Sydney Girls High School

MATHEMATICS

YEAR 11 Preliminary Course

ASSESSMENT TASK 3

2017

Reading time : 5 minutes

Writing time : 60 minutes

Total marks : 60

Topics: Basic Arithmetic and Algebra, Plane Geometry, Real Functions, Trigonometric Ratios, The Linear Function, Introductory Calculus: Tangent to a curve, differentiation of a function.

General Instructions:

- There are four (4) questions which are of equal value.
- Attempt all questions.
- Show all necessary working. Marks may be deducted for badly arranged work or incomplete working.
- Start each question on a new page.
- Write on one side of the paper only.
- Diagrams are NOT to scale.
- Board-approved calculators may be used.
- Write your student number clearly at the top of each question and clearly number each question.
- Write using a black or blue pen.

Student Name:________________________ **Teacher Name:**____________________

QUESTION 1 **MARKS**

a) Evaluate $(8.315\times10^{3})\div(2.5\times10^{-5})$ answering in scientific notation. **1**

b) Fully factorise $4m-4n-m^{2}+mn$ **1**

c) Solve $(x+2)^{2}-25=0$ **2**

d) If $\dfrac{4}{2-\sqrt{3}}=a+b\sqrt{3}$ find the values of a and b. **2**

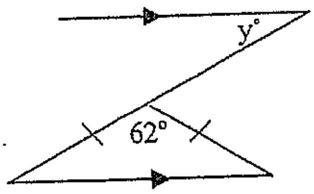
e) Given that $\sin\theta=\dfrac{5}{13}$ and $\tan\theta<0$, what is the exact value of $\cos\theta$? **2**

f) State the domain and range of the following function **2**

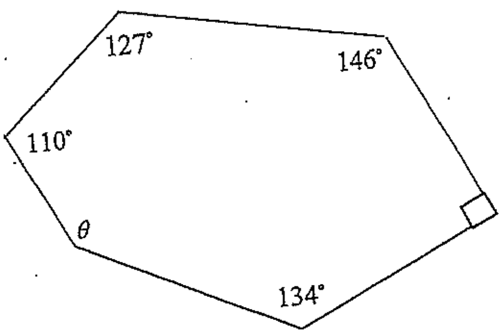
$$y=\sqrt{9-x^{2}}$$

g) Find the value of the pronumeral in each case

i) **1**



ii) **2**



h) The curve $y=ax^{2}-6x+3$ has a stationary point at $x=1$. **2**
What is the value of a?

QUESTION 2

MARKS

a) Solve $2^{3x-1} = \frac{1}{4}$ **2**

b) Differentiate the following:

i) $\sqrt{1-x}$ **1**

ii) $\dfrac{7x}{3-x}$ **2**

c) Given $f(x) = \begin{cases} x+1 & x \geq 3 \\ x^2+2x-1 & -1 < x < 3 \\ 3^x & x \leq -1 \end{cases}$, **2**

find the value of $f(2)+f(-2)-f(6)$.

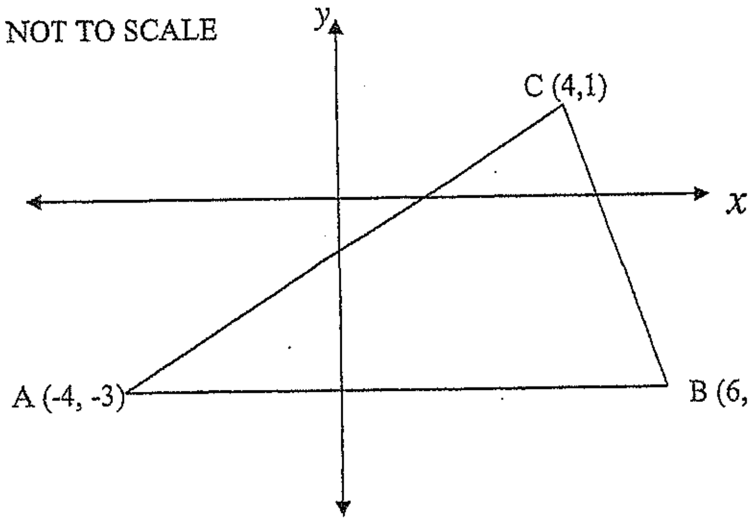
d) Find the equation of the tangent to the curve $y = 3x^2 - x^3$ at $x = -1$. **3**

e) Solve $|8-2m| \leq 70$ **2**

f) Show that $\dfrac{(1+\tan^2\theta)\cot\theta}{\operatorname{cosec}^2\theta} = \tan\theta$ **3**

QUESTION 3 **MARKS**

a) The diagram below shows ΔABC with the vertices $A(-4,-3)$, $B(6,-3)$ and $C(4,1)$.



i) Find the gradient of the interval BC. **1**

ii) Prove A, B and C are the vertices of a right-angled triangle. **1**

iii) Find the distance BC in exact form. **1**

iv) Find the size of $\angle CAB$ to the nearest minute. **2**

v) Show that the coordinates of M, the midpoint of AB are (1,-3). **1**

vi) Hence, show that the equation of the circle with diameter AB is **2**

$$x^2 + y^2 - 2x + 6y - 15 = 0$$

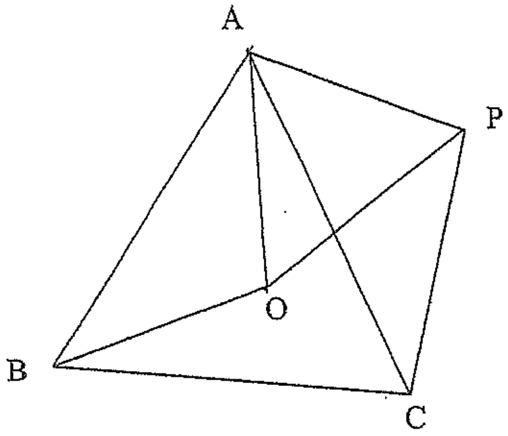
b) Find the derivative of $y = (3x^2 + 1)(4 - x)^4$, expressing your answer in factorised form. **2**

c) Given $f(x) = x^2 + 4$, $g(x) = x - x^3$ and $h(x) = f(x) \times g(x)$,

show that $h(x)$ is an odd function. **1**

QUESTION 3 (continued) **MARKS**

d)



In the figure above, triangles ACB and APO are equilateral.

i) Copy this diagram onto your answer sheet and label all information given.

ii) Explain why $\angle BAO = \angle PAC$. **1**

(Hint: Let $\angle OAC = x$)

iii) Prove $\Delta AOB \equiv \Delta APC$. **2**

iv) Hence, prove $OB = CP$. **1**

QUESTION 4

MARKS

a) Differentiate $y = 5x^2 + 1$ from first principles. **2**

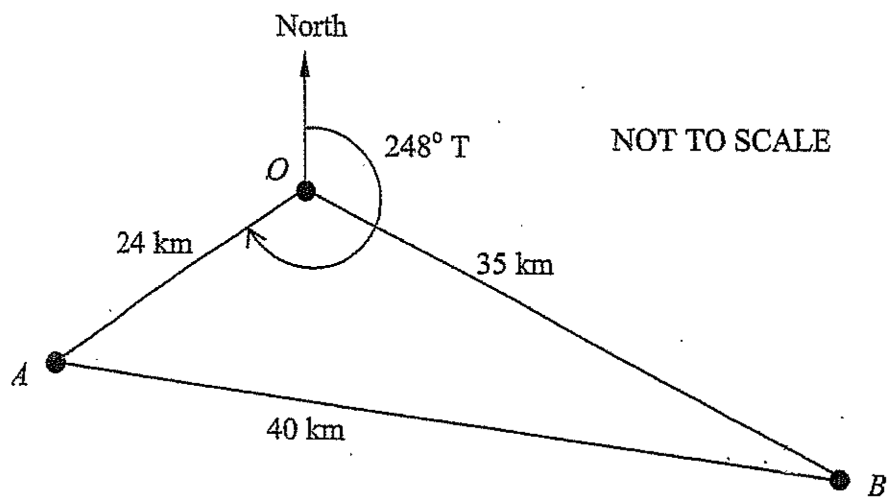
b) Draw a neat sketch showing the region on the number plane where the inequalities $y < x^2 - 4x + 3$ and $y \le x + 3$ hold simultaneously. **2**

c) Consider the curve $y = x^3 - 12x^2 + 36x$.

i) Find any stationary points. **2**

ii) Determine the nature of the stationary points. **1**

d) A triangular section of rainforest is to be designated for a species count. The shape is represented below by ΔAOB. The bearing of landmark A from landmark O is 248° and is 24km in distance. The distance from landmark A to B is 40km and from landmark B to O is 35km.



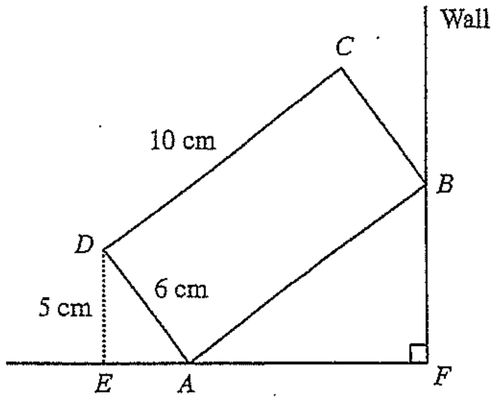
i) Show that $\angle AOB$ is 83°. **1**

ii) Hence, calculate the area of this section of the rainforest. **1**

iii) What is the bearing of landmark O from landmark B? **2**

QUESTION 4 (continued) **MARKS**

e)



In the diagram above, $ABCD$ is the cross section of a rectangular block which has been placed against a wall. The block is 10cm long and 6cm wide. The outermost edge (indicated by the point D in the diagram) of the block is 5cm above the ground.

i) Prove that ΔEAD is similar to ΔFBA **2**

ii) At what height does the block touch the wall (indicated by the point B in the diagram)? **2**
Give your answer in simplest surd form.

THE END

Question One Yr 11 Task 2 Adv.

a) 3.326×10^{8} ✓ (no other answers accepted) (1)

b) $4m - 4n - m^2 + mn$

$4(m-n) - m(m-n)$

$(m-n)(4-m)$ ✓ (1)

c) $(x+2)^2 - 25 = 0$

$(x+2)^2 - 5^2 = 0$

$(x+2+5)(x+2-5) = 0$

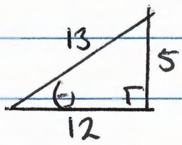
$(x+7)(x-3) = 0$

$x = -7$ ✓, $x = 3$ ✓ (2)

d) $\frac{4}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{8+4\sqrt{3}}{4-3}$

$= 8 + 4\sqrt{3}$ ✓✓ or $a = 8$ ✓, $b = 4$ ✓ (2)

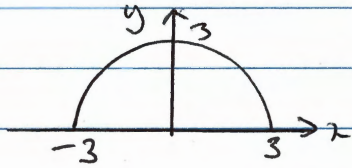
e) $\sin\theta = \frac{5}{13}$



$\tan\theta < 0$

$\cos\theta = -\frac{12}{13}$ ✓✓ (2)

f)



$-3 \le x \le 3$ ✓, $0 \le y \le 3$ ✓ (2)

g) i) $y = \frac{180° - 62}{2}$

$= 59°$ (1) ✓

ii) $S_n = (n-2)180°$, $n = 6$

$S_n = 4 \times 180° = 720°$ ✓

$\theta = 720° - (110° + 127° + 146° + 90° + 134°)$

$= 113°$ ✓ (2)

h) $y = ax^2 - 6x + 3$, $\frac{dy}{dx} = 2ax - 6$ ✓

for sty pt $\frac{dy}{dx} = 0$ ie $2a(1) - 6 = 0$

$2a = 6$ (2)

$a = 3$ ✓

Assessment Task 3
Year 11 Preliminary course
Mathematics

2 a)

$$2^{3x-1} = \frac{1}{4}$$

$$2^{3x-1} = 2^{-2}$$

$$3x-1 = -2$$

$$3x = -1$$

$$x = -\frac{1}{3}$$

Majority of students got it correct.

b) i)

$$f(x) = \sqrt{1-x}$$

$$f'(x) = \frac{1}{2}(1-x)^{-\frac{1}{2}}(-1)$$

$$= -\frac{(1-x)^{-\frac{1}{2}}}{2}$$

$$= -\frac{1}{2\sqrt{1-x}}$$

Some students made mistakes while attempting the chain rule. $\left(\frac{1}{2}(1-x)^{-\frac{1}{2}} \frac{d}{dx}(1-x)\right]$ so marks were deducted.

(ii)

$$f(x) = \frac{7x}{3-x}$$

This is an example of quotient rule. Some students made the mistake when attempting to sub values in the final formula.

$u = 7x \qquad u' = 7 \qquad f(x) = \frac{u}{v}$

$v = 3-x \qquad v' = -1$

$$f'(x) = \frac{vu' - uv'}{v^2}$$

$$f'(x) = \frac{(3-x)7 - 7x(-1)}{(3-x)^2} = \frac{21 - \cancel{7x} + \cancel{7x}}{(3-x)^2}$$

$$f'(x) = \frac{21}{(3-x)^2}$$

Assessment Task-3
Year 11 Preliminary course.
Mathematics.

2 c)

$f(2) = 4+4-1 = 7$

$f(-2) = 3^{-2} = \frac{1}{9}$

$f(6) = 7$

$f(2) + f(-2) - f(6) = \cancel{7} + \frac{1}{9} - \cancel{7} = \frac{1}{9}$

Some students were confused when choosing the correct function value for the given ~~ran~~ domain. So they were penalised.

2 d)

$Y = 3x^2 - x^3$ at $x = -1$, $Y = +3+1 = 4$

$y' = 6x - 3x^2$

gradient m at $x = -1$

$m = -6 - 3 = -9$

Some students ~~miscalcu~~ miscalculated the gradient.

Equation of tangent

$Y - 4 = -9(x+1)$

$Y - 4 = -9x - 9$

$9x + Y + 5 = 0$

e)

$|8 - 2m| \leq 70$

~~Ma~~ Many students made the mistake in this question while processing the values.

$-70 \leq 8 - 2m \leq 70$

$-78 \leq -2m \leq 62$

$39 \geq m \geq -31$

$-31 \leq m \leq 39$

2

L.H.S

f) $= \dfrac{\sec^2\theta \times \cot\theta}{\operatorname{cosec}^2\theta} = \dfrac{\dfrac{1}{\cos^{\cancel{2}}\theta} \times \dfrac{\cancel{\cos\theta}}{\sin\theta}}{\dfrac{1}{\sin^2\theta}}$

$= \dfrac{\sin^{\cancel{2}}\theta}{\cos\theta} \times \dfrac{1}{\cancel{\sin\theta}} = \tan\theta$

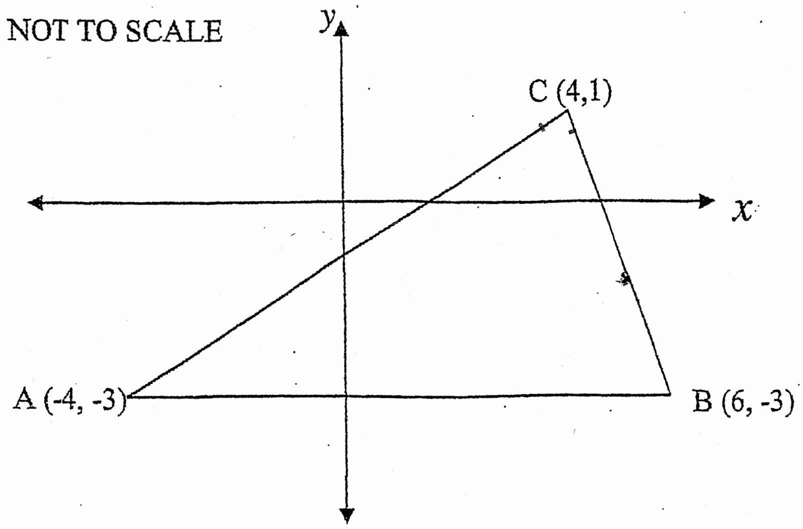
$=$ R.H.S

Hence proved.

Some students took a complicated and unnecessary long way to prove the above relation. Always look for the most economical way which will save your time.

Question 3

a)



i) $m_{BC} = \frac{-3-1}{6-4}$

$= \frac{-4}{2}$

$m_{BC} = -2$ (1)

ii) $m_{AC} = \frac{-3-1}{-4-4}$

$= \frac{-4}{-8}$

$= \frac{1}{2}$

$m_{BC} \times m_{AC} = -1 \quad (AC \perp BC)$ (1)

$\therefore \Delta ABC$ is right-angled at $\angle ACB$

iii) $BC = \sqrt{(6-4)^2 + (-3-1)^2}$

$= \sqrt{4+16}$

$BC = 2\sqrt{5}$ (1)

iv) $\sin \angle CAB = \frac{BC}{AB}$

$\sin \angle CAB = \frac{2\sqrt{5}}{10}$ (2)

$\therefore \angle CAB = 26°34'$

v) $M = \left(\frac{-4+6}{2}, \frac{-3+-3}{2}\right)$

$M = (1,-3)$. (1)

vi) Equation of circle:

$(x-a)^2 + (y-b)^2 = r^2$

$(x-1)^2 + (y+3)^2 = 5^2$ (2)

$x^2 - 2x + 1 + y^2 + 6y + 9 = 25$

$x^2 + y^2 - 2x + 6y - 15 = 0$

as required

Q3

b) $y = (3x^2+1)(4-x)^4$

$\frac{dy}{dx} = (3x^2+1) \times 4(4-x)^3 \times -1 + 6x(4-x)^4$

$= -4(3x^2+1)(4-x)^3 + 6x(4-x)^4$

$= (4-x)^3 [-12x^2 - 4 + 6x(4-x)]$

$= (4-x)^3 [-12x^2 - 4 + 24x - 6x^2]$

$= (4-x)^3 (-18x^2 + 24x - 4)$

$= 2(4-x)^3 (-9x^2 + 12x - 2)$ ✓ 2

c) $h(x) = f(x) \times g(x)$

$= (x^2+4)(x-x^3)$

$= x^3 - x^5 + 4x - 4x^3$

$= -x^5 - 3x^3 + 4x$

$h(-x) = -(-x)^5 - 3(-x)^3 + 4(-x)$

$= x^5 + 3x^3 - 4x$

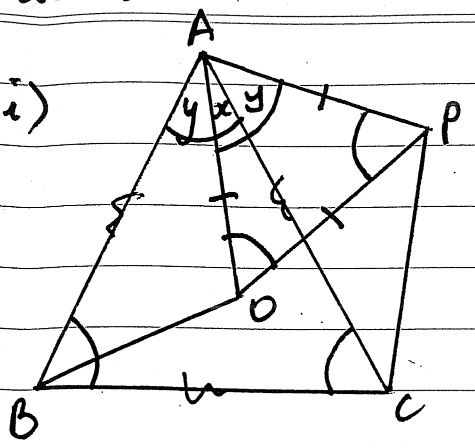
$= -(-x^5 - 3x^3 + 4x)$

$= -h(x)$ 1

$\therefore h(x)$ is odd.

Q3d).

i)



ii) $\angle BAC = 60°$ (ΔACB is equilateral).

$\angle BAO + \angle OAC = 60°$

$\angle BAO + x = 60°$

$\therefore \angle BAO = 60° - x$.

$\angle PAO = 60°$ (ΔAPO is equilateral).

$\angle PAC + \angle OAC = 60°$

$\angle PAC + x = 60°$

$\therefore \angle PAC = 60° - x$. (1)

$\therefore \angle BAO = \angle PAC$.

iii) In ΔAOB and ΔAPC

$AB = AC$ (ΔACB is equilateral).

$AO = AP$ (ΔAPO is equilateral)

$\angle BAO = \angle PAC$ (as above).

$\therefore \Delta AOB \equiv \Delta APC$ (SAS) (2)

iv) $\therefore OB = CP$ (corresponding sides in congruent Δs).

(1)

Question 4 (15 marks)

a) $y = 5x^2 + 1$

let $f(x) = 5x^2 + 1$

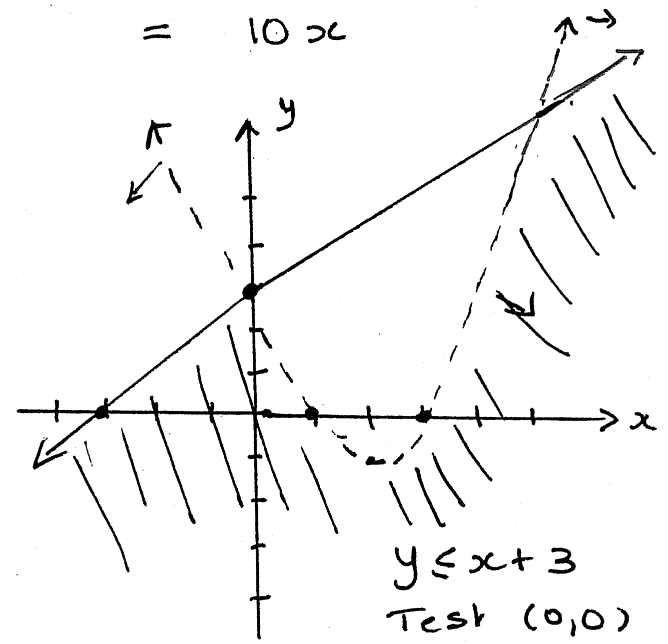
$$f(x+h) = 5(x+h)^2 + 1$$
$$= 5(x^2 + 2xh + h^2) + 1$$
$$= 5x^2 + 10xh + 5h^2 + 1$$

(overall question was completed very well, with a few making careless errors)

$$\therefore \frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h \to 0} \frac{\cancel{5x^2} + 10xh + 5h^2 \cancel{+1} - \cancel{5x^2} \cancel{-1}}{h}$$
$$= \lim_{h \to 0} \frac{10xh + 5h^2}{h}$$
$$= \lim_{h \to 0} \frac{\cancel{h}(10x + 5h)}{\cancel{h}}$$
$$= \lim_{h \to 0} 10x + 5h$$
$$= 10x$$

/2

b)



$y \le x + 3$

Test $(0,0)$

$0 \le 3$ true

(Graphs were drawn poorly and incomplete. Most found correct region)

$y = x^2 - 4x + 3$

$y = (x-3)(x-1)$

$x = 3, \; x = 1$

$y = x + 3$

at $x = 0$, $y = 3$

$x = -3 \quad y = 0$

$y < x^2 - 4x + 3$

Test $(2,0)$

$0 \not< -1$ false

/2

4 c) $y = x^3 - 12x^2 + 36x$

$y' = 3x^2 - 24x + 36$

(Most students found stationary pts)

i) Stationary points at $y' = 0$

$3x^2 - 24x + 36 = 0$

$x^2 - 8x + 12 = 0$

$(x-6)(x-2) = 0$

$x = 6$ or $x = 2$

$y = 0$ $\quad y = 32$

$P_1 (6,0)$ $\quad P_2 (2,32)$

(Full marks were given if found $x=6$ and $x=2$ without y-values however, this will not happen in the next assessment!)

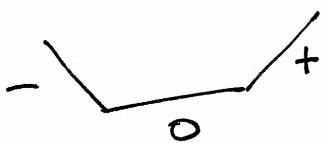
Both x & y values must <u>always</u> be found.

2

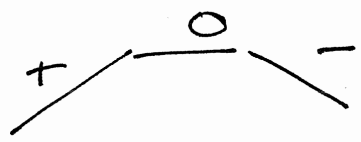
ii) Nature of stationary points

x	5	6	7
y'	$-$	0	+

x	1	2	3
y'	+	0	$-$



Minimum



Maximum

$\therefore$ $P_1 (6,0)$ is a minimum stationary point

$P_2 (2,32)$ is a maximum stationary point.

(Overall question answered extremely well).

1

4 d) i) $\cos \angle AOB = \dfrac{b^2+a^2-o^2}{2ab}$

(Most students answered very well)

$= \dfrac{24^2+35^2-40^2}{2\times 24\times 35}$

$\cos \angle AOB = 0\cdot 11964...$

$\angle AOB = \cos^{-1}(0\cdot 11964...)$

$\therefore \angle AOB = 83^\circ$

$\overline{}\,/1$

ii) Area $= \frac{1}{2} ab \sin O$

$= \frac{1}{2} \times 24 \times 35 \times \sin 83^\circ$

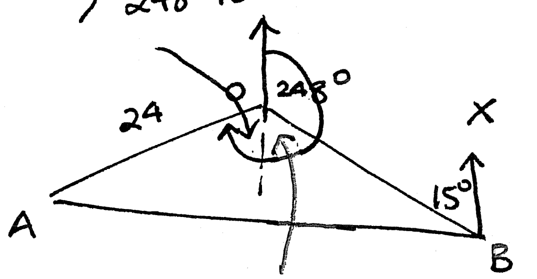
$= 416\cdot 8693...$

$= 417 \text{ km}^2$

$\overline{}\,/1$

iii) $248-180 = 68^\circ$

(Most students answered extremely well).



$83-68 = 15^\circ$

$\therefore X\hat{B}O = 15^\circ$

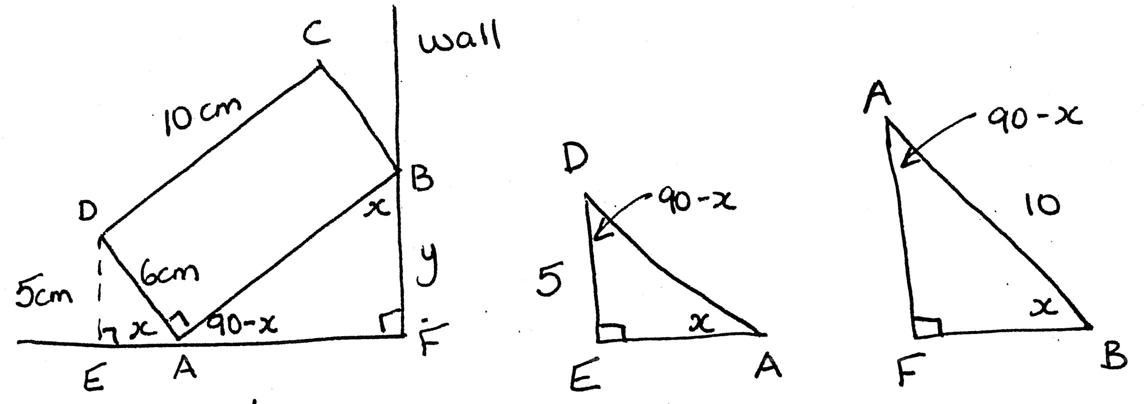
(co-interior $\angle$'s) (1)

$\therefore$ Bearing of O from B is: N 15° W or (1)

$360-15^\circ = 345^\circ$

(A lot of students had issues determining correct bearing). $\overline{}\,/2$

4 e)



In ΔEDA and ΔFBA

i) let $x = \angle DAE$

$$\angle BAF = 180 - (90 + x) \quad \text{(straight } \angle)$$
$$= 180 - 90 - x$$
$$= 90 - x$$

(Question completed poorly)

$$\angle ABF = 180 - 90 - (90 - x) \quad \text{(Angle sum of } \Delta)$$
$$= 180 - 90 - 90 + x$$
$$= x$$

$\therefore \angle DAE = \angle ABF = x$ (as shown)

/2

1 mark if stated *

* $\angle DEA = \angle AFB = 90°$ (given)

$\angle EDA = \angle FAB = 90 - x$ (angle sum of Δ)

* $\therefore \Delta EAD \,|||\, \Delta FBA$ (equiangular)

ii) Find FB

Since Δ's are similar corresponding sides are in the same ratio

(This was completed very well)

$$\frac{AF}{DE} = \frac{AB}{DA}$$

$$\frac{AF}{5} = \frac{10}{6}$$

$$\therefore AF = \frac{10 \times 5}{6} = \frac{25}{3}$$

$\therefore$ In ΔAFB,

$$(BF)^2 = (AB)^2 - (AF)^2$$
$$= 10^2 - \left(\frac{25}{3}\right)^2$$
$$(BF)^2 = 100 - \frac{625}{9}$$
$$= \frac{275}{9}$$
$$\therefore BF = \sqrt{\frac{275}{9}} = \frac{5\sqrt{11}}{3} \text{ cm}$$

$\therefore$ Block touches the wall at a height of $\frac{5\sqrt{11}}{3}$ cm above floor

/2