Students' Name: ----------Teacher Code: ----------

Saint Ignatius' College, Riverview
Mathematics Assessment Task
2022

Year 11
Mathematics
Task 3
PRELIMINARY COURSE EXAMINATION
SEPTEMBER 15 2022

General Instructions:

- **Reading time: 5 Minutes**
- **Time Allowed: 120 Minutes**
- Write using blue or black pen only
- Board approved calculators may be used
- A NESA formula sheet is provided
- Attempt all questions in the answer booklet that is provided.
- Write **your name** and **your teacher's name** in the positions indicated
- Marks may not be awarded for missing or carelessly arranged working

Teachers:	**Codes**
Mr P Collins	PPC
Mr J Newey	JPN
Ms J Barlow	JMB
Dr M Furtado	MXF

Topics Examined:

Question 1: Algebra, Equations, and Inequalities
Question 2: Functions and Further Functions
Question 3: Trigonometry
Question 4: Calculus
Question 5: Probability and Calculus
Question 6: Exponential and Logarithmic Functions
Question 7: Trigonometric Functions

Total **84 Marks**

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Question 1 **Algebra, Equations, and Inequations** **Total 12 Marks**

Please use a separate writing booklet.

a. Simplify $\dfrac{x^2 - 5xy}{x^2 - 25y^2}$ **2**

b. Find the value of n if $\sqrt{108} - \sqrt{12} = \sqrt{n}$. **2**

c. Solve $-2 < 3y + 1 \leq 10$ and plot the solution on a number line. **2**

d. Solve $2^{3y-2} = 16$ **2**

e. Solve $x^2 = |x|$. **2**

f. In a right-angled triangle, one of the sides adjacent to the right angle is 6cm longer than the other adjacent side. **2**

Find the length of the hypotenuse of this triangle if it's area is 216cm^2.

Question 2 Functions and Further Functions **Total 12 Marks**

Please use a separate writing booklet.

a. A function is defined as:

$$f(x)\begin{cases} 36 - x^2 & x > 3 \\ x^3 & 1 < x \leq 3 \\ |x| & x \leq 1 \end{cases}$$

i. Find $f(-2)$. **1**

ii. Find $f(3)$. **1**

b. Functions f and g are defined on suitable domains by: **2**

$f(x) = \sqrt{9 - x}$ and $g(x) = (1 + x)^2$

Find an expression for $f(g(x))$ in the simplest form.

c. **i.** Sketch the function $|x - 3|$. **1**

ii. Hence, solve graphically, $|x - 3| = 2$. **2**

d. **i.** Given $f(x) = (x + 1)^3$. Find $f(-x)$. **1**

ii. Hence, sketch $f(-x)$. Showing all intercepts. **1**

e. **i.** Show whether the function $f(x) = \dfrac{-2}{x + 1}$ is odd,even or neither.

(need to show all steps for full marks) **2**

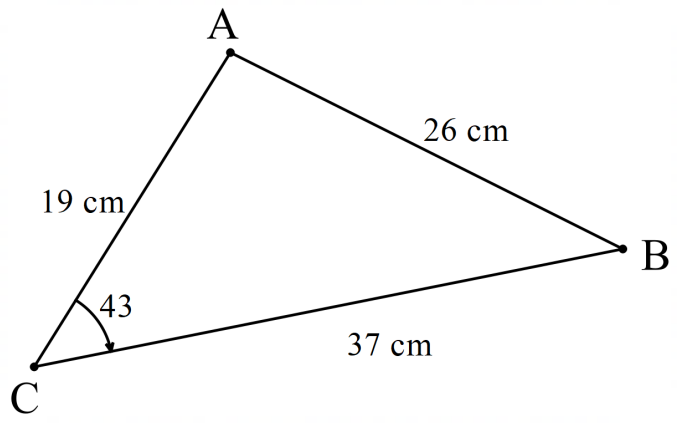
ii. Hence, sketch $f(x)$ showing all features of the graph. **1**

Question 3 **Trigonometry** **Total 12 Marks**

Please use a separate writing booklet.

a. In the diagram given below: **2**

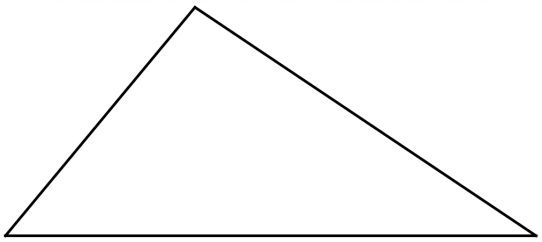
Find $\angle ABC$ in $\Delta\ ABC$ to the nearest minute.



b. Find the exact value of cos 225° + sec 60°? **2**

(single fraction form)

c. Aria and Ben are flying kites in the local park when their kites collide mid-air. The angles of elevation of Aria and Ben's kites are 50° and 31° respectively. Aria's kite string was 13 metres and Ben's kite string was 19 metres long at the time of impact.



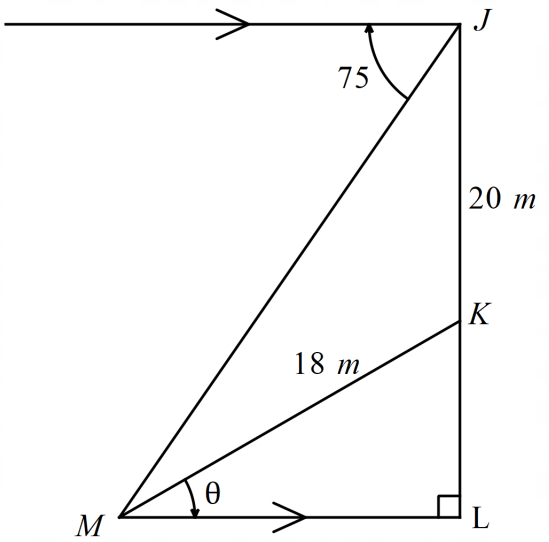
i. Show all the above information with the diagram on your answer booklet. **1**

ii. Using cosine rule or otherwise find distance between Aria and Ben when their kites collide. **2**

Answer correct to the nearest whole number.

d. A circle of area 200 cm^2 has a sector cut off by an angle of $\frac{3\pi}{4}$ at the centre. Find the area of the sector. **2**

e. The angle of depression from J to M is 75°. The length of JK is 20 m and the length of MK is 18 m. Copy this diagram into your booklet and calculate the angle of elevation θ from M to K. Give your answer to the nearest degree. **3**



Question 4 Calculus **Total 12 Marks**

Please use a separate writing booklet.

a. Differentiate each of the following functions:

i. $y = 7x^2 - 2x + 4$. **2**

ii. $y = \sqrt{4 - x}$. (Solution in positive index form) **2**

iii. $y = 8x(x^2 - 4)^4$. (Solution fully factorised) **2**

iv. $y = \dfrac{2x + 3}{4x + 9}$. (Solution fully factorised) **2**

b. A student was asked to differentiate $f(x) = x^2 + 3x + 4$ from first principles. **2**

The student began the solution as shown below. Complete the solution.

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} .$$

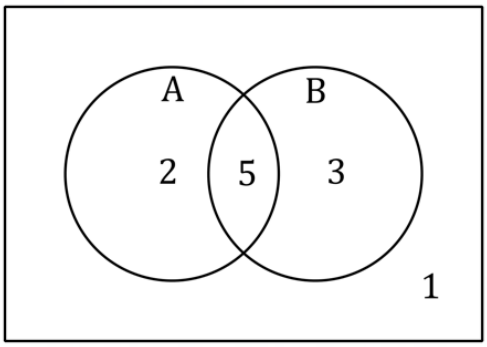
c. The displacement x cm of a particle over time t seconds is given by $x = 5 + 6t - 3t^2$.

i. Find the initial displacement of the particle. **1**

ii. When is the particle at rest? **1**

Question 5 **Probability and Calculus** (Six marks each) **Total 12 Marks**

Please use a separate writing booklet.



a. What is the value of $P(A \cap B)$ in the above diagram? **1**

b. There are two containers of identically sized balls. One container contains 5 blue and 6 green balls while the other container contains 3 blue and 2 green balls. Oscar randomly chooses one ball from each container.

i. What is the probability that both balls are green? **1**

ii. What is the probability that at least one ball is blue? **1**

iii. What is the probability that both balls not the same colour? **2**

c. A curve has the cubic equation $y = \frac{1}{3}x^3 - 4x^2 + 8x + 5$.

The point P lies on this curve and has the coordinates $(3,2)$.
Find the equation of the tangent at P. **3**

d. **i.** Find the gradient of the normal to the curve $f(x) = e^{2x}$ at $x = 5$, in exact form. **2**

ii. Hence find the equation of the normal to the curve $f(x)$ at $x = 5$. **2**

Question 6 **Exponential and Logarithmic Functions** **Total 12 Marks**

Please use a separate writing booklet.

a. Solve $\log x^2 - \log 3x = \log 9$. **2**

b. Evaluate $\log_6 12 + \frac{1}{3}\log_6 27$. **2**

c. Solve $e^{2\ln x} = x + 12$. $(x > 0)$. **2**

d. The number of birds, N, in a park at time t days, is modelled by the formula: $N = 100000e^{-0.15t}$

i. What is the number of birds in the park when $t = 40$? **2**

ii. For what value of t are there 100 birds in the park? Answer correct to the nearest day. **2**

e. Sketch the curve $y = 3\ln(2x) + 4$. **2**
Clearly stating the asymptote and x intercept if any.

Question 7 Further Trigonometric Functions **Total 12 Marks**

Please use a separate writing booklet.

a. What is the exact value of $\sin(-480°)$. **2**

b. Show that $(\sin x + \cos x)^2 + (\sin x - \cos x)^2 = 2$ **2**

c. Given $\tan\theta = \frac{3}{4}$ and $\cos\theta < 0$, find $\sin\theta$. **2**

d. Graph the following functions, showing clearly all points of intersections, domain and range on the graph.

i. $y = 2\sin 2x \quad \text{for } 0 \leq x \leq \pi.$ **2**

ii. $y = 2\cos x + 1 \quad \text{for } 0 \leq x \leq 2\pi.$ **2**

e. Solve the equation $\sin\left(2x + \frac{\pi}{3}\right) = \frac{1}{2} \quad \text{for } 0 \leq x \leq 2\pi.$ **2**

END OF TASK 3

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QUESTION ONE

Start your answer here.

1 a. Simplify $\frac{x^2 - 5xy}{x^2 - 25y^2}$

$= \frac{x(x-5y)}{(x-5y)(x+5y)}$

$= \frac{x}{x+5y}$ ← 1mk

← 1mk.

b. Find n: $\sqrt{108} - \sqrt{12} = \sqrt{n}$

$\sqrt{9 \times 12} - \sqrt{12}$

$3\sqrt{12} - \sqrt{12}$

$2\sqrt{12} = \sqrt{n}$.

$\therefore \sqrt{48} = \sqrt{n}$ ← 1mk

$n = 48$ ← 1mk

c. $-2 < 3y + 1 \leq 10$

$-2 - 1 < 3y \leq 10 - 1$

$-3 < 3y \leq 9$

$\div 3 \quad -1 < y \leq 3$ ← 1mk

← 1mk

d. $2^{3y-2} = 16$

$2^{3y-2} = 2^4$

$\therefore 3y - 2 = 4$ ← 1mk

$3y = 6 \qquad y = 2$ ← 1mk

e. Solve $x^2 = |x|$

$x^2 = x$ $\qquad$ $x^2 = -x$

$x^2 - x = 0$ $\qquad$ $x^2 + x = 0$

$x(x-1) = 0$ $\qquad$ $x(x+1) = 0$

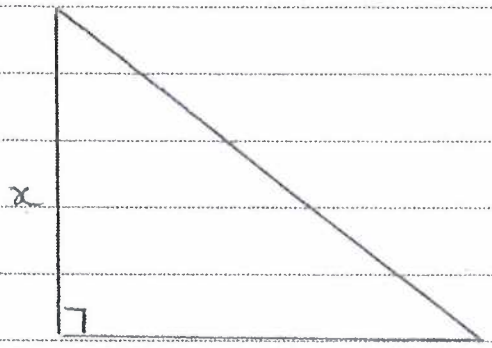
$x = 0, x = 1$ $\qquad$ $x = 0, x = -1$

$\therefore \quad x = 0, \; x = 1, \; x = -1$

1mk for two correct

2mks for three

f.



$A = \frac{1}{2} \times b \times h = \frac{1}{2} \times x(x+6) = 216$

$x(x+6) = 432$

$x^2 + 6x - 432 = 0$

$x = \frac{-6 \pm \sqrt{(6)^2 - 4(1)(-432)}}{2(1)}$ Quad. formula.

$= \frac{-6 \pm \sqrt{1764}}{2} = 18, -24$ 1mk for $x = 18$

$x \neq -24 \quad \therefore \quad x = 18$ 1mk for $h = 30$

$\therefore$ 3 Sides are 18, 24, and 30.

Hypotenuse = 30 cms.

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Question 2 marking

Start your answer here.

2a. (i) $f(-2) = |x| = |-2| = 2$ ✓ — 1 mark

(ii) $f(3) = x^3 = (3)^3 = 27$ ✓ — 1 mark

b. $f(x) = \sqrt{9-x}$

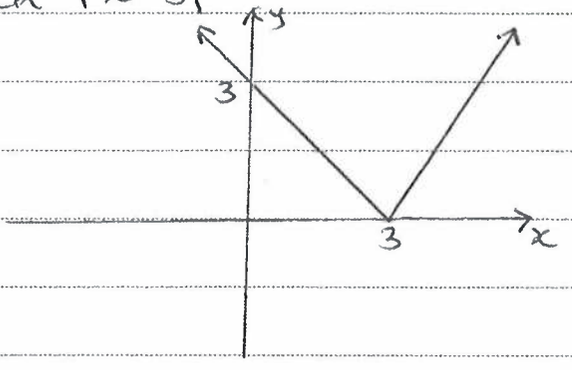
$g(x) = (1+x)^2$

$f[g(x)] = \sqrt{9-(1+x)^2}$ ✓ — 1 mark

$= \sqrt{(3-1-x)(3+1+x)}$

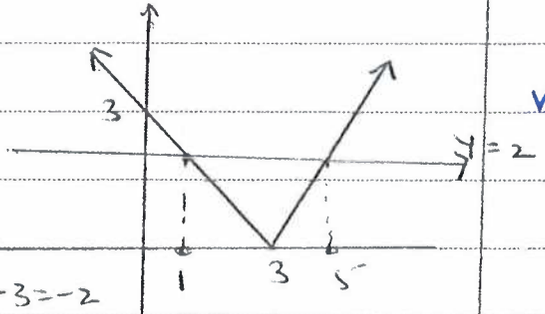
$= \sqrt{(2-x)(4+x)}$ OR $\sqrt{8-2x-x^2}$ — 1 mark

c. (i) Sketch $|x-3|$



✓ 1 mark graph & intercepts

(ii)



✓ 1 mark for graphing $y=2$

$x-3=2$ $\quad x-3=-2$

$x=5$ $\quad x=1$

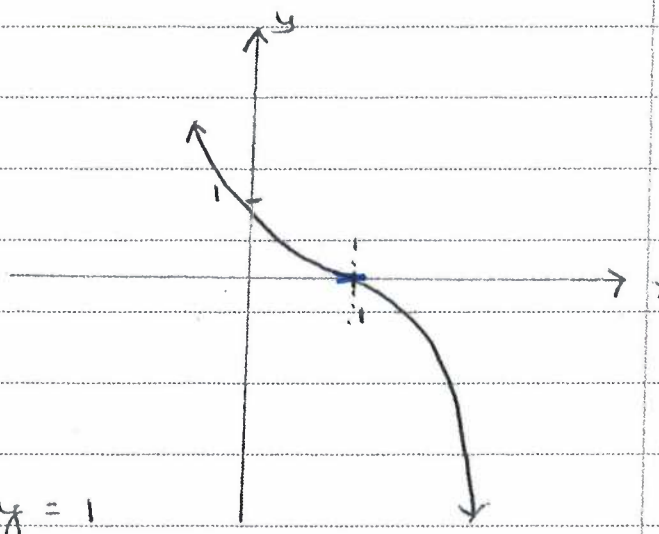
✓ 1 mark for solutions

d. i) $f(x) = (x+1)^3$

$f(-x) = (-x+1)^3 = -(x-1)^3$ → or $(1-x)^3$

√1 mark

(ii)



√ 1 mark

y intercept + shape

& 1 mark

saddle at $x=1$

$f(-x)$:

$x = 0$, $y = 1$

$y = 0$ $x = 1$

e. $f(x) = \frac{-2}{x+1}$

2 marks

* Needed enough working & conclusion 'neither'

$f(-x) = \frac{-2}{-x+1} = \frac{-2}{-(x-1)} = \frac{2}{x-1}$

$\neq f(x)$ Not even.

$-f(+x) = -\left(\frac{-2}{x+1}\right) = \frac{2}{x+1}$

$\neq f(x)$ Not Odd.

∴ Neither.

→ If attempted this test

√ 1 mark

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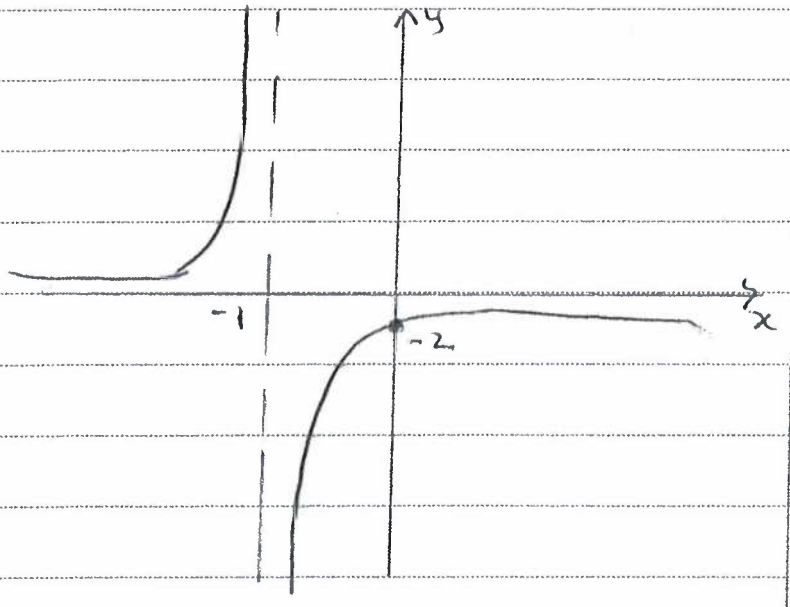
ii. Sketch $f(x) = \frac{-2}{x+1}$



✓ 1 mark

required asymptote $x = -1$

& shape in correct quadrants

You may ask for an extra Writing Booklet if you need more space to answer this question.

Q3 Marker Comments GJA.

Start your answer here.

3 a. $\frac{\sin B}{19} = \frac{\sin C}{26}$ ✓ application of 'sine rule' to connect angles and sides.

$\therefore \sin B = \frac{\sin 43 \times 19}{26}$

$= 0.498\ldots$

$\angle ABC = \sin^{-1}(0.498\ldots$

$= 29^0 54'$. ✓

b. $\cos 225^- + \sec 60$

$-\cos 45 + \frac{1}{\cos 60}$

	45^-
225^-	

✓ $-\frac{1}{\sqrt{2}} + 2$ ✓

1 mark each.

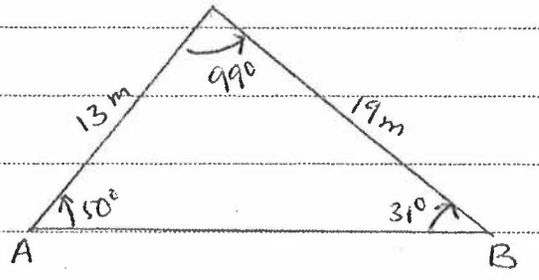
$\cos 225 = -\frac{1}{\sqrt{2}}$

$\sec 60^\circ = 2$.

$= \frac{-1+2\sqrt{2}}{\sqrt{2}}$

or $\frac{-\sqrt{2}+4}{2} = \frac{4-\sqrt{2}}{2}$

c. (i)



✓ all information.

ii. Cosine rule:

$c^2 = a^2 + b^2 - 2ab\cos C$

$= 13^2 + 19^2 - 2 \times 13 \times 19 \times \cos 99$ ✓ correct application of cosine rule to find a missing side.

$= 607.278...$

$c = 24.643...$

$c = 25$ m (Nearest whole no) ✓

d. $A_{\text{Circle}} = \pi r^2 = 200\ cm^2$ ✓

$r^2 = \frac{200}{\pi}$ OR $r = \sqrt{\frac{200}{\pi}}$ ✓ an expression to find the value of 'r'

$\theta = \frac{3\pi}{4}$

$A_{\text{sector}} = \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \times \frac{200}{\pi} \times \frac{3\pi}{4}$

$= 75\ cm^2$. ✓

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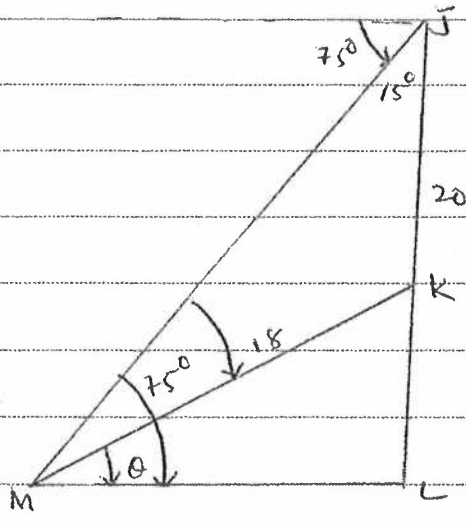
e.



✓ diagram information

In △ JMK : by Sine Rule.

$$\frac{\sin\alpha}{20} = \frac{\sin 15}{18}$$

$$\sin\alpha = \frac{20 \times \sin 15}{18}$$

$$= 0.2875\ldots$$ ✓

$$\alpha = 16.71 = 17^0$$

$\therefore \theta = 75 - 17$

$= 58^0$

75 - answer ✓

'correct from previous answer'

You may ask for an extra Writing Booklet if you need more space to answer this question.

QUESTION 4

Start your answer here.		Marker's Comments
(a) (i) $y = 7x^2 - 2x + 4$		one mark for each part
$\frac{dy}{dx} = \boxed{14x - 2}$	✓✓	(ie two easy marks!!)
(ii) $y = \sqrt{4-x}$		
$y = (4-x)^{\frac{1}{2}}$	✓	for changing to index form
$y' = \frac{1}{2}(4-x)^{-\frac{1}{2}} \times (-1)$		
$= \boxed{\frac{-1}{2(4-x)^{\frac{1}{2}}}}$ OR $\frac{-1}{2\sqrt{4-x}}$	✓	for differentiating and giving answer with positive index
(iii) $y = 8x(x^2-4)^4$		This is a "Product Rule"
let $u = 8x$ $\quad v = (x^2-4)^4$		
$u' = 8$ $\quad v' = 4(x^2-4)^3 \cdot 2x$		
$= 8x(x^2-4)^3$	✓	for differentiating u and v correctly
$y' = 8(x^2-4)^4 + 64x^2(x^2-4)^3$		
$= 8(x^2-4)^3(x^2-4+8x^2)$		for correctly substituting into product rule formula and factorising.
$= 8(x^2-4)^3(9x^2-4)$	✓	
$= \boxed{8(x-2)^3(x+2)^3(3x-2)(3x+2)}$		(not many fully factorised)
(iv) $y = \frac{2x+3}{4x+9}$		This is a Quotient rule question.
let $u = 2x+3$ $\quad$ let $v = 4x+9$		Many had no idea of formula.
$u' = 2$ $\quad \therefore v' = 4$	✓	1 mark for differentiating u and v correctly
$y' = \frac{2(4x+9) - 4(2x+3)}{(4x+9)^2}$		
$= \frac{8x+18-8x-12}{(4x+9)^2}$		
$= \boxed{\frac{6}{(4x+9)^2}}$	✓	1 mark for simplifying correctly.

Start your answer here.

Q4 Continued.

Marker Comments

(b) $f(x) = x^2 + 3x + 4$

$f(x+h) = (x+h)^2 + 3(x+h) + 4$

$= x^2 + 2xh + h^2 + 3x + 3h + 4$

many students had no idea how to set out a "1st principles" differentiation.

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 3x + 3h + 4 - (x^2 + 3x + 4)}{h}$ ✓

$= \lim_{h \to 0} \frac{2xh + h^2 + 3h}{h}$ ✓

$= \lim_{h \to 0} 2x + h + 3$

need to show $\lim_{h \to 0}$ until you have substituted $h = 0$

$= 2x + 0 + 3$

$= 2x + 3$

(c)(i) $x = 5 + 6t - 3t^2$

when $t = 0$

"initial" displacement when $t = 0$

$x = 5 + 6 \times 0 - 3 \times 0^2$

$\boxed{x = 5 \text{ cm}}$ ✓

(ii) $v = \frac{dx}{dt} = 6 - 6t$

a particle is "at rest" when it is still.

at rest when $v = 0$

ie when velocity $= 0$.

ie $6 - 6t = 0$

$6 = 6t$

$\boxed{t = 1 \text{ second}}$ ✓

QUESTION 5

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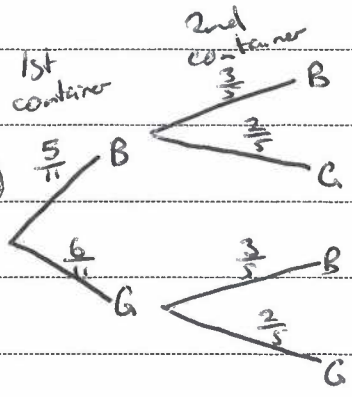
(a) $P(A \cap B) = \frac{5}{11}$ ✓

many didn't recognise that we were looking for the probability of $A \cap B$ occurring.

(b)



Those that drew a probability tree had an advantage.

(i) $P(GG) = \frac{6}{11} \times \frac{2}{5} = \boxed{\frac{12}{55}}$ ✓

- only GG.

(ii) $P(\text{at least 1B}) = 1 - P(\text{no blues})$

$= 1 - \frac{12}{55}$

$= \boxed{\frac{43}{55}}$ ✓

- "at least one blue" is the complementary event of "no blues"

(iii) $P(\text{diff. colours}) = P(BG) + P(GB)$

$= \frac{5}{11} \times \frac{2}{5} + \frac{6}{11} \times \frac{3}{5}$ ✓

$= \boxed{\frac{28}{55}}$ ✓

There are two outcomes ∴ add the probabilities

(c) $y = \frac{1}{3}x^3 - 4x^2 + 8x + 5$

$y' = x^2 - 8x + 8$ ✓

for differentiating correctly.

when $x = 3$ $(3, 2)$

$m_{\text{tangent}} = 3^2 - 8 \times 3 + 8$

$= -7$ ✓

finding gradient of tangent.

equation of tangent

$y - y_1 = m(x - x_1)$

$y - 2 = -7(x - 3)$

$y - 2 = -7x + 21$

$\boxed{y = -7x + 23}$ ✓

or

$7x + y - 23 = 0$

a tangent is a straight line.

finding equation of tangent

Q5 continued.

(d) (i) $f(x) = e^{2x}$

$f'(x) = 2e^{2x}$ ✓ differentiating

when $x = 5$ $\quad y = e^{10}$

$m_{target} = 2e^{2\times 5}$

$= 2e^{10}$

$\therefore m_{normal} = \boxed{\frac{-1}{2e^{10}}}$ ✓ finding gradient of normal

(ii) equation of normal

$y - y_1 = m(x - x_1)$

$y - e^{10} = -\frac{1}{2e^{10}}(x - 5)$ ✓ correctly substituting into gradient-point formula.

$y - e^{10} = \frac{-x}{2e^{10}} + \frac{5}{2e^{10}}$

$\boxed{y = \frac{-x}{2e^{10}} + \frac{5}{2e^{10}} + e^{10}}$ ✓ Finding equation of normal

Alternatives

$\boxed{y = \frac{-x + 5 + 2e^{20}}{2e^{10}}}$

or

$2e^{10}y - 2e^{20} = -x + 5$

$\Downarrow$

$\boxed{x + 2e^{10}y - 2e^{20} - 5 = 0}$

Several other equivalent equations were allowed.

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Question 6

Start your answer here.

(a) $\log x^2 - \log 3x = \log 9$.

$2\log x - \log 3 - \log x = 2\log 3$ ✓

$\log x = 3\log 3$

$\therefore x = 3^3$

$= 27$ ✓ /2

(b) $\log_6 12 + \frac{1}{3}\log_6 27 = \log_6 6 + \log_6 2 + \log_6 3$ ✓

$= 1 + 1$

$= 2.$ ✓ /2

(c) $e^{2\ln x} = x + 12 \quad (x > 0)$

$x^2 - x - 12 = 0$ ✓

$(x-4)(x+3) = 0$

$\therefore x = -3$ or 4

$x = 4$ ✓ /2

(d) (i) $N = 100\,000e^{-0.15t}$

$t = 40, \quad N = 100\,000e^{-0.15\times 40}$ ✓

$= 247.8...$ ✓ accept 247.

$\doteqdot 248$ /2

(ii) $1\text{ø}\text{ø} = 100\,\text{ø}\text{ø}\text{ø}\,e^{-0.15t}$ ✓

$\frac{1}{1000} = e^{-0.15t}$

$\ln \frac{1}{1000} = -0.15t$ → $t = \ln\frac{1}{1000} \div (-0.15)$

$= 46.05$

$\doteqdot 46$ days. ✓

/2

$y = 3\ln(2x) + 4$

For x intercept let $y = 0$.

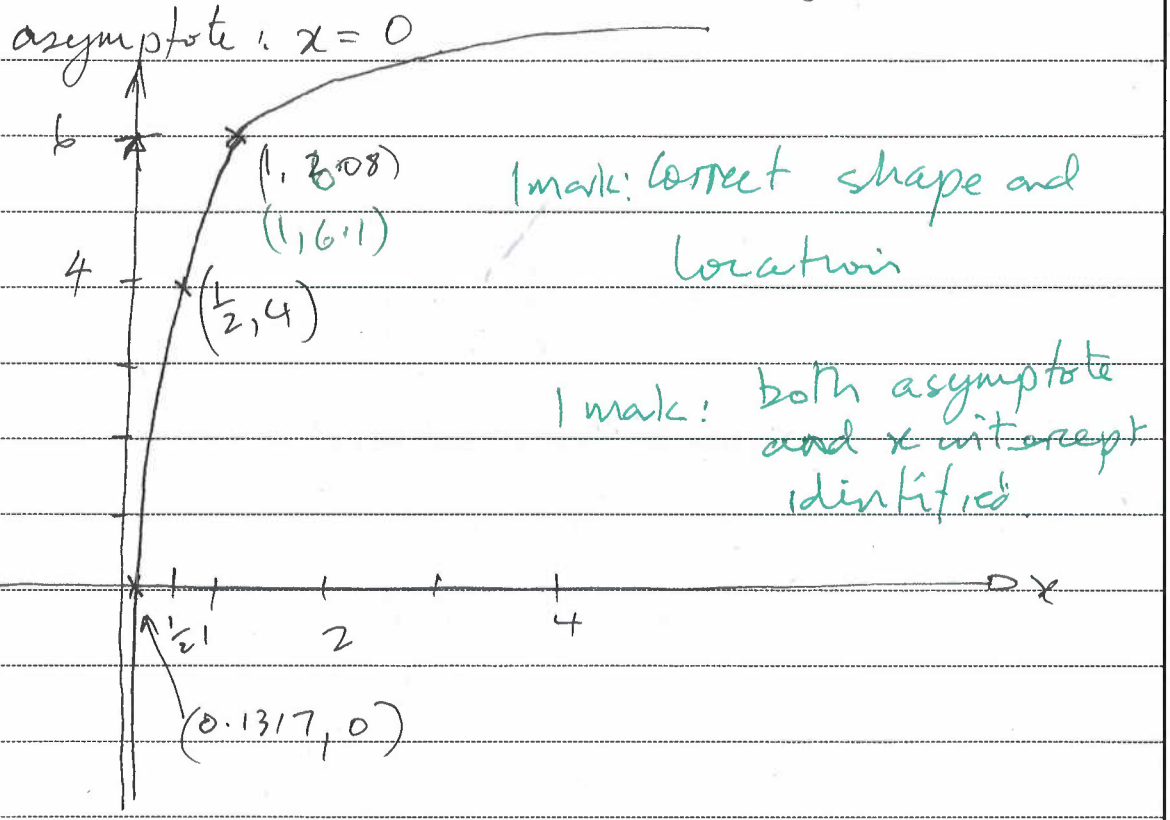
$\therefore 3\ln(2x) = -4$

$\ln(2x) = -\frac{4}{3}$

$2x = e^{-\frac{4}{3}}$

$x = \frac{1}{2} e^{-\frac{4}{3}}$

$= 0.1317$ (4 sig fig).



1 mark: correct shape and location

1 mark: both asymptote and x intercept identified.

x	1	2	3	4	$\frac{1}{2}$
y	6.1	8.2	9.4	10.2	4

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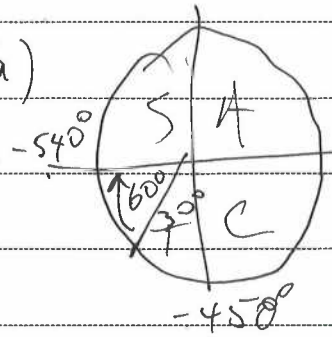
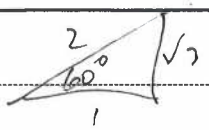
Q7

Start your answer here.

(a)





related angle = 60°

$\therefore \sin(-480°) = -\sin 60°$

$= -\frac{\sqrt{3}}{2}$

1 mark: size
1 mark: sign. /2

(b) LHS $= (\sin x + \cos x)^2 + (\sin x - \cos x)^2$

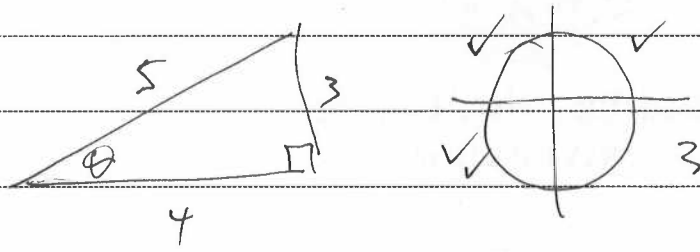
$= \sin^2 x + 2\sin x\cos x + \cos^2 x + \sin^2 x - 2\sin x\cos x + \cos^2 x$ ✓

$= 2(\sin^2 x + \cos^2 x)$ ✓

$= 2$

$=$ RHS. /2

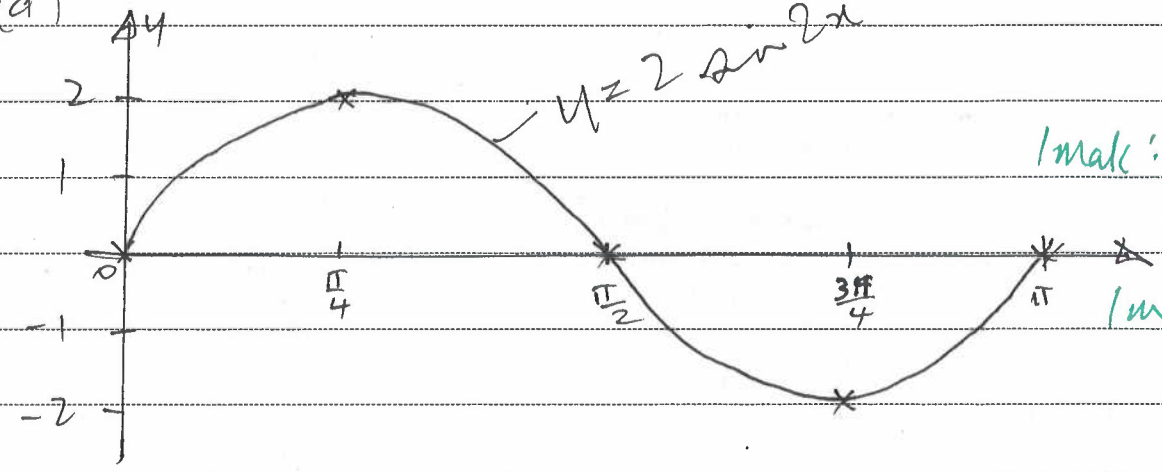
(c)



$\sin\theta = -\frac{3}{5}$

1 mark: size
1 mark: sign. /2

(d)

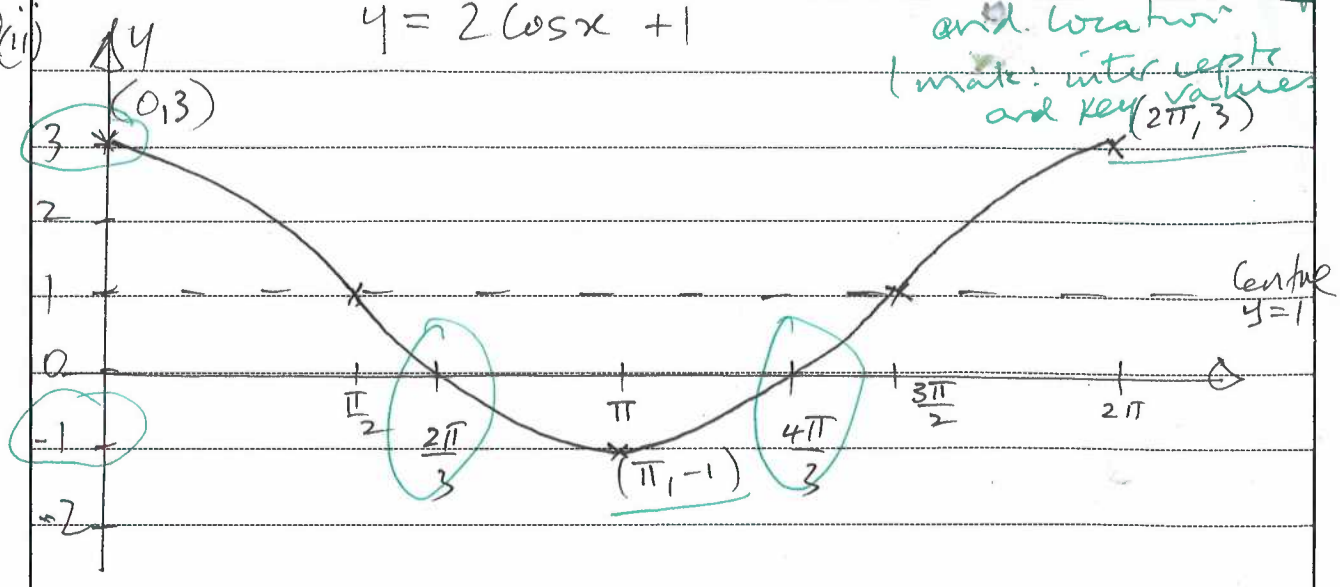


1 mark: correct sh
and locatio

1 mark: inter
and scal

1 mark: correct shape and location
1 mark: intercepts and key values

d(ii) $y = 2\cos x + 1$

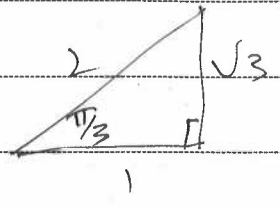


for x intercepts, let $y = 0$.

$\therefore 2\cos x + 1 = 0$

$2\cos x = -1$

$\cos x = -\frac{1}{2}$

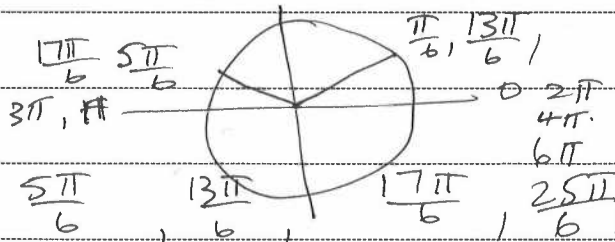


$\therefore x = \pi - \frac{\pi}{3}, \pi + \frac{\pi}{3}$

$= \frac{2\pi}{3}, \frac{4\pi}{3}$

(e) $\sin\left(2x + \frac{\pi}{3}\right) = \frac{1}{2}$ $\quad 0 \le x \le 2\pi$

related angle $= \frac{\pi}{6}$ $\quad 0 \le 2x \le 4\pi$



$\therefore 2x + \frac{\pi}{3} = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \frac{25\pi}{6}, \ldots$

$2x = -\frac{\pi}{6}, \frac{3\pi}{6}, \frac{11\pi}{6}, \frac{15\pi}{6}, \frac{23\pi}{6}, \ldots$

$x = \cancel{-\frac{\pi}{12}}, \frac{\pi}{4}, \frac{11\pi}{12}, \frac{5\pi}{4}, \frac{23\pi}{12}$

$x = \frac{\pi}{4}, \frac{11\pi}{12}, \frac{5\pi}{4}, \frac{23\pi}{12}$ $\left(\frac{3\pi}{12}, \frac{11\pi}{12}, \frac{15\pi}{12}, \frac{23\pi}{12}\right)$

at least 2 correct, all 4 correct.

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