

Students' Name: _____ Teacher: _____



Saint Ignatius' College, Riverview

Mathematics Assessment Task

2021

Year 11
Mathematics
Task 3
PRELIMINARY COURSE EXAMINATION
SEPTEMBER 2021

General Instructions:

- **Reading time: 5 Minutes**
- **Time Allowed: 90 Minutes**
- Write using blue or black pen only
- Board approved calculators may be used
- Attempt all questions in the space provided on the paper
- Write **your name** and **your teacher's name** in the positions indicated
- Marks may not be awarded for missing or carelessly arranged working

Teachers:

Mr R Maxwell	REM
Mr A Chan	AXC
Dr M Furtado	MXF
Mr J Newey	JPN
Ms K Mullan	KXM

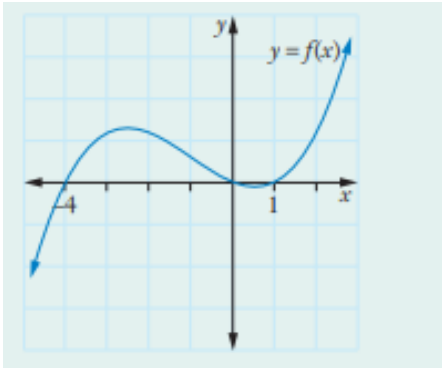
Topics Examined:

Question 1: Functions and Further Functions
Question 2: Trigonometry and Trigonometric Functions
Question 3: Introductory Calculus
Question 4: Exponential and Logarithmic Functions
Question 5: Probability

Total

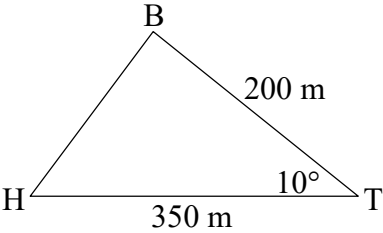
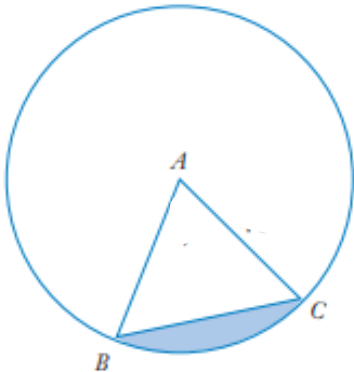
75 Marks

Question 1 Functions and Further Functions**Total 15 Marks***Please use a separate writing booklet.*

a)	If $f(x) = x^2 - x$, find in simplest form. i) $f(-x)$ ii) $f(1-x)$	2
b)	The gradient of the line through $(2, -1)$ and $(x, 0)$ is 3. Find the value of x.	2
c)	Consider the following function $f(x) = x^2 - x - 2$ State i) The Domain ii) The Range	2
d)	Make a neat sketch of the following function. $y = x - 2 $ Hence solve the following equation graphically $ x - 2 = 1$	3
e)	Given the following function  Sketch the graph of $y = -f(-x)$	2

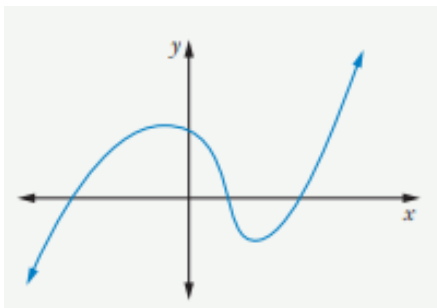
f)	Given the two functions listed below $f(x) = 4x^2 + 3$ $g(x) = 2x - 3$ Find the combined function i) $y = g(x) + f(x)$ ii) $y = g(f(x))$	4
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Question 2 Trigonometry and Trigonometric Functions**Total 15 Marks***Please use a separate writing booklet.*

a)	If $\cos \theta = 0.7$ and $\sin \theta < 0$, find the exact value for $\operatorname{Cosec} \theta$.	2
b)	 On a golf course, the distance from a tee T to the hole H is 350 m. A golfer's ball comes to rest at point B, 200 m from T. Angle HTB is 10°, as shown in the diagram. How far is B from H? Answer to nearest metre.	2
c)	Sketch the following function over the domain $[0, 2\pi]$ $y = 3 \sin 2x + 1$	3
d)	Given the following diagram  The area of a sector ABC is $\frac{3\pi}{10} \text{ cm}^2$ and the arc length BC cut off by the sector is $\frac{\pi}{5} \text{ cm}$. Find - The radius of the circle. - The angle subtended at the centre of the circle	3

e)	Solve the following equation for $[0, 2\pi]$ $\tan x + \tan x \cos x = 0$	2
f)	Prove the following $\frac{2 \sin^2 \theta}{1 - \cos \theta} = 2 + 2 \cos \theta$	3

Question 3 Introductory Calculus**Total 15 Marks***Please use a separate writing booklet.*

a)	The tangent to the curve $y = f(x)$ at the point P has the equation $y = 2x - 1$. What is the gradient of the normal at the point P?	1
b)	Sketch the gradient function of the curve shown below 	2
c)	Given the function $f(x) = x^2 - 2x$ Express $f(x+h) - f(x)$ in simplest form.	2
d)	Differentiate the following i) $f(x) = \frac{x}{\sqrt{x}}$ ii) $f(x) = x(x-1)^3$. (Answer in fully factored form).	4
e)	Find the equation of the tangent to the curve $f(x) = \frac{x}{x+1}$ at the point where $x = 1$. Express your answer in general form.	3
f)	The displacement x metres from the origin at time t seconds of a particle travelling in a straight line is given by $x = t^3 - 9t$, where $t \geq 0$. i. Find the velocity at time t seconds. ii. Calculate the velocity when $t = 2$. iii. Find the time when the particle is stationary.	3

Question 4 Exponential and Logarithmic Functions**Total 15 Marks***Please use a separate writing booklet.*

a)	Find the exact gradient of the normal to the curve $y = e^{2x-1}$ at the point where $x = 2$	2
b)	Solve for x , correct to 1 decimal place where necessary. i) $\log_8 x = \frac{2}{3}$ ii) $\log_x 3 = \frac{1}{2}$	2
c)	Given that $\log_a b = 2.75$ and $\log_a c = 0.25$, find the value of: $\log_a (bc)^2$	2
d)	Consider the function $y = \ln x + 1$. State the following i) Domain ii) Coordinates of the point P, where the curve intersects with the x axis .	3
e)	The population of a community is given by $P = 20000e^{0.02t}$ where t is time in years. i) Find the initial population. ii) Find in how many years will the population will reach 90000. (Answer to the nearest year)	3
f)	The line $y = px$ is a tangent to the curve $y = e^{3x}$. Find the value of p .	3

Question 5 Probability**Total 15 Marks***Please use a separate writing booklet.*

a)	A factory assembles torches. Each torch requires one battery and one bulb. It is known that 6% of all batteries and 4% of all bulbs are defective. Find the probability that, in a torch selected at random, both the battery and the bulb are NOT defective.	2												
b)	Neil and Fiona each throw a die. i. Find the probability that they throw the same number. ii. Find the probability that the number thrown by Neil is greater than the number thrown by Fiona.	3												
c)	A class of 32 students puts on a musical with all class members performing. If 16 dance and 21 sing in the performance, find the probability that any one student chosen at random from the class will i) Only dance ii) Both sing and dance	2												
d)	The table below shows the relative frequency of selecting each of the different coloured “Smarties” in a packet. <table border="1"><thead><tr><th>Colour</th><th>Relative frequency</th></tr></thead><tbody><tr><td>Green</td><td>0.32</td></tr><tr><td>Yellow</td><td>0.13</td></tr><tr><td>Black</td><td>0.14</td></tr><tr><td>Red</td><td></td></tr><tr><td>White</td><td>0.24</td></tr></tbody></table> i) What is the relative frequency of selecting a red “Smartie”? ii) If I select one “Smartie” at random, what is the probability that I select a green or yellow “Smartie”?	Colour	Relative frequency	Green	0.32	Yellow	0.13	Black	0.14	Red		White	0.24	2
Colour	Relative frequency													
Green	0.32													
Yellow	0.13													
Black	0.14													
Red														
White	0.24													
e)	A certain breed of sheep has a 30% chance of producing a male lamb. If a sheep has three lambs what is the probability that the sheep will produce i) Three male lambs. ii) At least one male lamb	2												

f)	At Saint Ignatius College students may decide to study Drama and Music. For a group of 40 students it is known that • 18 students study Music • 20 students study Drama • 7 students study neither Music or Drama i) A student is chosen at random. Find the probability that the student studies both Music and Drama. ii) A student is chosen at random. Given that the student studies Music, what is the probability that the student DOES NOT study Drama iii) Two different students are chosen at random, one after the other. What is the probability that the first student studies Drama and the second student DOES NOT study drama.	4
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SOLUTIONS

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75 Marks

SOLUTIONS

Question One

(a) $f(x) = x^2 - x$

i) $f(-x) = (-x)^2 - (-x)$
 $= x^2 + x$

ii) $f(1-x) = (1-x)^2 - (1-x)$
 $= 1 - 2x + x^2 - 1 + x$
 $= x^2 - x$

b) $(2, -1) (x, 0) \quad m = 3$

$$\frac{0 + 1}{x - 2} = \frac{3}{1}$$

$$1 = 3x - 6$$

$$3x = 7$$

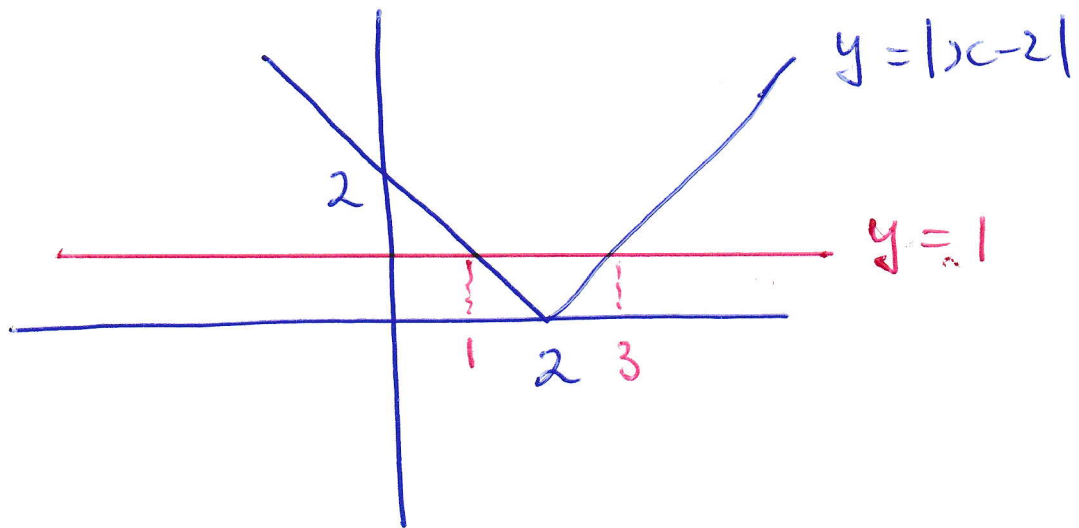
$$x = \frac{7}{3}$$

c) $f(x) = x^2 - x - 2$

i) Domain : $(-\infty, \infty)$

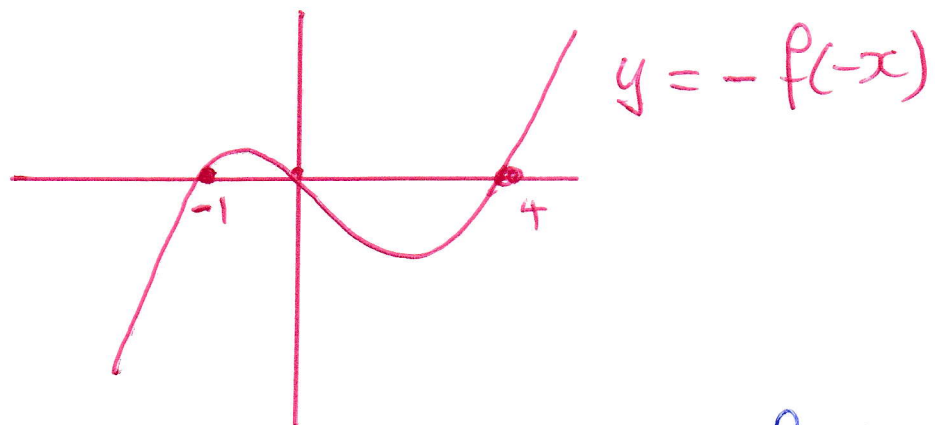
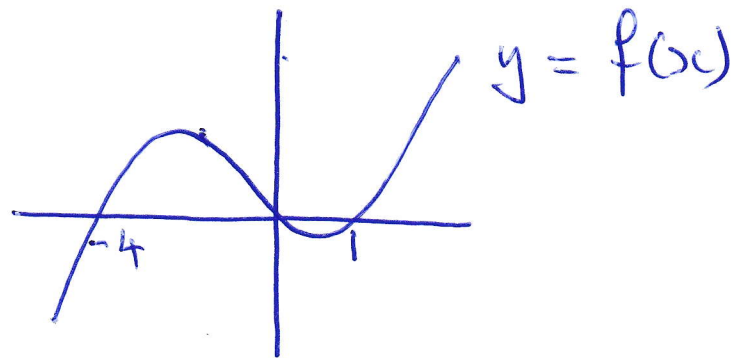
ii) Range : $x = -\frac{b}{2a} \quad y = \left(\frac{1}{2}\right)^2 - \frac{1}{2} - 2$
 $= -\frac{9}{4}$
 $= -\frac{1}{2} \therefore \text{Range } \left[-\frac{9}{4}, \infty\right)$

d)



$$x = 1 \quad x = 3$$

e)



f)

$$f(x) = 4x^2 + 3$$

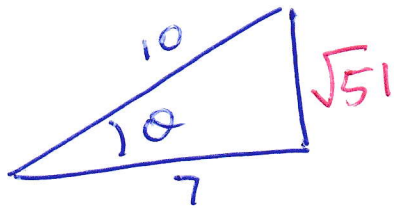
$$g(x) = 2x - 3$$

$$\begin{aligned} \text{i) } y &= g(x) + f(x) \\ &= 4x^2 + 2x \end{aligned}$$

$$\begin{aligned} \text{ii) } y &= g(f(x)) \\ &= 2(4x^2 + 3) - 3 \\ &= 8x^2 + 3 \end{aligned}$$

Question Two

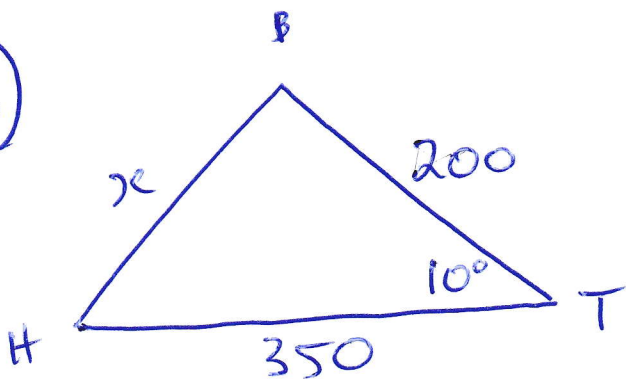
a) $\cos \theta = 0.7 = \frac{7}{10}$



$$\frac{\sqrt{}}{\sqrt{}}$$

$$\therefore \operatorname{cosec} \theta = \frac{-10}{\sqrt{51}} = \frac{-10\sqrt{51}}{51}$$

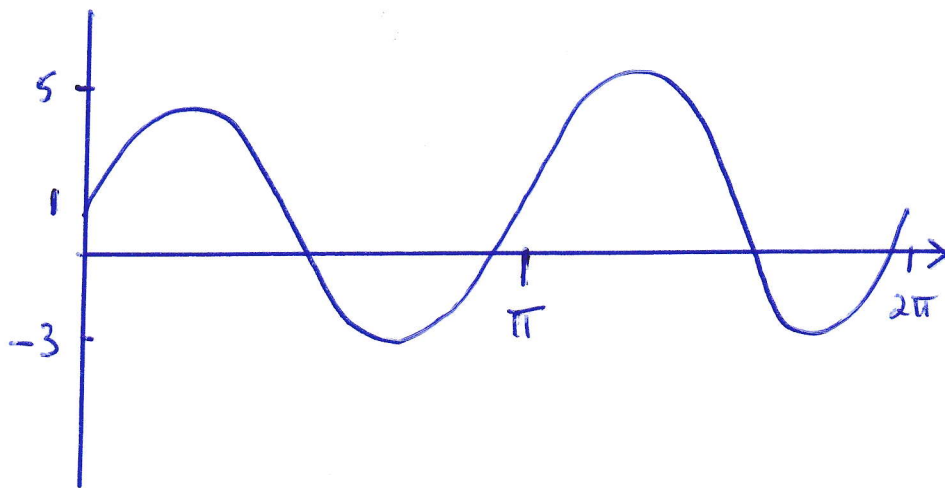
b)



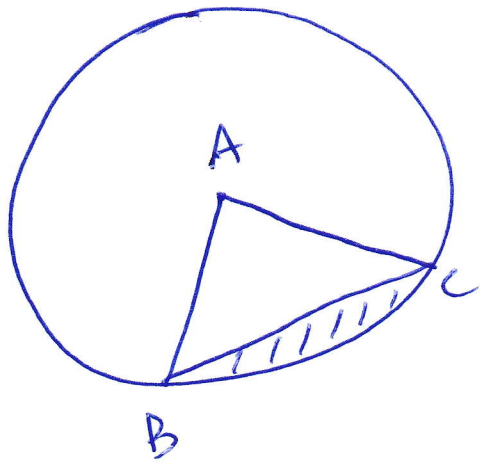
$$x^2 = 350^2 + 200^2 - 2 \times 350 \times 200 \cos 10$$

$$x = 157 \text{ m}$$

c)



d)



$$\text{Area} = \frac{3\pi}{10}$$

$$\text{Length} = \frac{\pi}{5}$$

$$\therefore \frac{1}{2} r^2 \theta = \frac{3\pi}{10}$$

$$\therefore r\theta = \frac{\pi}{5}$$

$$\therefore \theta = \frac{\pi}{5r}$$

$$\begin{aligned} \text{Now } \frac{1}{2} r^2 \theta &= \frac{1}{2} r^2 \frac{\pi}{5r} \\ &= \frac{1}{10} r\pi \end{aligned}$$

$$\text{So } \frac{1}{10} r\pi = \frac{3\pi}{10}$$

$$\text{i) } \therefore r = 3 \text{ cm}$$

$$\begin{aligned} \text{ii) } r\theta &= \frac{\pi}{5} \\ \theta &= \frac{\pi}{5} \div 3 \\ &= \frac{\pi}{15} \end{aligned}$$

$$\text{e) } \tan x + \tan x \cos x = 0 \quad [0, 2\pi]$$

$$\tan x [1 + \cos x] = 0$$

$$\therefore \tan x = 0 \quad \cos x = -1$$

$$x = 0, \pi, 2\pi \quad x = \pi$$

$$\therefore x = 0, \pi, 2\pi$$

f) Prove $\frac{2\sin^2\theta}{1-\cos\theta} = 2 + 2\cos\theta$

$$\text{LHS} = \frac{2\sin^2\theta}{1-\cos\theta}$$

$$= \frac{2(1-\cos^2\theta)}{1-\cos\theta}$$

$$= \frac{2(1-\cancel{\cos\theta})(1+\cos\theta)}{1-\cancel{\cos\theta}}$$

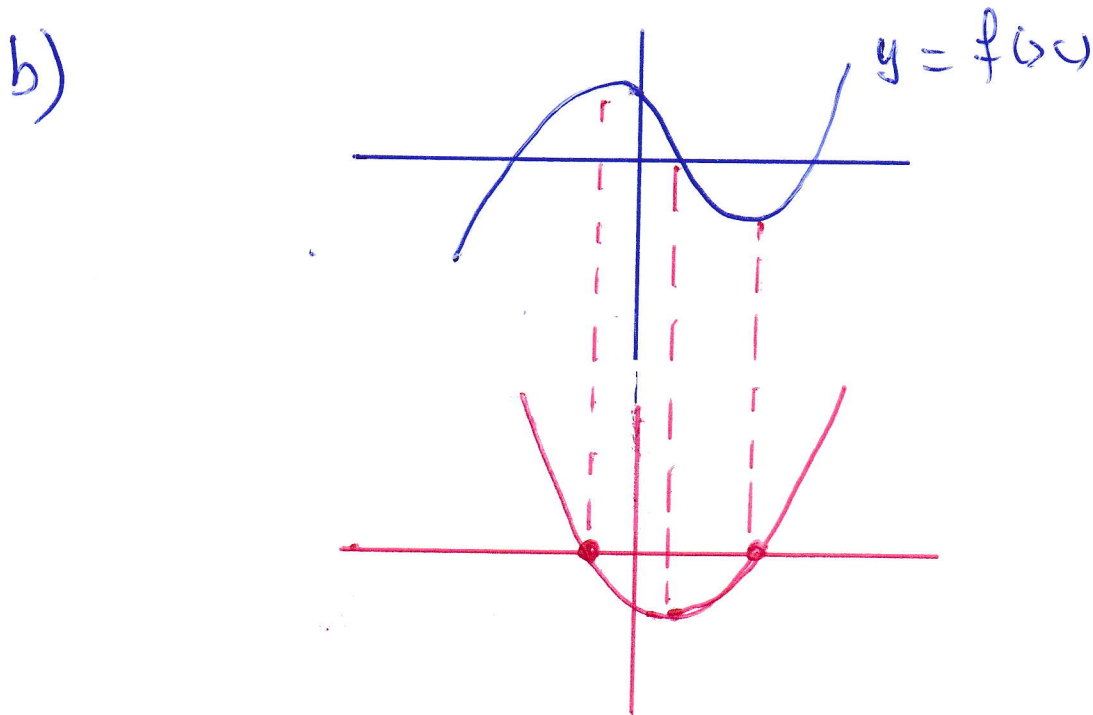
$$= 2 + 2\cos\theta$$

$$= \text{RHS}$$

Question Three

a) $y = 2x - 1$ $M = 2$

∴ M of Normal $= -\frac{1}{2}$



c) $f(x) = x^2 - 2x$

$$\begin{aligned} f(x+h) - f(x) &= (x+h)^2 - 2(x+h) - [x^2 - 2x] \\ &= \cancel{x^2} + 2xh + h^2 - \cancel{2x} - 2h - \cancel{x^2} + \cancel{2x} \\ &= 2xh + h^2 - 2h \end{aligned}$$

$$\begin{aligned}
 d) \quad i) \quad f(x) &= \frac{x}{\sqrt{x}} \\
 &= \sqrt{x} \\
 &= x^{\frac{1}{2}}
 \end{aligned}
 \quad \therefore f'(x) = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

$$\begin{aligned}
 ii) \quad f(x) &= x(x-1)^3 \\
 f'(x) &= x \cdot 3(x-1)^2 + (x-1)^3 \cdot 1 \\
 &= (x-1)^2(3x + x - 1) \\
 &= (x-1)^2(4x-1)
 \end{aligned}$$

$$\begin{aligned}
 e) \quad f(x) &= \frac{x}{x+1} \\
 f'(x) &= \frac{(x+1) \cdot 1 - x \cdot 1}{(x+1)^2} \\
 &= \frac{1}{(x+1)^2}
 \end{aligned}$$

$$\text{at } x = 1 \quad m_T = \frac{1}{4} \quad y = \frac{1}{2}$$

$$\begin{aligned}
 \text{Tangent} \quad y - \frac{1}{2} &= \frac{1}{4}(x-1) \\
 4y - 2 &= x - 1 \\
 0 &= x - 4y + 1
 \end{aligned}$$

f) $x = t^3 - 9t$

i) $v = 3t^2 - 9$

ii) $t = 2 \quad v = 3(4) - 9$
 $= 3 \text{ m/s.}$

iii) $v = 0$ when $t = ??$

$$\begin{aligned} 0 &= 3t^2 - 9 \\ &= t^2 - 3 \\ &= (t - \sqrt{3})(t + \sqrt{3}) \end{aligned}$$

$\therefore t = \sqrt{3}$ seconds ($t = -\sqrt{3}$
Not a sol'n)

Question Four

$$a) \quad y = e^{2x-1} \quad x = 2$$

$$y' = 2e^{2x-1} \quad M_T = 2e^3$$

$$\therefore M_n = \frac{-1}{2e^3}$$

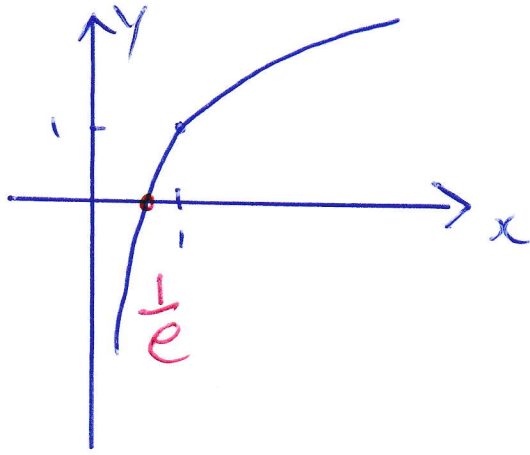
$$b) \quad i) \quad \log_8 x = \frac{2}{3} \quad \therefore x = 8^{\frac{2}{3}} = 4.$$

$$ii) \quad \log_x 3 = \frac{1}{2} \quad \therefore x^{\frac{1}{2}} = 3 \\ x = 9.$$

$$c) \quad \log_a b = 2.75 \quad \log_a c = 0.25$$

$$\begin{aligned} \log_a (bc)^2 &= 2 \log_a (bc) \\ &= 2 [\log_a b + \log_a c] \\ &= 2 [2.75 + 0.25] \\ &= 6. \end{aligned}$$

d)



i) $D: (0, \infty)$

ii) Let $y = 0$

$$0 = \ln x + 1$$

$$\ln x = -1$$

$$x = e^{-1}$$

$$= \frac{1}{e}$$

$$P\left(\frac{1}{e}, 0\right)$$

e) $P = 20000 e^{0.02t}$

i) $t = 0 \quad P = 20000$

ii) $90000 = 20000 e^{0.02t}$

$$\frac{9}{2} = e^{0.02t}$$

$$0.02t = \ln \frac{9}{2}$$

$$t = \frac{\ln \frac{9}{2}}{0.02}$$

$$= 75 \text{ yrs.}$$

$$p) \quad y = px$$
$$y' = p$$

$$y = e^{3x}$$
$$y' = 3e^{3x}$$

at point where $y = px$ is tangent to
 $y = e^{3x}$

$$\textcircled{1} \quad p = 3e^{3x}$$

$$\textcircled{2} \quad px = e^{3x}$$



$$p = 3px$$

$$\therefore x = \frac{1}{3}$$

$$\text{at } x = \frac{1}{3} \quad y = e^{3(\frac{1}{3})}$$
$$= e$$

$\therefore$ Pt at Tangent is $(\frac{1}{3}, e)$

$$y = px \quad \therefore e = p \cdot \frac{1}{3}$$
$$p = 3e.$$

Question Five

a) $P(\text{Batt det}) = \frac{6}{100}$ $P(\text{Bulb det}) = \frac{4}{100}$

$$P(E) = \frac{94}{100} \times \frac{96}{100}$$
$$= \frac{564}{625}$$

b) i) $P(\text{same}) = 1 \times \frac{1}{6}$

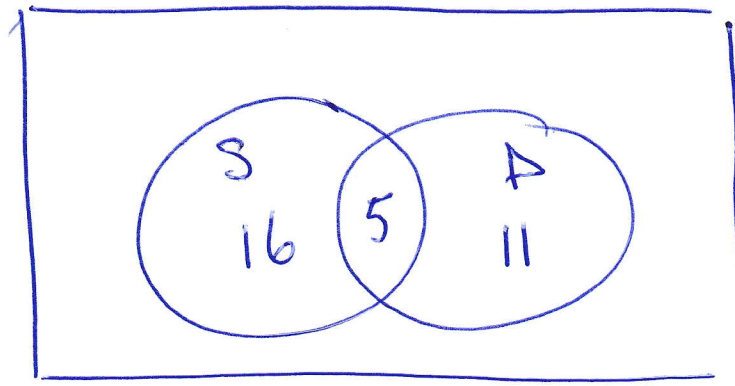
$$= \frac{1}{6}$$

ii)

		NEIL				
	1	2	3	4	5	6
FIONA	1	12	13	14	15	16
	2	21	22	23	24	25 26
	3	31	32	33	34	35 36
	4	41	42	43	44	45 46
	5	51	52	53	54	55 56
	6	61	62	63	64	65 66

$$P = \frac{15}{36}$$
$$= \frac{5}{12}$$

c)



$$i) P(\text{only } D) = \frac{11}{32}$$

$$ii) P(S \cap D) = \frac{5}{32}$$

$$d) \quad i) 1 - [0.32 + 0.13 + 0.14 + 0.24] \\ = 0.17.$$

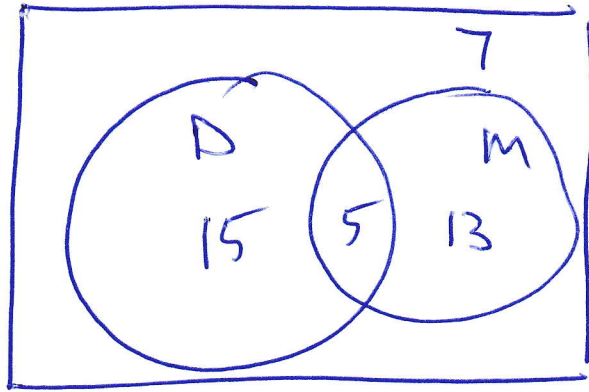
$$ii) P(\text{G or Y}) = 0.45$$

$$e) P(m) = 0.3$$

$$i) P(m m m) = (0.3)^3 \\ = 0.027$$

$$ii) P(\text{at least 1 male}) = 1 - P(\text{no male}) \\ = 1 - (0.7)^3 \\ = 0.657$$

p)



$$\begin{aligned} \text{i) } P(M \cap D) &= \frac{5}{40} \\ &= \frac{1}{8} \end{aligned}$$

$$\begin{aligned} \text{ii) } P(\bar{D} | M) &= \frac{P(\bar{D} \cap M)}{P(M)} \\ &= \frac{\frac{13}{40}}{\frac{18}{40}} \\ &= \frac{13}{18} \end{aligned}$$

$$\begin{aligned} \text{iii) } P(D \text{ then } \bar{D}) &= \frac{20}{40} \times \frac{20}{39} \\ &= \frac{10}{39} \end{aligned}$$