 5}{2x + 1}$ **2**

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(d) $y = 3x^2(2x-3)^4$ (Give answer in factored form) **3**

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Question 14

$P(1, 4)$ lies on the curve $y = x^3 - 4x^2 + x + 6$

(a) Find the equation of the **tangent** to the curve at P. **2**

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(b) Find the equation of the **normal** to the curve at P. **2**

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(c) The tangent at P intersects the x-axis at T. **2**
What are the coordinates of T?

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Extra Working Space.

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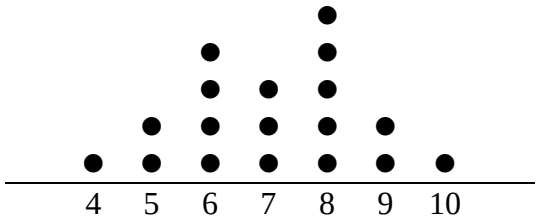
Probability (10 marks)

Question 15

The dot plot below shows the results of a Year 11 class quiz out of 10.



(a) Find the relative frequency of the score 8. (Express answer as a decimal correct to 2 d.p) **1**

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(b) A student from this class is selected at random. **1**
Find the probability that his score is greater than 7.
(Give answer as a simplified fraction)

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Question 16

A group of 50 Year 11 students were asked if they studied History and/or Geography. It was found that 26 students study Geography and 29 students study History. 7 students study neither History nor Geography.

(a) Draw a Venn Diagram to represent this group of students. **1**

(b) Find the number of students who study both History and Geography. **1**

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(c) Find the probability of selecting a student from this group that studies History given that he studies Geography. **1**

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Question 17

A bag contains 7 blue and 3 white marbles. Three marbles are selected from the bag at random and not replaced.

(a) Draw a probability tree to represent this experiment and list the sample space. **2**

(b) Find the probability of selecting 3 blue marbles. **1**

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(c) Find the probability of selecting exactly 2 blue marbles. **1**

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(d) Find the probability of selecting at least one white marble. **1**

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Functions (17 Marks)

Question 18

The line l_1 passes through the points $P(-1, 2)$ and $Q(11, 8)$.

(a) Find an equation for l_1 in the form $y = mx + c$, where m and c are constants. **2**

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(b) Find the angle of inclination of l_1. (The angle between l_1 and the positive x axis.) Give answer to nearest degree. **1**

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(c) The line l_2 passes through the point $R(10, 0)$ and is perpendicular to l_1. The lines l_1 and l_2 intersect at the point S. Find the coordinates of S. **3**

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Question 19 (2 marks)

A parabola has the equation $y = 5 + 4x - x^2$

(a) Find the equation of the axis of symmetry of $y = 5 + 4x - x^2$. **1**

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(b) Find the maximum value of the expression $5 + 4x - x^2$ **1**

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Question 20 (2 marks)

$$f(x) = \begin{cases} x^2 - 5 & \text{when } x > 2 \\ 2x + 1 & \text{when } -2 < x \leq 2 \\ 3 & \text{when } x \leq -2 \end{cases}$$

Evaluate: $3f(2) - 6f(-4)$ **2**

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Question 21 (3 marks)

A circle has equation $x^2 + y^2 + 8x - 6y - 24 = 0$.

(a) Write the equation of the circle in the form $(x-h)^2 + (y-k)^2 = r^2$ and hence find the coordinates of the centre and the radius of the circle. **2**

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(b) Find the domain of the circle. **1**

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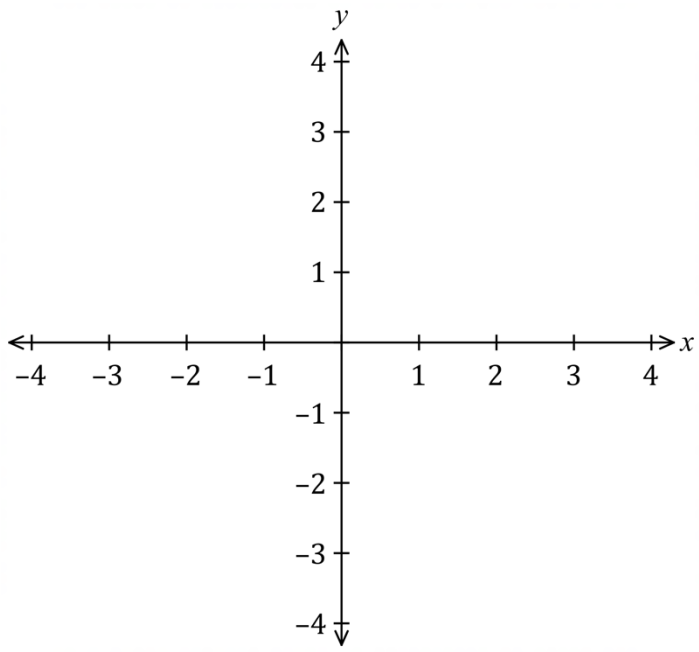
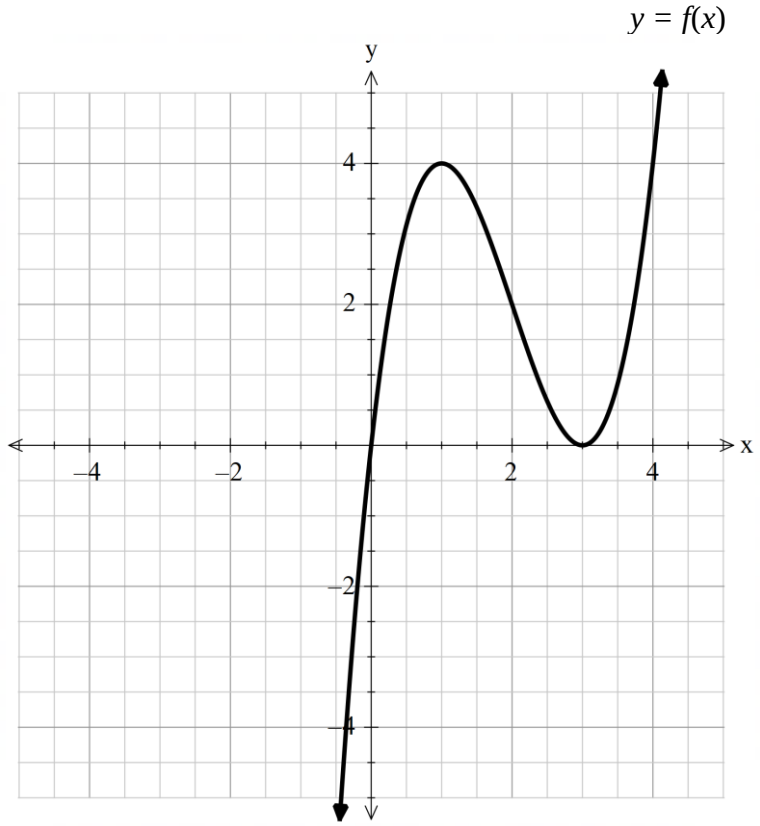
Question 22 (2 marks)

Sketch the graph of the function $y = \dfrac{3}{x-1}$ on the axes below. Show all asymptotes and intercepts. **2**



Question 23 (2 marks)

The graph of $y = f(x)$ is sketched below.
On the same set of axes, sketch the graph of $y = f(-x)$ showing all features. **2**



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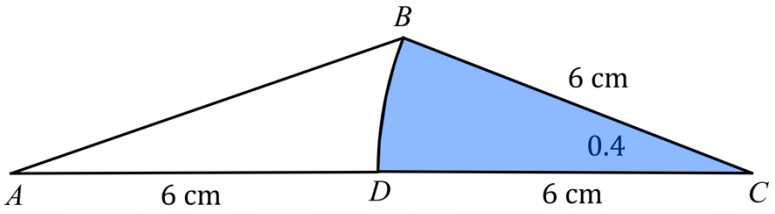
Trigonometry (13 marks)

Question 24



Not to scale

ΔABC has D as the midpoint of AC and $BC = CD = AD = 6$ cm. $\angle BCD$ is 0.4 radians. The curve BD is an arc of a circle with centre C and radius 6 cm.

(a) What is the length of arc BD? **1**

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(b) Find the area of sector BCD. Answer correct to one decimal place. **1**

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(c) Find the area of Triangle ABC. Answer correct to three significant figures. **1**

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Question 25

Find the exact value of $\cot 330°$ **1**

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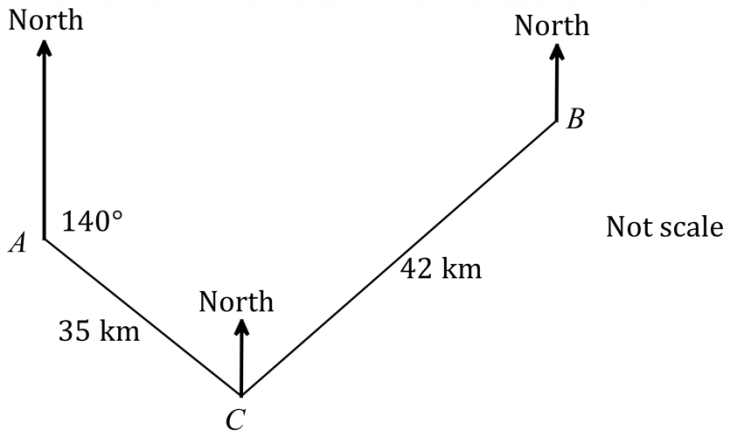
Question 26



Tyler drives 35km from town *A* on a true bearing of $140°T$ to town *C*. He then drives 42km on a true bearing of $038°T$ to town *B*.

(a) Show that $\angle ACB = 78°$ **1**

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(b) Find the distance from *A* to *B*. Answer correct to the nearest kilometre. **2**

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Question 27

Solve the equation $2\sin\theta + \sqrt{3} = 0$ in the interval $0° \le \theta \le 360°$. **2**

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Question 28

Prove the identity $\dfrac{\sin x}{1-\cos x} = \dfrac{1+\cos x}{\sin x}$ **2**

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Question 29

Solve: $2\cos^2 x + 3\cos x + 1 = 0 \qquad [0, 2\pi]$ **2**

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Extra working

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Exponential & Logarithmic Functions (10 marks)

Question 30

Differentiate: $y = 2e^{3x}$ **1**

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Question 31

The population of a city is changing over time according to the formula $P = 36000e^{0.03t}$ where P is the population and t is the time in years.

(a) Find the population after 5 years. **1**

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(b) Find the rate at which the population is changing when t = 10 years. **2**

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Question 32

Solve:

(a) $16^x = 120$ (Answer correct to 3 decimal places) **1**

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(b) $\log_3 x = -2$ **1**

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Question 33

Solve: $2\log_3 x - \log_3(x-2) = 2$ where $x > 2$ **2**

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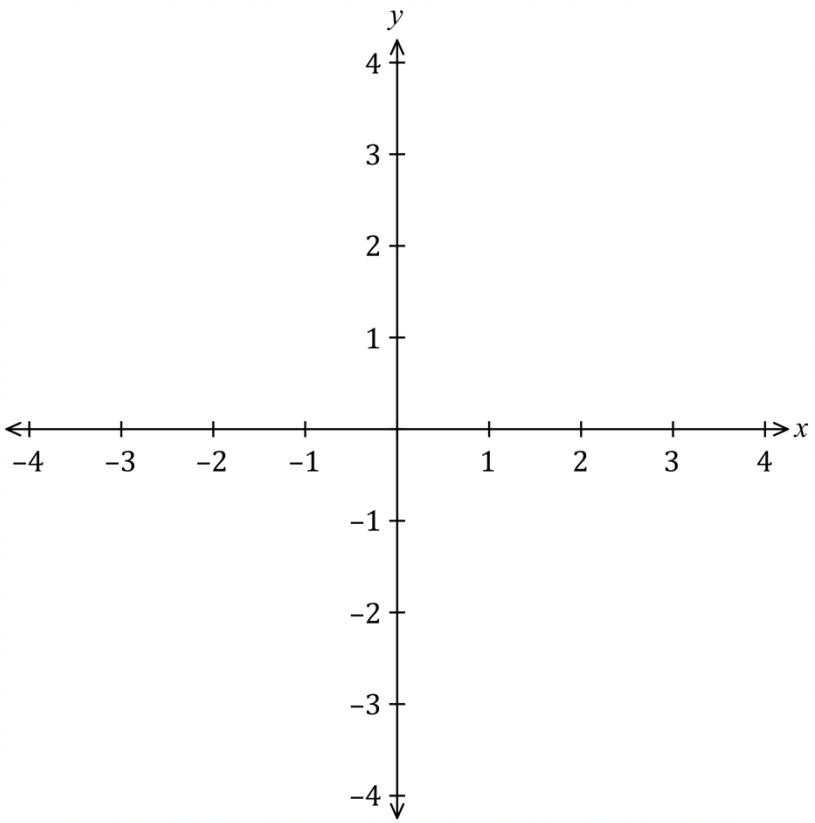
Question 34

Sketch the graph of $y = e^{-x}$ showing all important features. **2**



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NSW Education Standards Authority

2020 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $$\alpha\beta\gamma = -\frac{d}{a}$$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, \; r \neq 1$$

$$S = \frac{a}{1 - r}, \; |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

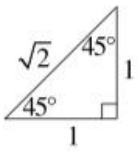
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

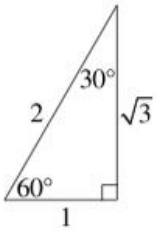
$$A = \frac{1}{2}ab\sin C$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1 - t^2}{1 + t^2}$$

$$\tan A = \frac{2t}{1 - t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

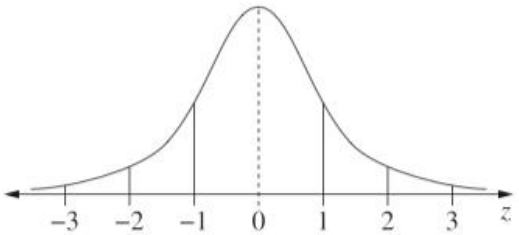
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\text{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \leq x) = \int_a^x f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r(1 - p)^{n-r}$$

$$X \sim \text{Bin}(n, p)$$

$$\Rightarrow \ P(X = x) = \binom{n}{x}p^x(1 - p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\text{Var}(X) = np(1 - p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)\left[f(x)\right]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+\left[f(x)\right]^2}$

Integral Calculus

$$\int f'(x)\left[f(x)\right]^n dx = \frac{1}{n+1}\left[f(x)\right]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln\left|f(x)\right| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-\left[f(x)\right]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+\left[f(x)\right]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f\left(x_1\right) + \cdots + f\left(x_{n-1}\right)\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

$$(x+a)^{n} = x^{n} + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^{r} + \cdots + a^{n}$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^{2}+y^{2}}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_{1}x_{2} + y_{1}y_{2},$$

where $\underset{\sim}{u} = x_{1}\underset{\sim}{i} + y_{1}\underset{\sim}{j}$

and $\underset{\sim}{v} = x_{2}\underset{\sim}{i} + y_{2}\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$z = a + ib = r(\cos\theta + i\sin\theta)$$
$$= re^{i\theta}$$

$$\left[r(\cos\theta + i\sin\theta)\right]^{n} = r^{n}(\cos n\theta + i\sin n\theta)$$
$$= r^{n}e^{in\theta}$$

Mechanics

$$\frac{d^{2}x}{dt^{2}} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^{2}\right)$$

$$x = a\cos(nt+\alpha) + c$$

$$x = a\sin(nt+\alpha) + c$$

$$\ddot{x} = -n^{2}(x-c)$$

Name: SOLUTIONS. Teacher's Code : PPC MXF REM HBH JPN

Saint Ignatius' College, Riverview
Mathematics Assessment Task
2020

Year 11
Mathematics Advanced
Task 4
Preliminary HSC Exam
Date: Monday 21st September 2020

General Instructions:

- **Time Allowed: 2 hours (+5 minutes reading)**
- Write using blue or black pen only
- Board approved calculators may be used
- Attempt all questions on separate answer sheets
- Write **your name** and circle **your teacher's code** in the positions indicated
- Marks may be deducted for missing or carelessly arranged working

Teachers :

• Mr P Collins	**PPC**
• Dr M Furtado	**MXF**
• Mr Maxwell	**REM**
• Ms Haxby	**HBH**
• Mr Newey	**JPN**

Topics Examined:

Section I – 10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II – 70 marks

- Attempt all questions 11-34
- Allow about 1 hour and 45 minutes for this section

Total **80 Marks**

Section I

10 marks
Attempt questions 1 - 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for questions 1-10

1. What is the value of $f'(1)$ if $f(x) = x^2(x-2)$?

(A) –2
(B) –1
(C) 1
(D) 7

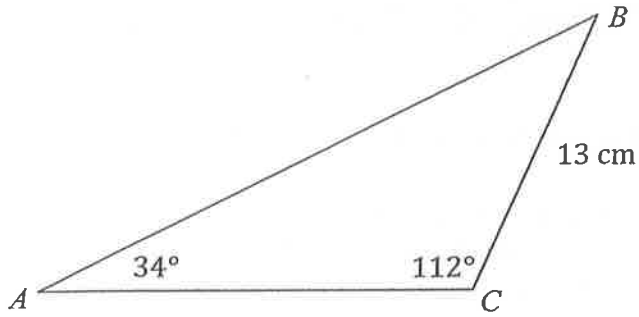
$f(x) = x^3 - 2x^2$
$f'(x) = 3x^2 - 4x$
$f'(1) = 3 - 4$
$= -1$

2. What is the value of x if $\log_5(x^2 + 16) = 2$?

(A) ± 3
(B) ± 4
(C) –4
(D) 3

$5^2 = x^2 + 16$
$x^2 = 9$
$x = \pm 3$

3.



Not to scale

What is the length of AB, correct to one decimal place?

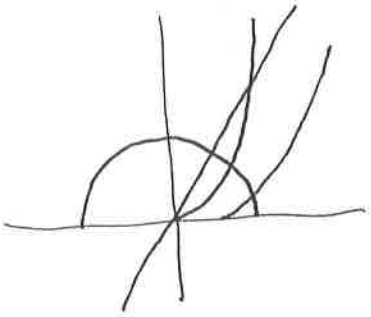
(A) 13.0 cm
(B) 20.9 cm
(C) 21.6 cm
(D) 40.6 cm

$\frac{c}{\sin C} = \frac{a}{\sin A}$
$\frac{c}{\sin 112^\circ} = \frac{13}{\sin 34^\circ}$
$c = \frac{13 \sin 112^\circ}{\sin 34^\circ}$
$= 21.5549\ldots$

4. Which of the following is *not* a one-to-one function?

(A) $y = 3x^2 - 3x, x \geq 1$
(B) $y = \sqrt{3 - x^2}$
(C) $y = 3x^2, x \geq 0$
(D) $y = 3x$

(must pass vertical line & horizontal line tests)



5. The line $5x - y - 4 = 0$ passes through the point $(k, 26)$. What is the value of k?

$5k - 26 - 4 = 0$

$5k = 30$

$k = 6$

(A) 3
(B) 4
(C) 5
(D) 6

6. Which of the following statements about the graph of the function $y = 2\sin 4x$ is correct?

$\text{period} = \frac{2\pi}{n} = \frac{2\pi}{4} = \frac{\pi}{2}$

(A) The amplitude is 2 and the period is $\frac{\pi}{4}$

(B) The amplitude is 2 and the period is $\frac{\pi}{2}$

(C) The amplitude is 4 and the period is $\frac{\pi}{2}$

(D) The amplitude is 4 and the period is π

7. What is the derivative of $x^{-2} - \frac{1}{x}$?

$y = x^{-2} - x^{-1}$

$y' = -2x^{-3} + x^{-2}$

$= \frac{-2}{x^3} + \frac{1}{x^2}$

(A) $\frac{-2}{x} - \frac{1}{x^2}$

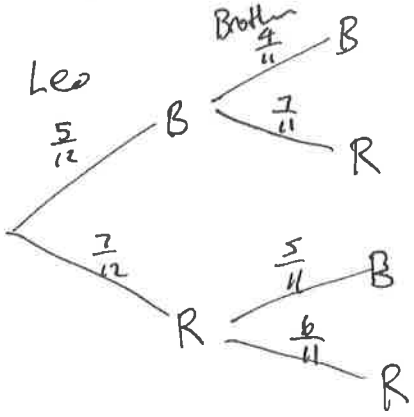
(B) $\frac{-2}{x} + \frac{1}{x^2}$

(C) $\frac{-2}{x^3} - \frac{1}{x^2}$

(D) $\frac{-2}{x^3} + \frac{1}{x^2}$

8. Leo owns five blue and seven red ties. He chooses a tie at random for himself and puts it on. He then chooses another tie at random, from the remaining ties, and gives it to his brother. What is the probability that both of the ties, are the same colour?

(A) $\frac{1}{2}$
(B) $\frac{5}{33}$
(C) $\frac{31}{66}$
(D) $\frac{31}{72}$



$P(\text{same}) = P(BB) + P(RR)$

$= \frac{5}{12} \times \frac{4}{11} + \frac{7}{12} \times \frac{6}{11}$

$= \frac{31}{66}$

9. Which of the following expressions is equal to $4 + 4\cot^2 x$?

(A) $4\sec^2 x$

(B) $4\operatorname{cosec}^2 x$

(C) $\cot^2 x$

(D) $8\cot^2 x$

$= 4(1 + \cot^2 x)$

$= 4\operatorname{cosec}^2 x$

10. If $\frac{4}{x-3} + \frac{2}{x} = 1, \quad x > 0$ then x is equal to:

(A) $\frac{9 \pm \sqrt{57}}{2}$

(B) $\frac{9 + \sqrt{57}}{2}$

(C) 2

(D) 7

$\frac{4x + 2(x-3)}{x(x-3)} = 1$

$4x + 2x - 6 = x^2 - 3x$

$0 = x^2 - 9x + 6$

$x = \frac{9 \pm \sqrt{81 - 24}}{2}$

$= \frac{9 \pm \sqrt{57}}{2}$

$= \frac{9 + \sqrt{57}}{2}$ or $\frac{9 - \sqrt{57}}{2}$

$= 8.27\ldots$ or 0.725

both are greater than 0.

Student's Name: _____ Teacher's Code: _____

Year 11 Mathematics Advanced– Multiple Choice Answer Sheet

Select the alternative A, B, C or D that best answers the question.
Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

- If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

- If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.

correct

A ● B ● C ○ D ○

---

1. A ○ B ● C ○ D ○
2. A ● B ○ C ○ D ○
3. A ○ B ○ C ● D ○
4. A ○ B ● C ○ D ○
5. A ○ B ○ C ○ D ●
6. A ○ B ● C ○ D ○
7. A ○ B ○ C ○ D ●
8. A ○ B ○ C ● D ○
9. A ○ B ● C ○ D ○
10. A ● B ○ C ○ D ○

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Section II **70 marks**
Attempt all questions (Question 11 – 31)
Allow about 1 hour and 45 minutes for this section

Answer the questions in the spaces provided. Your responses should include relevant mathematical reasoning and/or calculations.
Extra writing space is provided at the back of each section of the examination paper.

Algebraic Techniques (6 Marks)

Marks

Question 11

(a) Write in index form $\dfrac{1}{\sqrt[3]{x^2}}$ **1**

$x^{-\frac{2}{3}}$

(b) Express $\dfrac{3x+2}{x+1} - \dfrac{1}{x(x+1)}$ as a single fraction in simplest form. **3**

$= \dfrac{x(3x+2)-1}{x(x+1)}$

$= \dfrac{3x^2+2x-1}{x(x+1)}$

$= \dfrac{(3x-1)(x+1)}{x(x+1)}$

$= \dfrac{3x-1}{x}$

$3x^2+2x-1$
$= 3x^2+3x-x-1$
$= 3x(x+1)-1(x+1)$
$= (3x-1)(x+1)$

Question 12

Express $(2\sqrt{3}+1)(2-\sqrt{3})$ in the form $a+b\sqrt{3}$, and hence find the value of a and b. **2**

$= 4\sqrt{3} - 6 + 2 - \sqrt{3}$

$= -4 + 3\sqrt{3}$

ie $a = -4 \quad b = 3$

Introductory Calculus (14 Marks)

Question 13

Differentiate with respect to x.

(a) $y = 4x^{-3}$ **1**

$y' = -12x^{-4}$ or $\frac{-12}{x^4}$

(b) $y = \sqrt{5+x^2}$ **2**

$y = (5+x^2)^{\frac{1}{2}}$

$y' = \frac{1}{2}(5+x^2)^{-\frac{1}{2}} \cdot 2x$

$= x(5+x^2)^{-\frac{1}{2}}$

or

$= \frac{x}{\sqrt{5+x^2}}$

(c) $f(x) = \frac{3x^2-5}{2x+1}$ **2**

let $u = 3x^2-5$ let $v = 2x+1$

$u' = 6x$ $v' = 2$

$y' = \frac{vu' - uv'}{v^2}$

$= \frac{6x(2x+1) - 2(3x^2-5)}{(2x+1)^2}$

$= \frac{12x^2 + 6x - 6x^2 + 10}{(2x+1)^2}$

$= \frac{6x^2 + 6x + 10}{(2x+1)^2}$

(d) $y = 3x^2(2x-3)^4$ (Give answer in factored form) 3

let $u = 3x^2$ let $v = (2x-3)^4$

$u' = 6x$ $v' = 4(2x-3)^3 \cdot 2$

$= 8(2x-3)^3$

$y' = 6x(2x-3)^4 + 24x^2(2x-3)^3$

$= 6x(2x-3)^3[(2x-3) + 4x]$

$= 6x(2x-3)^3(6x-3)$

$= \boxed{18x(2x-3)^3(2x-1)}$

Question 14

$P(1, 4)$ lies on the curve $y = x^3 - 4x^2 + x + 6$

(a) Find the equation of the **tangent** to the curve at P. 2

$y' = 3x^2 - 8x + 1$

when $x = 1$

$m_{tangent} = 3(1)^2 - 8(1) + 1$

$= -4$

eq of tangent

$y - y_1 = m(x - x_1)$

$y - 4 = -4(x - 1)$

$y - 4 = -4x + 4$

$\boxed{y = -4x + 8}$

or

$4x + y - 8 = 0$

(b) Find the equation of the **normal** to the curve at P. 2

$m_{tangent} = -4$

$\therefore m_{normal} = \frac{1}{4}$

$y - y_1 = m(x - x_1)$

$y - 4 = \frac{1}{4}(x - 1)$

$y - 4 = \frac{1}{4}x - \frac{1}{4}$

$\boxed{y = \frac{1}{4}x + 3\frac{3}{4}}$

or

$y = \frac{x}{4} + \frac{15}{4}$

$4y - 16 = x - 1$

$\boxed{x - 4y + 15 = 0}$

or

(c) The tangent at P intersects the x-axis at T. 2
What are the coordinates of T?

tangent: $y = -4x + 8$

for x intercept let $y = 0$

$0 = -4x + 8$

$4x = 8$

$x = 2$

$\therefore \boxed{T(2, 0)}$

Extra Working Space.

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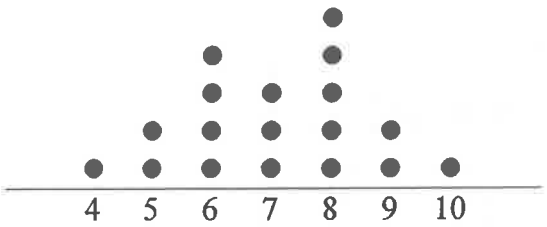
Probability (10 marks)

Question 15

The dot plot below shows the results of a Year 11 class quiz out of 10.



(a) Find the relative frequency of the score 8. (Express answer as a decimal correct to 2 d.p) **1**

$\frac{5}{18}$ = 0·28 (2d.p)

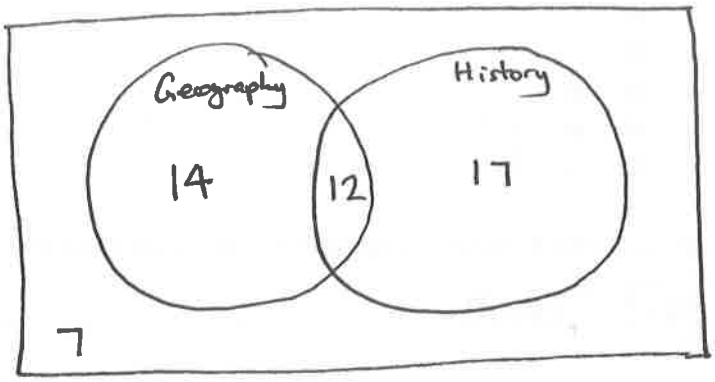
(b) A student from this class is selected at random. **1**
Find the probability that his score is greater than 7.
(Give answer as a simplified fraction)

$\frac{8}{18} = \frac{4}{9}$

Question 16

A group of 50 Year 11 students were asked if they studied History and/or Geography. It was found that 26 students study Geography and 29 students study History. 7 students study neither History nor Geography.

(a) Draw a Venn Diagram to represent this group of students. **1**



(b) Find the number of students who study both History and Geography. **1**

12

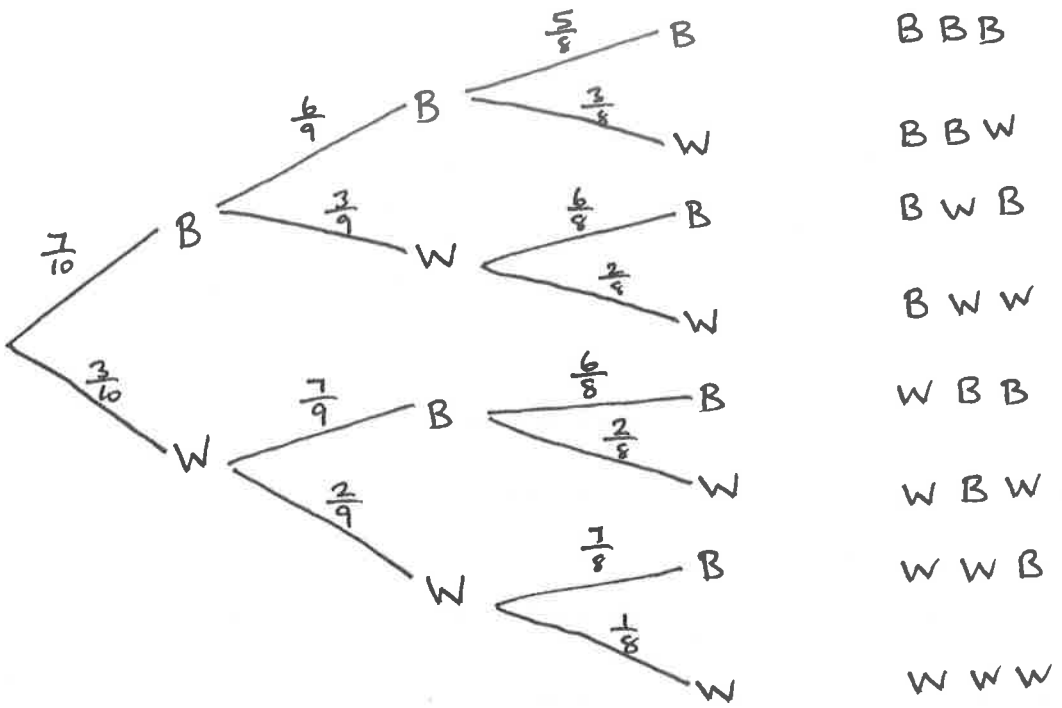
(c) Find the probability of selecting a student from this group that studies History given that he studies Geography. **1**

$\frac{12}{26} = \frac{6}{13}$

Question 17

A bag contains 7 blue and 3 white marbles. Three marbles are selected from the bag at random and not replaced.

(a) Draw a probability tree to represent this experiment and list the sample space. **2**



(b) Find the probability of selecting 3 blue marbles. **1**

$P(BBB) = \frac{7}{10} \times \frac{6}{9} \times \frac{5}{8}$

$= \boxed{\frac{7}{24}}$

(c) Find the probability of selecting exactly 2 blue marbles. **1**

$P(\text{exactly 2 Blue}) = P(BBW) + P(BWB) + P(WBB)$

$= \frac{7}{10} \times \frac{6}{9} \times \frac{3}{8} + \frac{7}{10} \times \frac{3}{9} \times \frac{6}{8} + \frac{3}{10} \times \frac{7}{9} \times \frac{6}{8}$

$= \boxed{\frac{21}{40}}$

(d) Find the probability of selecting at least one white marble. **1**

$P(\text{at least 1 white}) = 1 - P(BBB)$

$= 1 - \frac{7}{24} = \boxed{\frac{17}{24}}$

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Functions (17 Marks)

Question 18

The line l_1 passes through the points $P(-1, 2)$ and $Q(11, 8)$.

(a) Find an equation for l_1 in the form $y = mx + c$, where m and c are constants. **2**

$m = \frac{8-2}{11+1}$

$= \frac{6}{12}$

$= \frac{1}{2}$

equation of l_1

$y - y_1 = m(x - x_1)$

$y - 2 = \frac{1}{2}(x + 1)$

$y - 2 = \frac{1}{2}x + \frac{1}{2}$

$\boxed{y = \frac{1}{2}x + \frac{5}{2}}$

(b) Find the angle of inclination of l_1. (The angle between l_1 and the positive x axis.) **1**
Give answer to nearest degree.

$m = \tan\theta$

$\frac{1}{2} = \tan\theta \qquad \theta = 26.565...^\circ \qquad \boxed{\theta = 27^\circ}$ nearest degree

(c) The line l_2 passes through the point $R(10, 0)$ and is perpendicular to l_1. The lines l_1 and l_2 intersect at the point S. Find the coordinates of S. **3**

$m_1 = \frac{1}{2} \therefore m_2 = -2$

eq of l_2

$y - y_1 = m(x - x_1)$

$y - 0 = -2(x - 10)$

$y = -2x + 20$

solve l_1 and l_2 simultaneously

$y = \frac{1}{2}x + \frac{5}{2} \quad ....①$

$y = -2x + 20 \quad ....②$

$\therefore \frac{1}{2}x + \frac{5}{2} = -2x + 20$

$\frac{5}{2}x = 17\frac{1}{2}$

$5x = 35$

$x = 7$

$y = -2 \times 7 + 20$

$= 6$

$\therefore \boxed{S(7, 6)}$

Question 19 (2 marks)

A parabola has the equation $y = 5 + 4x - x^2$

(a) Find the equation of the axis of symmetry of $y = 5 + 4x - x^2$. 1

axis $x = -\frac{b}{2a} = \frac{-4}{-2}$

$\therefore$ $x = 2$

(b) Find the maximum value of the expression $5 + 4x - x^2$ 1

max value $= 5 + 4(2) - (2)^2$

$= 9$

Question 20 (2 marks)

$$f(x) = \begin{cases} x^2 - 5 & \text{when } x > 2 \\ 2x + 1 & \text{when } -2 < x \leq 2 \\ 3 & \text{when } x \leq -2 \end{cases}$$

Evaluate: $3f(2) - 6f(-4)$ 2

$f(2) = 2(2) + 1 = 5$

$f(-4) = 3$

$\therefore 3f(2) - 6f(-4)$

$= 3 \times 5 - 6 \times 3 = -3$

Question 21 (3 marks)

A circle has equation $x^2 + y^2 + 8x - 6y - 24 = 0$.

(a) Write the equation of the circle in the form $(x-h)^2 + (y-k)^2 = r^2$ and hence find the coordinates of the centre and the radius of the circle. 2

$x^2 + 8x + y^2 - 6x = 24$

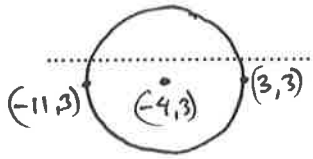
$x^2 + 8x + 16 + y^2 - 6x + 9 = 24 + 16 + 9$

$(x+4)^2 + (y-3)^2 = 49$

$\therefore$ centre $(-4, 3)$

radius $= 7$ units

(b) Find the domain of the circle. 1

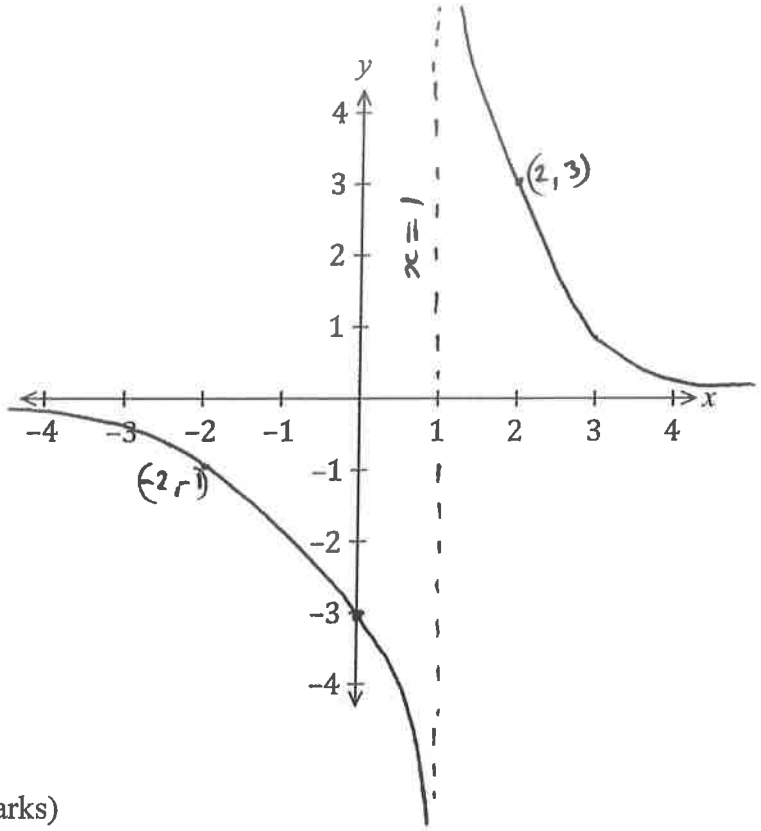


Domain $[-11, 3]$ or $-11 \leq x \leq 3$

Question 22 (2 marks)

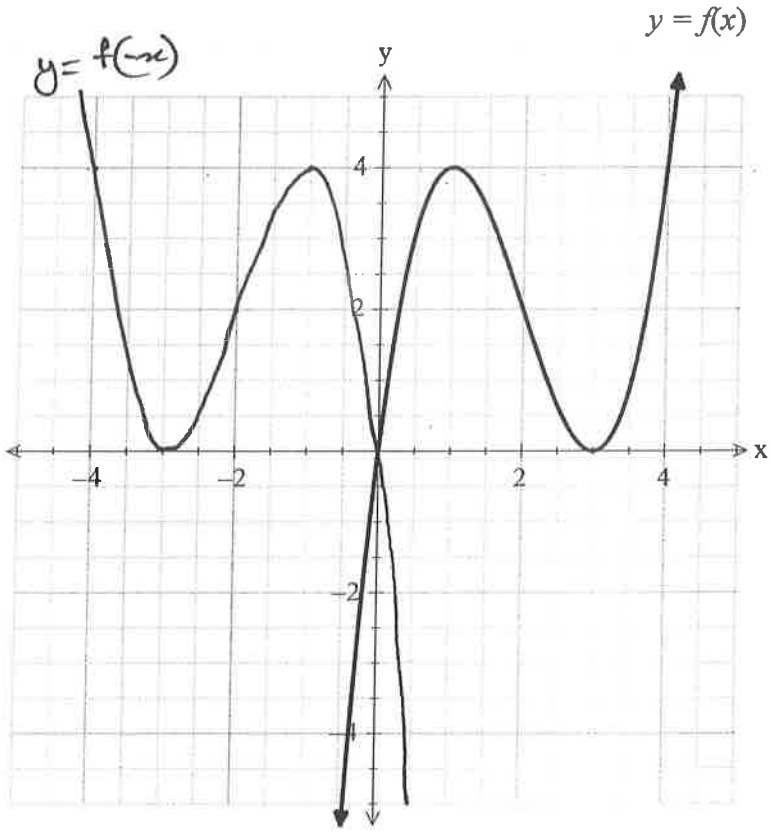
Sketch the graph of the function $y = \dfrac{3}{x-1}$ on the axes below. Show all asymptotes and intercepts. **2**

when $x=0$
$y = \frac{3}{0-1}$
$= -3$



Question 23 (2 marks)

The graph of $y = f(x)$ is sketched below.
On the same set of axes, sketch the graph of $y = f(-x)$ showing all features. **2**



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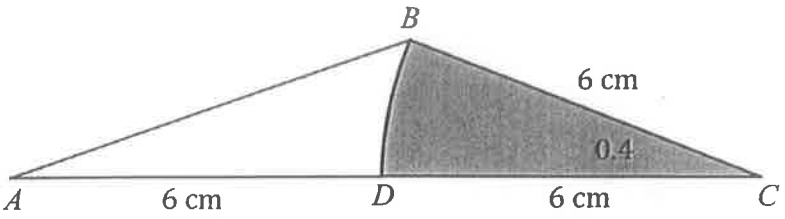
Trigonometry (13 marks)

Question 24



Not to scale

ΔABC has D as the midpoint of AC and $BC = CD = AD = 6$ cm. $\angle BCD$ is 0.4 radians. The curve BD is an arc of a circle with centre C and radius 6 cm.

(a) What is the length of arc BD? **1**

$L = r\theta$

$= 6 \times 0.4$

$= \boxed{2.4 \text{ cm}}$

(b) Find the area of sector BCD. Answer correct to one decimal place. **1**

$A = \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \times 6^2 \times 0.4$

$= \boxed{7.2 \text{ cm}^2}$

(c) Find the area of Triangle ABC. Answer correct to three significant figures. **1**

$A = \frac{1}{2} ab \sin C$

$= \frac{1}{2} \times 6 \times 12 \times \sin 0.4^c$

$= 14.01906....$

$= \boxed{14.0 \text{ cm}^2}$ (3 s.f)

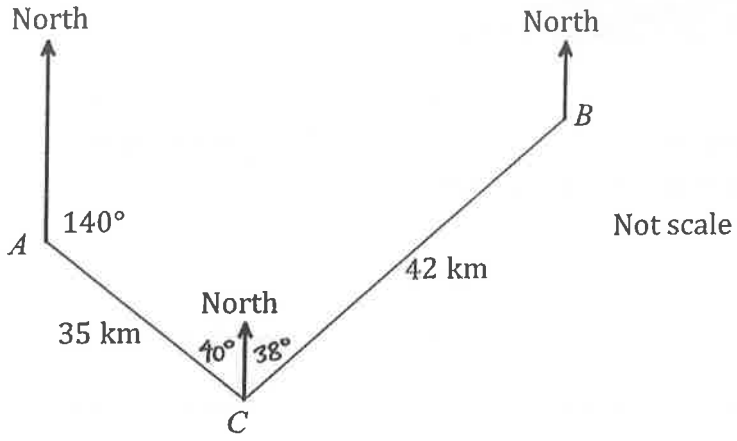
Question 25

Find the exact value of $\cot 330°$ 1

$= \frac{1}{\tan 330°}$

$= \frac{1}{-\tan 30°} = \frac{1}{-\frac{1}{\sqrt{3}}} = \boxed{-\sqrt{3}}$

Question 26



Tyler drives 35km from town A on a true bearing of $140°T$ to town C. He then drives 42km on a true bearing of $038°T$ to town B.

(a) Show that $\angle ACB = 78°$ 1

$40° + 38° = 78°$

(b) Find the distance from A to B. Answer correct to the nearest kilometre. 2

$c^2 = a^2 + b^2 - 2ab\cos C$

$= 42^2 + 35^2 - 2 \times 42 \times 35 \times \cos 78°$

$= 2377.739....$

$c = 48.76207...$

$\therefore AB = \boxed{49\text{ km}}$ nearest km

Question 27

Solve the equation $2\sin\theta + \sqrt{3} = 0$ in the interval $0° \leq \theta \leq 360°$. 2

$2\sin\theta = -\sqrt{3}$

$\sin\theta = -\frac{\sqrt{3}}{2}$

$\theta = 240°, 300°$

Question 28

Prove the identity $\frac{\sin x}{1-\cos x} = \frac{1+\cos x}{\sin x}$ 2

LHS $= \frac{\sin x}{1-\cos x}$

$= \frac{\sin x}{1-\cos x} \times \frac{1+\cos x}{1+\cos x}$

$= \frac{\sin x(1+\cos x)}{1-\cos^2 x}$

$= \frac{\sin x(1+\cos x)}{\sin^2 x}$

$= \frac{1+\cos x}{\sin x}$

= RHS as required.

Question 29

Solve: $2\cos^2 x + 3\cos x + 1 = 0 \qquad [0, 2\pi]$ **2**

let $m = \cos x$

$2m^2 + 3m + 1 = 0$

$2m^2 + 2m + m + 1 = 0$

$2m(m+1) + 1(m+1) = 0$

$(m+1)(2m+1) = 0$

$m = -1, -\frac{1}{2}$

ie $\cos x = -1$ $\qquad \cos x = -\frac{1}{2}$

$x = \pi$ $\qquad x = \frac{2\pi}{3}, \frac{4\pi}{3}$

ie $\boxed{x = \pi, \frac{2\pi}{3}, \frac{4\pi}{3}}$

Extra working

Name:_____ Teacher's Code : PPC MXF REM HBH JPN

Exponential & Logarithmic Functions (10 marks)

Question 30

Differentiate: $y = 2e^{3x}$ **1**

$y' = 2e^{3x} \cdot 3$

$= 6e^{3x}$

Question 31

The population of a city is changing over time according to the formula $P = 36000e^{0.03t}$ where P is the population and t is the time in years.

(a) Find the population after 5 years. **1**

$P = 36000e^{0.03 \times 5}$

$= 41826.032...$ ie $P = 41826$

(b) Find the rate at which the population is changing when $t = 10$ years. **2**

$P' = 36000e^{0.03t} \times 0.03$

$= 1080e^{0.03t}$

when $t = 10$

$P' = 1080e^{0.3}$

$= 1457.84...$

ie growing at rate of 1458 people/year

Question 32

Solve:

(a) $16^x = 120$ (Answer correct to 3 decimal places) **1**

$\ln 16^x = \ln 120$

$x \ln 16 = \ln 120$

$x = \frac{\ln 120}{\ln 16}$

$= \boxed{1.727}$ (3dp)

(b) $\log_3 x = -2$ **1**

$3^{-2} = x$

$\boxed{x = \frac{1}{9}}$

Question 33

Solve: $2\log_3 x - \log_3(x-2) = 2$ where $x > 2$ **2**

$\log_3 x^2 - \log_3(x-2) = 2$

$\log_3 \frac{x^2}{x-2} = 2$

$\frac{x^2}{x-2} = 9$

$x^2 = 9x - 18$

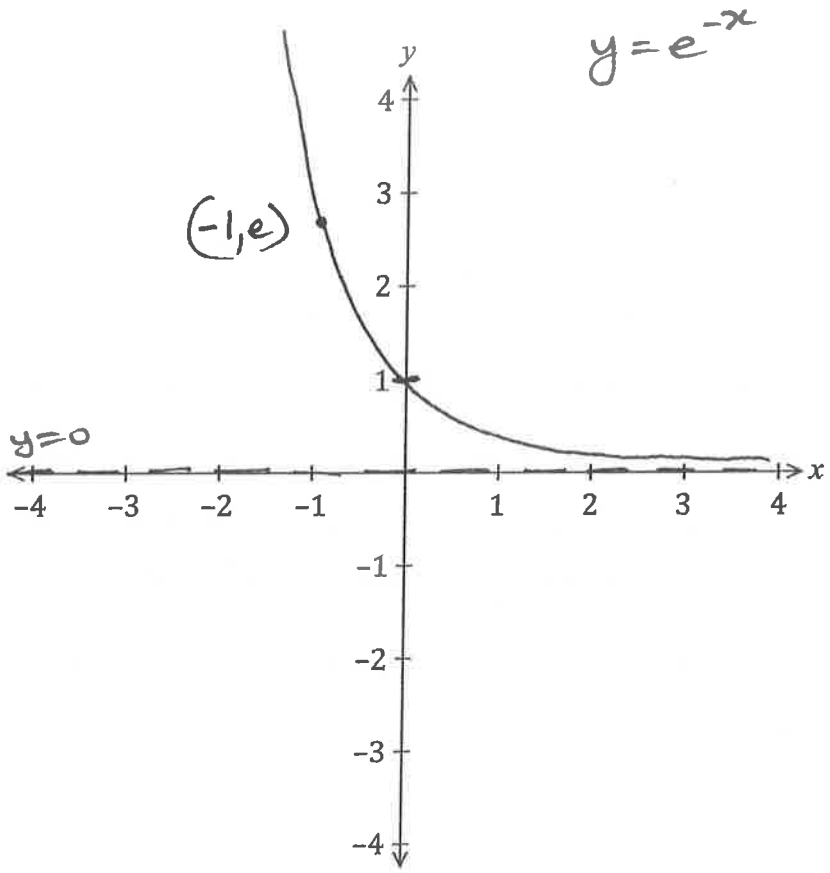
$x^2 - 9x + 18 = 0$

$(x-6)(x-3) = 0$

$\boxed{x = 6, 3}$

Question 34

Sketch the graph of $y = e^{-x}$ showing all important features. **2**



End of paper