Student's Name:_____ Teacher's Code:_____

Saint Ignatius'College, Riverview

Mathematics Assessment Task

2025

Year 11
Advanced Mathematics
Task 3
PRELIMINARY COURSE EXAMINATION
Date: Monday 22nd September

General Instructions:

- **Reading Time: 5 minutes**
- **Time Allowed: 120 minutes**
- Write using blue or black pen only
- NESA approved calculators may be used
- Attempt all questions **in the space provided on the paper in the booklets**
- Write **your name and your teacher's code in the positions indicated**
- Marks may not be awarded for **missing or carelessly arranged working**

Teachers:

- Mr S. Maher **SJM**
- Mr J Newey **JPN**
- **Mr D Reidy DPR**
- Ms K Mullan **KXM**
- Ms M Kim **MYK**

Topics Examined:

Question 1: Algebra, Equations and Inequalities
Question 2: Functions and Further Functions
Question 3: Trigonometry
Question 4: Calculus
Question 5: Probability
Question 6: Trigonometric Functions

Six questions each worth **14 marks**.

Total 84 marks

Name: Circle Teacher: SJM KXM MYK DPR

Question 1 – Algebra, Equations and Total /14

a) MULTIPLE CHOICE QUESTION **(1)**

Which of the following is **equivalent** to $\left(\frac{x+1}{x}\right)^2 - \left(\frac{x-1}{x}\right)^2$? (*Circle One)*

A. $\frac{2}{x}$

B. $\frac{4}{x}$

C. $\frac{2}{x^2}$

D. $\frac{4}{x^2}$

b) **Factorise** $ac + bc - ad - bd$. **(1)**

---

c) If $3\sqrt{27} - 2\sqrt{12} = \sqrt{b}$. Find $\boldsymbol{b}$. **(2)**

---

d) Solve for $\boldsymbol{x}$ where $|2x + 1| = |x|$. **(2)**

---

e) Solve for $\boldsymbol{x}$ where $\frac{3x-5}{4} + 1 = \frac{x-7}{3}$ **(3)**

---

f) Using the **quadratic formula** or by **completing the square:**

Solve for $\boldsymbol{x}$ where $5x^2 + 11x + 3 = 0$. (Answer in Surd Form) **(3)**

---

g) Solve for $\boldsymbol{x}$ **and** $\boldsymbol{y}$ where:

$3x - 2y = 14$

$2x + 3y = 17$ **(2)**

End Of Question One

Name: Circle Teacher: SJM KXM MYK DPR

Question 1 extra writing space.

If you use this space, clearly indicate which question you are answering.

Name: Circle Teacher: SJM KXM MYK DPR

Question 2 – Functions & Further Functions

Total /14

a) MULTIPLE CHOICE QUESTION **(1)**

What is the **centre of the circle** given by the **equation**: $x^2 - 8x - 9 = -y^2$?

(*Circle One Answer)*

A. (-4,0)

B. (0,4)

C. (0,-4)

D. (4,0)

b) If $f(x) = -x^2 - 2x + 3$ *and* $g(x) = 2x + 1$:

(i) Write an expression for $f\big(g(x)\big)$ **fully simplified.** **(2)**

Question b) continued over page….

b) (ii) What is the **axis of symmetry** of $f\big(g(x)\big)$? **(1)**

---

c) Consider the two functions: $y = x^2 - 2x - 3, and\ y = 2x - 6.$

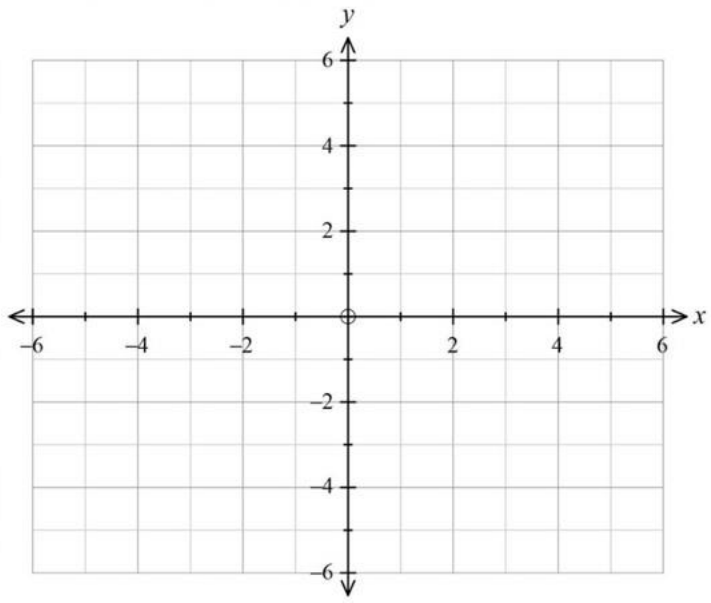
Find the **points of intersection** between the two functions. **(2)**

---

d) For the function:

$$f(x) = \begin{cases} -\frac{1}{x+1}, & x < -1 \\ x + 1, & x \geq -1 \end{cases}$$

(i) Sketch the graph of $f(x)$ below showing ***x* and *y* intercepts** **(2)**



(ii) State the **range** of $f(x)$. **(1)**

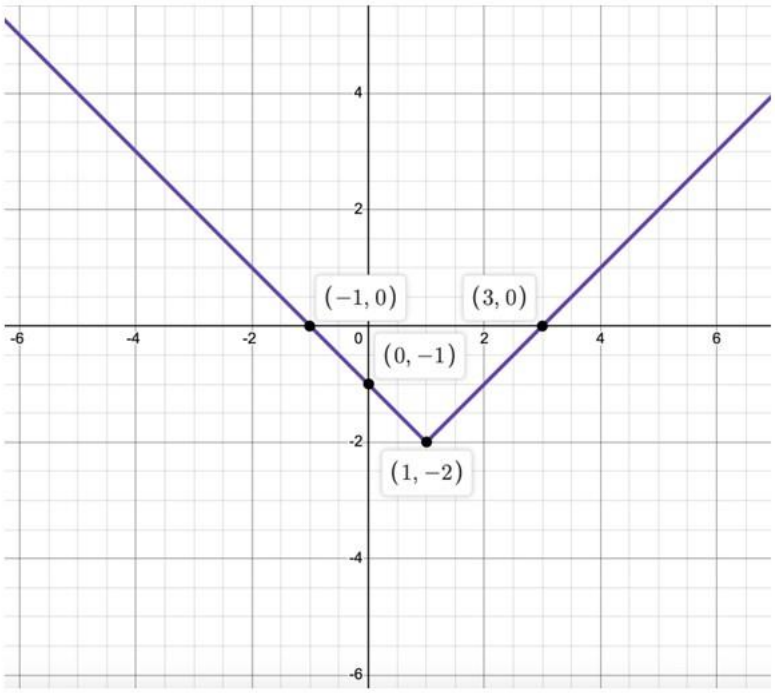
---

e) The function $f(x)$ satisfies the condition:

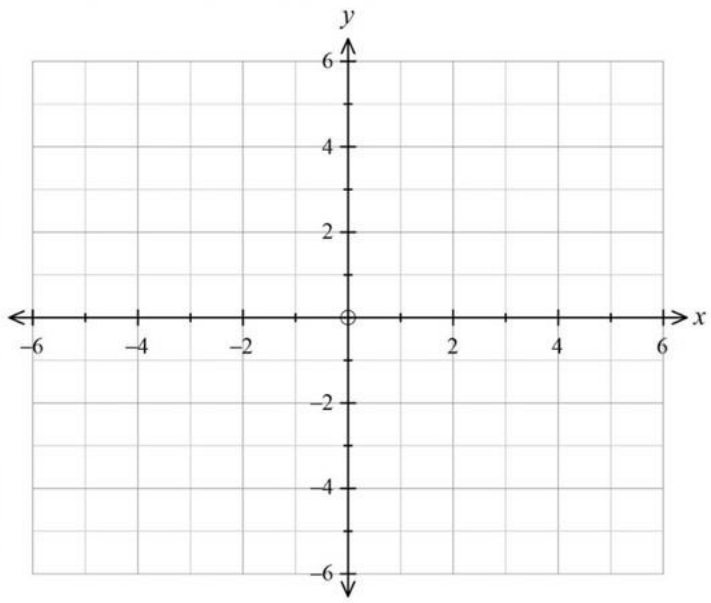
$2f(x) + 3f\left(\frac{2x+3}{x-2}\right) = 3x + 1 \; for \; all \; x, x \neq 2.$

Find the **value of** $\boldsymbol{f(9.)}$ **(3)**

f) The graph of the function $f(x)$ has been shown below.



Sketch the graph of $-f(x) - 2$ below and show **all intercepts.** **(2)**



End Of Question Two

Question 2 extra writing space.

If you use this space, clearly indicate which question you are answering.

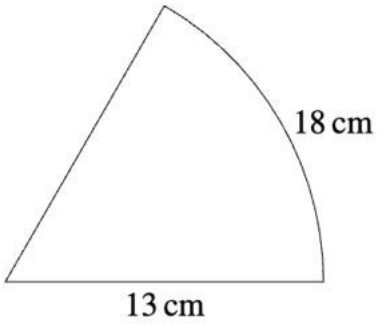
Name: Circle Teacher: SJM KXM MYK DPR

Question 3 – Trigonometry

Total /14

a) MULTIPLE CHOICE QUESTION **(1)**

The sector of **radius** 13cm shown below has an arc length of 18cm.



What is the **area of the sector**? (*Circle One Answer)*

A. $84.5cm^2$

B. $117cm^2$

C. $162cm^2$

D. $234cm^2$

b) The triangle ABC has $AC=32cm$, $BC=45cm$ and $\angle BAC=58^\circ$.

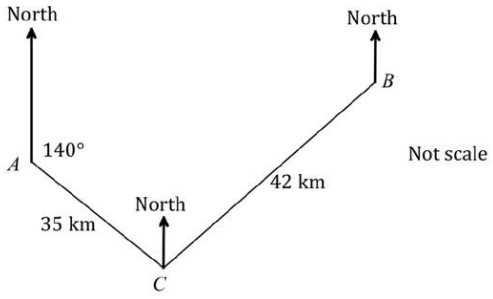
(i) Draw a **labelled diagram** of the **triangle** $\boldsymbol{ABC}$ in the space below **(1)**

(ii) Find the value of $\angle ABC$, showing your answer to the **nearest degree**. **(2)**

---

c) Tyler drives 35km from town *A* on a true bearing of $140^{\circ}T$ to town *C*.

He then drives 42km on a true bearing of $038^{\circ}T$ to town *B*.

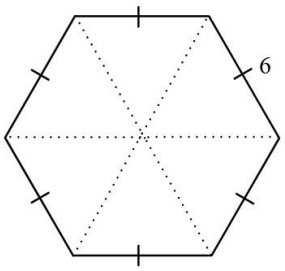


(i) Calculate the **distance** between *A* and *B* and round your answer to **2 decimal places.** **(2)**

(ii) Find the **bearing** of *A* from *B* to the **nearest degree** **(2)**

---

d) Find the **area** of the regular hexagon below in **surd form** given it is **divided into six equilateral triangles** **(2)**



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e) Given that $cot\beta = -\frac{5}{9}$ and that $sin\beta < 0$, find the **value of** $\boldsymbol{cos\beta}$. **(2)**

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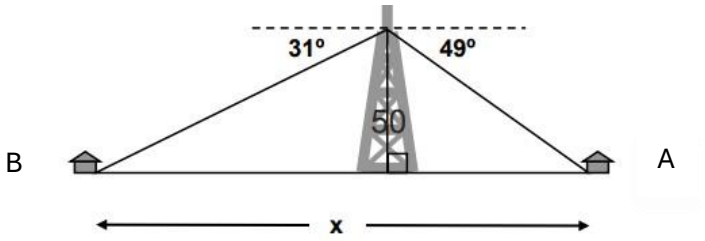
f) An observer at the top of a **50m high** tower notes that the angle of depression of a house lying due east is 49° and the angle of depression of a house lying due west is 31°.



Find the **distance** between the houses at points *A and B* to the **nearest metre**. **(2)**

End of Question Three

Name: Circle Teacher: SJM KXM MYK DPR

Question 3 extra writing space.

If you use this space, clearly indicate which questions you are answering.

Name: Circle Teacher: SJM KXM MYK DPR

Question 4 – Calculus

Total /14

a) MULTIPLE CHOICE QUESTION **(1)**

Find the **equation of the normal** to the curve $y = -x^2 + 3x - 4$ at the point on the graph **where $x = 2$**. (*circle one answer)*

A. $y = 2x - 1$

B. $y = -x + 2$

C. $y = x$

D. $y = x - 4$

b) Differentiate: $y = -2x^3 - \frac{5x}{2} + 3$. **(1)**

---

c) Differentiate: $y = \frac{2x-5}{-x^3+1}$. **(2)**

---

d) Differentiate: $y = \frac{1}{2}\sqrt{x^3 - 2x}$. **(2)**

---

e) Find the **gradient function** of $x(t) = 4t^2(t^2 - 10)^4$ in **factored form.** **(2)**

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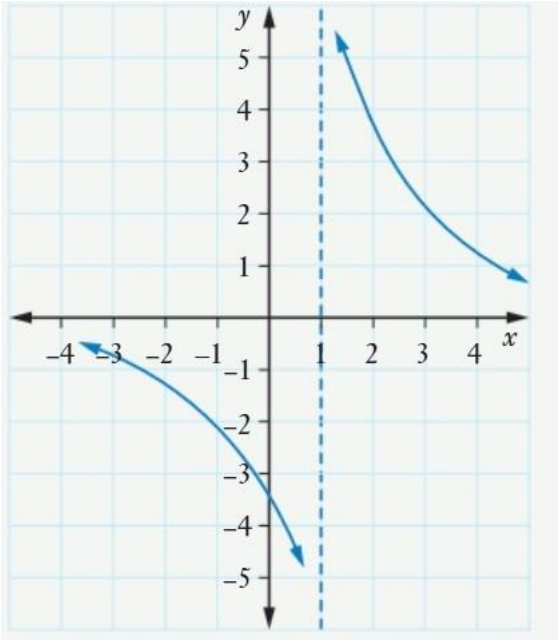
f) Consider the graph of the function below:



Find all the values of x where the function is **not differentiable**. **(1)**

---

g) The formula for **differentiation by first principles** is:

$\frac{d(f(x))}{dx} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$.

Given the function $f(x) = x^2 - 2x + 3$, use **differentiation by first principles** to show that $\frac{d(f(x))}{dx} = 2x - 2$. **(3)**

---

h) A function $f(x)$ satisfies $f(5) = 25 \; and \; f'(5) = 20$.

Find the **equation of the tangent to $y = \sqrt{f(x)}$ at $x = 5$**. **(2)**

End of Question Four

Name: Circle Teacher: SJM KXM MYK DPR

Question 4 extra writing space.

If you use this space, clearly indicate which question you are answering.

Name: Circle Teacher: SJM KXM MYK DPR

Question 5 – Probability Total /14

a) MULTIPLE CHOICE QUESTION **(1)**

One letter is chosen randomly from each of the two words 'THIRTY' and 'EIGHTY'. What is the probability that the **two letters chosen are the same**? (*circle one answer)*

A. $\frac{1}{12}$

B. $\frac{1}{9}$

C. $\frac{5}{36}$

D. $\frac{1}{6}$

b) A standard deck of **52** cards is placed on the table after being shuffled. Two cards are picked **with replacement.**

What is the probability that the first card is a **Heart** and the second card is a **King**?

(Note: A deck of cards, has four suits, hearts, diamonds, clubs and spades. Each suit comprises 13 cards with numbers from 1-10 and picture cards king, queen and jack.) **(2)**

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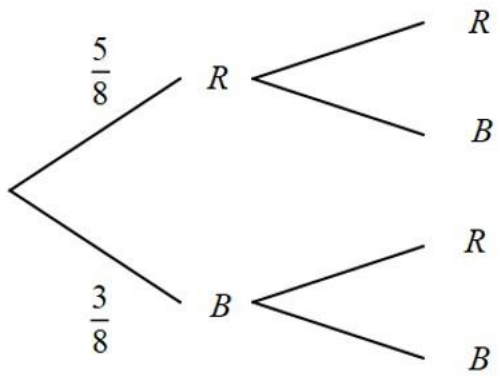
c) Two numbers are chosen randomly from the set $\{1,2,3,4,5\}$ **with replacement**.
What is the probability that **multiplication** of the two numbers
results in a **multiple of 3**? **(3)**

---

d) A bag contains **5 red marbles and 3 blue marbles**. Two marbles are drawn **without replacement**.

(i) Complete the probability tree diagram below. **(1)**



(ii) Find the probability that both marbles are the same colour. **(1)**

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e) A country could have a president, a queen, both a president, and a queen or be self-governed. 60 countries were surveyed, and their results can be categorized into three groups: 35 have a **President,** 27 have a **Queen**, and 8 are **self-governed.**

(i) Using the information above complete the two-way table below:

	President	No President	Total
Queen			
No Queen			
Total			60

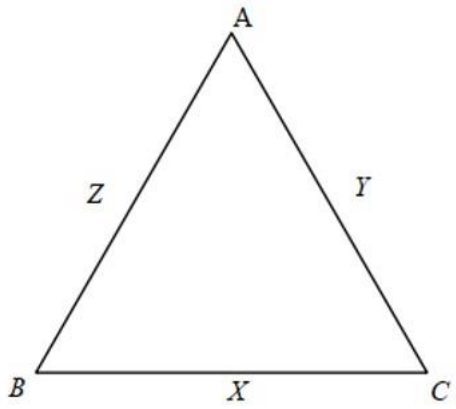
(2)

(ii) What is the probability that a country will have a **President** only? **(1)**

(iii) What is the probability that a country will have **a President if we know it has a Queen**? **(1)**

---

f) Three telecommunications towers, *A,B,* and *C* are connected via cables *X,Y,* and *Z.* as shown below. *A* and *B* remain in contact only if *Z* is operative or if both cables *X* and *Y* are operative. Cables *X,Y,* and *Z* are subject to failure in any one day with probabilities of failure 0.01, 0.025 and 0.03 respectively. Such failures occur independently.



Find the probability that towers ***A* and *B* are unable to make contact** on a given day. (Do Not Round Off Your Answer At The End) **(2)**

End of Question Five

Question 5 extra writing space.

If you use this space, clearly indicate which question you are answering.

Name: Circle Teacher: SJM KXM MYK DPR

Question 6 – Trigonometric Functions

Total /14

a) MULTIPLE CHOICE QUESTION **(1)**

What is the **correct expression** for $(\sin\theta + \cos\theta)^2 - 1$. (*circle one answer)*

A. 0

B. $2cos^2\theta$

C. $2sin^2\theta$

D. $2\sin\theta\cos\theta$

b) Given $-\pi \leq x \leq \pi$: find all **suitable solutions** to the equation:

$2cos(2x) - 0.5 = 0.5.$ **(2)**

---

c) Show that $\frac{1+sin\theta}{1-sin\theta}+\frac{1-sin\theta}{1+sin\theta}=2(sec^2\theta+tan^2\theta)$. **(3)**

---

d) Given $0 \leq \theta \leq 2\pi$

solve the equation $2cos^2\theta+cos\theta-1=0$, giving answers in **terms of π**. **(2)**

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e) Consider the function $f(x) = 2sin\left(\frac{2}{3}x - \frac{\pi}{3}\right)$:

What are the **amplitude, period and phase** of this function? **(3)**

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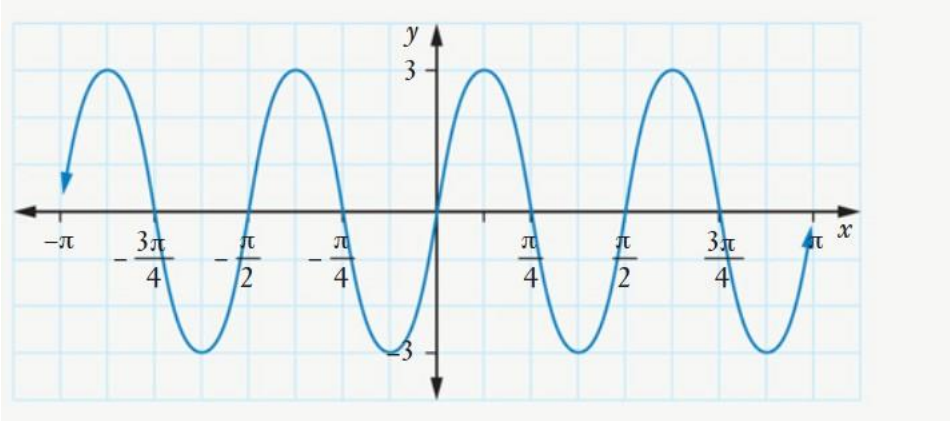
f) What is the equation of the graph below: **(2)**



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g) Solve $sin\theta = cos\frac{2\pi}{3}$ for $-\pi \leq \theta \leq \pi$. **(1)**

End of Examination

Name: Circle Teacher: SJM KXM MYK DPR

Question 6 extra writing space.

If you use this space, clearly indicate which question you are answering.

Student's Name:_____ Teacher's Code:_____

SUGGESTED SOLUTIONS

Saint Ignatius'College, Riverview

Mathematics Assessment Task

2025

Year 11
Advanced Mathematics
Task 3
PRELIMINARY COURSE EXAMINATION
Date: Monday 22nd September

General Instructions:

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Teachers:

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Topics Examined:

Question 1: Algebra, Equations and Inequalities
Question 2: Functions and Further Functions
Question 3: Trigonometry
Question 4: Calculus
Question 5: Probability
Question 6: Trigonometric Functions

Six questions each worth **14 marks**.

Total **84 marks**

Name: Circle Teacher: SJM KXM MYK DPR

Question 1 – Algebra, Equations and

Total /14

a) MULTIPLE CHOICE QUESTION **(1)**

Which of the following is **equivalent** to $\left(\frac{x+1}{x}\right)^2 - \left(\frac{x-1}{x}\right)^2$? *(Circle One)*

A. $\frac{2}{x}$

B. $\frac{4}{x}$ ✓

C. $\frac{2}{x^2}$

D. $\frac{4}{x^2}$

b) **Factorise** $ac + bc - ad - bd$. **(1)**

$c(a+b) - d(a+b) = (a+b)(c-d)$ ✓

c) If $3\sqrt{27} - 2\sqrt{12} = \sqrt{b}$. Find b. **(2)**

$3\sqrt{9}\sqrt{3} - 2\sqrt{4}\sqrt{3} = 9\sqrt{3} - 4\sqrt{3} = 5\sqrt{3}$ ✓

$= \sqrt{25} \times \sqrt{3} = \sqrt{75}$

$b = 75$. ✓

d) Solve for x where $|2x + 1| = |x|$. **(2)**

$2x+1 = x$ | $2x+1 = -x$

$x = -1$ ✓ | $3x = -1$

$x = -\frac{1}{3}$ ✓

e) Solve for x where $\frac{3x-5}{4} + 1 = \frac{x-7}{3}$ **(3)**

$12\left(\frac{3x-5}{4}\right) + 12\left(\frac{1}{1}\right) = 12\left(\frac{x-7}{3}\right)$ ✓

$3(3x-5) + 12 = 4(x-7)$

$9x - 15 + 12 = 4x - 28$ ✓

$5x = -25$

$x = -5$ ✓

d) Solve for x where $|2x + 1| = |x|$. **(2)**

$2x+1 = x$ | $2x+1 = -x$

$x = -1$ ✓ | $3x = -1$

$x = -\frac{1}{3}$ ✓

e) Solve for x where $\frac{3x-5}{4} + 1 = \frac{x-7}{3}$ **(3)**

$\frac{3(3x-5)}{12} + 12\left(\frac{1}{1}\right) =$

f) Using the **quadratic formula** or by **completing the square:**

Solve for x where $5x^2 + 11x + 3 = 0$. (Answer in Surd Form) **(3)**

$x = \frac{-11 \pm \sqrt{121 - 60}}{10}$ ✓

$x = \frac{-11 \pm \sqrt{61}}{10}$ ✓

$x = \frac{-11 + \sqrt{61}}{10}, \frac{-11 - \sqrt{61}}{10}$ ✓

$5\left(x^2 + \frac{11}{5}x + \frac{3}{5}\right) = 0$

$x^2 + \frac{11}{5}x + \frac{3}{5} = 0$ ✓

$\left(x^2 + \frac{11}{5}x + \frac{121}{100}\right) = -\frac{3}{5} + \frac{121}{100}$

$\left(x + \frac{11}{10}\right)^2 = \frac{61}{100}$ ✓

$x + \frac{11}{10} = \pm\frac{\sqrt{61}}{10}$

$x = -\frac{11}{10} \pm \frac{\sqrt{61}}{10}$ ✓

g) Solve for x ***and*** y where:

$3x - 2y = 14$ (1)

$2x + 3y = 17$ (2) **(2)**

$9x - 6y = 42$ (1) $\times 3$ } Add

$4x + 6y = 34$ (2) $\times 2$ }

$13x = 76$

$x = \frac{76}{13}$

$3\left(\frac{76}{13}\right) - 2y = 14$

$\frac{228}{13} - 14 = 2y$

$\frac{228 - 182}{13} = 2y$

$\frac{46}{26} = y$

$\frac{23}{13} = y$

End Of Question One

Name: Circle Teacher: SJM KXM MYK DPR

Question 1 extra writing space.

If you use this space, clearly indicate which question you are answering.

Name: Circle Teacher: SJM KXM MYK DPR

Question 2 – Functions & Further Functions

Total /14

a) MULTIPLE CHOICE QUESTION **(1)**

What is the **centre of the circle** given by the **equation**: $x^2 - 8x - 9 = -y^2$?

(Circle One Answer)

A. (-4,0)

B. (0,4)

C. (0,-4)

D. (4,0)

b) If $f(x) = -x^2 - 2x + 3$ *and* $g(x) = 2x + 1$:

(i) Write an expression for $f(g(x))$ **fully simplified.** **(2)**

$f(g(x)) = -(2x+1)^2 - 2(2x+1) + 3$

$f(g(x)) = -(4x^2+4x+1) - 4x - 2 + 3$ ✓

$= -4x^2 - 8x$

$= -4x(x+2)$ ✓

Question b) continued over page....

b) (ii) What is the **axis of symmetry** of $f(g(x))$? **(1)**

$f(g(x)) = -4x^2 - 8x$

$x = \frac{-6}{2a} = \frac{-8}{-8} = -1$

$x = -1$ ✓

c) Consider the two functions: $y = x^2 - 2x - 3, and\ y = 2x - 6.$

Find the **points of intersection** between the two functions. **(2)**

$y = x^2 - 2x - 3 \qquad y = 2x - 6$

$x^2 - 2x - 3 = 2x - 6$

$x^2 - 4x + 3 = 0$

$(x - 3)(x - 1) = 0$

$x = 3$, $x = 1$ ✓

$y = 6 - 6$ | $y = 2 - 6$

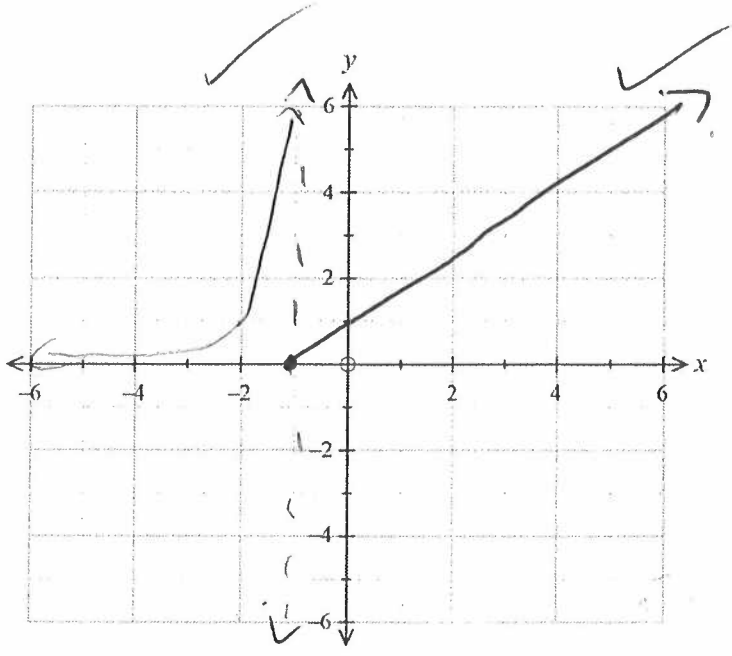
$y = 0$ | $y = -4$ ✓

$(3, 0)$ & $(1, -4)$ are the points of intersection.

d) For the function:

$$f(x) = \begin{cases} -\frac{1}{x+1}, & x < -1 \\ x + 1, & x \geq -1 \end{cases}$$

(i) Sketch the graph of $f(x)$ below showing ***x* and *y* intercepts** **(2)**



(ii) State the **range** of $f(x)$. **(1)**

$y \geq 0$ or $y \in [0, \infty)$

e) The function $f(x)$ satisfies the condition:

$2f(x) + 3f\left(\frac{2x+3}{x-2}\right) = 3x + 1 \; for \; all \; x, x \neq 2.$

Find the **value of** $f(9.)$ **(3)**

$2f(9) + 3f\left(\frac{21}{7}\right) = 28$

$2f(9) + 3f(3) = 28$ ✓

$2f(3) + 3f(9) = 10$ ✓ (×3)

$2f(9) + 3f(3) = 28$ (×2)

$6f(3) + 9f(9) = 30$

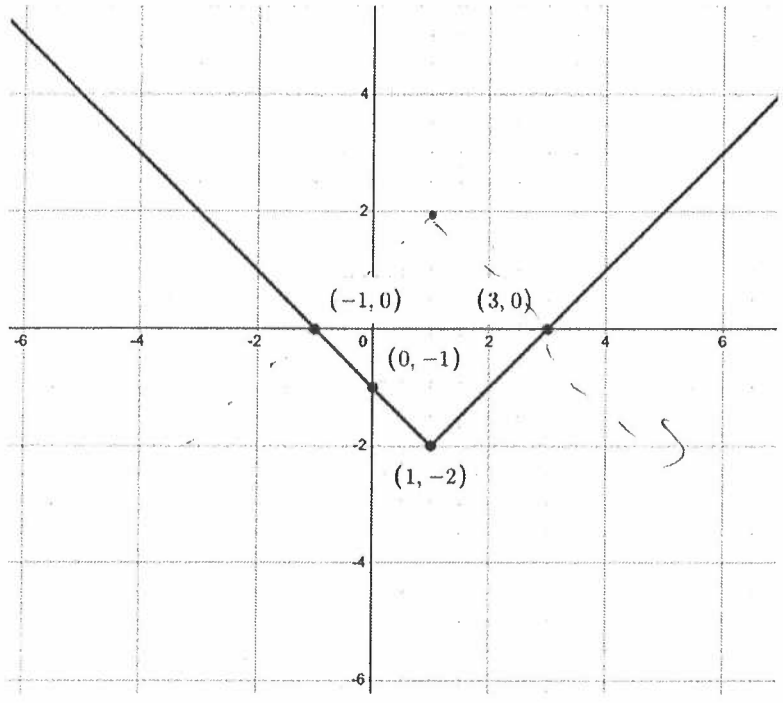
$-6f(3) - 4f(9) = -56$ } Add

$5f(9) = -26$

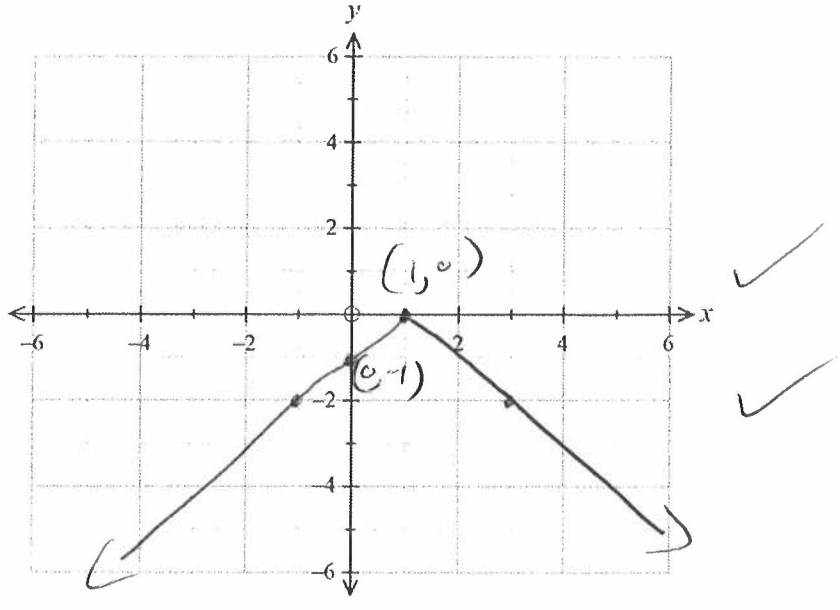
$f(9) = -26$ ✓

5.

f) The graph of the function $f(x)$ has been shown below.



Sketch the graph of $-f(x)-2$ below and show **all intercepts.** **(2)**



End Of Question Two

Question 2 extra writing space.

If you use this space, clearly indicate which question you are answering.

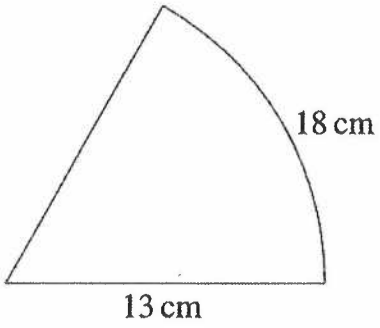
Name: Circle Teacher: SJM KXM MYK DPR

Question 3 – Trigonometry

Total /14

a) MULTIPLE CHOICE QUESTION **(1)**

The sector of **radius** 13cm shown below has an arc length of 18cm.



What is the **area of the sector**? *(Circle One Answer)*

A. $84.5cm^2$

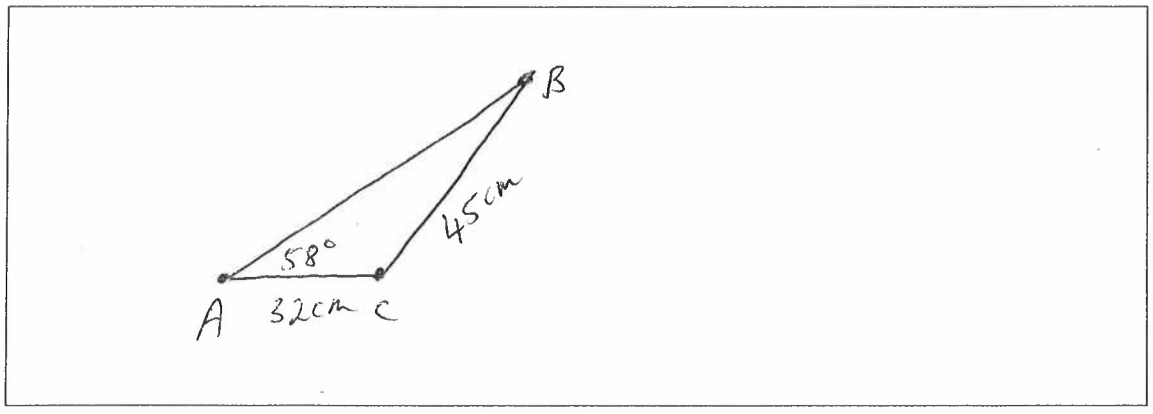
B. $117cm^2$

C. $162cm^2$

D. $234cm^2$

b) The triangle ABC has $AC=32cm$, $BC=45cm$ and $\angle BAC=58^\circ$.

(i) Draw a **labelled diagram** of the **triangle** ***ABC*** in the space below **(1)**



(ii) Find the value of $\angle ABC$, showing your answer to the **nearest degree**. **(2)**

Let $\angle ABC = \theta$

$$\frac{\sin\theta}{32} = \frac{\sin 58^\circ}{45}$$

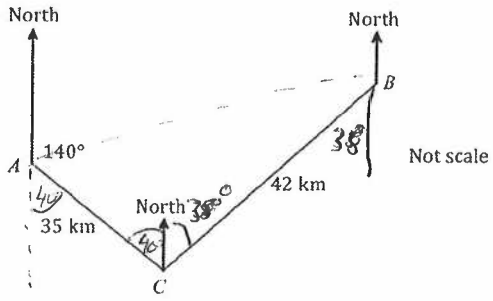
$$\sin\theta = 32 \times \frac{\sin 58^\circ}{45}$$ ✓

$$\theta = \sin^{-1}\left(32 \times \frac{\sin 58^\circ}{45}\right)$$

$$\theta \approx 37^\circ$$ ✓

c) Tyler drives 35km from town A on a true bearing of $140^\circ T$ to town C.

He then drives 42km on a true bearing of $038^\circ T$ to town B.



(i) Calculate the **distance** between A and B and round your answer to **2 decimal places.** **(2)**

$\angle ACB = 38 + 40 = 78^\circ$

$|AB|^2 = 35^2 + 42^2 - 2(35)(42)\cos 78^\circ$ ✓

$= 2377.739\ldots$

$AB \approx 49\text{km}$ ✓

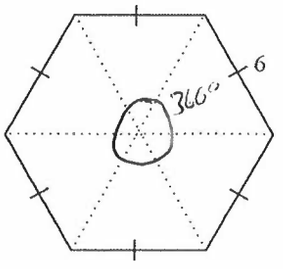
(ii) Find the **bearing** of A from B to the **nearest degree** **(2)**

$\cos \angle ABC = \dfrac{2377.739 + 42^2 - 35^2}{2 \times 49 \times 42} = 0.708\ldots$

$\angle ABC = \cos^{-1} 0.708\ldots \approx 45^\circ$ ✓

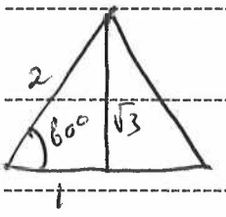
Bearing $\approx 180^\circ + 38^\circ + 45^\circ = 263^\circ T$ ✓

d) Find the **area** of the regular hexagon below in **surd form** given it is **divided into six equilateral triangles** **(2)**



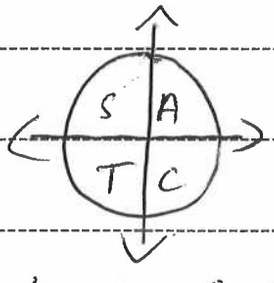
Area of one equilateral triangle $= \frac{1}{2} \times 6^2 \times \sin 60°$

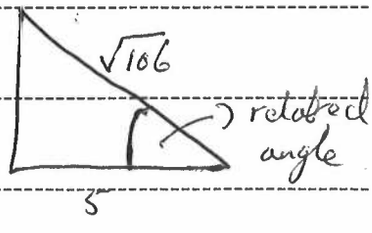
$= 18 \times \frac{\sqrt{3}}{2} = 9\sqrt{3}\ u^2$ ✓



Area of hexagon $= 6 \times 9\sqrt{3}\ u^2 = 54\sqrt{3}\ u^2$ ✓

e) Given that $\cot\beta = -\frac{5}{9}$ and that $\sin\beta < 0$, find the **value of** $\cos\beta$. **(2)**



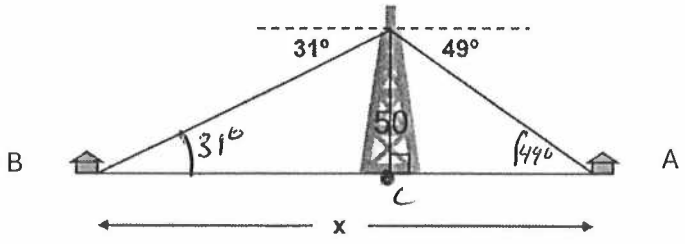


$\tan\beta = -\frac{9}{5} < 0$ } 4th quad ✓

$\sin\beta < 0$

$\cos\beta = \frac{5}{\sqrt{106}}$ ✓

f) An observer at the top of a **50m high** tower notes that the angle of depression of a house lying due east is 49° and the angle of depression of a house lying due west is 31°.



Find the **distance** between the houses at points *A and B* to the **nearest metre**. **(2)**

$\tan 31^\circ = \frac{50}{|BC|}$

$|BC| = \frac{50}{\tan 31^\circ}$

$\tan 49^\circ = \frac{50}{|AC|}$

$|AC| = \frac{50}{\tan 49^\circ}$

Ans: $\frac{50}{\tan 31} + \frac{50}{\tan 49} \approx 127\text{m}$.

End of Question Three

Question 3 extra writing space.

If you use this space, clearly indicate which questions you are answering.

Name: Circle Teacher: SJM KXM MYK DPR

Question 4 – Calculus

Total /14

a) MULTIPLE CHOICE QUESTION **(1)**

Find the **equation of the normal** to the curve $y = -x^2 + 3x - 4$ at the point on the graph **where** $x = 2$. *(circle one answer)*

A. $y = 2x - 1$

B. $y = -x + 2$

C. $y = x$

D. $y = x - 4$

b) Differentiate: $y = -2x^3 - \frac{5x}{2} + 3$. **(1)**

$y' = -6x^2 - \frac{5}{2}$ ✓

c) Differentiate: $y = \frac{2x-5}{-x^3+1}$. $= \frac{u}{v}$. **(2)**

$y' = \frac{v\,u' - uv'}{v^2}$ $u = 2x-5$ $u' = 2$

$v = -x^3+1$ $v' = -3x^2$ ✓

$y' = \frac{(-x^3+1)(2) + 3x^2(2x-5)}{(1-x^3)^2} = \frac{4x^3 - 15x^2 + 2}{(1-x^3)^2}$ ✓

d) Differentiate: $y = \frac{1}{2}\sqrt{x^3 - 2x}$. **(2)**

$y = \frac{1}{2}(x^3-2x)^{\frac{1}{2}}$.

$y' = \frac{1}{2} \times \frac{1}{2}(x^3-2x)^{-\frac{1}{2}}(3x^2-2)$ ✓

$y' = \frac{3x^2-2}{4\sqrt{x^3-2x}}$ ✓

e) Find the **gradient function** of $x(t) = 4t^2(t^2 - 10)^4$ in **factored form.** **(2)**

$u = 4t^2 \quad v = (t^2-10)^4$

$u' = 8t \quad v' = 4(t^2-10)^3(2t)$

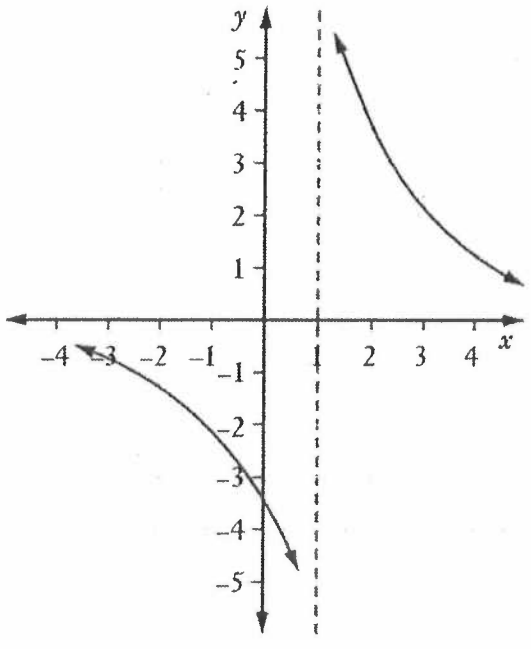
$x'(t) = 32t^3(t^2-10)^3 + 8t(t^2-10)^4$ ✓

$x'(t) = 8t(t^2-10)^3(4t^2 + t^2 - 10)$

$x'(t) = 8t(t^2-10)^3(5t^2-10)$

$x'(t) = 40t(t^2-10)^3(t^2-2)$ ✓

f) Consider the graph of the function below:



Find all the values of x where the function is **not differentiable**. **(1)**

$x = 1$ (discontinuous) ✓

g) The formula for **differentiation by first principles** is:

$\frac{d(f(x))}{dx} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$.

Given the function $f(x) = x^2 - 2x + 3$, use **differentiation by first principles** to show that $\frac{d(f(x))}{dx} = 2x - 2$. **(3)**

$f(x) = x^2 - 2x + 3$

$f(x+h) = (x+h)^2 - 2(x+h) + 3 = x^2 + 2hx + h^2 - 2x - 2h + 3$

$f(x+h) - f(x) = x^2 + 2hx + h^2 - 2x - 2h + 3 - x^2 + 2x - 3$

$f(x+h) - f(x) = 2hx + h^2 - 2h$

$\frac{f(x+h) - f(x)}{h} = 2x - 2 + h$ ✓

$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = 2x - 2 + 0 = 2x - 2$ ✓

h) A function $f(x)$ satisfies $f(5) = 25$ *and* $f'(5) = 20$.

Find the **equation of the tangent to** $y = \sqrt{f(x)}$ **at** $x = 5$. **(2)**

$y' = \frac{1}{2} f(x)^{-\frac{1}{2}} (f'(x))$ $\quad x = 5 \quad y = \sqrt{f(5)} = \sqrt{25} = 5$

y' @ $x = 5$ = $\quad (5, 5)$ point on tangent. ✓

$\frac{1}{2} \times \frac{1}{\sqrt{25}} \times 20 = 2$ ✓

equation = $y - 5 = 2(x - 5)$

$y = 2x - 5$ ✓

End of Question Four

Question 4 extra writing space.

If you use this space, clearly indicate which question you are answering.

Name: Circle Teacher: SJM KXM MYK DPR

Question 5 – Probability

Total /14

a) MULTIPLE CHOICE QUESTION **(1)**

One letter is chosen randomly from each of the two words 'THIRTY' and 'EIGHTY'. What is the probability that the **two letters chosen are the same**? *(circle one answer)*

A. $\frac{1}{12}$

B. $\frac{1}{9}$

C. $\frac{5}{36}$

D. $\frac{1}{6}$

b) A standard deck of **52** cards is placed on the table after being shuffled. Two cards are picked **with replacement.**

What is the probability that the first card is a **Heart** and the second card is a **King**?

(Note: A deck of cards, has four suits, hearts, diamonds, clubs and spades. Each suit comprises 13 cards with numbers from 1-10 and picture cards king, queen and jack.) **(2)**

$\frac{13}{52} \times \frac{4}{52} = \frac{1}{4} \times \frac{1}{13} = \frac{1}{52}$ ✓ ✓

c) Two numbers are chosen randomly from the set $\{1,2,3,4,5\}$ **with replacement**.

What is the probability that **multiplication** of the two numbers results in a **multiple of 3**? **(3)**

Pick 2

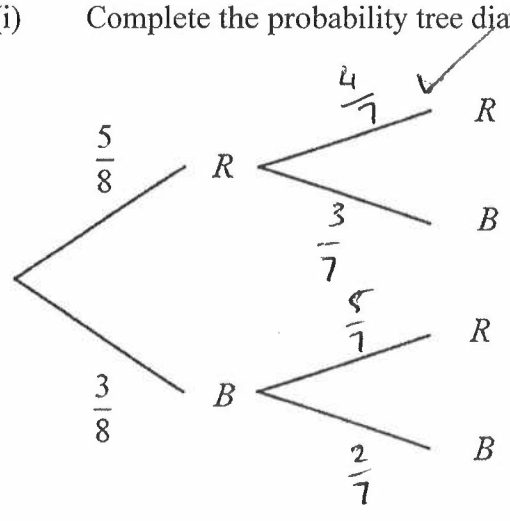
X	1	2	3	4	5
1	1	2	3	4	5
2	2	4	6	8	10
3	3	6	9	12	15
4	4	8	12	16	20
5	5	10	15	20	25

Pick 1

$\frac{9}{25}$

d) A bag contains **5 red marbles and 3 blue marbles**. Two marbles are drawn **without replacement**.

(i) Complete the probability tree diagram below. **(1)**



(ii) Find the probability that both marbles are the same colour. **(1)**

$P(R,R) = \frac{5}{8} \times \frac{4}{7} = \frac{5}{14}$

$P(B,B) = \frac{3}{8} \times \frac{2}{7} = \frac{3}{28}$

Ans $= \frac{5}{14} + \frac{3}{28} = \frac{13}{28}$

e) A country could have a president, a queen, both a president, and a queen or be self-governed. 60 countries were surveyed, and their results can be categorized into three groups: 35 have a **President,** 27 have a **Queen**, and 8 are **self-governed.**

(i) Using the information above complete the two-way table below:

	President	No President	Total
Queen	10	17	27
No Queen	25	8	33
Total	35	25	60

(2)

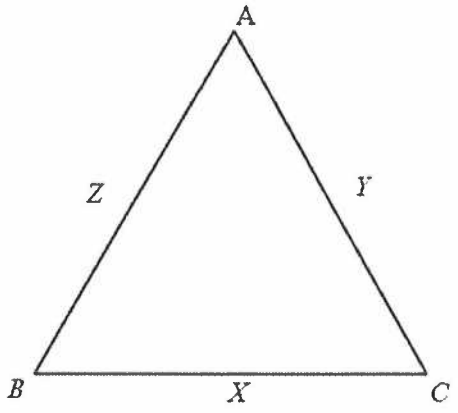
(ii) What is the probability that a country will have a **President** only? **(1)**

$\frac{25}{60} = \frac{5}{12}$

(iii) What is the probability that a country will have **a President if we know it has a Queen**? **(1)**

$\frac{10}{27}$

f) Three telecommunications towers, $A, B,$ and C are connected via cables $X, Y,$ and $Z.$ as shown below. A and B remain in contact only if Z is operative or if both cables X and Y are operative. Cables $X, Y,$ and Z are subject to failure in any one day with probabilities of failure 0.01, 0.025 and 0.03 respectively. Such failures occur independently.



Find the probability that towers ***A* and *B* are unable to make contact** on a given day. (Do Not Round Off Your Answer At The End) **(2)**

P(Z fail and X or Y fail)

= P(Z fail) × (1 − P(neither X or Y fail)) ✓

= 0.03 × (1 − (0.99 × 0.975))

= 0.00110425 ✓

End of Question Five

Question 5 extra writing space.

If you use this space, clearly indicate which question you are answering.

or P(Z fails and x fails)

or

P(Z fails and x passes and y fails) =

$(0.03 \times 0.01) + (0.03 \times 0.99 \times 0.025) =$

0.0010425

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Question 6 – Trigonometric Functions

Total /14

a) MULTIPLE CHOICE QUESTION **(1)**

What is the **correct expression** for $(\sin\theta + \cos\theta)^2 - 1$. *(circle one answer)*

A. 0

B. $2cos^2\theta$

C. $2sin^2\theta$

D. $2\sin\theta\cos\theta$

b) Given $-\pi \leq x \leq \pi$: find all **suitable solutions** to the equation:

$2cos(2x) - 0.5 = 0.5$. **(2)**

$2\cos 2x = 1$

$\cos 2x = \frac{1}{2}$

$-2\pi \leq 2x \leq 2\pi$ ✓

$2x = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{-\pi}{3}, \frac{-5\pi}{3}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{-\pi}{6}, \frac{-5\pi}{6}$ ✓

c) Show that $\frac{1+\sin\theta}{1-\sin\theta} + \frac{1-\sin\theta}{1+\sin\theta} = 2(\sec^2\theta + \tan^2\theta)$. **(3)**

L.H.S $= \frac{(1+\sin\theta)^2 + (1-\sin\theta)^2}{(1-\sin\theta)(1+\sin\theta)}$

$\frac{2 + 2\sin^2\theta}{1-\sin^2\theta} = \frac{2 + 2\sin^2\theta}{\cos^2\theta}$ ✓

$= \frac{2}{\cos^2\theta} + \frac{2\sin^2\theta}{\cos^2\theta} = 2\sec^2\theta + 2\tan^2\theta$

$= 2(\sec^2\theta + \tan^2\theta)$ ✓

d) Given $0 \leq \theta \leq 2\pi$

solve the equation $2\cos^2\theta + \cos\theta - 1 = 0$, giving answers in **terms of** π. **(2)**

let $\cos\theta = m$

$2m^2 + m - 1 = 0$

$(2m - 1)(m + 1) = 0$

$m = \frac{1}{2}$ $m = -1$ ✓

$\cos\theta = \frac{1}{2}$ $\cos\theta = -1$

$\theta = \pi$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$ ✓

e) Consider the function $f(x) = 2sin\left(\frac{2}{3}x - \frac{\pi}{3}\right)$:

What are the **amplitude, period and phase** of this function? **(3)**

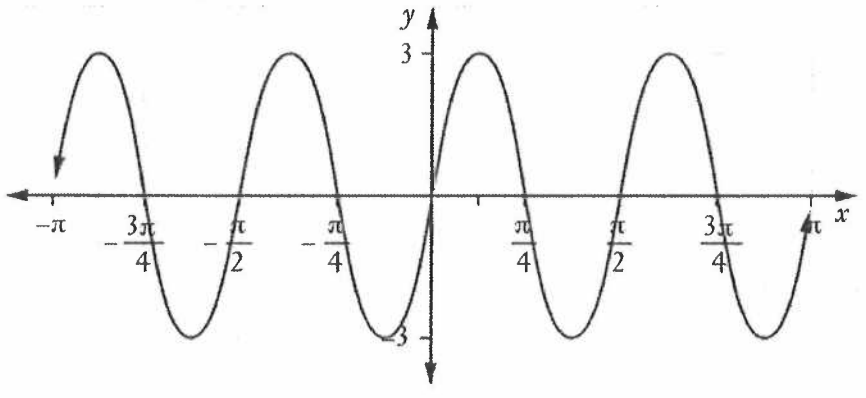
Amplitude = 2 ✓

Period = $2\pi \div \frac{2}{3} = \frac{2\pi}{1} \times \frac{3}{2} = 3\pi$ ✓

Phase: $f(x) = 2\sin\frac{2}{3}\left(x - \frac{\pi}{2}\right)$.

Phase = $-\frac{\pi}{2}$ or $\frac{\pi}{2}$ units to the right ✓

f) What is the equation of the graph below: **(2)**



Amplitude = 3 $y = 3\sin 4x$ ✓

Centre of motion = 0.

Sine function

Period = $\frac{\pi}{2}$ ✓

$\frac{2\pi}{n} = \frac{\pi}{2}$ $4\pi = n\pi$

$4 = n$

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g) Solve $sin\theta = cos\frac{2\pi}{3}$ for $-\pi \leq \theta \leq \pi$. **(1)**

$\sin\theta = \sin\left(\frac{\pi}{2} - \frac{2\pi}{3}\right)$

$\sin\theta = \sin\left(\frac{3\pi - 4\pi}{6}\right)$

$\sin\theta = \sin\left(\frac{\pi}{6}\right)$

$\theta = -\frac{\pi}{6}$.

$\left(\sin\frac{5\pi}{6} = \sin\frac{\pi}{6} = \cos\frac{\pi}{3} \neq \cos\frac{2\pi}{3}\right)$ disregard

$\sin(\pi - \theta) = \sin\theta$

$\pi - \theta = \frac{\pi}{2} - \frac{2\pi}{3}$.

$\pi - \frac{\pi}{2} + \frac{2\pi}{3} = \theta$.

$\frac{\pi}{2} + \frac{2\pi}{3} = \theta$

$\frac{3\pi + 4\pi}{6} = \theta$

$\frac{7\pi}{6} = \theta$

$-\frac{5\pi}{6} = \theta$

Ans $= -\frac{\pi}{6}, -\frac{5\pi}{6}$. ✓

End of Examination

Question 6 extra writing space.

If you use this space, clearly indicate which question you are answering.