Sydney Girls High School

2016

YEAR 11 Preliminary Course

ASSESSMENT TASK 3

MATHEMATICS

Time Allowed: 60 minutes + 5 minutes reading time

Total: 60 marks

Topics: Basic Arithmetic and Algebra, Plane Geometry, Real Functions, Trigonometric Ratios, The Linear Function, Introductory Calculus: Tangent to a curve, differentiation of a function, The Quadratic Polynomial and Locus.

General Instructions:

- There are Four (4) Questions which are of equal value.
- Attempt all questions.
- Show all necessary working. Marks may be deducted for badly arranged work or incomplete working.
- Start each Question on a new page.
- Write on one side of the paper only.
- Diagrams are NOT to scale.
- Board-approved calculators may be used.

Student Name :__________________ **Teacher Name :__________________**

QUESTION 1 **Marks**

(a) Evaluate $255(0.0024)^2$ giving your answer correct to 3 significant figures. **2**

(b) Solve $x^2 + 9x - 10 = 0$. **2**

(c) Solve $|3-2x| > 5$. **2**

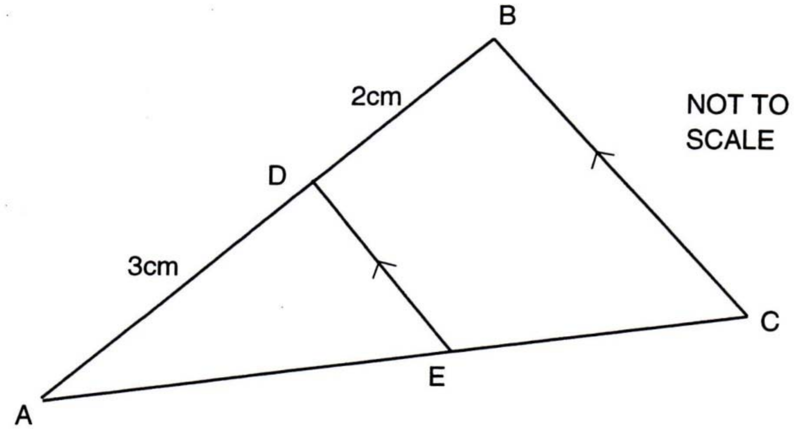
(d) Factorise $p^3 + 8$. **1**

(e) Solve $\sin\beta = \dfrac{\sqrt{3}}{2}$ for $0^\circ \le \beta \le 360^\circ$. **2**

(f) Express $\dfrac{9}{5-\sqrt{7}}$ in the form $a + b\sqrt{7}$, where a and b are rational. **2**

(g) Simplify $\sqrt{(a-5)(a+5)+25}$ where $a > 0$. **2**

(h)



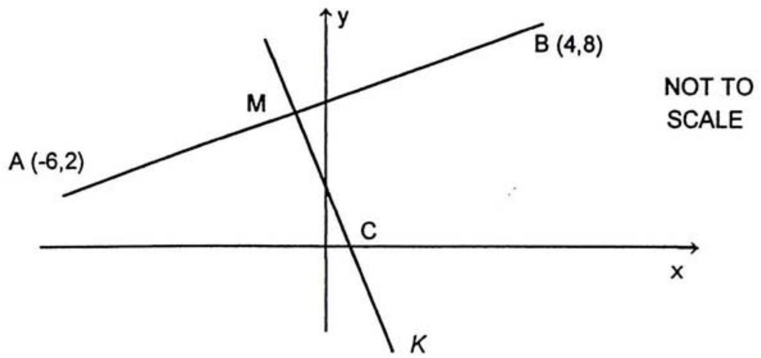
Copy or trace the diagram onto your worksheet.

In ΔABC, points D and E lie on lines AB and AC respectively, such that DE is parallel to BC. If AD is 3 centimetres and DB is 2 centimetres, find the ratio DE:BC, giving reasons. **2**

QUESTION 2 (Start a new page) **Marks**

(a) If $f(x) = 3x^4 - 5$, find $f'(-3)$. **2**

(b)



In the diagram above, A is the point $(-6, 2)$ and B $(4, 8)$.

(i) Find the point M, the midpoint of AB. **1**

(ii) Find the gradient of the line AB. **1**

(iii) Line K is the perpendicular bisector of line AB. Find the equation of line K. **2**

(iv) Find the coordinates of C, the point at which line K cuts the x-axis. **1**

(c) Find the equation of the normal to the curve $y = \dfrac{3}{x} + x$ at the point where $x = 2$. **2**

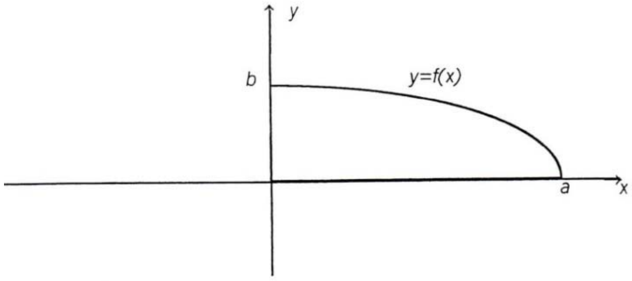
(d) Find the value(s) of k for which $x^2 + (k-1)x + 4 = 0$ has equal roots. **2**

(e) State the domain and range of the function $y = \sqrt{36 - x^2}$. **2**

(f) Draw a neat sketch showing the region on the number plane where the inequalities $x^2 + y^2 \leq 4$ and $y \geq x$ hold simultaneously. **2**

QUESTION 3 (Start a new page) **Marks**

(a) The diagram below shows part of the graph of the function $y = f(x)$. **1**



Copy the diagram and complete the graph if $y = f(x)$ is an odd function.

(b) Form a quadratic equation with roots $\left(1+\sqrt{3}\right)$ and $\left(1-\sqrt{3}\right)$. **2**

(c) Differentiate each of the following:

(i) $\dfrac{3x}{x^2-2}$. **2**

(ii) $\left(2-3x\right)^7$. **1**

(d) Consider the parabola $y = \dfrac{1}{2}x^2 - x + \dfrac{3}{2}$.

(i) Find the coordinates of the vertex. **1**

(ii) Find the coordinates of the focus **1**

(iii) Find the equation of the directrix. **1**

(e) Given that α and β are the roots of the quadratic equation $3x^2 - 5x - 1 = 0$, find the values of:

(i) $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$. **2**

(ii) $\left(\alpha^2 + \beta^2\right)^2$ **2**

(f) Prove the following identity: **2**

$$\sec\theta + \tan\theta = \frac{\cos\theta}{1-\sin\theta}.$$

QUESTION 4 (Start a new page) **Marks**

(a) If $\tan\beta = -2$ and $0^\circ < \beta < 270^\circ$, find the value of $\operatorname{cosec}\beta$ in surd form. **2**

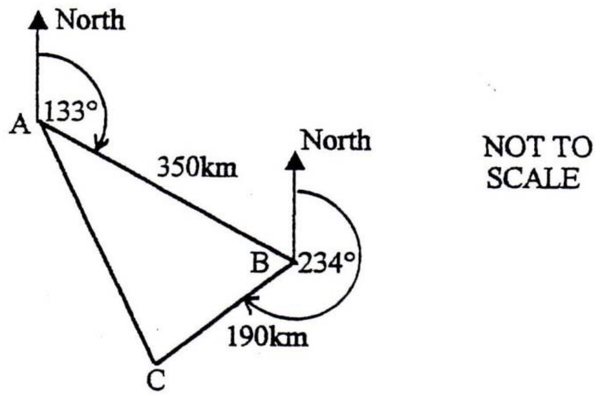
(b) The equation of a locus is given by $x^2 + y^2 - 2x + 4y - 11 = 0$.

(i) Show that this is the equation of a circle and that the centre is the point $(1, -2)$. **2**

(ii) Find the length of its radius. **1**

(iii) Does the line $2x - y + 8 = 0$ intersect the circle? Explain. **2**

(c) A plane leaves airport *A* and flies on a bearing of 133° a distance of 350 kilometres to point *B*. It then flies on a bearing of 234° for 190 kilometres to point *C*, as shown in the diagram.



Copy the diagram onto your worksheet.

(i) Show that $\angle ABC = 79^\circ$. **1**

(ii) Find the distance from *C* to *A*, to the nearest kilometre. **2**

(iii) Find the bearing of *A* from *C*, to the nearest degree. **2**

(d) The tangent to $y = ax + \dfrac{b}{x^2}$ at $(2, 0)$ makes an angle of 60° with the positive direction of the *x*-axis. Find the values of *a* and *b*. **3**

END OF PAPER

Question 1

a) $255(0.0024)^2 \doteq 0.0014688\ldots$

$= 1.47 \times 10^{-3}$ (3 sig figs)

or 0.00147 (3 sig figs)

* one mark awarded for the correct answer, 1 mark for converting correctly to 3 sig figs.

b) $x^2 + 9x - 10 = 0$

$(x+10)(x-1) = 0$

$\therefore x + 10 = 0 \qquad x - 1 = 0$

$x = -10 \qquad x = 1$

c) $|3 - 2x| > 5$

$3 - 2x > 5 \qquad 3 - 2x < -5$

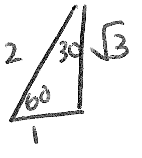
$-2x > 2 \qquad -2x < -8$

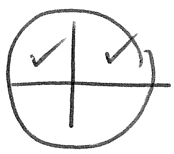
$x < -1 \qquad x > 4$

d) $p^3 + 8 = p^3 + 2^3$

$= (p+2)(p^2 - 2p + 4)$

e) $\sin \beta = \frac{\sqrt{3}}{2}$





$\beta = 60°, 180 - 60°$

$= 60°, 120°$

f) $\frac{9}{5-\sqrt{7}} \times \frac{5+\sqrt{7}}{5+\sqrt{7}} = \frac{45 + 9\sqrt{7}}{25-7}$

$= \frac{9(5+\sqrt{7})}{18}$

$= \frac{5}{2} + \frac{\sqrt{7}}{2}$

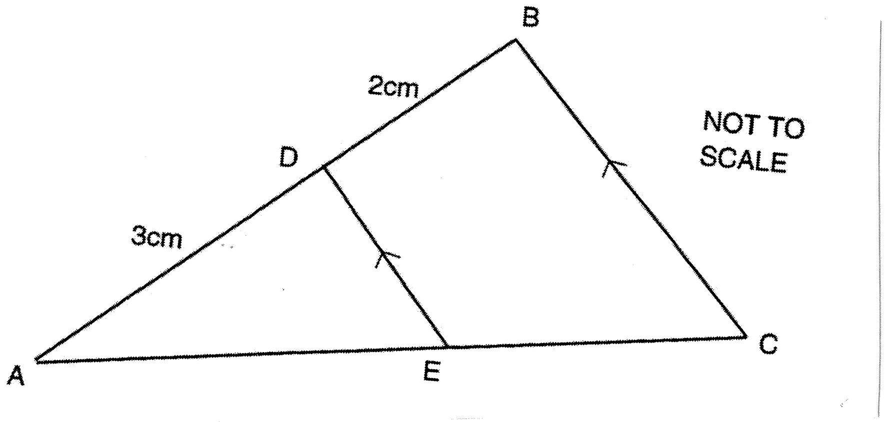
g) $\sqrt{(a-5)(a+5)+25}$

$= \sqrt{a^2-25+25}$

$= \sqrt{a^2}$

$= a \quad , a > 0$

h)



In $\triangle ABC$ and $\triangle ADE$

$\angle A$ is common

$\angle ADE = \angle ABC$ (corresponding $\angle$s $DE \parallel BC$)

$\therefore \triangle ABC \,|||\, \triangle ADE$ (equiangular)

$\therefore \quad \frac{DE}{BC} = \frac{AD}{AB}$ (matching sides similar $\triangle$s in same proportion)

$\frac{DE}{BC} = \frac{3}{5}$

$DE : BC = 3:5$

Question 2 (15 Marks)

a) $f(x) = 3x^4 - 5$

$f'(x) = 12x^3$

$\therefore f'(-3) = 12(-3)^3$

$= 12 \times -27$

$= -324$ (2)

b) i) $M = \left(\frac{-6+4}{2}, \frac{2+8}{2}\right) = (-1, 5)$ (1)

ii) $m_{AB} = \frac{8-2}{4+6} = \frac{3}{5}$ (1)

iii) $m_K = -\frac{5}{3}$ and $M = (-1, 5)$

$\therefore$ Equation of line k:

$y - 5 = -\frac{5}{3}(x+1)$

$y = -\frac{5}{3}x - \frac{5}{3} + 5$

(2)

$y = -\frac{5}{3}x + \frac{10}{3}$ OR $5x + 3y - 10 = 0$

iv) when $y=0$: $\frac{5}{3}x = \frac{10}{3}$ $\therefore C = (2, 0)$ (1)

$x = 2$

* Many students substituted $x=0$ (instead of $y=0$) to find the x-intercept.
* A few students did not give their answer as a point.

c) $y = 3x^{-1} + x$

$\frac{dy}{dx} = -3x^{-2} + 1$

at $x = 2$: $m = -\frac{3}{2^2} + 1 = \frac{1}{4}$; $y = \frac{7}{2}$

$\therefore$ Equation of normal:

$y - \frac{7}{2} = -4(x-2)$ ②

$\underline{\underline{y = -4x + \frac{23}{2}}}$ OR $\underline{\underline{8x + 2y - 23 = 0}}$.

d) $x^2 + (k-1)x + 4 = 0$.

For equal roots: $\Delta = 0$

$b^2 - 4ac = 0$

$(k-1)^2 - 4(1)(4) = 0$

$(k-1)^2 - 16 = 0$

$(k-1)^2 = 16$

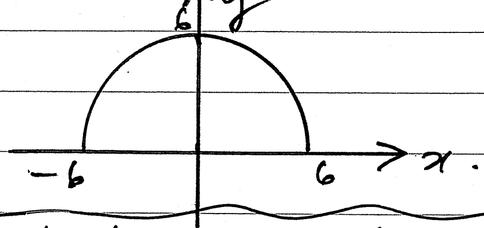
$k - 1 = \pm 4$ ②

$\therefore \underline{\underline{k = 5 \text{ or } -3}}$.

e) $y = \sqrt{36 - x^2} \rightarrow$ graph of semi-circle

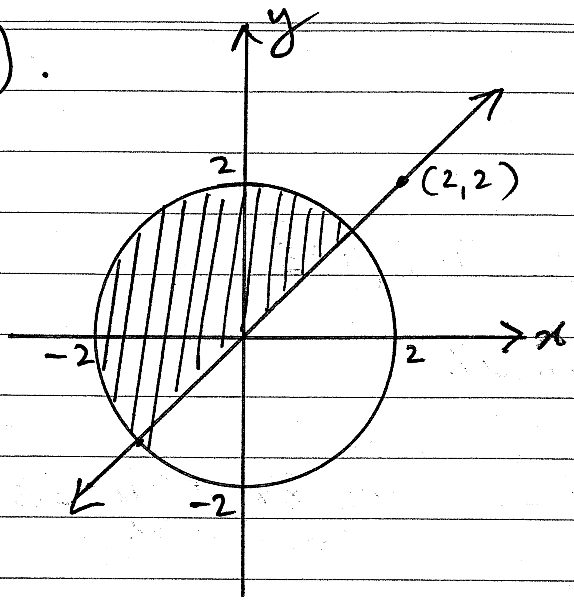
D: $-6 \le x \le 6$

R: $0 \le y \le 6$ ②



* Many students lost marks with the range being incorrectly stated

f).



② .

region ✓
graphs ✓

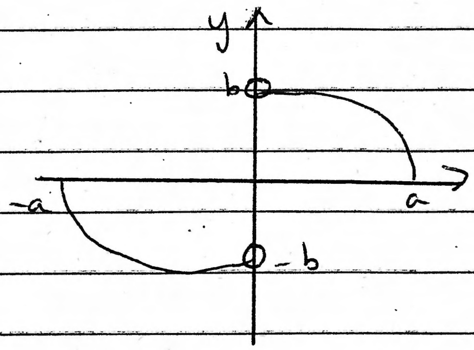
☆ Many students were unsure whether to dot the lines or not. Solid lines are drawn with equality.

☆ Diagrams need to be drawn in PEN

Question 3

1.

a)



1

b) $x^2 - (1+\sqrt{3} + 1-\sqrt{3})x + (1+\sqrt{3})(1-\sqrt{3})$

$x^2 - 2x - 2 = 0$ 2

c) i) $\frac{d}{dx}\left(\frac{3x}{x^2-2}\right) = \frac{(x^2-2)3 - 3x(2x)}{(x^2-2)^2}$

$= \frac{3x^2 - 6 - 6x^2}{(x^2-2)^2}$ 2

$= \frac{-3x^2 - 6}{(x^2-2)^2} \Rightarrow \frac{-3(x^2-2)}{(x^2-2)^2}$

$= \frac{-3}{x^2-2}$

ii) $\frac{d}{dx}(2-3x)^7 = 7(2-3x)^6(-3)$

$= -21(2-3x)^6$ ✓ 1

d) $y = \frac{1}{2}x^2 - x + \frac{3}{2}$

$2y = x^2 - 2x + 3$

$x^2 - 2x + 3 = 2y$

$x^2 - 2x + 1 = 2y - 2$

$(x-1)^2 = 2(y-1)$

$\therefore$ i) $V(1, 1)$ ✓ ii) $4a = 2 \Rightarrow a = \frac{1}{2}$ $\therefore$ F at $(1, \frac{3}{2})$ ✓

1 directrix $y = \frac{1}{2}$ ✓

CEPA 1 1 3

applies here.

e) i) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}$, $\alpha+\beta = \frac{5}{3}$, $\alpha\beta = \frac{-1}{3}$

$= \frac{5}{3} \div -\frac{1}{3}$

$= -5$ 2

ii) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= (\frac{5}{3})^2 - 2(-\frac{1}{3})$

$= \frac{31}{9}$

$\therefore (\alpha^2+\beta^2)^2 = \frac{961}{81}$ $(11\frac{70}{81})$ 2

f) $\sec\theta + \tan\theta$

$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$

$= \frac{1+\sin\theta}{\cos\theta} \times \frac{1-\sin\theta}{1-\sin\theta}$

$= \frac{1-\sin^2\theta}{\cos\theta(1-\sin\theta)}$

$= \frac{\cos^2\theta}{\cos\theta(1-\sin\theta)}$

$= \frac{\cos\theta}{1-\sin\theta}$ 2

OR $\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$

$= \frac{\cos\theta + \cos\theta\sin\theta}{\cos^2\theta}$

$= \frac{\cos\theta(1+\sin\theta)}{1-\sin^2\theta}$

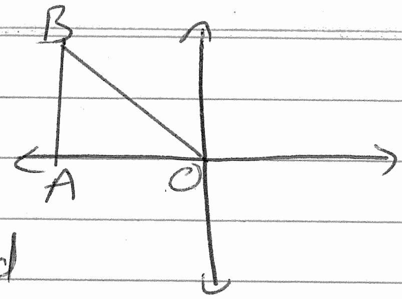
$= \frac{\cos\theta(1+\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$

$= \frac{\cos\theta}{1-\sin\theta}$

/15

4

a)



$\tan\beta = -2$

$0^\circ < \beta < 270^\circ$

tan is −ve in II and IV quardant, as β is restricted to be in the range 0° to 270°, so it is in II quardant

$$\tan\beta = \frac{AB}{OA} = -2$$

$$\Rightarrow AB = 2, \quad OA = -1 \quad OB = \sqrt{4+1} = \sqrt{5}$$

$$\operatorname{cosec}\beta = \frac{OB}{AB} = \frac{\sqrt{5}}{2}$$

Alternative $\quad \tan\beta = -2$

$$\beta = 116^\circ 33' 54''$$

$$\operatorname{cosec}\beta = \frac{1}{\sin\beta} = \frac{\sqrt{5}}{2}$$

Note: most students got it right, however, some were confused about the use of quardants.

b) (i), (ii)

$$x^2 + y^2 - 2x + 4y - 11 = 0$$

$$x^2 - 2x + 1 - 1 + y^2 + 4y + 4 - 11 - 4 = 0$$

$$(x-1)^2 + (y+2)^2 = 16$$

centre $(1, -2)$ radius $= 4$

Some were confused about the process of completing the square

(iii)

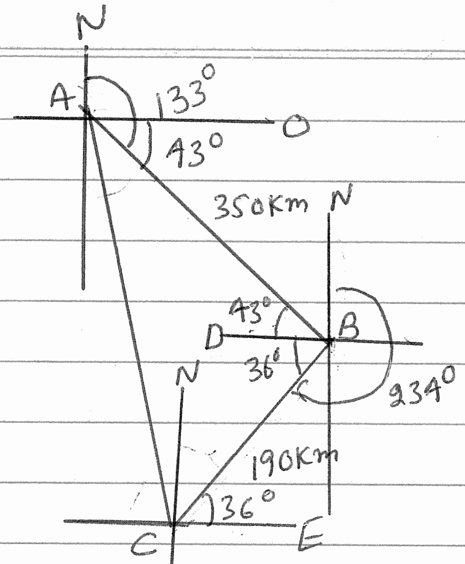
$\perp$ distance of the centre of the circle from the given line $= \dfrac{|2+2+8|}{\sqrt{4+1}} = \dfrac{12}{\sqrt{5}}$

As $\dfrac{12}{\sqrt{5}} > 4$

Hence $\perp$ distance is greater than the radius of the circle, so the given line does not touch the circle.

Different approaches, however, most of the students got it right.

c)



Majority got it correct.

i. $\angle OAB = \angle ABD =$ alt. angles

$\angle DBC = 36°$

$\angle ABC = \angle ABD + \angle DBC = 43° + 36° = 79°$

(ii) using cosine rule

$AC^2 = AB^2 + BC^2 - 2AB \cdot BC \cos 79°$

$AC^2 = 350^2 + 190^2 - 2 \times 350 \times 190 \cos 79°$

$AC \approx 365\ km$

(iii) $\angle BCE = 36°$

$\angle NCB = 90° - 36 = 54°$

using sine rule

$$\frac{\sin 79°}{AC} = \frac{\sin C}{350}$$

$$\sin C = \frac{350 \times \sin 79°}{AC}$$

$$C = \sin^{-1}\left(\frac{350 \times \sin 79°}{365}\right) = 70° 16' 8''$$

$\angle NCA = 70° 16' 8'' - 54 = 16° 16' 8''$

Bearing of A from C $\quad 360° - 16° 16' 8''$

$= 343° 43' 52''$

$\approx 344°$

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d) $y = ax + \frac{b}{x^2}$

gradient of tangent $m = \frac{dy}{dx} = a - 2bx^{-3}$

$m = a - \frac{2b}{x^3}$

Also $m = \tan 60^\circ$

$\Rightarrow a - \frac{2b}{x^3} = \sqrt{3}$

point $(2, 0)$

$a - \frac{2b}{8} = \sqrt{3}$ — ①

As $(2,0)$ lies on the curve

$2a + \frac{b}{4} = 0$

$8a + b = 0$ — ②

From ①

$8a - 2b = 8\sqrt{3}$ — ③

③ − ②

$-3b = +8\sqrt{3}$

$b = -\frac{8}{\sqrt{3}}$

Sub in ②

$8a + \frac{-8}{\sqrt{3}} = 0$

$a - \frac{1}{\sqrt{3}} = 0$

$a = \frac{1}{\sqrt{3}}$

Some students made the process of solving of simultaneous equations very complicated. Some did not simplify their answers. Few had done differentiation wrong.