

Students' Name: _____ Teacher: _____



Saint Ignatius' College, Riverview Mathematics Assessment Task 2023

Year 11
Advanced Mathematics
Task 3
PRELIMINARY COURSE EXAMINATION
SEPTEMBER 13, 2023

General Instructions:

- **Reading time:** 5 Minutes
- **Time Allowed:** 120 Minutes
- Write using blue or black pen only
- NESA approved calculators may be used
- Attempt all questions in the booklets provided
- Use a NEW answer booklet for each question.
- Write **your name** and **your teacher's name** in the positions indicated
- Marks may not be awarded for missing or carelessly arranged working

Teachers:

Mr P Collins	PPC
Mrs K Mullan	KXM
Dr M Furtado	MXF
Mr J Newey	JPN

Topics Examined:

- Question 1: Algebra, Equations, and Inequalities
Question 2: Functions and Further Functions
Question 3: Trigonometry
Question 4: Calculus
Question 5: Probability
Question 6: Exponential and Logarithmic Functions
Question 7: Trigonometric Functions

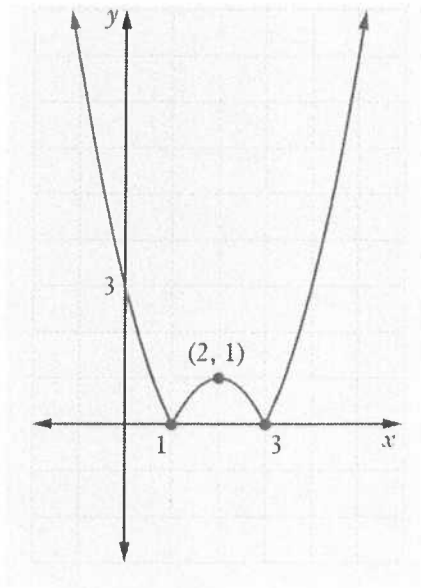
Total

84 Marks

Question 1 Algebra Equations and Inequations**Total 12 Marks***Please use a separate writing booklet.*

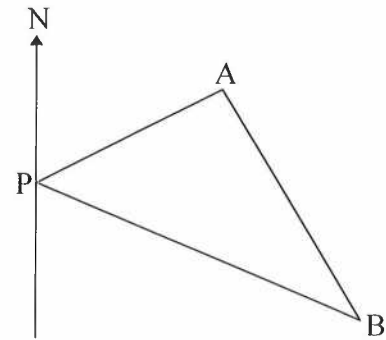
a)	If $2x^7 = 2000$, find x correct to 4 significant figures.	2
b)	Solve $ 4 - 2x = 9$	2
c)	Solve $12 - 5x < x + 3$	2
d)	If $(2\sqrt{5} - \sqrt{3})^2 = m - n\sqrt{15}$, find the value of m and n .	2
e)	If $P = \pi r^2 m$, find r correct to 1 decimal place when $P = 100$ and $m = 2.5$.	2
f)	Solve $15 - x^2 = 2x$.	2
End of Question 1		

Question 2 Functions and Further Functions**Total 12 Marks***Please use a separate writing booklet.*

a)	Given the function $f(x) = x^3 + x$, find in simplified form $f(x) - f(-x)$	2
b)	The gradient of the line through $(3, -2)$ and $(x, 1)$ is 5. Find the value of x .	2
c)	If the following lines are perpendicular, find the value of p . $3x - 2y + 5 = 0$ $y = px - 1$	2
d)	Given then graph of $y = f(x)$  Sketch the graph of $y = -f(x)$, (clearly indicate any key points)	2

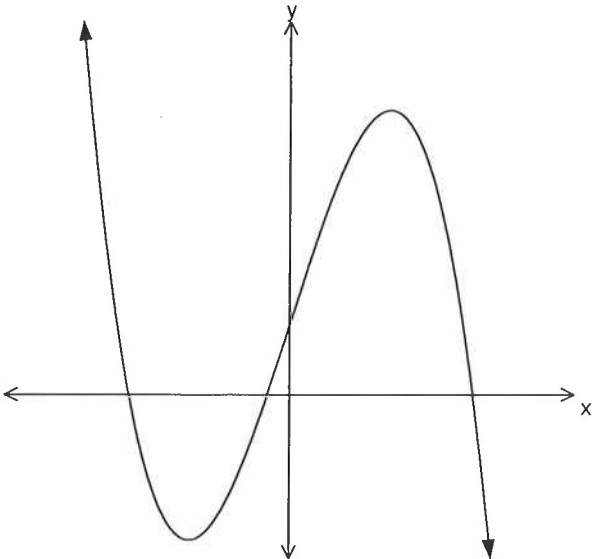
e)	Find the centre AND the radius of the circle with equation. $x^2 + y^2 + 6y - 1 = 0$	2
f)	Given $f(x) = 5x^2 + 2x$ and $g(x) = 2x - 1$. Find the following. i) The degree of $y = f(x)g(x)$ ii) The constant term of $y = f(x) - g(x)$	2
End of Question 2		

Question 3 Trigonometry**Total 12 Marks***Please use a separate writing booklet.*

a)	Solve the following equation and find the value of θ correct to the nearest minute. $7 \sin \theta = 2$	2
b)	 FIGURE NOT TO SCALE. Ship A is 20 nautical miles from a port P and is on a bearing of 055. Ship B is 27 nautical miles from P and is on a bearing of 115. i. Show that $\angle APB = 60$. ii. Use the cosine rule to determine the distance between the two ships, giving your answer correct to one decimal place.	1 2
c)	Parallelogram ABCD has sides 15cm and 12cm. One of its interior angles is equal to $75^{\circ}15'$. Find the area of Parallelogram ABCD (Answer to the nearest sq cm)	2

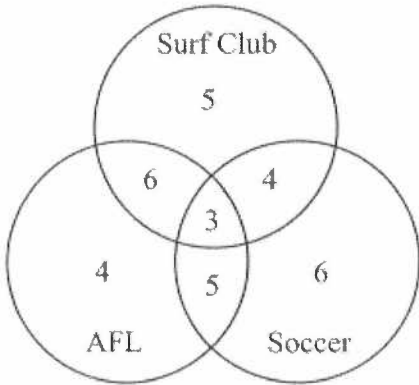
d)	Note: $\angle ROS = \frac{\pi}{3}$ Find the area of the shaded region RSTU. (Leave your answer in terms of π) Please note: TOU is a sector of a circle AND SOR is a sector of a circle.	2
e)	From a point A, 90m due south of the base of a tower, the angle of elevation to the top of the tower is 30°. Point B is 125m due east of the tower. i) Find the height of the tower correct to the nearest metre. ii) Find the angle of elevation of the top of the tower from point B. (answer to the nearest degree).	1 2
End of Question 3		

Question 4 Calculus**Total 12 Marks***Please use a separate writing booklet.*

a)	Consider the function $f(x) = 4x^3 - 3x^2 + 1$. Find the value of $f'(-3)$	2
b)	Consider the graph of the function $y = f(x)$  Copy the diagram into your answer booklet. On the same axes, sketch the gradient function i.e. $y = f'(x)$	2
c)	Given $f(x) = 5 - 2x - 3x^2$ Find the equation of the normal to the curve $y = f(x)$, at the point where $x = 1$.	2

d)	Differentiate the following	
i)	$y = x\sqrt{x}$	2
ii)	$f(x) = \frac{3x}{2x-1}$	2
iii)	$y = \sqrt{2x-7}$	2
End of Question 4		

Question 5 Probability**Total 12 Marks***Please use a separate writing booklet.*

a)	A pack of playing cards contains 52 different cards. If one card is chosen at random, find the probability that i) It is a ten of clubs ii) A king or a seven iii) A King or a Heart.	1 1 1
b)	 In classroom VA209, students are asked what sports club they are members of and the results are shown in the Venn diagram. Note: All students are members of at least one sporting club i) A student is chosen at random from the class. What is the probability that the student is a member of all three clubs? ii) A student who is a member of a soccer club is chosen at random. What is the probability that he is also a member of a surf club ?	1 2

c)	When William plays chess, he has a probability of 0.4 of winning a match and a probability of 0.1 of a draw. If he plays 2 matches, find the probability that it will win at least one match.	2
d)	Mrs Yates has four different pairs of running shoes. If two shoes are randomly selected, what is the probability that they will be a matching pair?	2
e)	A coin is tossed “n” times. Find the probability in terms of n of tossing at least one tail.	2
End of Question 5		

Question 6 Exponential and Logarithmic Functions**Total 12 Marks***Please use a separate writing booklet.*

a)	Solve for x , correct to 1 decimal place where necessary. i) $\log_8 x = \frac{-1}{3}$ ii) $25^x = 125$	1 1
b)	Solve the equation correct to four significant figures. $84 = 7e^{x+1}$	2
c)	Find the value of m . $\log_x 40 = \log_x m + 3\log_x 2$	2
d)	The population (B) of “Bin Chickens” living inside the grounds of Riverview after t years can be calculated using $B = 60 + 2e^{0.5t}$ i) How many “Bin Chickens” will be living at Riverview in five years? (Answer to nearest whole number) ii) How many years until the population of Bin Chickens reaches 500? (Answer to the nearest year)	1 2
e)	Sketch the graph of $y = \ln(x - 1)$ showing all important features. Clearly indicate on your sketch any asymptotes or intercepts on the x or y axis.	3
End of Question 6		

Question 7 Trigonometric Functions**Total 12 Marks***Please use a separate writing booklet.*

a)	Find the exact value of $\sec 120^\circ$	1
b)	Solve the equation $2 \cos x = \sqrt{3}$, where $0 \leq x \leq 2\pi$	2
c)	Find the exact value of $1 + \cot^2 \frac{\pi}{6}$.	2
d)	Solve the following equation for $[0, 2\pi]$ $\sin x \cos x + \cos x = 0$	2
e)	Given $f(x) = 3 \cos 2x$ i) State the amplitude of the function. ii) Sketch the graph of $y = f(x)$ for $0 \leq x \leq \pi$. Clearly indicate all intercepts	1 2
f)	Prove the following $\sec x - \sec x \sin^2 x = \cos x$	2
End of Question 7		

End of Examination

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Saint Ignatius' College, Riverview
Mathematics Assessment Task
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SOLUTIONS.

QUESTION ONE

a) $2x^2 = 2000$

$$x^2 = 1000$$

$$x = 2.683$$

b) $|4 - 2x| = 9$

$$4 - 2x = 9 \quad 4 - 2x = -9$$

$$-2x = 5 \quad -2x = -13$$

$$x = -\frac{5}{2} \quad x = \frac{13}{2}$$

✓

✓

c) $12 - 5x < x + 3$

$$-6x < -9$$

$$x > \frac{3}{2}$$

d) $(2\sqrt{5} - \sqrt{3})^2 = 20 - 4\sqrt{15} + 3$

$$= 23 - 4\sqrt{15}$$

$$M = 23$$

$$n = 4$$

$$e) V = \pi r^2 h$$

$$100 = \pi \cdot r^2 \cdot 2.5$$

$$r^2 = \frac{40}{\pi}$$

$$r^2 = 12.73239$$

$$r = 3.6$$

$$f) 15 - x^2 = 2x$$

$$x^2 + 2x - 15 = 0$$

$$(x+5)(x-3) = 0$$

$$\therefore x = -5 \quad x = 3$$

QUESTION TWO

$$a) f(x) = x^3 + x$$

$$\begin{aligned} f(-x) &= (-x)^3 + (-x) \\ &= -x^3 - x \end{aligned}$$

$$\begin{aligned} \therefore f(x) - f(-x) &= x^3 + x - [-x^3 - x] \\ &= 2x^3 + 2x \end{aligned}$$

$$b) m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$5 = \frac{-2 - 1}{3 - x}$$

$$15 - 5x = -3$$

$$-5x = -18$$

$$x = \frac{18}{5}$$

$$c) 3x - 2y + 5 = 0$$

$$\therefore 2y = -3x - 5$$

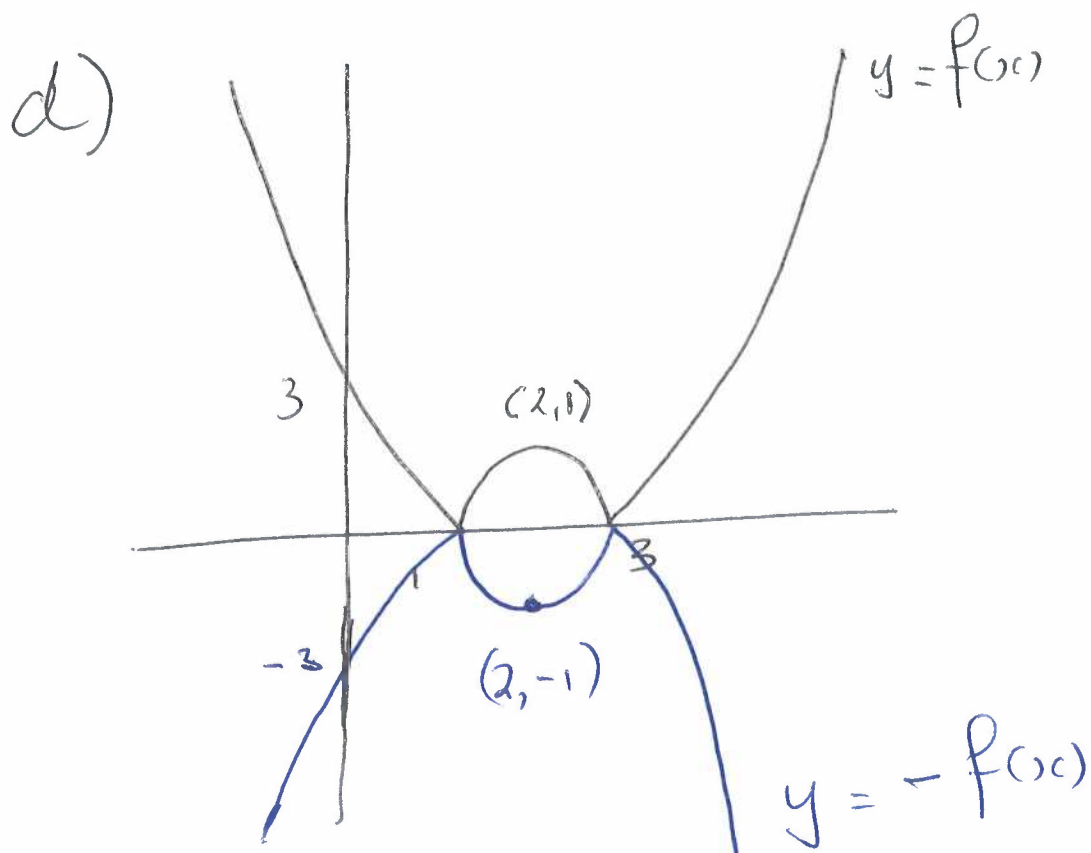
$$y = \frac{3}{2}x + \frac{5}{2}$$

$$\therefore m = \frac{3}{2}$$

$$y = px - 1$$

$$m = p$$

$$\therefore p = -\frac{2}{3}$$



e)

$$x^2 + y^2 + 6y - 1 = 0$$

$$x^2 + y^2 + 6y + 9 = 1 + 9$$

$$x^2 + (y+3)^2 = 10$$

$\therefore C (0, -3)$

$$R = \sqrt{10}$$

f) $f(x) = 5x^2 + 2x$ $g(x) = 2x - 1$

i) $f(x)g(x) = 10x^3 \dots \dots \dots$ DEGREE = 3

ii) $f(x) - g(x) = 5x^2 + 2x - 2x + 1$

CONSTANT = 1

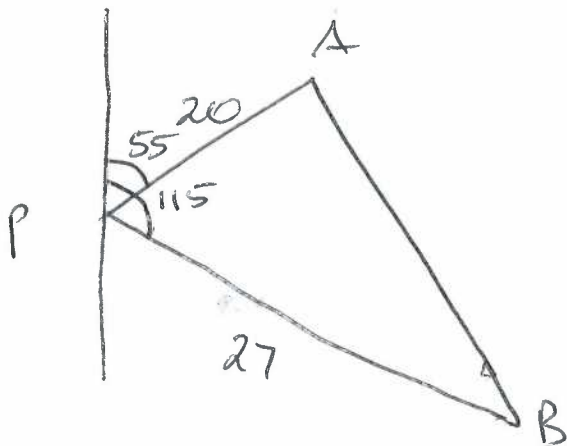
QUESTION THREE

a) $7 \sin \theta = 2$

$$\sin \theta = \frac{2}{7}$$

$$\theta = 16^\circ 36'$$

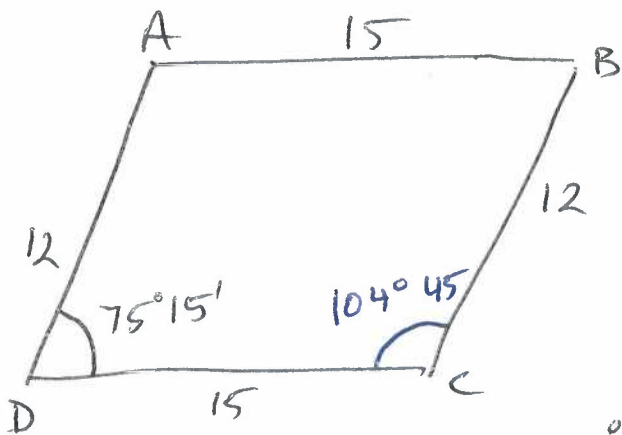
b)



i) $\angle APB = 115 - 55$
 $= 60$

ii) $AB^2 = 20^2 + 27^2 - 2 \cdot 20 \cdot 27 \cos 60$
 $AB = 24.3 \text{ km}$

c)



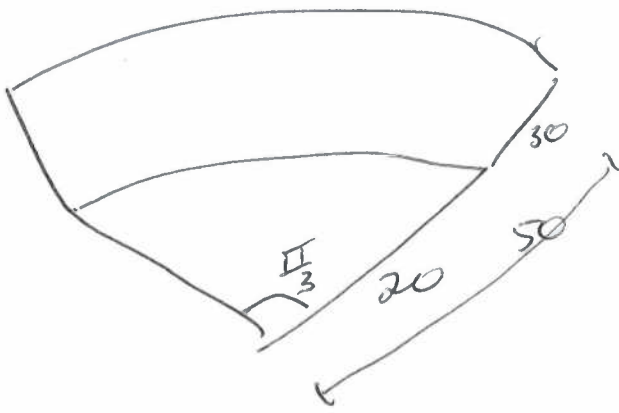
$$A_{\Delta} = \frac{1}{2} ab \sin c$$

$$= \frac{1}{2} 12 \cdot 15 \cdot \sin 75^\circ 15'$$

$$= 87.034 \text{ cm}^2$$

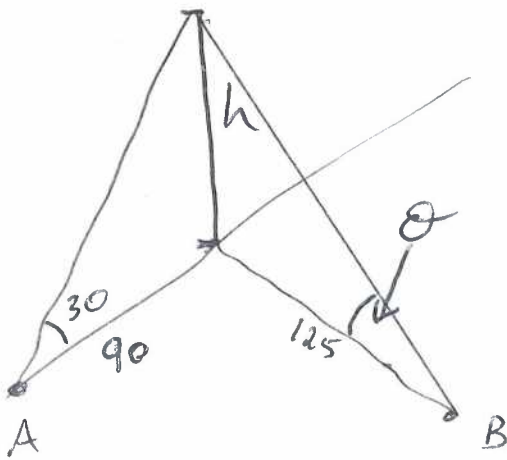
$$\therefore \text{AREA PARM} = 174.07 \text{ cm}^2$$

d)



$$\begin{aligned}
 A &= \frac{1}{2} 50^2 \cdot \frac{\pi}{3} - \frac{1}{2} 20^2 \cdot \frac{\pi}{3} \\
 &= \frac{1250\pi}{3} - \frac{200\pi}{3} \\
 &= \frac{1050\pi}{3} \\
 &= 350\pi \text{ cm}^2
 \end{aligned}$$

e)



$$\begin{aligned}
 \text{i)} \quad \frac{h}{90} &= \tan 30 \\
 h &= 90 \tan 30 \\
 h &= 52 \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 \text{ii)} \quad \tan \theta &= \frac{52}{125} \\
 \theta &= 23^\circ
 \end{aligned}$$

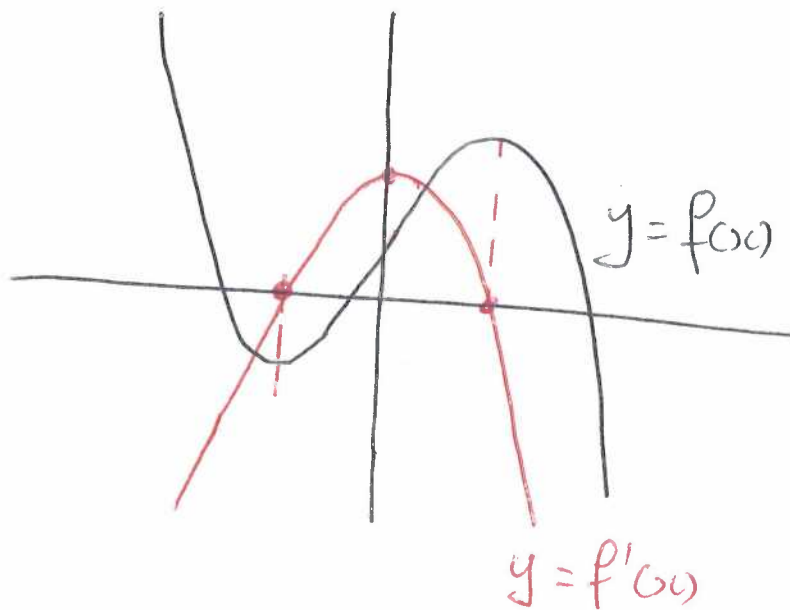
QUESTION FOUR

a) $f(x) = 4x^3 - 3x^2 + 1$

$$f'(x) = 12x^2 - 6x$$

$$\begin{aligned} f'(-3) &= 12(-3)^2 - 6(-3) \\ &= 12(9) + 18 \\ &= 126 \end{aligned}$$

b)



c) $f(x) = 5 - 2x - 3x^2$

$$f'(x) = -2 - 6x$$

at $x=1$ $m_T = -8$

∴ $m_N = \frac{1}{8}$ $y=0$

$$y - 0 = \frac{1}{8}(x - 1)$$

$$8y = x - 1$$

$$0 = x - 8y - 1$$

$$\begin{aligned} \text{d) i} \quad y &= x \sqrt{x} \\ &= x^{3/2} \end{aligned}$$

$$\begin{aligned} y' &= \frac{3}{2} x^{\frac{1}{2}} \\ &= \frac{3\sqrt{x}}{2} \end{aligned}$$

$$\text{ii) } f(x) = \frac{3x}{2x-1}$$

$$\begin{aligned} f'(x) &= \frac{(2x-1) \cdot 3 - 3x \cdot 2}{(2x-1)^2} \\ &= \frac{6x - 3 - 6x}{(2x-1)^2} \\ &= \frac{-3}{(2x-1)^2} \end{aligned}$$

$$\begin{aligned} \text{iii) } y &= \sqrt{2x-7} \\ &= (2x-7)^{\frac{1}{2}} \end{aligned}$$

$$\begin{aligned} y' &= \frac{1}{2} (2x-7)^{-\frac{1}{2}} \cdot 2 \\ &= \frac{1}{\sqrt{2x-7}} \end{aligned}$$

QUESTION FIVE

a) i) $P(e) = \frac{1}{52}$

ii)
$$P(e) = \frac{4}{52} + \frac{4}{52}$$
$$= \frac{8}{52} \left(\frac{2}{13} \right)$$

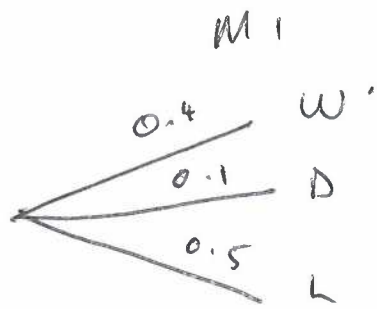
iii)
$$P(e) = \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$$
$$= \frac{16}{52}$$
$$= \frac{4}{13}$$

b) i) $\text{TOTAL} = 33$

$$P(\text{all 3}) = \frac{3}{33}$$
$$= \frac{1}{11}$$

ii)
$$P(\text{Surf} | \text{SOCCER}) = \frac{P(\text{Surf} \cap \text{SOCCER})}{P(\text{SOCCER})}$$
$$= \frac{7}{18}$$

(c)



$$\begin{aligned} P(\text{at least 1 } W) &= 1 - P(\tilde{W} \tilde{W}) \\ &= 1 - (0.6 \times 0.6) \\ &= 1 - 0.36 \\ &= 0.64 \end{aligned}$$

d)

A A

B B

C C

D D

$$\begin{aligned} P(\text{match}) &= 1 \times \frac{1}{7} \\ &= \frac{1}{7} \end{aligned}$$

e) Toss 3 times

$$\begin{aligned} P(\text{at least 1 tail}) &= 1 - P(HHH) \\ &= 1 - \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= 1 - \left(\frac{1}{2}\right)^3 \end{aligned}$$

Toss "n" times.

$$P(\text{at least 1 Tail}) = 1 - \left(\frac{1}{2}\right)^n$$

QUESTION SIX

a) i) $\log_8 x = -\frac{1}{3}$

$$\therefore x = 8^{-\frac{1}{3}}$$

$$x = \frac{1}{2}$$

ii) $25^x = 125$

$$5^{2x} = 5^3$$

$$\therefore 2x = 3$$

$$x = \frac{3}{2}$$

b) $84 = 7e^{x+1}$

$$e^{x+1} = 12$$

$$x+1 = \ln 12$$

$$x = \ln 12 - 1$$

$$x = 1.485$$

c) $\log_x 40 = \log_x m + 3 \log_x 2$
 $= \log_x 8m$

$$\therefore 40 = 8m$$

$$m = 5$$

d) $B = 60 + 2e^{0.5t}$

i) $t = 5$

$$B = 60 + 2e^{2.5}$$

$$= 84.4 \quad (84)$$

ii) $500 = 60 + 2e^{0.5t}$

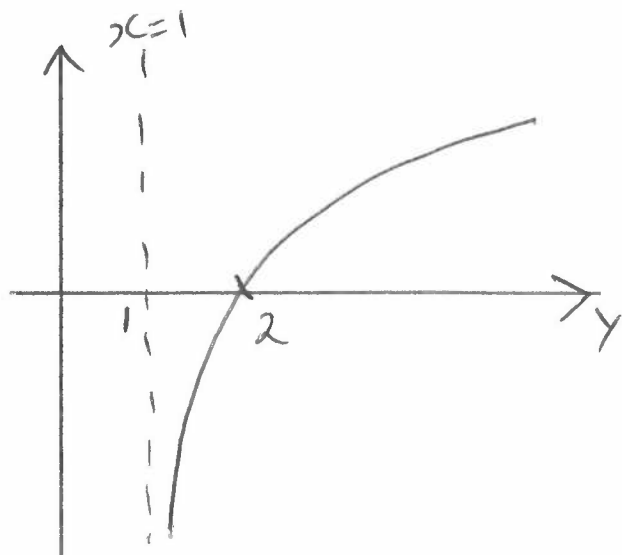
$$220 = e^{0.5t}$$

$$0.5t = \ln 220$$

$$t = \frac{\ln 220}{0.5}$$

$$= 11 \text{ yrs.}$$

e)



QUESTION SEVEN

$$\begin{aligned} \text{a) } \sec 120^\circ &= \frac{1}{\cos 120} \\ &= \frac{1}{-\frac{1}{2}} \\ &= -2 \end{aligned}$$

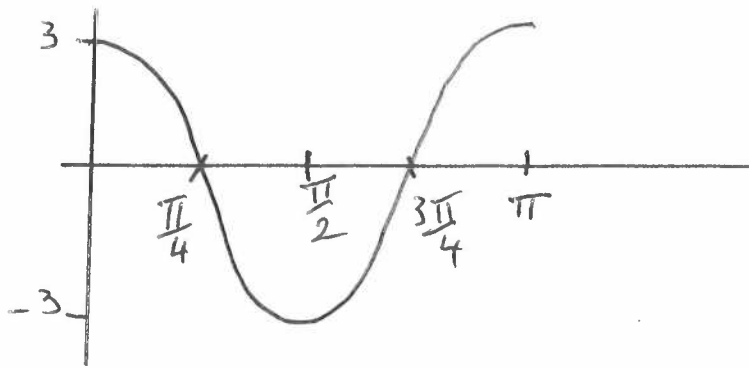
$$\begin{aligned} \text{b) } 2 \cos x &= \sqrt{3} \\ \cos x &= \frac{\sqrt{3}}{2} \\ x &= \frac{\pi}{6}, \frac{5\pi}{6} \end{aligned}$$

$$\begin{aligned} \text{c) } 1 + \cot^2 \frac{\pi}{6} &= 1 + \frac{1}{\tan^2 \frac{\pi}{6}} \\ &= 1 + \frac{1}{\frac{1}{3}} \\ &= 1 + 3 \\ &= 4 \end{aligned}$$

$$\begin{aligned} \text{d) } \sin x \cos x + \cos x &= 0 \\ \cos x (\sin x + 1) &= 0 \\ \therefore \cos x &= 0 \quad \sin x = -1 \\ x &= \frac{\pi}{2}, \frac{3\pi}{2} \end{aligned}$$

e) i) Amp = 3.

ii)



f) Prove $\sec x - \sec x \sin^2 x = \cos x$

$$\text{LHS} = \sec x - \sec x \sin^2 x$$

$$= \sec x (1 - \sin^2 x)$$

$$= \sec x (\cos^2 x)$$

$$= \frac{1}{\cos x} \cdot \cos^2 x$$

$$= \cos x$$

$$= \text{RHS}$$